Novel hadron physics by structure functions of spin-1 hadrons

Shunzo Kumano
High Energy Accelerator Research Organization (KEK)
J-PARC Center (J-PARC)
http://research.kek.jp/people/kumanos/

Collaborator on recent works:
Qin-Tao Song (Ecole Polytechnique, Zhengzhou University)

Light Cone 2021 Conference: Physics of Hadrons on the Light Front
(In-person/Online) Jeju Island, South Korea, November 29 - December 4, 2021
https://indico.cern.ch/event/938795/

November 30, 2021

Contents

1. Introduction

- Brief history, Tensor-polarized structure functions, gluon transversity, TMD

2. Tensor-polarized structure function b_1

- Convolution description for b_1, Comparison with HERMES data
- b_1 and tensor-polarized PDFs at lepton and hadron accelerator facilities

3. Gluon transversity

- Introduction to gluon transversity
- Measurements at lepton and hadron accelerator facilities

4. TMDs and PDFs of spin-1 hadrons

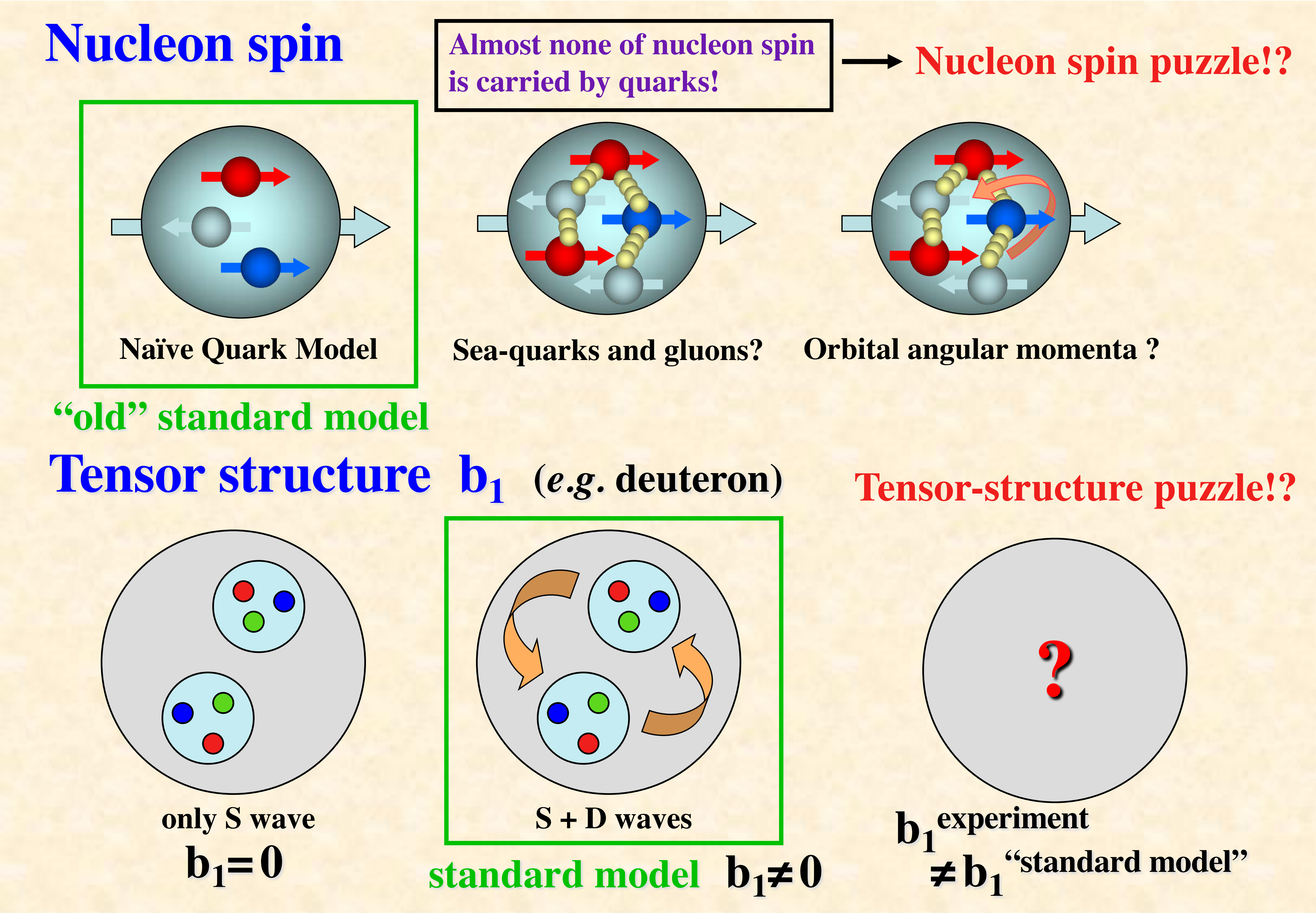
- TMDs and PDFs up to twist 4
- Twist-2 relation and sum rule for PDFs

5. Future prospects and summary

Note on our notations

Tensor-polarized gluon distribution: $\delta_T g$

Gluon transversity: $\Delta_T g$

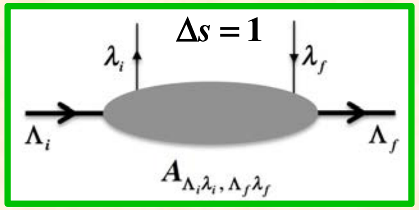

Nucleon spin
Almost none of nucleon spin is carried by quarks!
Nucleon spin puzzle!?
Naïve Quark Model
Sea-quarks and gluons?
Orbital angular momenta ?
"old" standard model
Tensor structure b_1 (*e.g.* deuteron)
Tensor-structure puzzle!?
?
only S wave
$b_1 = 0$
S + D waves
standard model $b_1 \neq 0$
$b_1^{\text{experiment}} \neq b_1^{\text{"standard model"}}$

Gluon transversity $\Delta_T g$

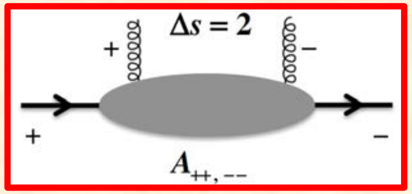
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, **conservation** $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

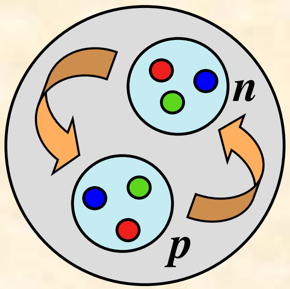
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ **quark spin flip** ($\Delta s = 1$)



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ **not possible for nucleon**



Note: Gluon transversity does not exist for spin-1/2 nucleons.



S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \iff$ **still** $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

TMDs of spin-1 hadrons

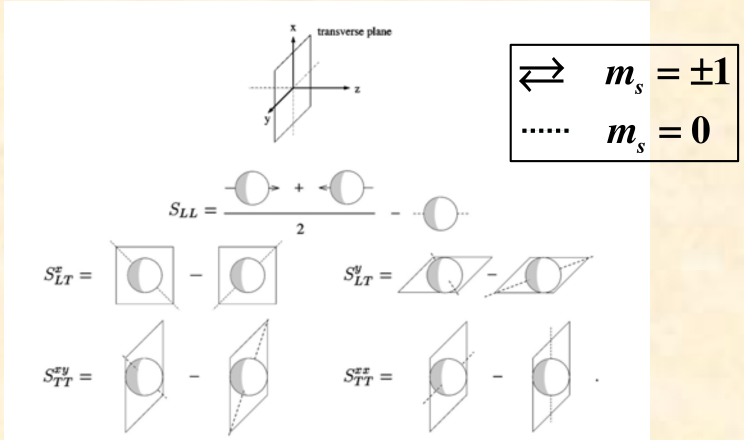
Twist 2: A. Bacchetta and P. J. Mulders, PRD 62 (2000) 114004.
Twist 3, 4: SK and Qin-Tao Song, PRD 103 (2021) 014025.

Twist-2 TMDs

Quark \ Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5$ / σ^{i+})	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1]$, $[h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}]$, $[h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}]$, $[h_{1TT}^\perp]$

Spin-1/2 nucleon, Spin-1 deuteron

Spin-1 deuteron



Twist-3 TMDs

Quark \ Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		f_T, $f_T^\perp$ $[e_T, e_T^\perp]$	$g_T, g_T^\perp$		$[h_T]$, $[h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	f_{LT}, $f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			$g_{LT}, g_{LT}^\perp$		$[h_{LT}]$, $[h_{LT}^\perp]$
TT	f_{TT}, $f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			$g_{TT}, g_{TT}^\perp$		$[h_{TT}]$, $[h_{TT}^\perp]$

Twist-4 TMDs

Quark \ Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}]$, $[h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}]$, $[h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}]$, $[h_{3TT}^\perp]$

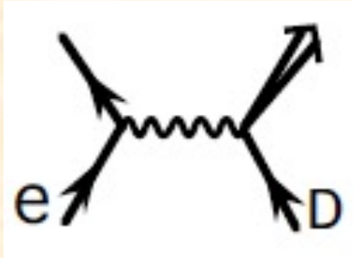
Tensor-polarized structure function b_1

for spin-1 hadrons

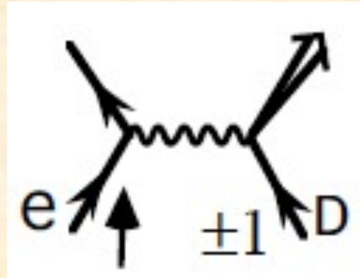
(deuteron)

Structure Functions

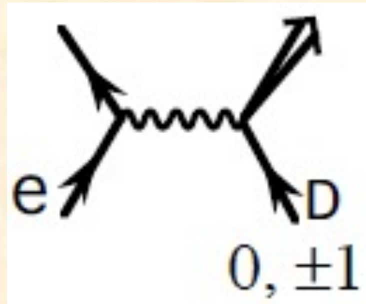
$$F_1 \propto \langle d\sigma \rangle$$



$$g_1 \propto d\sigma(\uparrow,+1) - d\sigma(\uparrow,-1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1)+\sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2}[\sigma(+1)+\sigma(-1)]$

Parton Model

$$F_1 = \frac{1}{2}\sum_i e_i^2 \left(q_i + \bar{q}_i\right) \qquad q_i = \frac{1}{3}\left(q_i^{+1} + q_i^{0} + q_i^{-1}\right)$$

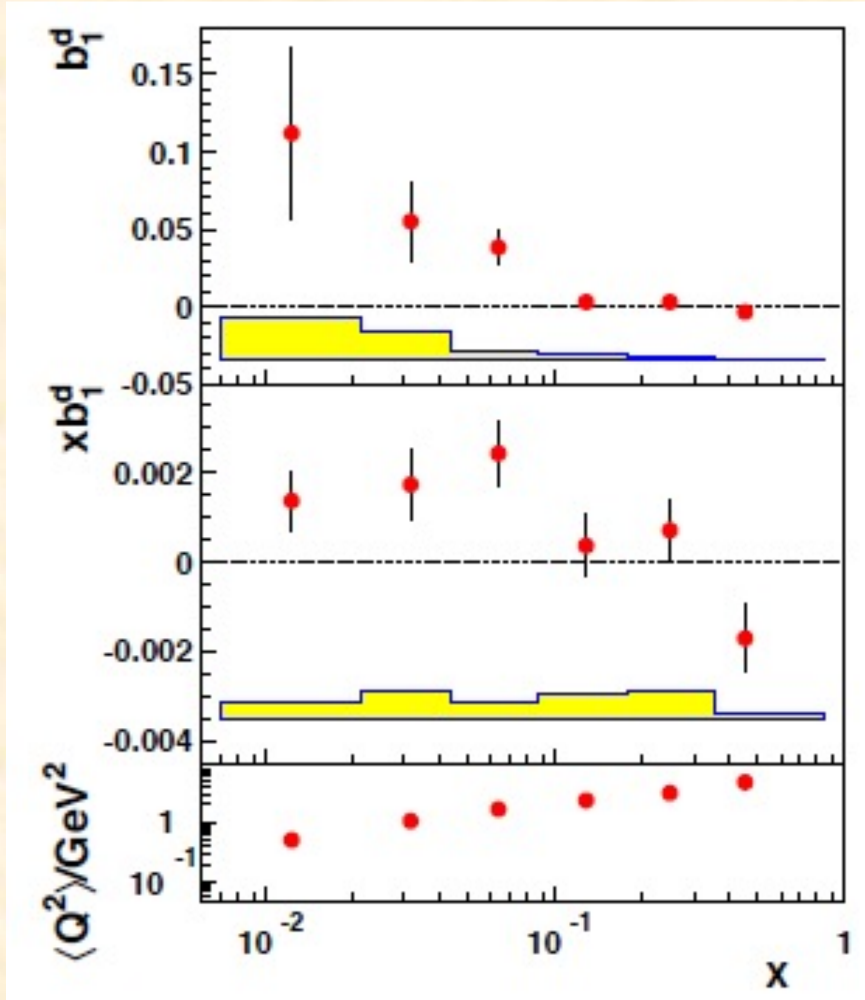
$$g_1 = \frac{1}{2}\sum_i e_i^2 \left(\Delta q_i + \Delta \bar{q}_i\right) \qquad \Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1}$$

$$\left[q_\uparrow^H\left(x,Q^2\right)\right]$$

$$b_1 = \frac{1}{2}\sum_i e_i^2 \left(\delta_T q_i + \delta_T \bar{q}_i\right) \qquad \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.



