Photon-to-nucleon transition distribution amplitudes and backward time-like Compton scattering.

K.M. Semenov-Tian-Shansky

National Research Centre Kurchatov Institute: Petersburg Nuclear Physics Institute, Gatchina, Russia,
National Research University Higher School of Economics, Saint Petersburg, Russia

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Outline

1. Introduction: Forward and backward kinematical regimes, DAs, GPDs, TDAs;
2. Nucleon-to-photon TDAs: definition and properties;
3. Physical contents of $\mathcal{MM}$ and γN TDAs;
4. Near-backward TCS within the TDA framework;
5. Cross section estimates for JLab, EIC and EicC;
6. Summary and Outlook.

In collaboration with: B. Pire, L. Szymanowski,

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Transition distribution amplitudes and hard exclusive reactions with baryon number transfer

B. Pire [a], K. Semenov-Tian-Shansky [b,c,*], L. Szymanowski [d]

Factorization regimes for hard meson production

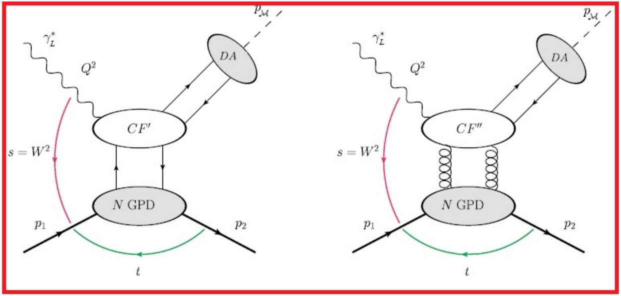
- J. Collins, L. Frankfurt and M. Strikman'97: the collinear factorization theorem for

$$\gamma^*(q) + N(p_1) \to N(p_2) + \mathcal{M}(p_{\mathcal{M}}).$$

Generalized Bjorken limit $t \sim 0$ (near-forward kinematics):

$$-q^2 = Q^2, \; W^2 \; - \; \text{large}; \quad x_B = \frac{Q^2}{2p_1 \cdot q} \; - \; \text{fixed}; \quad -t = -(p_2 - p_2)^2 \; - \; \text{small}.$$

- Description in terms of nucleon GPDs and meson DAs.



A complementary regime in the generalized Bjorken limit:

PHYSICAL REVIEW D, VOLUME 60, 014010

Hard exclusive pseudoscalar meson electroproduction and spin structure of the nucleon

L. L. Frankfurt,[1,2] P. V. Pobylitsa,[2,3] M. V. Polyakov,[2,3] and M. Strikman[2,4,*]

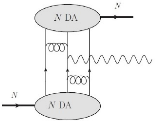
(Received 5 February 1999; published 4 June 1999)

"....Therefore the factorization theorem is valid also for the production of leading baryons

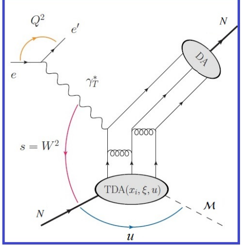
$$\gamma^*(q)+p\to B(q+\Delta)+M(p-\Delta)\text{....''}$$

- $u \sim 0$ (near-backward kinematics): nucleon-to-meson TDAs B. Pire, L. Szymanowski'05, 07 and nucleon DAs. Factorization theorem: same status as nucleon e.m. FF.

LO pQCD description of the nucleon e.m. FF:



Brodsky & Lepage'81
Efremov & Radyushkin'80



GPDs, DAs, GDAs and TDAs

- Main objects: matrix elements of QCD light-cone ($z^2 = 0$) operators.
- Quark-antiquark bilinear light-cone operator:

$$\langle A|\bar{\Psi}(0)[0;z]\Psi(z)|B\rangle$$

$\Rightarrow$ PDFs, meson DAs, meson-meson GDAs, GPDs, transition GPDs, etc.

