Un-binned Angular Analysis of $B \rightarrow D^*\ell\nu$ and the Right-handed Current

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Semileptonic $B \rightarrow D^{(*)} \ell \nu$ decays

- $R(D^{(*)})$ anomalies

$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\nu)}, \quad \text{with } \ell = \mu, e$$

- $V_{cb}$ puzzle

  inclusive decay $B \rightarrow X_c \ell \nu \ (X_c = D, D^*, D_0^* \ldots)$
  
  HQE, Optical theorem, OPE

  exclusive decay $B \rightarrow D^{(*)} \ell \nu$

  in. 42.16(50) vs ex. 39.70(60)

  $\sim 3\sigma$ deviation

  [M. Bordone et al. '21] [S. Iguro et al. '20]
Relation between the R.H. vector current and the $V_{cb}$ puzzle

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} [C_{V_L} \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R}] + \text{h.c.}$$

$$\mathcal{O}_{V_L} = (\bar{c}_L \gamma^\mu b_L) (\bar{\ell}_L \gamma_\mu \nu_L), \quad \mathcal{O}_{V_R} = (\bar{c}_R \gamma^\mu b_R) (\bar{\ell}_L \gamma_\mu \nu_L).$$

Considerable ex. uncertainties.
Theo. uncertainty from lattice QCD input.
More measurements needed!

$B \to D \ell \nu \text{ vs } B \to X_c \ell \nu$: $C_{V_R} \sim -5\%$
$B \to D^* \ell \nu \text{ vs } B \to X_c \ell \nu$: $C_{V_R} \sim 5\%$

$C_{V_L} = 1$ and $C_{V_R} = 0$ in the SM
$C_{V_R} \neq 0$ in the Left-Right symmetric model from $W_L - W_R$ mixing [E. Kou, et al ’13]
Differential decay rate \((m_{\mu,e} \rightarrow 0)\):

\[
\frac{\text{d} \Gamma(\bar{B} \rightarrow D^* (\rightarrow D\pi) \ell^- \bar{\nu}_\ell)}{\text{d}w \text{d} \cos \theta_V \text{d} \cos \theta_t \text{d} \chi} = \frac{6m_Bm_{D^*}^2}{8(4\pi)^4} \sqrt{w^2 - 1(1 - 2wr + r^2)} \left( V_{cb} \right)^2 B(D^* \rightarrow D\pi)
\]

\[
x \left\{ J_1 \sin^2 \theta_V + J_{1c} \cos^2 \theta_V + (J_{2s} \sin^2 \theta_V
\right.

+ J_{2c} \cos^2 \theta_V) \cos 2\theta_t

+ J_3 \sin^2 \theta_V \sin^2 \theta_t \cos 2\chi

+ J_4 \sin 2\theta_V \sin 2\theta_t \cos \chi + J_5 \sin 2\theta_V \sin \theta_t \cos \chi

+ (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_t

+ J_7 \sin 2\theta_V \sin \theta_t \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_t \sin \chi

+ J_9 \sin^2 \theta_V \sin^2 \theta_t \sin 2\chi \right\},
\]

\(J_i\) functions:

\[
J_{1s} = \frac{3}{2} (H_+^2 + H_-^2)(|CV_L|^2 + |CV_R|^2) - 6H_+H_-\text{Re}[CV_L C_{V_R}^*]
\]

\[
J_{1c} = 2H_0^2(|CV_L|^2 + |CV_R|^2 - 2\text{Re}[CV_L C_{V_R}^*])
\]

\[
J_{c} = 2H_0^2(|CV_L|^2 + |CV_R|^2 - 2\text{Re}[CV_L C_{V_R}^*])
\]

\[
J_{2s} = \frac{1}{2} (H_+^2 + H_-^2)(|CV_L|^2 + |CV_R|^2) - 2H_+H_-\text{Re}[CV_L C_{V_R}^*]
\]

\[
J_{2c} = -2H_0^2(|CV_L|^2 + |CV_R|^2 - 2\text{Re}[CV_L C_{V_R}^*])
\]

\[
J_3 = -2H_+H_- (|CV_L|^2 + |CV_R|^2) + 2(H_+^2 + H_-^2)\text{Re}[CV_L C_{V_R}^*]
\]