27.6 GeV/c $\rightleftarrows$, 0

positron → **deuteron**

b_1 **measurement in the kinematical region**

$$0.01 < x < 0.45,\ \ 0.5\ \text{GeV}^2 < Q^2 < 5\ \text{GeV}^2$$

b_1 **sum in the restricted** Q^2 **range** $Q^2 > 1\ \text{GeV}^2$

$$\int_{0.02}^{0.85} dx\, b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5\ \text{GeV}^2$

$$\int dx\, b_1^D(x) = \lim_{t\to 0} -\frac{5}{12}\frac{t}{M^2}F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x) = 0\ ?$$

b_1 sum rule: F. E. Close and SK, PRD 42 (1990) 2377.

$$\int \frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\int dx\left[u_v - d_v\right] + \frac{2}{3}\int dx\left[\bar{u} - \bar{d}\right] \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

“Standard” deuteron model prediction for b_1

Standard model prediction for b_1 of deuteron

$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2), \qquad y = \frac{M p\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y)+f^-(y)}{2}$$

$$= \int d^3p \, y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p)\phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3\cos^2\theta - 1) \delta\left(y - \frac{p\cdot q}{M_N \nu} \right)$$

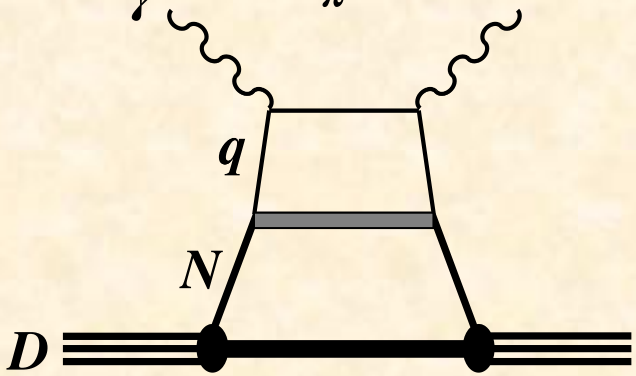
S-D term D-D term

Nucleon momentum distribution:

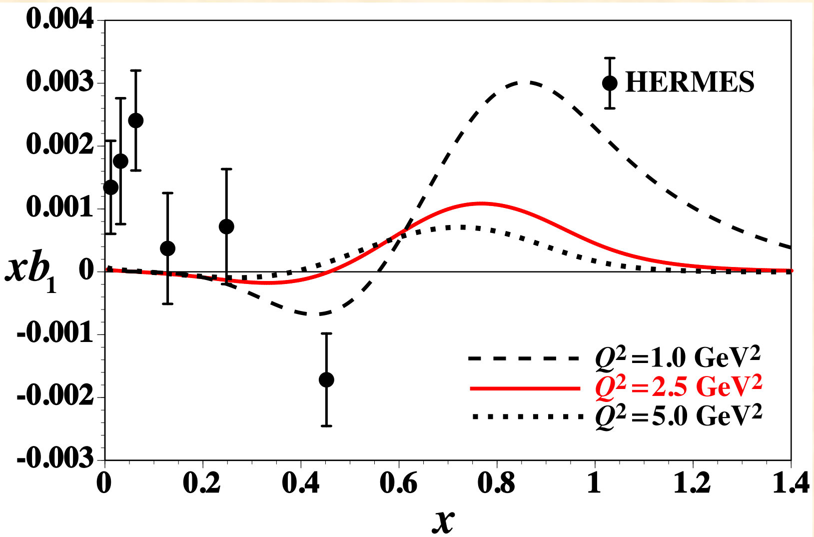
$$f^H(y) \equiv f_\uparrow^H(y) + f_\downarrow^H(y) = \int d^3p \, y \, |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N} \right)$$

D-state admixture: $\phi^H(\vec{p}) = \phi^H_{\ell=0}(\vec{p}) + \phi^H_{\ell=2}(\vec{p})$

$W_{\mu\nu} = \frac{1}{\pi} \text{Im}\, T_{\mu\nu}$



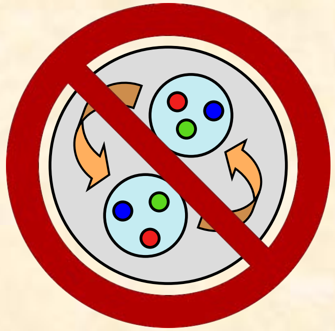
Standard model of the deuteron



W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, Phys. Rev. D 95 (2017) 074036.

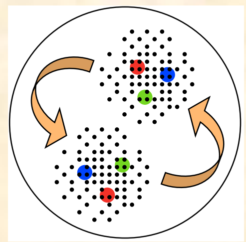
$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$ at $x < 0.5$

Standard convolution model does not work for the deuteron tensor structure!?



G. A. Miller, PRC 89 (2014) 045203,
Interesting suggestions:
hidden-color, 6-quark, $\cdots$

$$|6q\rangle = |NN\rangle + |\Delta\Delta\rangle + |CC\rangle + \cdots$$



?

JLab PAC-38 (Aug. 22-26, 2011) proposal, PR12-11-110

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson),
K. Allada, A. Camsonne, A. Deur, D. Gaskell,
C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

N. Kalantarians (co-spokesperson), O. Rondon (co-spokesperson)
Donal B. Day, Hovhannes Baghdasaryan, Charles Hanretty
Richard Lindgren, Blaine Norum, Zhihong Ye
University of Virginia, Charlottesville, VA 22903

K. Slifer[†](co-spokesperson), A. Atkins, T. Badman,
J. Calarco, J. Maxwell, S. Phillips, R. Zielinski
University of New Hampshire, Durham, NH 03861

J. Dunne, D. Dutta
Mississippi State University, Mississippi State, MS 39762

G. Ron
Hebrew University of Jerusalem, Jerusalem

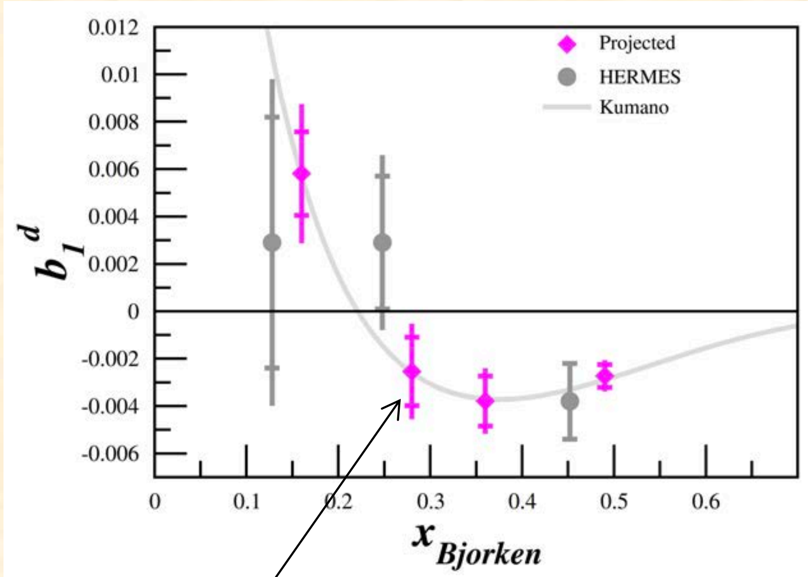
W. Bertozzi, S. Gilad,
A. Kelleher, V. Sulkosky
Massachusetts Institute of Technology, Cambridge, MA 02139

K. Adhikari
Old Dominion University, Norfolk, VA 23529

R. Gilman
Rutgers, The State University of New Jersey, Piscataway, NJ 08854

Seonho Choi, Hoyoung Kang, Hyekoo Kang, Yoomin Oh
Seoul National University, Seoul 151-747 Korea

Approved!



Expected errors by JLab

Experimental possibilities

JLab **Approved experiment!**

Fermilab

E1039 experiment (with deuteron target)

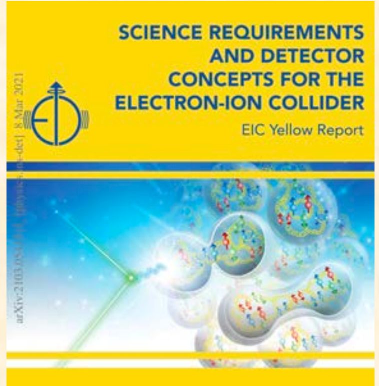
IHEP

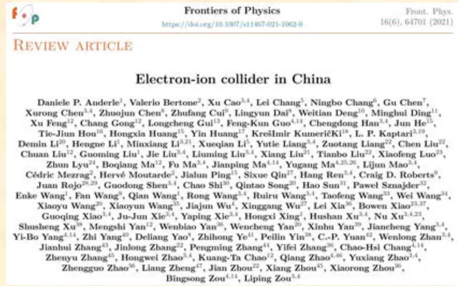
NICA has polarized-deuteron beam

Projects in 2020's

Projects in 2030's

EIC, EicC





Frontiers of Physics https://doi.org/10.1007/s11467-021-1062-0 Front. Phys. 16(6), 64701 (2021)

REVIEW ARTICLE

Electron-ion collider in China

Daniele P. Anderle[1], Valerio Bertone[2], Xu Cao[3,4], Lei Chang[5], Ningbo Chang[6], Gu Chen[7], Xurong Chen[3,4], Zhuojun Chen[8], Zhufang Cui[9], Lingyun Dai[8], Weitian Deng[10], Minghui Ding[11], Xu Feng[12], Chang Gong[12], Longcheng Gui[13], Feng-Kun Guo[4,14], Chengdong Han[3,4], Jun He[15], Tie-Jiun Hou[16], Hongxia Huang[15], Yin Huang[17], Krešimir Kumerički[18], L. P. Kaptari[3,19], Demin Li[20], Hengne Li[1], Minxiang Li[3,21], Xueqian Li[5], Yutie Liang[3,4], Zuotang Liang[22], Chen Liu[22], Chuan Liu[12], Guoming Liu[1], Jie Liu[3,4], Liuming Liu[3,4], Xiang Liu[21], Tianbo Liu[22], Xiaofeng Luo[23], Zhun Lyu[24], Boqiang Ma[12], Fu Ma[3,4], Jianping Ma[4,14], Yugang Ma[4,25,26], Lijun Mao[3,4], Cédric Mezrag[2], Hervé Moutarde[2], Jialun Ping[15], Sixue Qin[27], Hang Ren[3,4], Craig D. Roberts[9], Juan Rojo[28,29], Guodong Shen[3,4], Chao Shi[30], Qintao Song[20], Hao Sun[31], Paweł Sznajder[32], Enke Wang[1], Fan Wang[9], Qian Wang[1], Rong Wang[3,4], Ruiru Wang[3,4], Taofeng Wang[33], Wei Wang[34], Xiaoyu Wang[20], Xiaoyun Wang[35], Jiajun Wu[4], Xinggang Wu[27], Lei Xia[36], Bowen Xiao[23,37], Guoqing Xiao[3,4], Ju-Jun Xie[3,4], Yaping Xie[3,4], Hongxi Xing[1], Hushan Xu[3,4], Nu Xu[3,4,23], Shusheng Xu[38], Mengshi Yan[12], Wenbiao Yan[36], Wencheng Yan[20], Xinhu Yan[39], Jiancheng Yang[3,4], Yi-Bo Yang[4,14], Zhi Yang[40], Deliang Yao[8], Zhihong Ye[41], Peilin Yin[38], C.-P. Yuan[42], Wenlong Zhan[3,4], Jianhui Zhang[43], Jinlong Zhang[22], Pengming Zhang[44], Yifei Zhang[36], Chao-Hsi Chang[4,14], Zhenyu Zhang[45], Hongwei Zhao[3,4], Kuang-Ta Chao[12], Qiang Zhao[4,46], Yuxiang Zhao[3,4], Zhengguo Zhao[36], Liang Zheng[47], Jian Zhou[22], Xiang Zhou[45], Xiaorong Zhou[36], Bingsong Zou[4,14], Liping Zou[3,4]

Possibilities: other hadron facilities

BNL

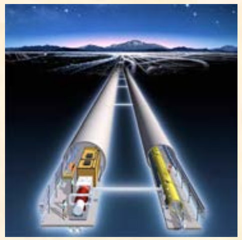
J-PARC

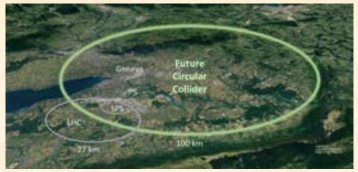
GSI

CERN-LHCspin, AMBER

Linear/Circular Colliders

(with fixed target)





Tensor-polarization asymmetry in Drell-Yan

Spin asymmetries in the parton model

unpolarized: q_a, longitudinally polarized: Δq_a,
transversely polarized: $\Delta_T q_a$, tensor polarized: δq_a

Unpolarized cross section

$$\left\langle \frac{d\sigma}{dx_A dx_B d\Omega} \right\rangle = \frac{\alpha^2}{4Q^2}(1+\cos^2\theta)\frac{1}{3}\sum_a e_a^2 \left[q_a(x_A)\bar{q}_a(x_B) + \bar{q}_a(x_A)q_a(x_B) \right]$$

Spin asymmetries

M. Hino and SK,
PRD 59 (1999) 094026;
60 (1999) 054018.

$$A_{LL} = \frac{\sum_a e_a^2 \left[\Delta q_a(x_A)\Delta\bar{q}_a(x_B) + \Delta\bar{q}_a(x_A)\Delta q_a(x_B) \right]}{\sum_a e_a^2 \left[q_a(x_A)\bar{q}_a(x_B) + \bar{q}_a(x_A)q_a(x_B) \right]}$$

$$A_{TT} = \frac{\sin^2\theta\cos(2\phi)}{1+\cos^2\theta}\frac{\sum_a e_a^2 \left[\Delta_T q_a(x_A)\Delta_T\bar{q}_a(x_B) + \Delta_T\bar{q}_a(x_A)\Delta_T q_a(x_B) \right]}{\sum_a e_a^2 \left[q_a(x_A)\bar{q}_a(x_B) + \bar{q}_a(x_A)q_a(x_B) \right]}$$

$$A_{UQ_0} = \frac{\sum_a e_a^2 \left[q_a(x_A)\delta_T\bar{q}_a(x_B) + \bar{q}_a(x_A)\delta_T q_a(x_B) \right]}{2\sum_a e_a^2 \left[q_a(x_A)\bar{q}_a(x_B) + \bar{q}_a(x_A)q_a(x_B) \right]}$$

$$A_{LT} = A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1}$$
$$= A_{LQ_1} = A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0$$

Advantage of the hadron reaction ($\delta\bar{q}$ measurement)

$$A_{UQ_0}(\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A)\delta_T\bar{q}_a(x_B)}{2\sum_a e_a^2 q_a(x_A)\bar{q}_a(x_B)}$$

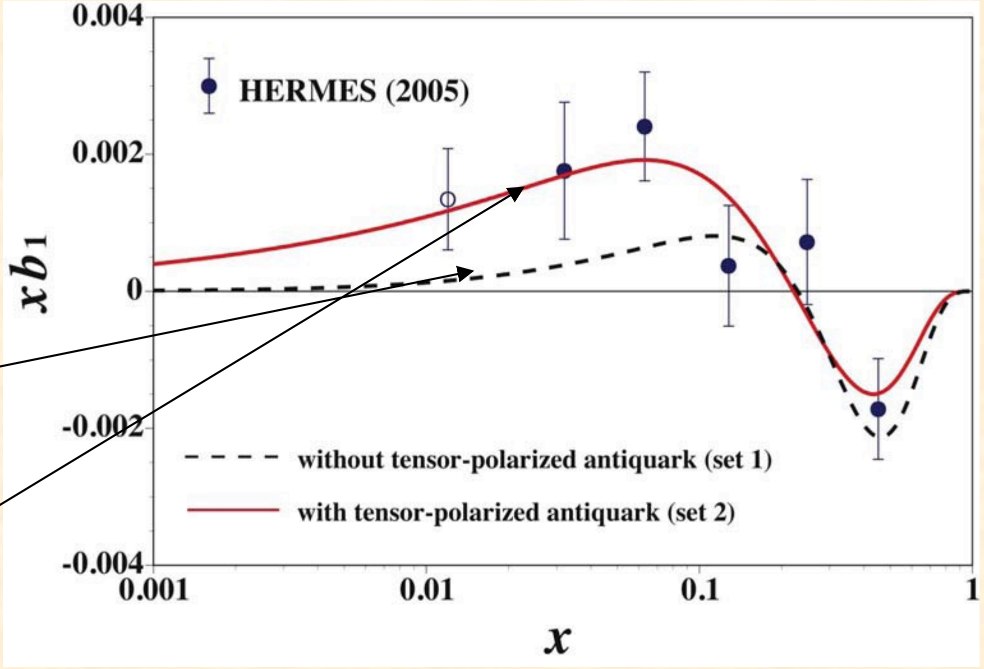
Note: $\delta \neq$ transversity in my notation

Tensor-polarized PDFs

SK, PRD 82 (2010) 017501.