- Three-quark trilinear light-cone ($z_i^2 = 0$) operator:

$$\langle A|\Psi(z_1)[z_1;z_0]\Psi(z_2)[z_2;z_0]\Psi(z_3)[z_3;z_0]|B\rangle$$

- $\langle A| = \langle 0|$; $|B\rangle$ - baryon; $\Rightarrow$ baryon DAs;
- Let $\langle A|$ be a meson state (π, η, ρ, ω, ...) $|B\rangle$ - nucleon; $\Rightarrow$ nucleon-to-meson TDAs.
- Let $\langle A|$ be a photon state $|B\rangle$ - nucleon; $\Rightarrow$ nucleon-to-photon TDAs.
- $\langle A| = \langle 0|$; $|B\rangle$ - baryon-meson state; $\Rightarrow$ baryon-meson GDAs.

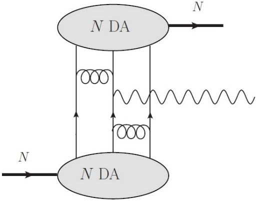
$\mathcal{M}N$ and γN TDAs have common features with:

- baryon DAs: same operator;
- GPDs: $\langle B|$ and $|A\rangle$ do not carry the same longitudinal momentum $\Rightarrow$ skewness:

$$\xi = -\frac{(p_A - p_B)\cdot n}{(p_A + p_B)\cdot n}.$$

Nucleon e.m. FF in QCD: a well known example

LO pQCD description of the nucleon e.m. FF:



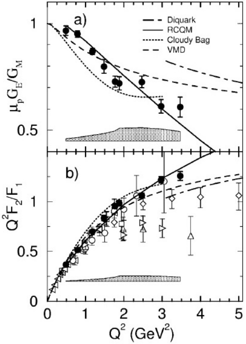
Brodsky & Lepage'81
Efremov & Radyushkin'80

Delayed scaling regime:

VOLUME 84, NUMBER 7 PHYSICAL REVIEW LETTERS 14 FEBRUARY 2000

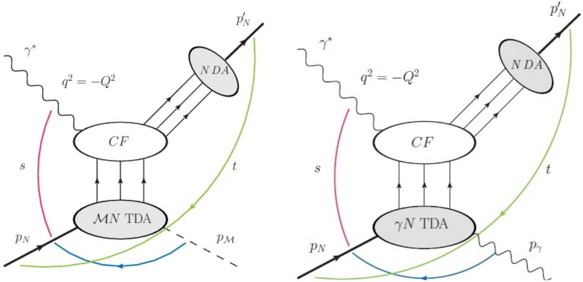
G_{E_p}/G_{M_p} Ratio by Polarization Transfer in $\vec{e}p \to e\vec{p}$

(The Jefferson Lab Hall A Collaboration)



- Importance of higher twist corrections!

Questions to address with $\mathcal{M}N$ and γN TDAs



Why this is interesting?

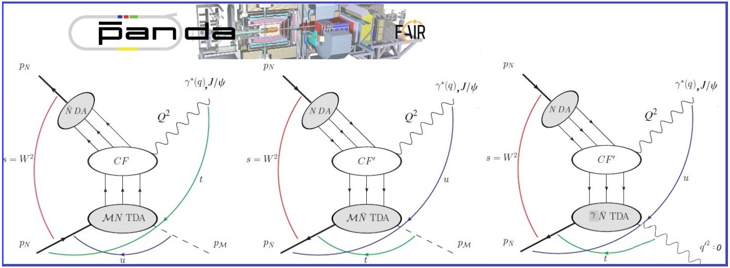
- γ and various mesons (π^0, $\pi^\pm$, η, η', ρ^0, $\rho^\pm$, ω, ϕ, $\ldots$) probe different spin-flavor combinations.
- Impact parameter picture: baryon charge distribution in the transverse plane.
- A view of the meson cloud and electromagnetic cloud inside a nucleon?
- Possible access to the 5-quark components of the nucleon LC WF?