\[
J_4 = (H_+H_0 + H_-H_0)(|CV_L|^2 + |CV_R|^2 - 2\text{Re}[CV_L C_{V_R}^*])
\]

\[
J_5 = -2(H_+H_0 - H_-H_0)(|CV_L|^2 - |CV_R|^2)
\]

\[
J_{6s} = -2(H_+^2 - H_-^2)(|CV_L|^2 - |CV_R|^2)
\]

\[
J_{6c} = 0
\]

\[
J_7 = 0
\]

\[
J_8 = 2(H_+H_0 - H_-H_0)\text{Im}[CV_L C_{V_R}^*]
\]

\[
J_9 = -2(H_+^2 - H_-^2)\text{Im}[CV_L C_{V_R}^*]
\]

\(J_i\) experimentally measurable, includes \(H_+\), \(H_-\), \(H_0\), \(CV_L\) and \(C_{V_R}\) (SM and BSM).
Kinematic variables in $B \rightarrow D^{*}(\rightarrow D\pi)\ell\nu$

$\theta_\ell$ the angle between the lepton and the direction opposite the B-meson in the virtual $W$-boson rest frame;

$\theta_\nu$ the angle between the $D$ meson and the direction opposite the $B$ meson in the $D^*$ rest frame;

$\chi$ the tilting angle between the two decay planes spanned by the $W$ and $D$ systems in the $B$ meson rest frame;

$\mathcal{W}$ the dimensionless four-momentum transfer.

[A. Abdesselam et al, Belle Collaboration ’17]
Helicity amplitudes in CLN and BGL parametrizations

\[ H_{\pm}(w) = m_B \sqrt{r} (w + 1) h_{A_1}(w) \]
\[ \times \left[ 1 \mp \sqrt{\frac{w - 1}{w + 1}} R_1(w) \right] \]

\[ H_0(w) = m_B^2 \sqrt{r} (w + 1) \frac{1 - r}{\sqrt{q^2}} h_{A_1}(w) \times \]
\[ \left[ 1 + \frac{w - 1}{1 - r} (1 - R_2(w)) \right] \]

CLN parametrization (HQE based)

\[ h_{A_1}(w) = h_{A_1}(1) (1 - 8 \rho_{D^*}^2 z + (53 \rho_{D^*}^2 - 15) z^2 \]
\[ - (231 \rho_{D^*}^2 - 91) z^3) \]
\[ R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2 \]
\[ R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2 \]

\[ H_{\pm}(w) = f(w) \mp m_B p_{D^*} |g(w)\]

\[ H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{q^2}} \]

BGL parametrization (analyticity based)

\[ g(z) = \frac{1}{P_g(z) \phi_g(z)} \sum_{n=0}^{N} a_n^g z^n \]

Blaschke factors: \( P_g, P_f, P_{F_1} \)

outer functions: \( \phi_g, \phi_f, \phi_{F_1} \)

\[ f(z) = \frac{1}{P_f(z) \phi_f(z)} \sum_{n=0}^{N} a_n^f z^n \]

\[ \mathcal{F}_1(z) = \frac{1}{P_{F_1}(z) \phi_{F_1}(z)} \sum_{n=0}^{N} a_n^{F_1} z^n \]
The experimental determination of $\langle g_i \rangle$ can be pursued by the maximum likelihood method:

$$\langle g_i \rangle \equiv \gamma \left( \frac{6\langle J'_{1s} \rangle + 3\langle J'_{1c} \rangle - 2\langle J'_{2s} \rangle - \langle J'_{2c} \rangle}{\sqrt{w^2 - 1(1 - 2wr + r^2)}} \right)$$

Angular observables allow to determine $C_{V_R}$ without the intervention of the $V_{cb}$ puzzle!
Pseudo data generation

Pseudo data generated using CLN parameters fitted by Belle [E. Waheed et al, ’18]

\[ N_{\text{event}} = (5306, 8934, 10525, 11241, 11392, 11132, 10555, 9726, 8693, 7497) \]

Pseudo data generated using BGL parameters fitted by Belle [E. Waheed et al, ’18]

\[ N_{\text{event}} = (5239, 8868, 10500, 11264, 11455, 11217, 10638, 9776, 8676, 7368) \]