Two-types of fit results:

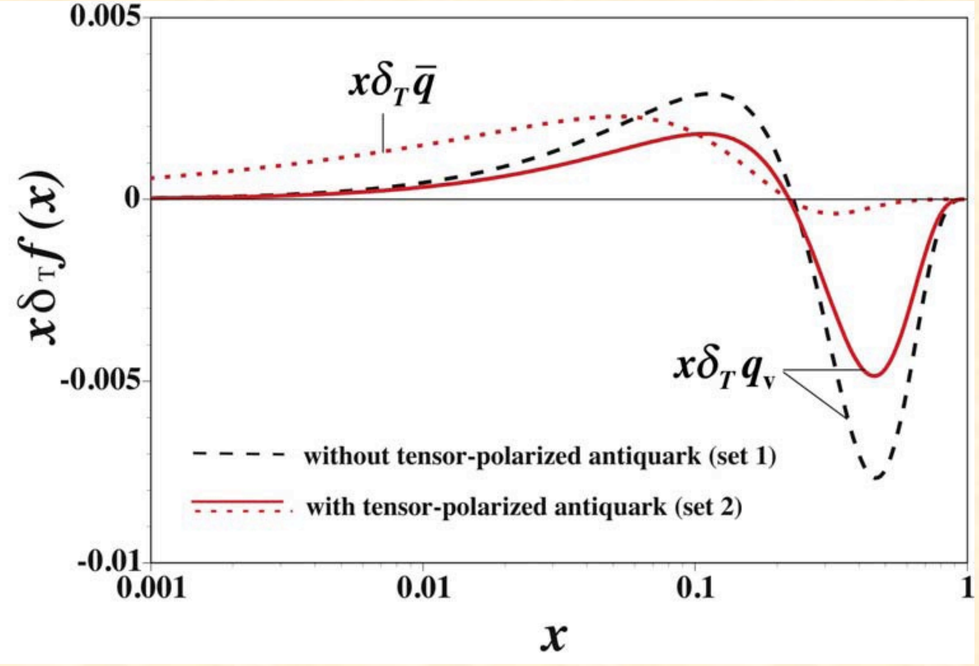
- set-1 ($\delta_T \bar{q} = 0$): $\chi^2 / \text{d.o.f.} = 2.83$
  Without $\delta_T \bar{q}$, the fit is not good enough.
- set-2 ($\delta_T \bar{q} \neq 0$): $\chi^2 / \text{d.o.f.} = 1.57$
  With finite $\delta_T \bar{q}$, the fit is reasonably good.



Obtained tensor-polarized distributions
$\delta_T q(x)$, $\delta_T \bar{q}(x)$ from the HERMES data.

→ They could be used for

- experimental proposals,
- comparison with theoretical models.

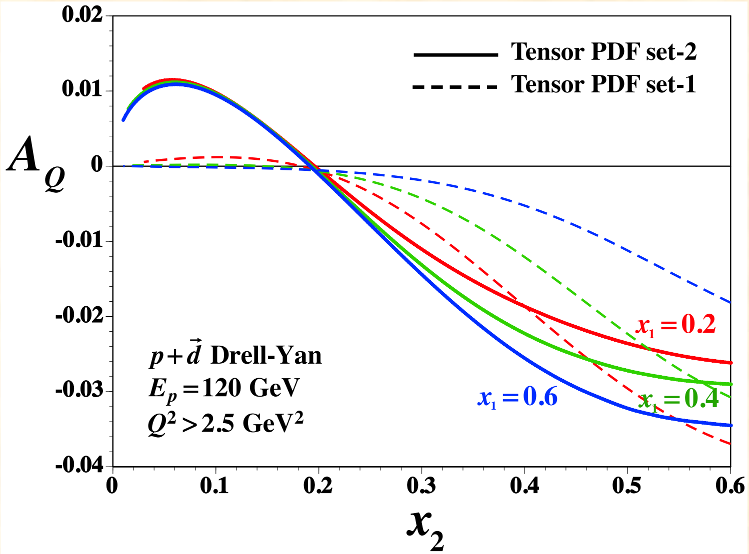


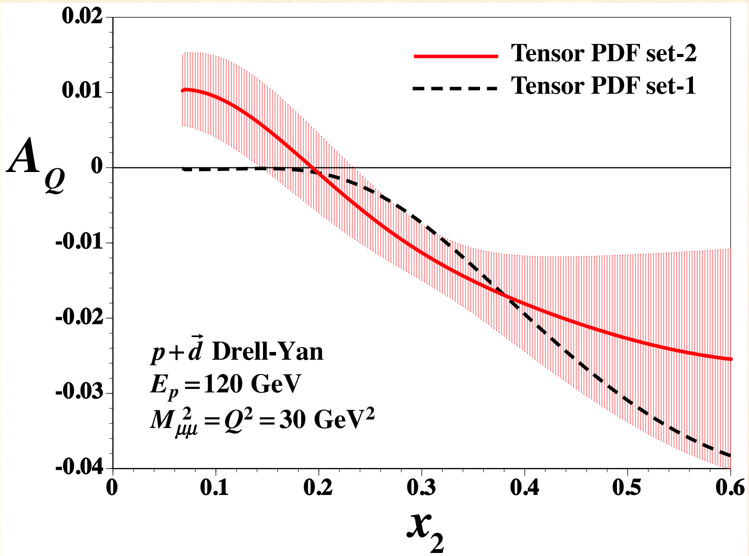
Finite tensor polarization for antiquarks:

$$\int_0^1 dx b_1(x) = 0.058$$
$$= \frac{1}{9} \int_0^1 dx \left[4\delta_T \bar{u}(x) + \delta_T \bar{d}(x) + \delta_T \bar{s}(x) \right]$$

Tensor-polarized spin asymmetry at Fermilab

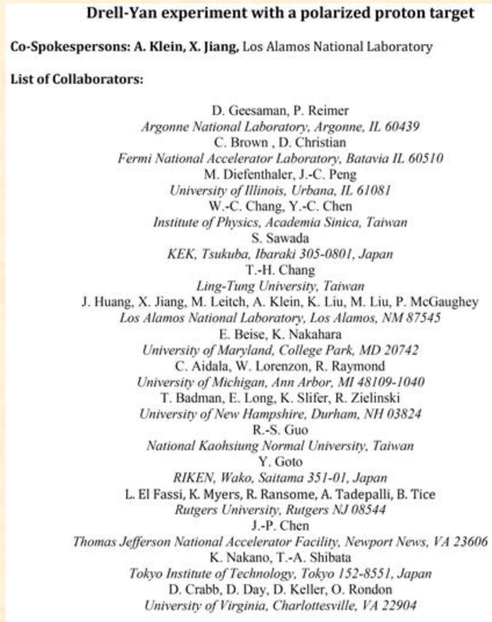
$$A_Q = \frac{\sum_a e_a^2 \left[q_a(x_A)\, \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A)\, \delta_T q_a(x_B) \right]}{\sum_a e_a^2 \left[q_a(x_A)\, \bar{q}_a(x_B) + \bar{q}_a(x_A)\, q_a(x_B) \right]}$$





Polarized fixed-target experiments at the Main Injector

E1039-SpinQuest



Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

List of Collaborators:

D. Geesaman, P. Reimer
Argonne National Laboratory, Argonne, IL 60439
C. Brown , D. Christian
Fermi National Accelerator Laboratory, Batavia IL 60510
M. Diefenthaler, J.-C. Peng
University of Illinois, Urbana, IL 61081
W.-C. Chang, Y.-C. Chen
Institute of Physics, Academia Sinica, Taiwan
S. Sawada
KEK, Tsukuba, Ibaraki 305-0801, Japan
T.-H. Chang
Ling-Tung University, Taiwan
J. Huang, X. Jiang, M. Leitch, A. Klein, K. Liu, M. Liu, P. McGaughey
Los Alamos National Laboratory, Los Alamos, NM 87545
E. Beise, K. Nakahara
University of Maryland, College Park, MD 20742
C. Aidala, W. Lorenzon, R. Raymond
University of Michigan, Ann Arbor, MI 48109-1040
T. Badman, E. Long, K. Slifer, R. Zielinski
University of New Hampshire, Durham, NH 03824
R.-S. Guo
National Kaohsiung Normal University, Taiwan
Y. Goto
RIKEN, Wako, Saitama 351-01, Japan
L. El Fassi, K. Myers, R. Ransome, A. Tadepalli, B. Tice
Rutgers University, Rutgers NJ 08544
J.-P. Chen
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606
K. Nakano, T.-A. Shibata
Tokyo Institute of Technology, Tokyo 152-8551, Japan
D. Crabb, D. Day, D. Keller, O. Rondon
University of Virginia, Charlottesville, VA 22904

SK and Qin-Tao Song, PRD 94 (2016) 054022.

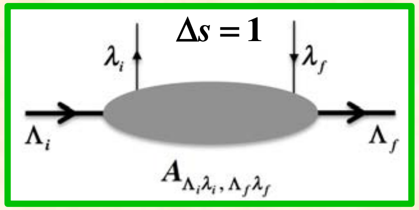
Gluon transversity
in deuteron

Gluon transversity $\Delta_T g$

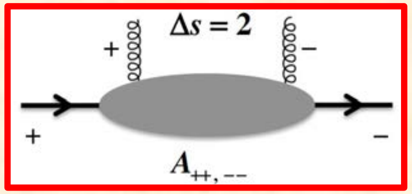
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, **conservation** $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

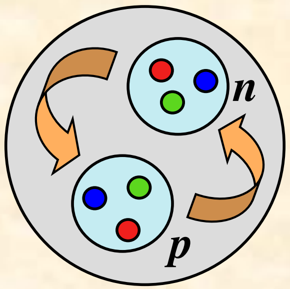
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ **quark spin flip** ($\Delta s = 1$)



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ **not possible for nucleon**



Note: Gluon transversity does not exist for spin-1/2 nucleons.

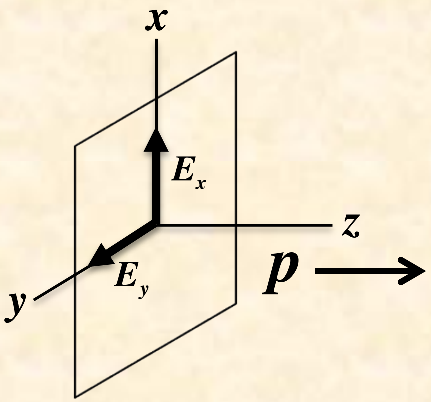


S + D waves

b_1 $(\delta_T q,\ \delta_T g) \neq 0$ $\Leftrightarrow$ **still** $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

Gluon transversity distribution in deuteron



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} x p^+ e^{ixp^+\xi^-} \left\langle pE_x \middle| A^x(0)A^x(\xi) - A^y(0)A^y(\xi) \middle| pE_x \right\rangle_{\xi^+=\vec{\xi}_T=0}$$
$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

$g_{\hat{y}/\hat{x}}$ = gluon distribution with the gluon linear polarization ε_y in the deuteron linear polarization E_x

Polarization vectors $\vec{E}_x = \vec{\varepsilon}_x = (1,\ 0,\ 0),\ \vec{E}_y = \vec{\varepsilon}_y = (0,\ 1,\ 0)$

Spin and tensor of the deuteron

$$S^\mu = \frac{1}{M}\varepsilon^{\mu\nu\alpha\beta} p_\nu \,\mathrm{Im}(E^*_\alpha E_\beta),\quad T^{\mu\nu} = -\frac{1}{3}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \mathrm{Re}(E^{\mu *} E^\nu)$$

$$E^\mu = (0,\ \vec{E}),\quad \vec{E}_\pm = \frac{1}{\sqrt{2}}(\mp 1,\ -i,\ 0),\quad \vec{E}_0 = (0,\ 0,\ 1)$$

- $\vec{E}_+, \vec{E}_0, \vec{E}_-$: Spin states with z-components of spin $s_z = +1,\ 0,\ -1$
- $\vec{E}_x = (1,\ 0,\ 0),\ \vec{E}_y = (0,\ 1,\ 0)$: Linear polarizations

  → to measure gluon transversity

(1) Prepare $s_x = 0$ [$\vec{E}_x = (1,\ 0,\ 0)$] by taking the quantization axis x and $s_y = 0$ [$\vec{E}_y = (0,\ 1,\ 0)$] by taking the quantization axis y.