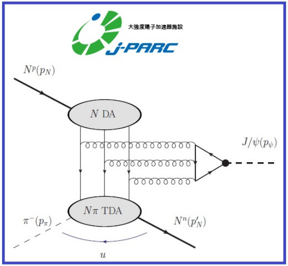
Learn more about QCD technique

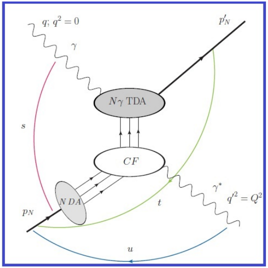
- A testbed for the QCD collinear factorization approach.
- A challenge for the lattice QCD & functional approaches based on DS/BS equations.

Cross channel counterpart reactions: P̄ANDA, JPARC and photoproduction at JLab

- Complementary experimental options and universality of TDAs.







A list of key issues:

- What are the properties and physical contents of nucleon-to-meson and nucleon-to-photon TDAs?
- What are the marking signs for the onset of the collinear factorization regime?
- Can we access backward reactions experimentally?

Leading twist-3 γN TDAs I

B.Pire, L.Szymanowski and K.S.'15 VN TDAs; $\left(n^2 = p^2 = 0;\; 2p \cdot n = 1;\ \text{LC gauge}\ \boxed{A \cdot n = 0}\right)$.

- $\frac{2^5}{2} = 16$ TDAs:
  $\left\{ V^{\gamma N}_{1\mathcal{E},1T,2\mathcal{E},2T},\ A^{\gamma N}_{1\mathcal{E},1T,2\mathcal{E},2T},\ T^{\gamma N}_{1\mathcal{E},1T,2\mathcal{E},2T,,3\mathcal{E},3T,4\mathcal{E},4T} \right\} (x_1,\, x_2,\, x_3,\, \xi,\, \Delta^2, \mu^2)$

Proton-to-γ TDAs:

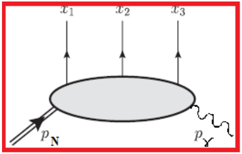
$$4(P\cdot n)^3 \int \left[\prod_{k=1}^{3} \frac{dz_k}{2\pi} e^{i\, x_k z_k (P\cdot n)}\right] \langle \gamma(p_\gamma, s_\gamma)|\, \varepsilon_{c_1 c_2 c_3} u^{c_1}_\rho(z_1 n) u^{c_2}_\tau(z_2 n) d^{c_3}_\chi(z_3 n)\, |N^p(p_N, s_N)\rangle$$

$$= \delta(x_1 + x_2 + x_3 - 2\xi)\, m_N \Big[\sum_{\substack{\Upsilon = 1\mathcal{E},1T\\ 2\mathcal{E},2T}} (v^{\gamma N}_\Upsilon)_{\rho\tau,\,\chi} V^{\gamma N}_\Upsilon(x_i, \xi, \Delta^2;\, \mu^2)$$

$$+ \sum_{\substack{\Upsilon = 1\mathcal{E},1T\\ 2\mathcal{E},2T}} (a^{\gamma N}_\Upsilon)_{\rho\tau,\,\chi} A^{\gamma N}_\Upsilon(x_i, \xi, \Delta^2;\, \mu^2) + \sum_{\substack{\Upsilon = 1\mathcal{E},1T,2\mathcal{E},2T\\ 3\mathcal{E},3T,4\mathcal{E},4T}} (t^{\gamma N}_\Upsilon)_{\rho\tau,\,\chi} T^{\gamma N}_\Upsilon(x_i, \xi, \Delta^2;\, \mu^2) \Big].$$

How to build Dirac structures?

- E.m. gauge invariance.
- Leading twist-3.
- $P = \frac{p_N + p_\gamma}{2}$; $\Delta = (p_\gamma - p_N)$; $\sigma_{P\mu} \equiv P^\nu \sigma_{\nu\mu}$;
  $\xi = -\frac{\Delta\cdot n}{2P\cdot n}$
- C: charge conjugation matrix;



Leading twist-3 γN TDAs II

- Just 4 (in fact 3) γN TDAs are relevant in the $\Delta_T = 0$ limit.