\( \langle g_i \rangle \) generated in 10 bins with covariance matrices by toy Monte-Carlo method

Total event number: 95k as in Belle analysis

Using pseudo data we fit theoretical formula including \( C_{VR} \) (on top of form factors). Note \( V_{cb} \) is not possible to fit any more because it cancels in \( g_i \)!
$\chi^2$ utilized in the CLN/BGL fit

$$\chi^2(\vec{v}) = \chi_{\text{angle}}(\vec{v}) + \chi_{\text{lattice}}(\vec{v})$$

$$\chi^2_{\text{angle}}(\vec{v}) = \sum_{w-\text{bin}=1}^{10} \left[ \sum_{ij} N_{\text{event}} \hat{V}_{ij}^{-1} (\langle g_i \rangle^{\text{exp}} - \langle g_i^{\text{th}}(\vec{v}) \rangle)(\langle g_j \rangle^{\text{exp}} - \langle g_j^{\text{th}}(\vec{v}) \rangle) \right]_{w-\text{bin}}$$

We include the lattice input by introducing

$$\chi^2_{\text{lattice}}(v_i) = \left( \frac{v_i^{\text{lattice}} - v_i}{\sigma_{v_i}^{\text{lattice}}} \right)^2$$

Notes:
1.) $C_{VR}$ and $V_{cb}$ are correlated in the fit using only $w$-dependence as the changes in both parameters directly impact $Br(B \rightarrow D^* \ell \nu)$

2.) the angular fit does not converge as $C_{VR}$ is not independent of the vector form factor

Lattice input of the vector form factor is crucial for determining $C_{VR}$!

$$R_1(1) \sim 4\% \text{ error} \quad h_V(1) \sim 7\% \text{ error}$$

[J.A. Bailey et al, '14]

with $h_{A_1}(1) = 0.906 \pm 0.013$ by Fermilab/MILC

[T. Kaneko et al, '19]
Fit of $C_{VR}$

**CLN fit:**

$$R_1(1) = \frac{h_V(1)}{h_A(1)}$$

$$\bar{v} = (\rho_{D^*}^2, R_1(1), R_2(1), C_{VR})$$

$$= (1.106, 1.229, 0.852, 0)$$

$$\sigma_{\bar{v}} = (3.177, 0.049, 0.018, 0.021)$$

$$\rho_{\bar{v}} = \begin{pmatrix}
1 & -0.016 & -0.763 & 0.095 \\
-0.016 & 1 & 0.006 & -0.973 \\
-0.763 & 0.006 & 1 & -0.117 \\
0.095 & -0.973 & -0.117 & 1
\end{pmatrix}$$

$C_{VR}$ can be determined to a precision of $\sim 2\%$ in CLN (BGL) parametrization.

**BGL fit:**

$$h_V(1) = \frac{m_B \sqrt{\tau}}{P_z(0) \phi_g(0)} g_0$$

$$\bar{v} = (a_{0}^{f}, a_{1}^{f}, a_{2}^{F_1}, a_{2}^{F_1}, g_0, C_{VR})$$

$$= (0.0132, 0.0169, 0.0070, -0.0852, 0.0241, 0.0024)$$

$$\sigma_{\bar{v}} = (0.0002, 0.0109, 0.0026, 0.0352, 0.0017, 0.0379)$$

$$\rho_{\bar{v}} = \begin{pmatrix}
1 & 0.022 & 0.039 & -0.035 & 0.000 & 0.189 \\
0.022 & 1 & 0.860 & -0.351 & 0.000 & 0.316 \\
0.039 & 0.860 & 1 & -0.762 & 0.000 & 0.283 \\
-0.035 & -0.351 & -0.762 & 1 & 0.000 & -0.119 \\
0.000 & 0.000 & 0.000 & 0.000 & 1 & -0.923 \\
0.189 & 0.316 & 0.283 & -0.119 & -0.923 & 1
\end{pmatrix}$$

$C_{VR}$ and the vector form factor are highly correlated!