(2) Combination of transverse polarizations.

$$\boldsymbol{S} = (S_T^x,\ S_T^y,\ S_L),$$
$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}$$

$S_{TT}^{xy} = S_{LT}^x = S_{LT}^y = 0$

Polarizations	$\vec{E}$	S_T^x	S_T^y	S_L	S_{LL}	S_{TT}^{xx}
Longitudinal $+z$	$\frac{1}{\sqrt{2}}(-1,\ -i,\ 0)$	0	0	+1	$+\frac{1}{2}$	0
Longitudinal $-z$	$\frac{1}{\sqrt{2}}(+1,\ -i,\ 0)$	0	0	−1	$+\frac{1}{2}$	0
Transverse $+x$	$\frac{1}{\sqrt{2}}(0,\ -1,\ -i)$	+1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $-x$	$\frac{1}{\sqrt{2}}(0,\ +1,\ -i)$	−1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $+y$	$\frac{1}{\sqrt{2}}(-i,\ 0,\ -1)$	0	+1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Transverse $-y$	$\frac{1}{\sqrt{2}}(-i,\ 0,\ +1)$	0	−1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Linear x	(1, 0, 0)	0	0	0	$+\frac{1}{2}$	−1
Linear y	(0, 1, 0)	0	0	0	$+\frac{1}{2}$	+1

Transverse polarization

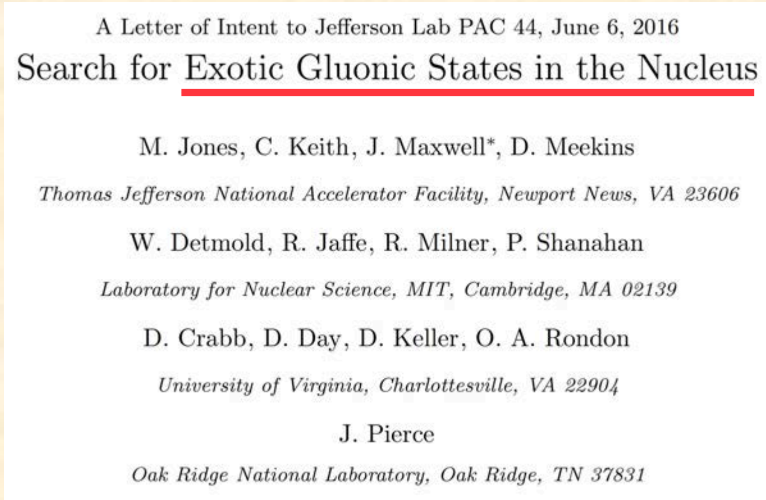
Linear polarization

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

Letter of Intent at Jefferson Lab (middle 2020's)

Jefferson Lab,
Electron accelerator ~12 GeV

LoI, arXiv:1803.11206



A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

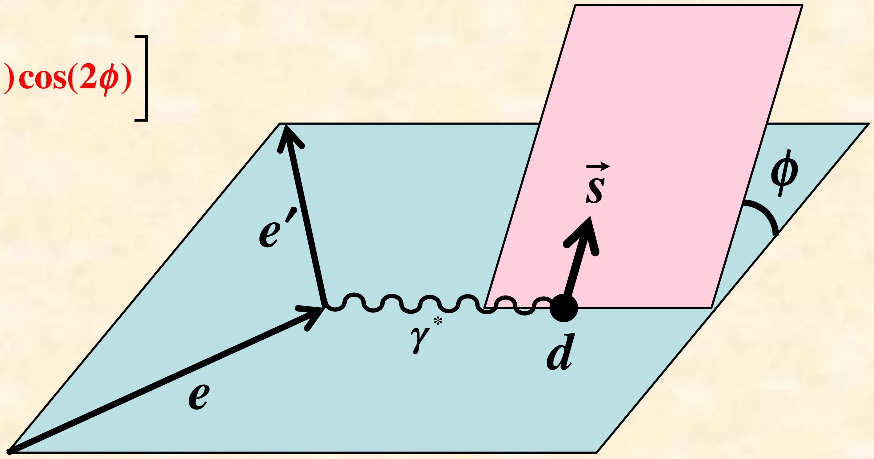
For development of polarized deuteron target,
see D. Keller, D. Crabb, D. Day
Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.

Electron scattering with polarized-deuteron target

$$\left.\frac{d\sigma}{dx\,dy\,d\phi}\right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{1}{2}x(1-y)\Delta(x,Q^2)\cos(2\phi)\right]$$

$$\Delta(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3}\Delta_T g(y,Q^2)$$

By looking at the deuteron-polarization angle ϕ,
the quark transversty $\Delta_T g$ can be measured.



Theory: J. P. Ma, C. Wang, and G. P. Zhang, arXiv:1306.6693.

Our motivation by considering the JLab experiment

We proposed to use hadron accelerator facilities for studying the gluon transversity.

Advantages:

- Independent experiment from JLab
- Different kinematical regions: larger Q^2, smaller x
- Hadron facilities are often useful for probing gluon distributions (namely a leading effect).
- Hadron cross sections are generally larger (not for Drell-Yan).
- The gluon transversity could be measured in a different form from the integral $\int_x^1 \frac{dy}{y^3} \Delta_T q(y,Q^2)$ in the JLab experiment.

→ In our PRD 101 (2020) 054011 & 094013 , we proposed proton-deuteron Drell-Yan process by considering the Fermilab-E1039.

However, our formalism is valided for Drell-Yan experiments at any other facilities.

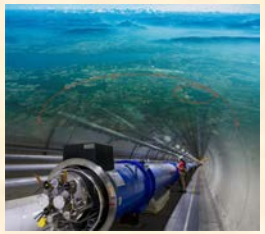
Fermilab-MI

NICA

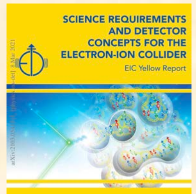
RHIC (fixed target)

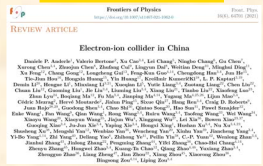
GSI-FAIR

J-PARC



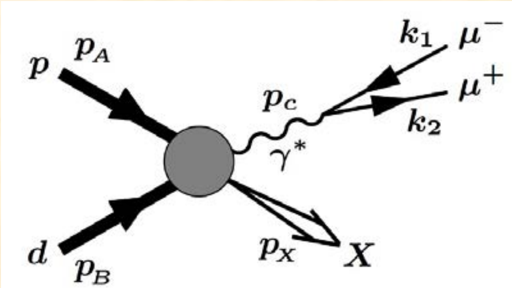
LHC (fixed target)
COMPASS/AMBER

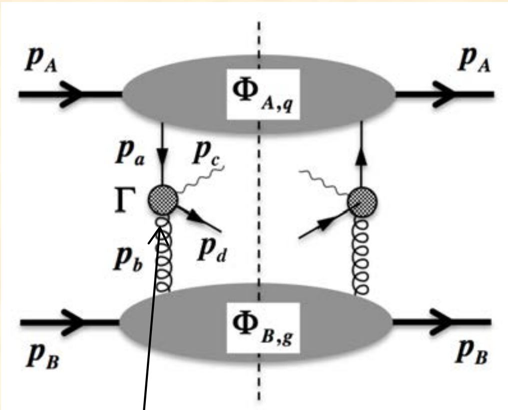


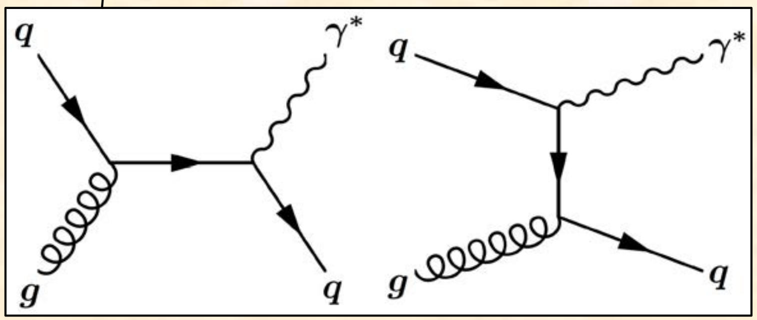


EIC
/EicC

Proton-deuteron Drell-Yan cross section







Drell-Yan cross section

$$d\sigma_{pd\to\mu^+\mu^-X} = \int_0^1 dx_a \int_0^1 dx_b f_a(x_a) f_b(x_b) d\hat{\sigma}_{ab\to\mu^+\mu^-d}, \quad M_{ab\to\mu^+\mu^-d} = eM^{\mu}_{\gamma^*\to\mu^+\mu^-}\frac{-1}{Q^2}eM_{ab\to\gamma^*d}$$

In terms of lepton tensor $L^{\mu\nu}$ and hadron tensor $W_{\mu\nu}$

$$\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy} = \frac{\alpha^2}{12\pi^2 Q^4}\left[\int d\Phi_2(q;\, k_1,\, k_2)\; 2L^{\mu\nu}\right]W_{\mu\nu}$$

dilepton phase space: $d\Phi_2(q;\, k_1,\, k_2) = \delta^4(q-k_1-k_2)\dfrac{d^3k_1}{2E_1(2\pi)^3}\dfrac{d^3k_2}{2E_2(2\pi)^3}$

$$L^{\mu\nu} = 2(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - k_1\cdot k_2 g^{\mu\nu})$$

$$W_{\mu\nu} = \overline{\sum_{\substack{\text{spin,}\\ \text{color}}}}\sum_q e_q^2 \int_{\min(x_a)}^1 dx_a \frac{\pi}{p_g^-(x_a - x_1)}\mathrm{Tr}\left[\Gamma_{\nu\beta}\left\{\Phi_{q/A}(x_a)+\Phi_{\bar{q}/A}(x_a)\right\}\hat{\Gamma}_{\mu\alpha}\Phi^{\alpha\beta}_{g/B}(x_b)\right], \quad \hat{\Gamma}_{\nu\beta} = \gamma^0\Gamma_{\nu\beta}\gamma^0$$

Collinear correlation functions

Refs. A. Bacchetta and P. J. Mulders, Phys. Rev. D 62 (2000) 114004,
D. Boer et al., JHEP 10 (2016) 013,
T. van Daal, arXiv:1812.07336 (Ph.D. Thesis).

$$\Phi_{q/A}(x_a) = \frac{1}{2}\left[\bar{\not{n}} f_{1,q/A}(x_a) + \gamma_5 \bar{\not{n}} S_{A,L} g_{1,q/A}(x_a) + \bar{\not{n}}\gamma_5 \not{S}_{A\perp} h_{1,q/A}(x_a)\right]$$

$$\Phi_{q/B}(x_b) = \frac{1}{2}\left[\not{n} f_{1,q/B}(x_b) + \gamma^5 \not{n}\, S_{B,L}\, g_{1,q/B}(x_b) + i\sigma_{\mu\nu}\gamma^5 n^\mu S^\nu_{B,T} h_{1,q/B}(x_b) + \not{n}\, S_{LL}\, f_{1LL,q/B}(x_b) + \sigma_{\mu\nu} n^\nu S^\mu_{B,LT} h_{1LT,q/B}(x_b)\right]$$

$$\Phi^{ij}_{g/B}(x_b) = \frac{1}{2}\left[-g_T^{ij} f_{1,g/B}(x_b) + i\varepsilon_T^{ij} S_{B,L} g_{1L,g/B}(x_b) - g_T^{ij} S_{B,LL} f_{1LL,g/B}(x_b) + S^{ij}_{B,TT} h_{1TT,g/B}(x_b)\right]$$

Gluon transversity: $\Delta_T g = h_{1TT,g}$
(Sorry to use two differenent notations in a talk.)

Proton-deuteron Drell-Yan cross section

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

Drell-Yan cross section

$$\frac{d\sigma_{pd\to\mu^+\mu^-X}(E_x-E_y)}{d\tau\, dq_T^2\, d\phi\, dy}=\frac{\alpha^2\alpha_s C_F q_T^2}{6\pi s^3}\cos(2\phi)\int_{\min(x_a)}^{1}dx_a\,\frac{1}{(x_a x_b)^2(x_a-x_1)(\tau-x_a x_2)^2}\sum_q e_q^2 x_a\left[q_A(x_a)+\bar{q}_A(x_a)\right]x_b\Delta_T g_B(x_b)$$

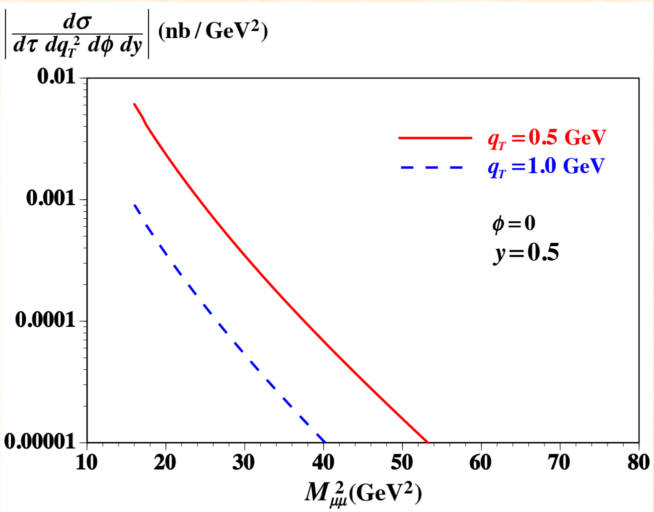
$$C_F=\frac{N_c^2-1}{2N_c},\quad \min(x_a)=\frac{x_1-\tau}{1-x_2},\quad x_b=\frac{x_a x_2-\tau}{x_a-\tau}$$

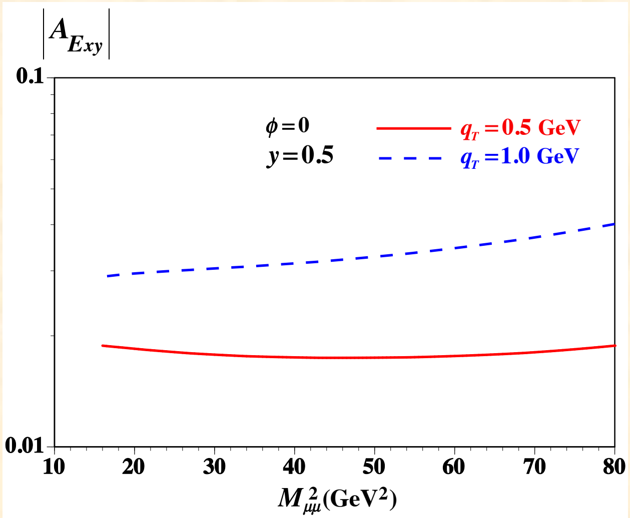
= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

- Consider the Fermilab-E1039 experiment with the proton beam of $p = 120$ GeV
- No available $\Delta_T g$, so we may tentatively assume $\Delta_T g = \Delta g_p + \Delta g_n$ $\left(\text{or } \frac{\Delta g_p+\Delta g_n}{2}, \frac{\Delta g_p+\Delta g_n}{4}\right)$
- CTEQ14 for $q(x)+\bar{q}(x)$, NNPDFpol1.1 for $\Delta g(x)$

Cross section: Dimuon mass squred ($M_{\mu\mu}^2 = Q^2$) dependence

Spin asymmetry: $A_{E_{xy}}=\dfrac{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x)-\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x)+\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}$





New proposal at Fermilab-PAC in January, 2022 (D. Keller) !