$$\boxed{(v_{1\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi}} = (\hat{p}C)_{\rho\tau}\left[\left(\gamma_5\hat{\mathcal{E}}^* U^+\right)_\chi - \frac{m_N}{1+\xi}(\mathcal{E}^*\cdot n)\left(\gamma_5 U^+\right)_\chi - \frac{2(\mathcal{E}^*\cdot n)}{1-\xi}\left(\gamma_5\hat{\Delta}_T U^+\right)_\chi\right];$$

$$(v_{1T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\hat{p}C)_{\rho\tau}\left(\gamma_5 U^+\right)_\chi;$$

$$(v_{2\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N}(\hat{p}C)_{\rho\tau}\left[\left(\gamma_5\sigma^{\Delta_T\mathcal{E}^*}U^+\right)_\chi - \frac{m_N(\mathcal{E}^*\cdot n)}{2(1+\xi)}\left(\gamma_5\hat{\Delta}_T U^+\right)_\chi\right];$$

$$(v_{2T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N^2}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\hat{p}C)_{\rho\tau}\left(\gamma_5\hat{\Delta}_T U^+\right)_\chi;$$

$$\boxed{(a_{1\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi}} = (\hat{p}\gamma^5 C)_{\rho\tau}\left[\left(\hat{\mathcal{E}}^* U^+\right)_\chi - \frac{m_N}{1+\xi}(\mathcal{E}^*\cdot n)\left(U^+\right)_\chi - \frac{2(\mathcal{E}^*\cdot n)}{1-\xi}\left(\hat{\Delta}_T U^+\right)_\chi\right];$$

$$(a_{1T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\hat{p}\gamma^5 C)_{\rho\tau}\left(U^+\right)_\chi;$$

$$(a_{2\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N}(\hat{p}\gamma^5 C)_{\rho\tau}\left[\left(\sigma^{\Delta_T\mathcal{E}^*}U^+\right)_\chi - \frac{m_N(\mathcal{E}^*\cdot n)}{2(1+\xi)}\left(\hat{\Delta}_T U^+\right)_\chi\right];$$

$$(a_{2T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N^2}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\hat{p}\gamma^5 C)_{\rho\tau}\left(\hat{\Delta}_T U^+\right)_\chi;$$

$$\boxed{(t_{1\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi}} = (\sigma_{p\mu}C)_{\rho\tau}\left[\left(\gamma_5\sigma^{\mu\mathcal{E}^*}U^+\right)_\chi - \frac{m_N(\mathcal{E}^*\cdot n)}{2(1+\xi)}\left(\gamma^5\gamma^\mu U^+\right)_\chi - \frac{2(\mathcal{E}^*\cdot n)}{(1-\xi)}\left(\gamma^5\sigma^{\mu\Delta_T}U^+\right)_\chi\right];$$

$$(t_{1T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\sigma_{p\mu}C)_{\rho\tau}\left(\gamma^5\gamma^\mu U^+\right)_\chi;$$

$$\boxed{(t_{2\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi}} = \left[(\sigma_{p\mathcal{E}^*}C)_{\rho\tau} - \frac{2(\mathcal{E}^*\cdot n)}{(1-\xi)}(\sigma_{p\Delta_T}C)_{\rho\tau}\right]\left(\gamma^5 U^+\right)_\chi;$$

$$(t_{2T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N^2}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\sigma_{p\mu}C)_{\rho\tau}\left(\gamma^5\sigma^{\mu\Delta_T}U^+\right)_\chi;$$

$$(t_{3\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N}(\sigma_{p\Delta_T}C)_{\rho\tau}\left[\left(\gamma^5\hat{\mathcal{E}}^* U^+\right)_\chi - \frac{m_N(\mathcal{E}^*\cdot n)}{(1+\xi)}\left(\gamma^5 U^+\right)_\chi - \frac{2(\mathcal{E}^*\cdot n)}{(1-\xi)}\left(\gamma^5\hat{\Delta}_T U^+\right)_\chi\right];$$

$$(t_{3T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N^2}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\sigma_{p\Delta_T}C)_{\rho\tau}\left(\gamma^5 U^+\right)_\chi;$$