$Im(C_{VR})$ can also be determined at precision of $0.7\%$ for both CLN and BGL!
If lattice results turn out to be different from the experimental fitted value (assuming SM), non-zero $C_{VR}$ can be hinted.
Fit of $C_{VR}$ using forward-backward asymmetry (FBA) only

$$FBA \sim <g_{6s}>$$

Advantage: one angle measurement

$$\langle A_{FB} \rangle \equiv \frac{\int_{0}^{1} \frac{d\tau}{d\cos\theta_{\ell}} d\cos\theta_{\ell} - \int_{-1}^{0} \frac{d\tau}{d\cos\theta_{\ell}} d\cos\theta_{\ell}}{\int_{0}^{1} \frac{d\tau}{d\cos\theta_{\ell}} d\cos\theta_{\ell} + \int_{-1}^{0} \frac{d\tau}{d\cos\theta_{\ell}} d\cos\theta_{\ell}}$$

$$= 3 \langle g_{6s} \rangle$$

$$\vec{v} = (\rho_{D^{*}}^{2}, R_{1}(1), R_{2}(1), C_{VR})$$

$$= (1.106, 1.229, 0.852, 0.000)$$

$$\sigma_{\vec{v}} = (2.200, 0.049, 0.031, 0.022)$$

$$\rho_{\vec{v}} = \begin{pmatrix}
1. & 0.008 & -0.873 & 0.262 \\
0.008 & 1. & -0.040 & -0.931 \\
-0.873 & -0.040 & 1. & -0.296 \\
0.262 & -0.931 & -0.296 & 1.
\end{pmatrix}$$

$C_{VR}$ can be determined at a precision of 2.2% using FBA alone! Almost as good as the full set of $<g_{i}>$!
The normalized angular observables \( g_i \) for \( B \to D^*(D\pi)\ell\nu \) determined in the un-binned angular analysis are useful for the precision measurement of \( C_{V_R} \) by circumventing the \( V_{cb} \) puzzle.

\( C_{V_R} \) is highly dependent on the vector form factor, thus it can only be determined with the vector form factor calculated by lattice.

The real (imaginary) part of \( C_{V_R} \) can be determined at precision of 2-4 \((1)\%\) using the full set of \( g_i \).

FBA (\( g_{6s} \)) can determine \( C_{V_R} \) at almost equally good precision, thus it is highly proposed to be measured in the near future.
Thank you!
SM fit including $V_{cb}$

$$\chi^2(v) = \chi_{\text{angle}}(v) + \chi_{\text{lattice}}(v) + \chi_{w-\text{bin}}(v)$$

w dependence in $\chi^2$:

$$\chi_{w-\text{bin}}^2(v) = \sum_{w-\text{bin}=1}^{10} \frac{([N]_{w-\text{bin}} - \alpha \langle \Gamma \rangle_{w-\text{bin}})^2}{[N]_{w-\text{bin}}}$$

The factor $\alpha$ is a constant, which relates the number of events and the decay rate:

$$\alpha \equiv \frac{4N_{\overline{B}B}}{1 + f_{+0}} \tau_{B^0} \times \epsilon_B(D^0 \rightarrow K^-\pi^+)$$

number of $B\overline{B}$ pairs produced from $\Upsilon(4S)$

$B^0$ lifetime

$B^+/B^0$ production ratio at Belle

$\alpha = 6.616(6.613) \times 10^{18}$ in CLN (BGL) parametrization

Experimental efficiency: $\epsilon = \sim 4.8 \times 10^{-2}$

SM fit results in CLN parametrization

$$\vec{v} = (h_{A_1}(1), \rho_{D^*}^2, R_1(1), R_2(1), V_{cb})$$

$$= (0.906, 1.106, 1.229, 0.852, 0.0387)$$

$$\sigma_{\vec{v}} = (0.013, 0.019, 0.011, 0.011, 0.0006)$$

SM fit results in BGL parametrization

$$\vec{v} = (a_0^f, a_1^f, a_2^f, a_0^g, V_{cb})$$

$$= (0.0132, 0.0169, 0.0070, -0.0853, 0.0242, 0.0384)$$

$$\sigma_{\vec{v}} = (0.0002, 0.0028, 0.0011, 0.0199, 0.0004, 0.0006)$$