Experimental possibility at Fermilab in 2020's

Polarized fixed-target experiments at the Main Injector, Proton beam = 120 GeV

© Fermilab

Fermilab-E1039

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

List of Collaborators:

D. Geesaman, P. Reimer
Argonne National Laboratory, Argonne, IL 60439
C. Brown , D. Christian
Fermi National Accelerator Laboratory, Batavia IL 60510
M. Diefenthaler, J.-C. Peng
University of Illinois, Urbana, IL 61081
W.-C. Chang, Y.-C. Chen
Institute of Physics, Academia Sinica, Taiwan
S. Sawada
KEK, Tsukuba, Ibaraki 305-0801, Japan
T.-H. Chang
Ling-Tung University, Taiwan
J. Huang, X. Jiang, M. Leitch, A. Klein, K. Liu, M. Liu, P. McGaughey
Los Alamos National Laboratory, Los Alamos, NM 87545
E. Beise, K. Nakahara
University of Maryland, College Park, MD 20742
C. Aidala, W. Lorenzon, R. Raymond
University of Michigan, Ann Arbor, MI 48109-1040
T. Badman, E. Long, K. Slifer, R. Zielinski
University of New Hampshire, Durham, NH 03824
R.-S. Guo
National Kaohsiung Normal University, Taiwan
Y. Goto
RIKEN, Wako, Saitama 351-01, Japan
L. El Fassi, K. Myers, R. Ransome, A. Tadepalli, B. Tice
Rutgers University, Rutgers NJ 08544
J.-P. Chen
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606
K. Nakano, T.-A. Shibata
Tokyo Institute of Technology, Tokyo 152-8551, Japan
D. Crabb, D. Day, D. Keller, O. Rondon
University of Virginia, Charlottesville, VA 22904

Fermilab experimentalists are interested in the gluon transversity by replacing the E1039 proton target for the deuteron one. (Spokesperson of E1039: D. Keller) However, there was no theoretical formalism until our work.

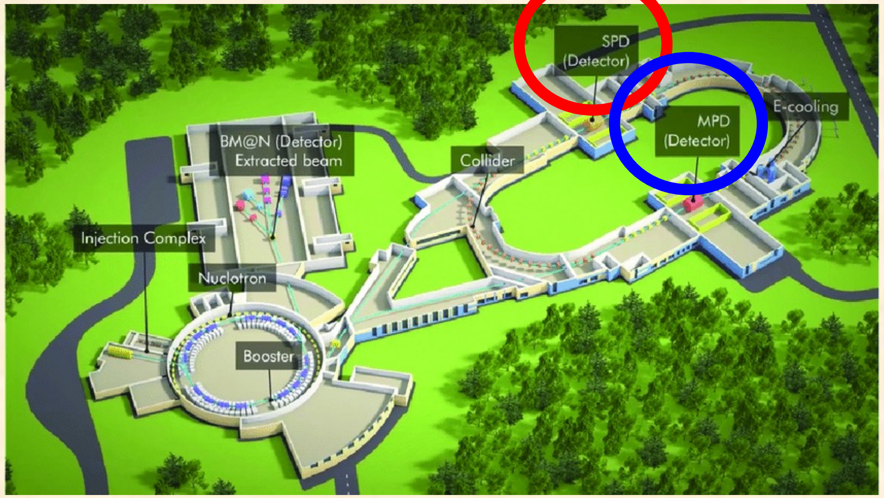
The Transverse Structure of the Deuteron with Drell-Yan

D. Keller[1]

[1] *University of Virginia, Charlottesville, VA 22904*

New proposal for a Fermilab-PAC in January, 2022.

Nuclotron-based Ion Collider fAcility (NICA)



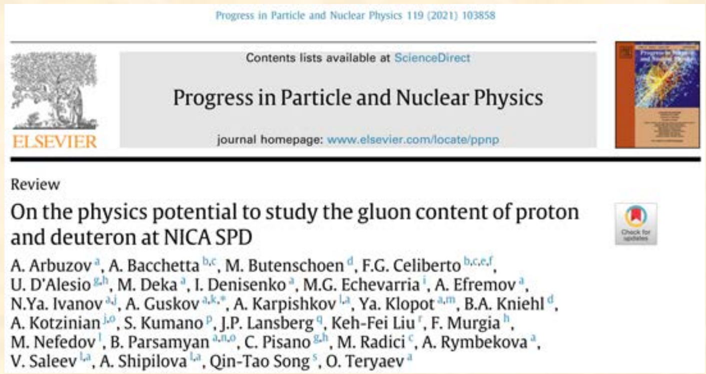
SPD (Spin Physics Detector for physics with polarized beams)
MPD (MultiPurpose Detector for heavy ion physics)

$\vec{p}+\vec{p}: \quad \sqrt{s_{pp}} = 12 \sim 27 \text{ GeV}$

$\vec{d}+\vec{d}: \quad \sqrt{s_{NN}} = 4 \sim 14 \text{ GeV}$

$\vec{p}+\vec{d}$ is also possilbe.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al*. (NICA project), Nucl. Part. Phys. 119 (2021) 103858.


Progress in Particle and Nuclear Physics 119 (2021) 103858
Contents lists available at ScienceDirect
Progress in Particle and Nuclear Physics
journal homepage: www.elsevier.com/locate/ppnp

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov[a], A. Bacchetta[b,c], M. Butenschoen[d], F.G. Celiberto[b,c,e,f], U. D'Alesio[g,h], M. Deka[a], I. Denisenko[a], M.G. Echevarria[i], A. Efremov[a], N.Ya. Ivanov[a,j], A. Guskov[a,k,*], A. Karpishkov[l,a], Ya. Klopot[a,m], B.A. Kniehl[d], A. Kotzinian[j,o], S. Kumano[p], J.P. Lansberg[q], Keh-Fei Liu[r], F. Murgia[h], M. Nefedov[l], B. Parsamyan[a,n,o], C. Pisano[g,h], M. Radici[c], A. Rymbekova[a], V. Saleev[l,a], A. Shipilova[l,a], Qin-Tao Song[s], O. Teryaev[a]

Unique opportunity in high-energy spin physics, especially on the deuteron spin physics.
→ Theoretical formalisms need to be developed.

Summary on transversity situation

may skip

- The quark-transversity distributions will be measured accurately in future by COMPASS/AMBER, JLab, and EicC/EIC projects.
- Accurate quark-transversity distributions can be used for EDM studies.
- There is no experiment and only a few theoretical papers on the gluon transversity $\Delta_T g$, which does not exist for spin-1/2 nucleons.
- Hadrons with spin≧1 are needed, for example, the deuteron for $\Delta_T g$.
- There is a plan to measure $\Delta_T g$ at JLab in 2020's.

- We proposed to use hadron facilities for measuring $\Delta_T g$.
  In particular, we showed the theoretical formalism and cross sections for the proton-deuteron Drell-Yan process with $\Delta_T g$.
- It will be proposed within the Fermilab-E1039 experiment (D. Keller).
- Our formalism can be used at any other hadron facilities, NICA, COMPASS/AMBER, RHIC, GSI-FAIR, J-PARC, LHC, (EicC/EIC/LHeC).
- $\Delta_T g \rightarrow$ "exotic" components in nuclei beyond bound states of nucleons.

TMDs and PDFs for spin-1 hadrons

Twsit-2 TMDs for spin-1/2 nucleons and spin-1 hadrons

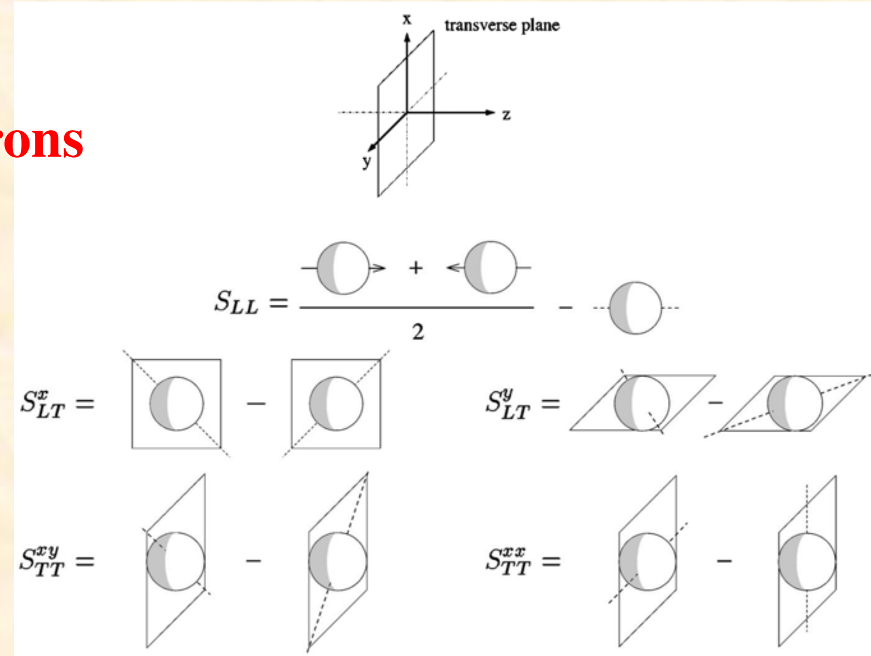
Twist-2 TMDs

Bacchetta-Mulders, PRD 62 (2000) 114004.

Quark \ Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Spin-1/2 nucleon
(also spin-1 hadrons)

Spin-1 hadrons



Twist-2 collinear PDFs

$[\cdot\cdot\cdot]$= chiral odd

Quark \ Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

*1 Because of the time-reversal invariance, the collinear PDF $h_{1LT}(x)$ vanishes. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function $H_{1LT}(z)$ should exist as a collinear fragmentation function. (see our PRD paper for the details)

TMD correlation functions for spin-1 hadrons

$$\Phi_{ij}(k,P,T)=\int\frac{d^4\xi}{(2\pi)^4}e^{ik\cdot\xi}\left\langle P,T\left|\bar\psi_j(0)W(0,\xi)\psi_i(\xi)\right|P,T\right\rangle$$
$$W(0,\xi)=P\exp\left[-ig\int_0^\xi d\xi\cdot A(\xi)\right]$$

Spin vector: $S^\mu=S_L\frac{P^+}{M}\bar n^\mu-S_L\frac{M}{2P^+}n^\mu+S_T^\mu$

Tensor: $T^{\mu\nu}=\frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar n^\mu\bar n^\nu+\frac{P^+}{M}\bar n^{\{\mu}S_{LT}^{\nu\}}-\frac{2}{3}S_{LL}\left(\bar n^{\{\mu}n^{\nu\}}-g_T^{\mu\nu}\right)+S_{TT}^{\mu\nu}-\frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}}+\frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu\right]$

Tensor part (twist-2): Bacchetta, Mulders, PRD 62 (2000) 114004

$$\Phi(k,P,T)=\left(\frac{A_{13}}{M}I+\frac{A_{14}}{M^2}\not P+\frac{A_{15}}{M^2}\not k+\frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu}+\left[A_{17}\gamma_\nu+\left(\frac{A_{18}}{M}P^\rho+\frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho}+\frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ dependent terms are added for up to twist 4.

[For the spin-1/2 nucleon: Goeke, Metzand, Schlegel, PLB 618 (2005) ,90; Metz, Schweitzer, Teckentrup, PLB 680 (2009) 141.]

Kumano-Song-2021, for the details see PRD 103 (2021) 014025

$$\Phi(k,P,T\,|\,n)=\left(\frac{A_{13}}{M}I+\frac{A_{14}}{M^2}\not P+\frac{A_{15}}{M^2}\not k+\frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu}+\left[A_{17}\gamma_\nu+\left(\frac{A_{18}}{M}P^\rho+\frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho}+\frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Bacchetta -Mulders

$$+\left(\frac{B_{21}M}{P\cdot n}k_\mu+\frac{B_{22}M^3}{(P\cdot n)^2}n_\mu\right)n_\nu T^{\mu\nu}+i\gamma_5\varepsilon_{\mu\tau\rho\sigma}P^\rho\left(\frac{B_{23}}{(P\cdot n)M}k^\tau n^\sigma k_\nu+\frac{B_{24}M}{(P\cdot n)^2}k^\tau n^\sigma n_\nu\right)T^{\mu\nu}$$
$$+\left[\frac{B_{25}}{P\cdot n}\not n k_\mu k_\nu+\left(\frac{B_{26}M^2}{(P\cdot n)^2}\not n+\frac{B_{28}}{P\cdot n}\not P+\frac{B_{30}}{P\cdot n}\not k\right)k_\mu n_\nu+\left(\frac{B_{27}M^4}{(P\cdot n)^3}\not n+\frac{B_{29}M^2}{(P\cdot n)^2}\not P+\frac{B_{31}M^2}{(P\cdot n)^2}\not k\right)n_\mu n_\nu+\frac{B_{32}M^2}{P\cdot n}\gamma_\mu n_\nu\right]T^{\mu\nu}$$
$$-\left[\varepsilon_{\mu\tau\rho\sigma}\gamma^\tau P^\rho\left(\frac{B_{34}}{P\cdot n}n^\sigma k_\nu+\frac{B_{33}}{P\cdot n}k^\sigma n_\nu+\frac{B_{35}M^2}{(P\cdot n)^2}n^\sigma n_\nu\right)+\varepsilon_{\lambda\tau\rho\sigma}k^\lambda\gamma^\tau P^\rho n^\sigma\left(\frac{B_{36}}{P\cdot nM^2}k_\mu k_\nu+\frac{B_{37}}{(P\cdot n)^2}k_\mu n_\nu+\frac{B_{38}M^2}{(P\cdot n)^3}n_\mu n_\nu\right)\right]\gamma_5 T^{\mu\nu}$$
$$+\varepsilon_{\mu\tau\rho\sigma}k^\tau P^\rho n^\sigma\left(\frac{B_{39}}{(P\cdot n)^2}k_\nu+\frac{B_{40}M^2}{(P\cdot n)^3}n_\nu\right)\not n\gamma_5 T^{\mu\nu}$$
$$+\sigma_{\rho\sigma}\left[P^\rho k^\sigma\left(\frac{B_{41}}{(P\cdot n)M}k_\mu n_\nu+\frac{B_{42}M}{(P\cdot n)^2}n_\mu n_\nu\right)+P^\rho n^\sigma\left(\frac{B_{43}}{(P\cdot n)M}k_\mu k_\nu+\frac{B_{44}M}{(P\cdot n)^2}k_\mu n_\nu+\frac{B_{45}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu}$$
$$+\sigma_{\rho\sigma}\left[k^\rho n^\sigma\left(\frac{B_{46}}{(P\cdot n)M}k_\mu k_\nu+\frac{B_{47}M}{(P\cdot n)^2}k_\mu n_\nu+\frac{B_{48}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu}+\sigma_{\mu\sigma}\left[n^\sigma\left(\frac{B_{49}M}{P\cdot n}k_\nu+\frac{B_{50}M^3}{(P\cdot n)^2}n_\nu\right)+\left(\frac{B_{51}M}{P\cdot n}P^\sigma+\frac{B_{52}M}{P\cdot n}k^\sigma\right)n_\nu\right]T^{\mu\nu}$$

New terms in our paper (2021)