$$(t_{4\mathcal{E}}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N}\left[(\sigma_{p\mathcal{E}^*}C)_{\rho\tau} - \frac{2(\mathcal{E}^*\cdot n)}{1-\xi}(\sigma_{p\Delta_T}C)_{\rho\tau}\right]\left(\gamma^5\hat{\Delta}_T U^+\right)_\chi;$$

$$(t_{4T}^{\gamma N})_{\rho\tau,\,\chi} = \frac{1}{m_N^3}\left[(\mathcal{E}^*\cdot\Delta_T) - \frac{2\Delta_T^2}{1-\xi}(\mathcal{E}^*\cdot n)\right](\sigma_{p\Delta_T}C)_{\rho\tau}\left(\gamma^5\hat{\Delta}_T U^+\right)_\chi.$$

Helicity contents I

- Light-front helicity amplitudes ($\lambda_N, \lambda_{u,u,d} = \uparrow, \downarrow$ and $\lambda_\gamma = \uparrow, \downarrow$):

$$\boxed{T^{\lambda_N, \lambda_\gamma}_{\lambda_u \lambda_u, \lambda_d}}$$

.

- 4 processes in which helicity is conserved:

$$3 \quad N^p(\uparrow) \to uud(\uparrow\downarrow\downarrow) + \gamma(\uparrow); \quad +1 \quad N^p(\uparrow) \to uud(\uparrow\uparrow\uparrow) + \gamma(\downarrow);$$

- This corresponds to 4 γN TDAs in the $\Delta_T = 0$ limit:

$$V^{\gamma p}_{1\mathcal{E}} = \frac{1}{2^{1/4}\sqrt{1+\xi}\,(P^+)^{3/2}} \frac{1}{m_N} \left[T^{\uparrow,\uparrow}_{\uparrow\downarrow,\downarrow} + T^{\uparrow,\uparrow}_{\downarrow\uparrow,\downarrow} \right];$$

$$A^{\gamma p}_{1\mathcal{E}} = -\frac{1}{2^{1/4}\sqrt{1+\xi}\,(P^+)^{3/2}} \frac{1}{m_N} \left[T^{\uparrow,\uparrow}_{\uparrow\downarrow,\downarrow} - T^{\uparrow,\uparrow}_{\downarrow\uparrow,\downarrow} \right];$$

$$T^{\gamma p}_{1\mathcal{E}} = -\frac{1}{2^{1/4}\sqrt{1+\xi}\,(P^+)^{3/2}} \frac{1}{m_N} \left[T^{\uparrow,\uparrow}_{\downarrow\downarrow,\uparrow} + T^{\uparrow,\downarrow}_{\uparrow\uparrow,\uparrow} \right];$$

$$T^{\gamma p}_{2\mathcal{E}} = -\frac{1}{2^{1/4}\sqrt{1+\xi}\,(P^+)^{3/2}} \frac{1}{m_N} \left[T^{\uparrow,\uparrow}_{\downarrow\downarrow,\uparrow} - T^{\uparrow,\downarrow}_{\uparrow\uparrow,\uparrow} \right].$$

Helicity contents II

- At $\Delta_T \neq 0$: counting Δ_T factors in the Dirac structure $\Leftrightarrow$ orbital angular momentum contribution to nucleon spin:
  - 1 power of Δ_T $\Leftrightarrow$ one unit of orbital angular momentum $L = 1$;
  - 2 powers of Δ_T $\Leftrightarrow$ two units of orbital angular momentum $L = 2$;
  - 3 powers of Δ_T $T^{\gamma N}_{4\mathcal{E}} \sim T^{\uparrow,\downarrow}_{\downarrow\downarrow,\downarrow}$
- New information on density probabilities for orbital angular momentum contributions when a proton emits a photon:

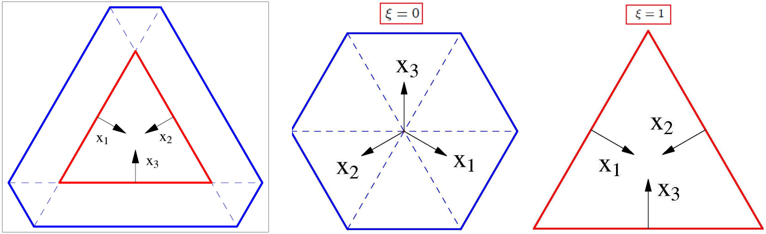
$$\frac{|V^{\gamma p}_{1\mathcal{E}}|^2 + |A^{\gamma p}_{1\mathcal{E}}|^2}{|T^{\gamma p}_{1\mathcal{E}}|^2 + |T^{\gamma p}_{2\mathcal{E}}|^2} \sim \frac{D_{h(u_1)=-h(u_2)}(x_i)}{D_{h(u_1)=+h(u_2)}(x_i)}.$$

- "Is the nucleon brighter when u-quarks have equal helicities?"

Fundamental properties I: support & polynomiality

B. Pire, L.Szymanowski, KS'10,11:

- Restricted support in x_1, x_2, x_3: intersection of three stripes $-1+\xi \leq x_k \leq 1+\xi$ $(\sum_k x_k = 2\xi)$; ERBL-like and DGLAP-like I, II domains.



- Mellin moments in $x_k \Rightarrow \gamma N$ matrix elements of local 3-quark operators

$$\left[i\vec{D}^{\mu_1} \ldots i\vec{D}^{\mu_{n_1}} \Psi_\rho(0)\right] \left[i\vec{D}^{\nu_1} \ldots i\vec{D}^{\nu_{n_2}} \Psi_\tau(0)\right] \left[i\vec{D}^{\lambda_1} \ldots i\vec{D}^{\lambda_{n_3}} \Psi_\chi(0)\right].$$

Can be studied on the lattice!

- Polynomiality in ξ of the Mellin moments in x_k:

$$\int_{-1+\xi}^{1+\xi} dx_1 dx_2 dx_3 \delta(\sum_k x_k - 2\xi) x_1^{n_1} x_2^{n_2} x_3^{n_3} H^{\gamma N}(x_1, x_2, x_3, \xi, \Delta^2)$$

$$= [\text{Polynomial of order } n_1 + n_2 + n_3\{+1\}](\xi).$$

Fundamental properties II: spectral representation

- Spectral representation A. Radyushkin'97 generalized for TDAs ensures polynomiality and support:

$$
\begin{aligned}
&H(x_1,\, x_2,\, x_3 = 2\xi - x_1 - x_2,\, \xi) \\
&= \left[\prod_{i=1}^{3} \int_{\Omega_i} d\beta_i d\alpha_i \right] \delta(x_1 - \xi - \beta_1 - \alpha_1 \xi)\, \delta(x_2 - \xi - \beta_2 - \alpha_2 \xi) \\
&\times \delta(\beta_1 + \beta_2 + \beta_3) \delta(\alpha_1 + \alpha_2 + \alpha_3 + 1) F(\beta_1,\, \beta_2,\, \beta_3,\, \alpha_1,\, \alpha_2, \alpha_3)\, ;
\end{aligned}
$$

- Ω_i: $\{|\beta_i| \leq 1,\ |\alpha_i| \leq 1 - |\beta_i|\}$ are copies of the usual DD square support;
- $F(\ldots)$: six variables that are subject to two constraints $\Rightarrow$ quadruple distributions;
- Can be supplemented with a D-term-like contribution (with pure ERBL-like support):

$$
\frac{1}{(2\xi)^2} \delta \left(x_1 + x_2 + x_3 - 2\xi \right) \left[\prod_{k=1}^{3} \theta \left(0 \leq x_k \leq 2\xi \right) \right] D \left(\frac{x_1}{2\xi}, \frac{x_2}{2\xi}, \frac{x_3}{2\xi} \right) .
$$