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

Terms associated with $n=\frac{1}{\sqrt{2}}(1,\,0,\,0,\,-1)$

Twist-3 TMDs for spin-1 hadrons

SK and Qin-Tao Song (2021)

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k, P, T\,|\,n)\,\Gamma\right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2} F^{\perp}(x, k_T^2)$$

$$\Phi^{[\gamma^i]}(x, k_T, T) = \frac{M}{P^+}\left[f_{LL}^{\perp}(x, k_T^2)\frac{S_{LL}k_T^i}{M} + f'_{LT}(x, k_T^2)S_{LT}^i - f_{LT}^{\perp}(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - f'_{TT}(x, k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + f_{TT}^{\perp}(x, k_T^2)\frac{k_T^i k_T \cdot S_{TT}\cdot k_T}{M^3} \right]$$

$$\Phi^{[1]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LL}(x, k_T^2)S_{LL} - e_{LT}^{\perp}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + e_{TT}^{\perp}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2} \right]$$

$$\Phi^{[i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} - e_{TT}(x, k_T^2)\frac{S_{TT\mu\rho}k_T^{\rho}\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2} \right]$$

$$\Phi^{[\gamma^i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[-g_{LL}^{\perp}(x, k_T^2)\frac{S_{LL}\varepsilon_T^{ij}k_{Tj}}{M} - g'_{LT}(x, k_T^2)\varepsilon_T^{ij}S_{LTj} + g_{LT}^{\perp}(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}S_{LT}\cdot k_T}{M^2} + g'_{TT}(x, k_T^2)\frac{\varepsilon_T^{ij}S_{TTjl}k_T^l}{M} - g_{TT}^{\perp}(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}k_T\cdot S_{TT}\cdot k_T}{M^3} \right]$$

$$\Phi^{[\sigma^{-+}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LL}(x, k_T^2)S_{LL} - h_{LT}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + h_{TT}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2} \right]$$

$$\Phi^{[\sigma^{ij}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LT}^{\perp}(x, k_T^2)\frac{S_{LT}^i k_T^j - S_{LT}^j k_T^i}{M} - h_{TT}^{\perp}(x, k_T^2)\frac{S_{TT}^{il}k_{Tl}k_T^j - S_{TT}^{jl}k_{Tl}k_T^i}{M^2} \right]$$

***2, *3 Because of the time-reversal invariance, the collinear PDFs $g_{LT}(x)$ and $h_{LL}(x)$ do not exist. However, the corresponding new collinear fragmentation functions $G_{LT}(z)$ and $H_{LL}(z)$ should exist. (see our PRD paper for the details)**

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^{\perp}$ $[e]$			$g^{\perp}$		$[h]$
L		$f_L^{\perp}$ $[e_L]$	$g_L^{\perp}$		$[h_L]$	
T		$f_T, f_T^{\perp}$ $[e_T, e_T^{\perp}]$	$g_T, g_T^{\perp}$		$[h_T], [h_T^{\perp}]$	
LL	$f_{LL}^{\perp}$ $[e_{LL}]$			$g_{LL}^{\perp}$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^{\perp}$ $[e_{LT}, e_{LT}^{\perp}]$			$g_{LT}, g_{LT}^{\perp}$		$[h_{LT}], [h_{LT}^{\perp}]$
TT	$f_{TT}, f_{TT}^{\perp}$ $[e_{TT}, e_{TT}^{\perp}]$			$g_{TT}, g_{TT}^{\perp}$		$[h_{TT}], [h_{TT}^{\perp}]$

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

New TMDs **[· · ·]= chiral odd** **New collinear PDFs**

Twist-4 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\text{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\text{Tr}\left[\int dk^- \Phi(k, P, T \mid n)\,\Gamma\right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2} F^{\perp}(x, k_T^2)$$

$$\Phi^{[\gamma^-]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[f_{3LL}(x, k_T^2) S_{LL} - f_{3LT}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + f_{3TT}(x, k_T^2)\frac{k_T \cdot S_{TT} \cdot k_T}{M^2}\right]$$

$$\Phi^{[\gamma^-\gamma_5]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[g_{3LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu} k_{T\nu}}{M} + g_{3TT}(x, k_T^2)\frac{S_{TT\mu\rho} k_T^{\rho} \varepsilon_T^{\mu\nu} k_{T\nu}}{M^2}\right]$$

$$\Phi^{[\sigma^{i-}]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[h_{3LL}^{\perp}(x, k_T^2)\frac{S_{LL} k_T^i}{M} + h'_{3LT}(x, k_T^2) S_{LT}^i - h_{3LT}^{\perp}(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - h'_{3TT}(x, k_T^2)\frac{S_{TT}^{ij} k_{Tj}}{M} + h_{3TT}^{\perp}(x, k_T^2)\frac{k_T^i k_T \cdot S_{TT} \cdot k_T}{M^3}\right]$$

Quark / Hadron	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					$[h_3^{\perp}]$
L			g_{3L}		$[h_{3L}^{\perp}]$	
T		$f_{3T}^{\perp}$	g_{3T}		$[h_{3T}], [h_{3T}^{\perp}]$	
LL	f_{3LL}					$[h_{3LL}^{\perp}]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^{\perp}]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^{\perp}]$

New TMDs

[· · ·]= chiral odd

*4 Because of the time-reversal invariance, $h_{3LT}(x)$ does not exist; however, the corresponding new collinear fragmentation function $H_{3LT}(z)$ should exist because the time-reversal invariance does not have to be imposed.

Quark / Hadron	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

New collinear PDFs

TMDs and their sum rules for spin-1 hadrons

Twist-2 TMDs

Quark \ Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Time-reversal invariance in colliear corrlation functions (PDFs)

$$\int d^2k_T \Phi_{\text{T-odd}}(x, k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T\, h_{1LT}(x, k_T^2) = 0, \qquad \int d^2k_T\, g_{LT}(x, k_T^2) = 0,$$

$$\int d^2k_T\, h_{LL}(x, k_T^2) = 0, \qquad \int d^2k_T\, h_{3LT}(x, k_T^2) = 0$$

For example, in the twist-4

$$\int d^2k_T\, h_{3LT}(x, k_T^2) \equiv \int d^2k_T \left[h'_{3LT}(x, k_T^2) - \frac{k_T^2}{2M^2} h_{3LT}(x, k_T^2) \right] = 0$$

$$\Phi^{[\sigma^{i-}]} = \frac{M^2}{P^{+2}} \left[h_{3LL}^\perp(x, k_T^2) S_{LL} \frac{k_T^i}{M} + h'_{3LT}(x, k_T^2) S_{LT}^i - h_{3LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} \right.$$
$$\left. - h'_{3TT}(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} + h_{3TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M} \right]$$

Twist-3 TMDs

Quark \ Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		$f_T, f_T^\perp$ $[e_T, e_T^\perp]$	$g_T, g_T^\perp$		$[h_T], [h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			$g_{LT}, g_{LT}^\perp$		$[h_{LT}], [h_{LT}^\perp]$
TT	$f_{TT}, f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			$g_{TT}, g_{TT}^\perp$		$[h_{TT}], [h_{TT}^\perp]$

Twist-4 TMDs

Quark \ Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}], [h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^\perp]$

New fragmentations for spin-1 hadrons

Corresponding fragmentation functions exist for the spin-1 haddrons
simply by changing function names and kinematical variables.

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ \ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$$\Downarrow$$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\ z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Collinear fragmentation functions:
X. Ji, Phys. Rev. D 49, 114 (1994).

Summary on Spin-1 TMDs and PDFs

SK and Qin-Tao Song,
PRD 103 (2021) 014025.

TMDs of spin-1 hadrons

- TMDs: interdisciplinary field of physics
- We proposed new 30 TMDs and 3 PDFs in twist 3 and 4.
- New sum rules for TMDs.
- New TMD fragmentation functions.

Twist-3 TMD: $f_{LL}^{\perp}, e_{LL}, f_{LT}, f_{LT}^{\perp}, e_{1T}, e_{1T}^{\perp}, f_{TT}, f_{TT}^{\perp}, e_{TT}, e_{TT}^{\perp},$
$g_{LL}^{\perp}, g_{LT}, g_{LT}^{\perp}, g_{TT}, g_{TT}^{\perp}, h_{1L}, h_{LT}, h_{LT}^{\perp}, h_{TT}, h_{TT}^{\perp}$

Twist-4 TMD: $f_{3LL}, f_{3LT}, f_{3TT}, g_{3LT}, f_{3TT}, h_{3LL}^{\perp}, h_{3LT}, h_{3LT}^{\perp}, h_{3TT}, h_{3TT}^{\perp}$

Twist-3 PDF: e_{LL}, f_{LT}

Twist-4 PDF: f_{3LL}

Sum rules: $\int d^2k_T\, g_{LT}(x, k_T^2) = \int d^2k_T\, h_{LL}(x, k_T^2) = \int d^2k_T\, h_{3LL}(x, k_T^2) = 0$

TMD distribution functions: $f,\ g,\ h,\ e;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$\Downarrow$

TMD fragmentation functions: $D,\ G,\ H,\ E;\quad z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Twist-2 relation and sum rule for PDFs of spin-1 hadrons

(analogous to the Wandzura-Wilczek relation and the Burkhardt-Cottingham sum rule)

PDFs for spin-1 hadrons

Twist-2 PDFs

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}(b_1)$					
LT						*1
TT						

We derived analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule for f_{LT} and f_{1LL}.

SK and Qin-Tao Song (2021)

Twist-3 PDFs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

Twist-4 PDFs

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

Wandzura-Wilczek and Burkhardt-Cottingham relations for g_1 and g_2

Structure functions: $\int \frac{d(P^+\xi^-)}{2\pi} e^{ixP^+\xi^-} \left\langle P,S \right| \bar\psi(0)\gamma^\mu\gamma_5\psi(\xi) \left| P,S \right\rangle_{\xi^+=\vec\xi_T=0} = 2M_N \left[g_{1L}(x)\bar n^\mu S\cdot n + g_T(x) S_T^\mu + g_{3L}(x)\frac{M_N^2}{(P^+)^2} n^\mu S\cdot n \right]$

$$S^\mu = S_L \frac{P^+}{M}\bar n^\mu - S_L\frac{M_N}{2P^+}n^\mu + S_T^\mu, \quad P^\mu = P^+\bar n^\mu + \frac{M_N^2}{2P^+}n^\mu, \quad S\cdot n = S_L\frac{P^+}{M_N}$$

$$g_1(x) = \frac{1}{2}\left[g_{1L}(x) + g_{1L}(-x)\right], \quad g_1(x)+g_2(x) = \frac{1}{2}\left[g_T(x)+g_T(-x)\right]$$

J. Kodaira and K. Tanaka, Prog. Theor. Phys. 101 (1999) 191.

Operators: $R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} = i^{n-1}\bar\psi\gamma^\sigma\gamma_5 D^{\{\mu_1}\cdots D^{\mu_{n-1}\}}\psi = R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} + R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} = \text{twist } 2 + \text{twist } 3$

$$R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} = \frac{1}{n}\left[S^\sigma P^{\{\mu_1}P^{\mu_2}\cdots P^{\mu_{n-1}\}} + S^{\mu_1}P^{\{\sigma}P^{\mu_2}\cdots P^{\mu_{n-1}\}} + S^{\mu_2}P^{\{\mu_1}P^{\sigma}\cdots P^{\mu_{n-1}\}} + \cdots\right]$$

$$R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} = \frac{1}{n}\left[(n-1)S^\sigma P^{\{\mu_1}P^{\mu_2}\cdots P^{\mu_{n-1}\}} - S^{\mu_1}P^{\{\sigma}P^{\mu_2}\cdots P^{\mu_{n-1}\}} - S^{\mu_2}P^{\{\mu_1}P^{\sigma}\cdots P^{\mu_{n-1}\}} - \cdots\right]$$

$$\left\langle P,S \right| R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} \left| P,S\right\rangle = \frac{2}{n}a_n M_N\left[S^\sigma P^{\mu_1}\cdots P^{\mu_{n-1}} + P^{\mu_1}S^\sigma\cdots P^{\mu_{n-1}} + \cdots\right]$$

$$\left\langle P,S \right| R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} \left| P,S\right\rangle = \frac{2}{n}d_n M_N\left[(S^\sigma P^{\mu_1} - P^\sigma S^{\mu_1})P^{\mu_2}\cdots P^{\mu_{n-1}} + (S^\sigma P^{\mu_2} - P^\sigma S^{\mu_2})P^{\mu_1}\cdots P^{\mu_{n-1}} + \cdots\right]$$

$$\frac{1}{2M_N(P^+)^{n-1}} n_{\mu_1}\cdots n_{\mu_{n-1}} \left\langle P,S \right| R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} \left| P,S\right\rangle = \bar n^\sigma (S\cdot n)\int_{-1}^1 dx x^{n-1} g_{1L}(x) + S_T^\sigma \int_{-1}^1 dx x^{n-1} g_T(x)$$

$$= \frac{1}{2M_N(P^+)^{n-1}} n_{\mu_1}\cdots n_{\mu_{n-1}} \left\langle P,S \right| R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} \left| P,S\right\rangle + \frac{1}{2M_N(P^+)^{n-1}} n_{\mu_1}\cdots n_{\mu_{n-1}} \left\langle P,S \right| R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} \left| P,S\right\rangle$$

$$\to \int_{-1}^1 dx x^{n-1} g_{1L}(x) = a_n, \quad \int_{-1}^1 dx x^{n-1} g_T(x) = \frac{1}{n}a_n + \frac{n-1}{n}d_n$$

$$\to \int_0^1 dx x^{n-1} g_1(x) = \int_{-1}^1 dx x^{n-1}\frac{1}{2}g_{1L}(x) = \frac{1}{2}a_n, \quad \int_0^1 dx x^{n-1}[g_1(x)+g_2(x)] = \int_{-1}^1 dx x^{n-1}\frac{1}{2}g_T(x) = \frac{1}{2n}a_n + \frac{n-1}{2n}d_n$$

$$\to \int_0^1 dx x^{n-1} g_2(x) = \int_0^1 dx x^{n-1}\left[-g_1(x) + \int_x^1 \frac{dy}{y}g_1(y)\right] + \frac{n-1}{2n}d_n$$

If we write $g_2(x) = g_2^{WW}(x) + \bar g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y}g_1(y) + \bar g_2(x)$

$\to$ $g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y}g_1(y)$ (Wandzura-Wilczek relation), $\int_0^1 dx x^{n-1}\bar g_2(x) = \frac{n-1}{2n}d_n$

$\to$ $\int_0^1 dx\, g_2(x) = 0$ (Burkhardt-Cottingham sum rule)

Note: Twist-3 operators $R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$ are obtained by the Tayler expansion of $\xi_\mu\bar\psi(0)(\partial^\mu\gamma^\sigma - \partial^\sigma\gamma^\mu)\gamma_5\psi(\xi)$, which needs to be investigated in details for finding the details of twist-3 terms.

Collinear PDFs for spin-1 hadrons

may skip

Tensor polarization: $T^{\mu\nu}=\frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar{n}^\mu\bar{n}^\nu-\frac{2}{3}S_{LL}\left(\bar{n}^\mu n^\nu+\bar{n}^\nu n^\mu-g_T^{\mu\nu}\right)+\frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu+\frac{P^+}{M}(\bar{n}^\mu S_{LT}^\nu+\bar{n}^\nu S_{LT}^\mu)-\frac{M}{2P^+}(n^\mu S_{LT}^\nu+n^\nu S_{LT}^\mu)+S_{TT}^{\mu\nu}\right]$

Collinear correlation function: $\Phi(x,P,T)=\frac{1}{2}\left[f_{1LL}(x)S_{LL}\bar{\not n}+\frac{M}{P^+}e_{LL}(x)S_{LL}+\frac{M}{P^+}f_{LT}(x)S_{LT}\right]$, up to twist-3

Matrix element of vector operator: $\left\langle P,T\left|\bar{\psi}(0)\gamma^\mu\psi(\xi^-)\right|P,T\right\rangle=\int_{-1}^{1}dx\,e^{-ixP^+\xi^-}P^+\mathrm{Tr}\left[\Phi_{ij}(x,P,T)(\gamma^\mu)_{ji}\right]=\int_{-}^{1}dx\,e^{-ixP^+\xi^-}2P^+\left[f_{1LL}(x)S_{LL}\bar{n}^\mu+\frac{M}{P^+}S_{LT}^\mu f_{LT}(x)\right]$

$\alpha=1,2=$ transverse: $\left\langle P,T\left|\bar{\psi}(0)(\partial^\mu\gamma^\alpha-\partial^\alpha\gamma^\mu)\psi(\xi)\right|P,T\right\rangle=2MS_{LT}^\alpha\int_{-}^{1}dx\,e^{-ixP^+\xi^-}\left[-\frac{3}{2}f_{1LL}(x)+f_{LT}(x)-\frac{d}{dx}\{xf_{LT}(x)\}\right]$

Note: Twist-3 operators $R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$ are obtained by the Tayler expansion of $\xi_\mu\bar{\psi}(0)(\partial^\mu\gamma^\sigma-\partial^\sigma\gamma^\mu)\psi(\xi)$, which needs to be investigated in details for finding the details of twist-3 terms.

$$\bar{\psi}(0)(\partial^\mu\gamma^\alpha-\partial^\alpha\gamma^\mu)\psi(\xi)=\bar{\psi}(0)(D^\mu\gamma^\alpha-D^\alpha\gamma^\mu)\psi(\xi)-\bar{\psi}(0)\gamma^\mu\psi(\xi)ig\int_0^1dt\,t\,\xi_\rho G^{\rho\alpha}(t\xi)$$

In the Fock-Schwinger gauge: $\xi_\mu A^\mu(\xi)=0$, we have $A^\nu(\xi)=\int_0^1dt\,t\xi_\mu G^{\mu\nu}(t\xi)$, $G^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu-ig[A^\mu,A^\nu]$

$$\bar{\psi}(0)(D^\mu\gamma^\alpha-D^\alpha\gamma^\mu)\psi(\xi)=-\frac{i}{2}\bar{\psi}(0)\sigma^{\alpha\mu}\vec{\not D}\psi(\xi)-\frac{i}{2}\bar{\psi}(0)\overleftarrow{\not D}\sigma^{\alpha\mu}\psi(\xi)-\frac{1}{2}g\int_0^1dt\,\xi_\nu G^{\rho\nu}(t\xi)\bar{\psi}(0)\gamma_\rho\sigma^{\alpha\mu}\psi(\xi)+\frac{i}{2}\bar{\partial}_\rho\left\{\bar{q}(0)W(0,\xi)\gamma^\rho\sigma^{\alpha\mu}q(\xi)\right\}$$

$$-\frac{1}{2}\xi_\mu g\int_0^1dt\,\xi_\nu G^{\rho\nu}(t\xi)\bar{\psi}(0)\gamma_\rho\sigma^{\alpha\mu}\psi(\xi)=\frac{1}{2}g\int_0^1dt\,\bar{\psi}(0)\left\{-i\xi_\mu G^{\alpha\mu}(t\xi)\not\xi+\xi_\mu\tilde{G}^{\alpha\mu}(tx)\not\xi\gamma_5-\left[\xi^2\tilde{G}^{\alpha\sigma}(t\xi)-\xi_\mu x^\alpha\tilde{G}^{\mu\sigma}(tx)\right]\gamma_\sigma\gamma_5\right\}\psi(x)$$

$$\bar{\partial}_\rho\left\{\bar{q}(0)W(0,\xi)\gamma^\rho\sigma^{\alpha\mu}q(\xi)\right\}=\bar{\psi}(0)\overleftarrow{\not D}\sigma^{\alpha\mu}\psi(\xi)+\bar{\psi}(0)\vec{\not D}\sigma^{\alpha\mu}\psi(\xi)-ig\int_0^1dt\,\xi^\nu G_{\rho\nu}(t\xi)\bar{\psi}(0)\gamma^\rho\sigma^{\alpha\mu}\psi(\xi)$$

$$\xi_\mu\left[\bar{\psi}(0)(\partial^\mu\gamma^\alpha-\partial^\alpha\gamma^\mu)\psi(\xi)\right]=g\int_0^1dt\,\bar{\psi}(0)\left\{i\left(t-\frac{1}{2}\right)G^{\alpha\mu}(t\xi)-\frac{1}{2}\tilde{G}^{\alpha\mu}(t\xi)\gamma_5\right\}\xi_\mu\not\xi\psi(x)+\frac{1}{2}g\int_0^1dt\,\bar{\psi}(0)\left[\xi_\mu\xi^\alpha\tilde{G}^{\mu\sigma}(t\xi)-\xi^2\tilde{G}^{\alpha\sigma}(t\xi)\right]\gamma_\sigma\gamma_5\psi(\xi)$$

$$-\frac{i}{2}\xi_\mu\bar{\psi}(0)\sigma^{\alpha\mu}(\vec{\not D}-m)\psi(\xi)-\frac{i}{2}\xi_\mu\bar{\psi}(0)(\overleftarrow{\not D}+m)\sigma^{\alpha\mu}\psi(\xi)+\frac{i}{2}\xi_\mu\bar{\partial}_\rho\left\{\bar{\psi}(0)W(0,\xi)\gamma^\rho\sigma^{\alpha\mu}\psi(\xi)\right\}$$

$$\simeq g\int_0^1dt\,\bar{\psi}(0)\left\{i\left(t-\frac{1}{2}\right)G^{\alpha\mu}(t\xi)-\frac{1}{2}\tilde{G}^{\alpha\mu}(t\xi)\gamma_5\right\}\xi_\mu\not\xi\psi(x)$$

Multiparton correlation function: $(\Phi_G^\nu)_{ij}(x_1,x_2)=\int\frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}e^{ix_1P^+\xi_1^-}e^{i(x_2-x_1)P^+\xi_2^-}\left\langle P,T\left|\bar{\psi}_j(0)\,gG^{+\nu}(\xi_2^-)\psi_i(\xi_1^-)\right|P,T\right\rangle$

Express Φ_G^ν in terms of possible Lorentz vectors and multiparton distribution functions with the conditions Hermiticity, parity invariance, and time-reversal invariance

$$\Phi_G^\nu(x_1,x_2)=\frac{M}{2}\left[iS_{LT}^\nu F_{G,LT}(x_1,x_2)-\varepsilon_\perp^{\nu\mu}S_{LT\mu}\gamma_5G_{G,LT}(x_1,x_2)+iS_{LL}\gamma^\alpha H_{G,LL}^\perp(x_1,x_2)+iS_{TT}^{\nu\mu}\gamma_\mu H_{G,TT}(x_1,x_2)\right]\bar{\not n}$$

$(\Phi_G^\nu)_{ij}(\not n)_{ji}$: $\quad S_{LT}^\nu F_{G,LT}(x_1,x_2)=-\frac{i}{2M}g\int\frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}e^{ix_1P^+\xi_1^-}e^{i(x_2-x_1)P^+\xi_2^-}\left\langle P,T\left|\bar{\psi}(0)\,\not n n_\mu G^{\mu\nu}(\xi_2^-)\psi(\xi_1^-)\right|P,T\right\rangle$

$(\Phi_G^\nu)_{ij}(i\gamma_5\not n)_{ji}$: $\quad S_{LT}^\nu G_{G,LT}(x_1,x_2)=\frac{i}{2M}g\int\frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}e^{ix_1P^+\xi_1^-}e^{i(x_2-x_1)P^+\xi_2^-}\left\langle P,T\left|\bar{\psi}(0)\,i\gamma_5\not n n_\mu\tilde{G}^{\mu\nu}(\xi_2^-)\psi(\xi_1^-)\right|P,T\right\rangle$

$$\int\frac{d(P\cdot\xi)}{2\pi}e^{ix_1P\cdot\xi}\left\langle P,T\right|g\int_0^1dt\,\bar{\psi}(0)\left\{i\left(t-\frac{1}{2}\right)G^{\mu\nu}(t\xi)-\frac{1}{2}\gamma_5\tilde{G}^{\mu\nu}(t\xi)\right\}\xi_\mu\not\xi\psi(\xi)\left|P,T\right\rangle_{\xi^+=\vec{\xi}_T=0}=-2MS_{LT}^\nu\mathcal{P}\int_0^1dx_2\frac{1}{x_1-x_2}\left[\frac{\partial}{\partial x_1}\left\{F_{G,LT}(x_1,x_2)+G_{G,LT}(x_1,x_2)\right\}+\frac{\partial}{\partial x_2}\left\{F_{G,LT}(x_2,x_1)+G_{G,LT}(x_2,x_1)\right\}\right]$$

Twist-2 relation and sum rule

SK and Qin-Tao Song, JHEP 09 (2021) 141.

- **Twist-3 matrix element in terms of tensor-polarized PDFs**

$$\left\langle P,T \left| \bar{\psi}(0)(\partial^\mu \gamma^\alpha - \partial^\alpha \gamma^\mu)\psi(\xi) \right| P,T \right\rangle = 2MS_{LT}^{\alpha} \int_{-}^{1} dx\, e^{-ixP^+\xi^-} \left[-\frac{3}{2} f_{1LL}(x) + f_{LT}(x) - \frac{d}{dx}\{x f_{LT}(x)\} \right]$$

- **Twist-3 operator in terms of gluon field tensor**

$$\xi_\mu \left[\bar{\psi}(0)(\gamma^\alpha \partial^\mu - \gamma^\mu \partial^\alpha)\psi(\xi) \right] = g \int_0^1 dt\, \bar{\psi}(0) \left\{ i\left(t - \frac{1}{2}\right) G^{\alpha\mu}(t\xi) - \frac{1}{2}\gamma_5 \tilde{G}^{\alpha\mu}(t\xi) \right\} \xi_\mu \not{\xi} \psi(\xi)$$

- **Matrix element of field tensor in terms of twist-3 multiparton distribution functions**

$$\int \frac{d(P\cdot\xi)}{2\pi} e^{ix_1 P\cdot\xi} \langle P,T | g \int_0^1 dt\, \bar{\psi}(0) \left\{ i\left(t - \frac{1}{2}\right) G^{\mu\nu}(t\xi) - \frac{1}{2}\gamma_5 \tilde{G}^{\mu\nu}(t\xi) \right\} \xi_\mu \not{\xi} \psi(\xi) | P,T \rangle_{\xi^+ = \vec{\xi}_T = 0}$$

$$= -2MS_{LT}^{\nu} \mathcal{P} \int_0^1 dx_2 \frac{1}{x_1 - x_2} \left[\frac{\partial}{\partial x_1} \{ F_{G,LT}(x_1,x_2) + G_{G,LT}(x_1,x_2) \} + \frac{\partial}{\partial x_2} \{ F_{G,LT}(x_2,x_1) + G_{G,LT}(x_2,x_1) \} \right]$$

$$x \frac{df_{LT}(x)}{dx} = -\frac{3}{2} f_{1LL}(x) - f_{LT}^{(HT)}(x),$$ **Higher-twist:** $$f_{LT}^{(HT)}(x) = -\mathcal{P} \int_0^1 dy \frac{1}{x-y} \left[\frac{\partial}{\partial x} \{ F_{G,LT}(x,y) + G_{G,LT}(x,y) \} + \frac{\partial}{\partial y} \{ F_{G,LT}(y,x) + G_{G,LT}(y,x) \} \right]$$

$$\to f_{LT}(x) = \frac{3}{2} \int_x^{\varepsilon(x)} \frac{dy}{y} f_{1LL}(y) + \int_x^{\varepsilon(x)} \frac{dy}{y} f_{LT}^{(HT)}(y), \quad \varepsilon(x) = \frac{i}{\pi} P \int_{-\infty}^{\infty} dy \frac{1}{y} e^{-ixy} = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

Define $f^+(x) = f(x) + \bar{f}(x) = f(x) - f(-x), \quad f = f_{1LL},\ f_{LT},\ f_{LT}^{(HT)}, \quad x > 0$

$$\to f_{LT}^+(x) = \frac{3}{2} \int_x^1 \frac{dy}{y} f_{1LL}^+(y) + \int_x^1 \frac{dy}{y} f_{LT}^{(HT)+}(y)$$ $\to$ **Twist-2 relation:** $$f_{LT}^+(x) = \frac{3}{2} \int_x^1 \frac{dy}{y} f_{1LL}^+(y)$$

If we define $f_{2LT}(x) = \frac{2}{3} f_{LT}(x) - f_{1LL}(x),$

$$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y) + \frac{2}{3} \int_x^1 \frac{dy}{y} f_{LT}^{(HT)+}(y)$$ $\to$ **Twist-2 relation:** $$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y),$$ **Wandzura-Wilczek like**

$\to$ **Sum rule:** $\int_0^1 dx\ f_{2LT}^+(x) = 0,$ **Burkhardt-Cottingham like**

If the parton-model sum rule without the tensor-polarized antiquark distributions $\int_0^1 dx\, f_{1LL}^+(x) = \frac{2}{3} \int_0^1 dx\ b_1^+(x) = 0$ **is valid,** $\to$ **Sum rule:** $\int_0^1 dx\ f_{LT}^+(x) = 0$

$$\int dx\, b_1^D(x) = \lim_{t\to 0} -\frac{5}{12} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x)$$
$$= 0\ ?$$