Fundamental properties III: evolution

- Evolution properties of 3-quark light-cone operator: V. M. Braun, S. E. Derkachov, G. P. Korchemsky, A. N. Manashov'99.
- Evolution equations for TDAs: B. Pire, L. Szymanowski'07.
- Conformal basis (Jacobi and Gegenbauer polynomials):

$$\Psi_{N,n}^{(12)3}(y_1,y_2,y_3)=(N+n+4)(y_1+y_2)^n P_{N-n}^{(2n+3,1)}(y_3-y_1-y_2)C_n^{\frac{3}{2}}\left(\frac{y_1-y_2}{y_1+y_2}\right).$$

- The conformal PWs:

$$p_{N,n}^{(12)3}(w,v,\xi)=\theta(|w|\leq\xi)\,\theta(|v|\leq\xi')\,\xi^{-N-2}\frac{1}{g_{N,n}}$$
$$\times\left(1-\frac{v^2}{\xi'^2}\right)C_n^{\frac{3}{2}}\left(-\frac{v}{\xi'}\right)\left(1-\frac{w}{\xi}\right)^{n+2}\left(1+\frac{w}{\xi}\right)P_{N-n}^{2n+3,1}\left(\frac{w}{\xi}\right).$$

- Conformal PW expansion for TDAs:

$$H(w,v,\xi,\Delta^2)=\sum_{N=0}^{\infty}\sum_{n=0}^{N}p_{N,n}^{(12)3}(w,v,\xi)h_{n,N}^{(12)3}(\xi,\Delta^2).$$

- SO(3) PW expansion of the conformal moments $h_{n,N}^{(12)3}$ $\Rightarrow$ cross-channel picture of baryon exchanges. Dual parametrization, see D. Müller, M.Polyakov, K.S.'15.

A connection to the quark-diquark picture

- Quark-diquark coordinates (one of 3 possible sets):

$$v_3 = \frac{x_1 - x_2}{2}; \quad w_3 = x_3 - \xi; \quad x_1 + x_2 = 2\xi_3'; \quad \left(\xi_3' \equiv \frac{\xi - w_3}{2} \right).$$

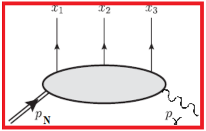
- The TDA support in quark-diquark coordinates:

$$\boxed{-1 \leq w_3 \leq 1; \quad -1 + |\xi - \xi_3'| \leq v_3 \leq 1 - |\xi - \xi_3'|}$$

- v_3-Mellin moment of γN TDAs: "diquark-quark" light-cone operator

$$\int_{-1+|\xi-\xi_3'|}^{1-|\xi-\xi_3'|} dv_3 H^{\gamma N}(w_3, v_3, \xi, \Delta^2)$$

$$\sim h^{-1}_{\rho\tau\chi} \int \frac{d\lambda}{4\pi} e^{i(w_3\lambda)(P\cdot n)} \langle \gamma(p_\gamma) | \underbrace{u_\rho(-\frac{\lambda}{2}n) u_\tau(-\frac{\lambda}{2}n) d_\chi(\frac{\lambda}{2}n)}_{\hat{O}^{\{uu\}d}_{\rho\tau\chi}(-\frac{\lambda}{2}n, \frac{\lambda}{2}n)} | N^p(p_1) \rangle.$$

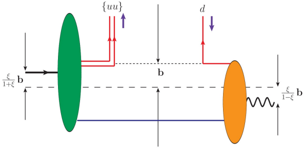


γN TDAs: an interpretation in the impact parameter space I

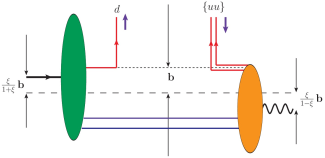
- A generalization of M. Burkardt'00,02; M. Diehl'02 for v_3-integrated TDAs.
- Fourier transform with respect to

$$\mathbf{D} = \frac{\mathbf{p}_\gamma}{1-\xi} - \frac{\mathbf{p}_N}{1+\xi}; \qquad \Delta^2 = -2\xi \left(-\frac{m_N^2}{1+\xi} \right) - (1-\xi^2)\mathbf{D}^2.$$