F. E. Close and SK, PRD 42 (1990) 2377.

Summary on the twist-2 relation and sum rule

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$ (Wandzura-Wilczek relation), $\int_0^1 dx\, g_2(x) = 0$ (Burkhardt-Cottingham sum rule)

For tensor-polarized spin-1 hadrons, we obtained

$$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y),$$ $\int_0^1 dx\ f_{2LT}^+(x) = 0,$ $\quad f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$

$\int_0^1 dx\ f_{LT}^+(x) = 0$ **if** $\int_0^1 dx\, f_{1LL}^+(x) = \frac{2}{3} \int_0^1 dx\ b_1^+(x) = 0$

Existence of multiparton distribution functions: $F_{G,LT}(x_1,x_2),\ G_{G,LT}(x_1,x_2),\ H_{G,LL}^{\perp}(x_1,x_2),\ H_{G,TT}(x_1,x_2)$

Summary on Twist-2 relation and sum rule

- **We derived twist-2 relation and sum rule analogous to Wandzura-Wilczek relation and Burkardt-Cottingham sum rule.**
- **We showed the existence of tensor-polarized multiparton distribution functions.**

For spin-1/2 nucleons,

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \text{ (Wandzura-Wilczek relation)}, \qquad \int_0^1 dx\, g_2(x) = 0 \text{ (Burkhardt-Cottingham sum rule)}$$

For tensor-polarized spin-1 hadrons, we obtained

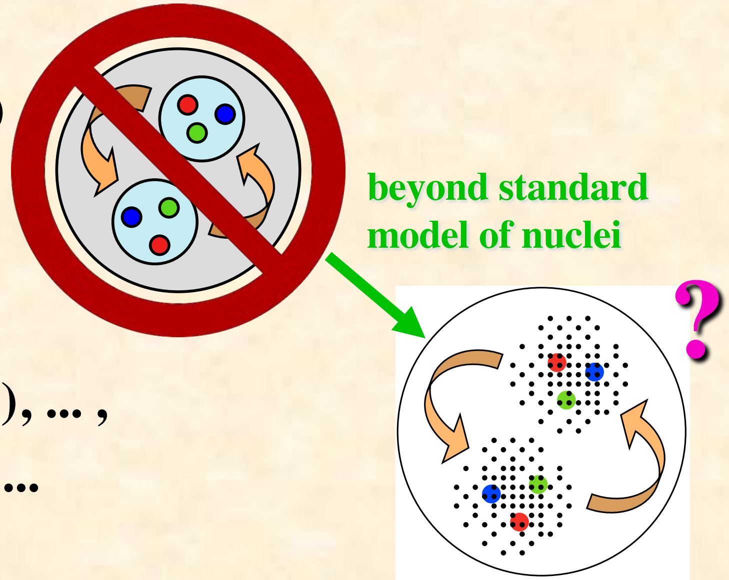
$$f_{2LT}^{+}(x) = -f_{1LL}^{+}(x) + \int_x^1 \frac{dy}{y} f_{1LL}^{+}(y), \qquad \int_0^1 dx\, f_{2LT}^{+}(x) = 0, \qquad f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$$

$$\int_0^1 dx\, f_{LT}^{+}(x) = 0 \text{ if } \int_0^1 dx\, f_{1LL}^{+}(x) = \frac{2}{3} \int_0^1 dx\, b_1^{+}(x) = 0$$

Existence of multiparton distribution functions: $F_{G,LT}(x_1,x_2)$, $G_{G,LT}(x_1,x_2)$, $H_{G,LL}^{\perp}(x_1,x_2)$, $H_{G,TT}(x_1,x_2)$

Spin-1 structure functions of the deuteron (new spin structure)

- tensor structure in quark-gluon degrees of freedom
- b_1, gluon transversity, new TMDs
- new signature beyond "standard" hadron physics?
- experiments: JLab (approved), Fermilab (proposal in 2022), ... , NICA (~2025), LHCspin (~2028), AMBER?, EIC, EicC, ...



Future prospects and summary

Spin-1 deuteron experiments from the middle of 2020's

JLab

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

Proposal (approved),
Experiment: middle of 2020's

Fermilab

The Transverse Structure of the Deuteron with Drell-Yan

D. Keller[1]

[1] *University of Virginia, Charlottesville, VA 22904*

Proposal,
Fermilab-PAC: January, 2022
Experiment: 2020's

NICA

Progress in Particle and Nuclear Physics 119 (2021) 103858

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov[a], A. Bacchetta[b,c], M. Butenschoen[d], F.G. Celiberto[b,c,e,f], U. D'Alesio[g,h], M. Deka[a], I. Denisenko[a], M.G. Echevarria[i], A. Efremov[a], N.Ya. Ivanov[a,j], A. Guskov[a,k,*], A. Karpishkov[l,a], Ya. Klopot[a,m], B.A. Kniehl[d], A. Kotzinian[j,o], S. Kumano[p], J.P. Lansberg[q], Keh-Fei Liu[r], F. Murgia[h], M. Nefedov[l], B. Parsamyan[a,n,o], C. Pisano[g,h], M. Radici[c], A. Rymbekova[a], V. Saleev[l,a], A. Shipilova[l,a], Qin-Tao Song[s], O. Teryaev[a]

Prog. Nucl. Part. Phys.
119 (2021) 103858,
Experiment: middle of 2020's

LHCspin

LHCspin

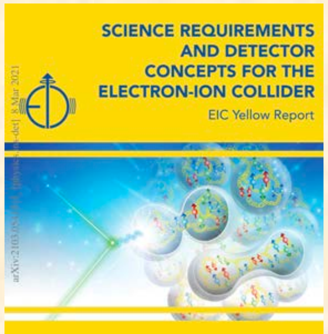
CERN-ESPP-Note-2018-111

The LHCSpin Project

C. A. Aidala[1], A. Bacchetta[2,3], M. Boglione[4,5], G. Bozzi[2,3], V. Carassiti[6,7], M. Chiosso[4,5], R. Cimino[8], G. Ciullo[6,7], M. Contalbrigo[6,7], U. D'Alesio[9,10], P. Di Nezza[8], R. Engels[11], K. Grigoryev[11], D. Keller[12], P. Lenisa[6,7], S. Liuti[12], A. Metz[13], P.J. Mulders[14,15], F. Murgia[10], A. Nass[11], D. Panzieri[5,16], L. L. Pappalardo[6,7], B. Pasquini[2,3], C. Pisano[9,10], M. Radici[3], F. Rathmann[11], D. Reggiani[17], M. Schlegel[18], S. Scopetta[19,20], E. Steffens[21], A. Vasilyev[22]

arXiv:1901.08002,
Experiment: ~2028

2030's EIC/EicC



R. Abdul Khalek *et al*.
arXiv:2103.05419.

D. P. Anderle *et al*.,
Front. Phys. 16 (2021) 64701.

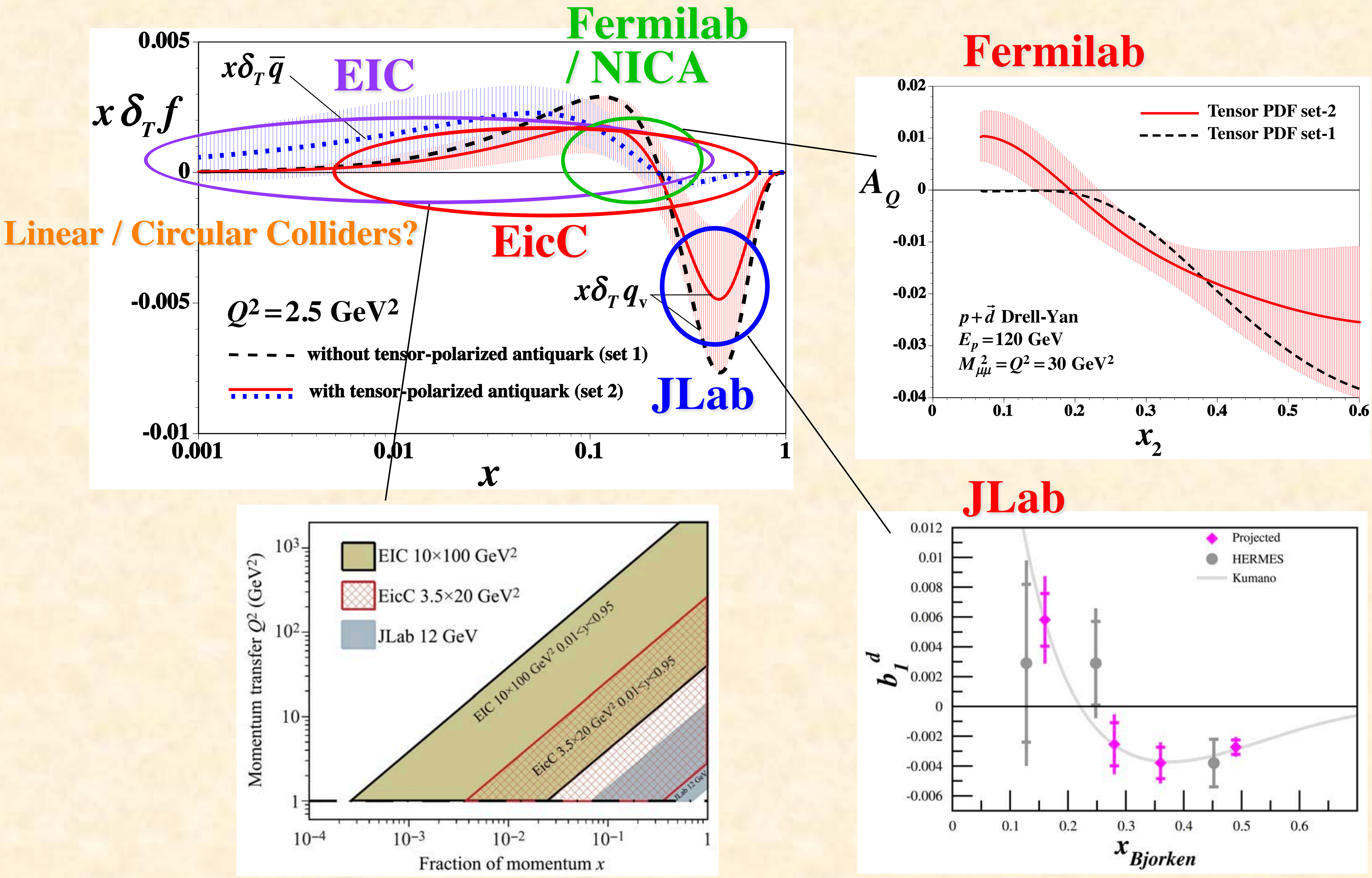
Frontiers of Physics
https://doi.org/10.1007/s11467-021-1062-0
Front. Phys. 16(6), 64701 (2021)

REVIEW ARTICLE

Electron-ion collider in China

Daniele P. Anderle[1], Valerio Bertone[2], Xu Cao[3,4], Lei Chang[5], Ningbo Chang[6], Gu Chen[7], Xurong Chen[3,4], Zhuojun Chen[8], Zhufang Cui[9], Lingyun Dai[8], Weitian Deng[10], Minghui Ding[11], Xu Feng[12], Chang Gong[12], Longcheng Gui[13], Feng-Kun Guo[4,14], Chengdong Han[3,4], Jun He[15], Tie-Jiun Hou[16], Hongxia Huang[15], Yin Huang[17], Krešimir Kumerički[18], L. P. Kaptari[3,19], Demin Li[20], Hengne Li[1], Minxiang Li[3,21], Xueqian Li[5], Yutie Liang[3,4], Zuotang Liang[22], Chen Liu[22], Chuan Liu[12], Guoming Liu[1], Jie Liu[3,4], Liuming Liu[3,4], Xiang Liu[21], Tianbo Liu[22], Xiaofeng Luo[23], Zhun Lyu[24], Boqiang Ma[12], Fu Ma[3,4], Jianping Ma[4,14], Yugang Ma[4,25,26], Lijun Mao[3,4], Cédric Mezrag[2], Hervé Moutarde[2], Jialun Ping[15], Sixue Qin[27], Hang Ren[3,4], Craig D. Roberts[9], Juan Rojo[28,29], Guodong Shen[3,4], Chao Shi[30], Qintao Song[20], Hao Sun[31], Paweł Sznajder[32], Enke Wang[1], Fan Wang[9], Qian Wang[1], Rong Wang[3,4], Ruiru Wang[3,4], Taofeng Wang[33], Wei Wang[34], Xiaoyu Wang[20], Xiaoyun Wang[35], Jiajun Wu[4], Xinggang Wu[27], Lei Xia[36], Bowen Xiao[23,37], Guoqing Xiao[3,4], Ju-Jun Xie[3,4], Yaping Xie[3,4], Hongxi Xing[1], Hushan Xu[3,4], Nu Xu[3,4,23], Shusheng Xu[38], Mengshi Yan[12], Wenbiao Yan[36], Wencheng Yan[20], Xinhu Yan[39], Jiancheng Yang[3,4], Yi-Bo Yang[4,14], Zhi Yang[40], Deliang Yao[8], Zhihong Ye[41], Peilin Yin[38], C.-P. Yuan[42], Wenlong Zhan[3,4], Jianhui Zhang[43], Jinlong Zhang[22], Pengming Zhang[44], Yifei Zhang[36], Chao-Hsi Chang[4,14], Zhenyu Zhang[45], Hongwei Zhao[3,4], Kuang-Ta Chao[12], Qiang Zhao[4,46], Yuxiang Zhao[3,4], Zhengguo Zhao[36], Liang Zheng[47], Jian Zhou[22], Xiang Zhou[45], Xiaorong Zhou[36], Bingsong Zou[4,14], Liping Zou[3,4]

x regions of b_1 in 2020's and 2030's



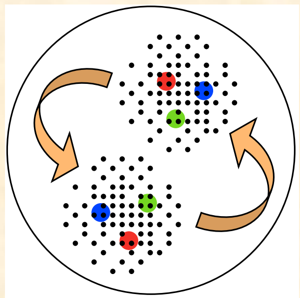
Summary

Spin-1 structure functions of the deuteron (additional spin structure to nucleon spin)

- **Tensor structure in quark-gluon degrees of freedom**
- **Tensor-polarized structure function b_1 and PDFs, gluon transversity**

  Experiments at JLab, Fermilab, NICA, LHCspin/AMBER, EIC/EicC, • • •
- **New signature beyond "standard" hadron physics?**

standard model



?

- **TMDs**
- **Not discussed: GPDs, GDAs (Generalized Distribution amplitudes = timelike GPDs),**

  Multiparton distribution functions, Fragmentation functions, • • •

There are various experimental projects on the polarized spin-1 deuteron in 2020's and 2030', and "exotic" hadron structure could be found by focusing on the spin-1 nature.

The End

The End