- A representation in the DGLAP-like I and II domains:



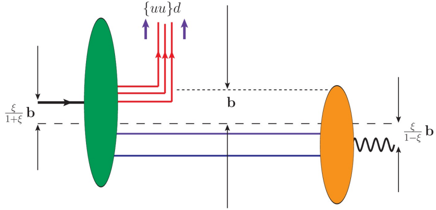
DGLAP I : $x_3 = w_3 - \xi \leq 0; \quad x_1 + x_2 = \xi - w_3 \geq 0$



DGLAP II : $x_3 = w_3 - \xi \geq 0; \quad x_1 + x_2 = \xi - w_3 \leq 0$

γN TDAs: an interpretation in the impact parameter space II

- A representation in the ERBL-like domain:



$$\text{ERBL}: \; x_3 = w_3 - \xi \geq 0; \quad x_1 + x_2 = \xi - w_3 \geq 0$$

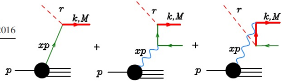
- A possible view on the inner light (e.m. cloud) within the nucleon.

A picture complementary to photon pdf of the nucleon?

PRL **117,** 242002 (2016) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2016

How Bright is the Proton? A Precise Determination of the Photon Parton Distribution Function

Aneesh Manohar,[1,2] Paolo Nason,[3] Gavin P. Salam,[2,*] and Giulia Zanderighi[2,4]



Building up a consistent model for TDAs

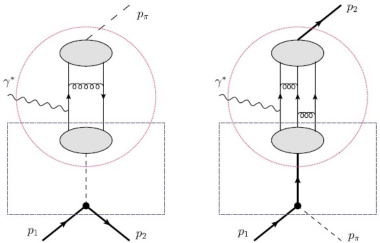
Key requirements:

1. support in x_ks and polynomialty;
2. isospin + permutation symmetry;
3. crossing πN TDA $\leftrightarrow$ πN GDA and chiral properties: soft pion theorem;

How to model quadruple distributions?

- No enlightening $\xi = 0$ limit as for GPDs.
- $\xi \to 1$ limit fixed from chiral dynamics.
- A factorized Ansatz with input at $\xi = 1$ designed in J.P. Lansberg, B. Pire, K.S. and L. Szymanowski'12

- N and $\Delta(1232)$ cross-channel exchanges $\Rightarrow$ D-term-like contribution: $\tilde{E}$ GPD v.s. TDA



How to check that the TDA-based reaction mechanism is relevant?

Distinguishing features

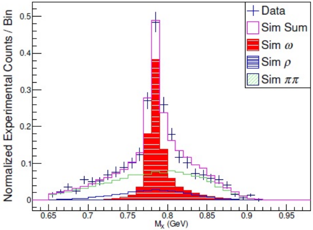
- Characteristic backward peak of the cross section. Special behavior in the near-backward region.
- Scaling behavior of the cross section in Q^2 and specific counting rules
- Dominance of the transverse cross section σ_T
- For time-like reactions: specific angular distribution of the lepton pair $\sim (1 + \cos^2 \theta_\ell)$.

- Pioneering analysis of backward $\gamma^* p \to \pi^0 p$. A. Kubarovsky, CIPANP 2012.
- Analysis of JLab @ 6 GeV data (Oct.2001-Jan.2002 run) for the backward $\gamma^* p \to \pi^+ n$ K. Park et al. (CLAS Collaboration), PLB 780 (2018).
- Backward ω-production at JLab Hall C. W. Li, G. Huber (The JLab F_π Collaboration), PRL 123, 2019
- S. Diehl et al. (CLAS collaboration), PRL 125 (2020) : extraction of BSA in $\gamma^* p \to \pi^+ n$.
- Feasibility studies for PANDA and JPARC.

More in Stefan Diehl's talk today!

Backward ω-production at JLab Hall C I

- TDA formalism for the case of light vector mesons (ρ, ω, ϕ) B. Pire, L. Szymanowski and K.S'15. 24 VN TDAs at the leading twist.
- The analysis W. Li, G. Huber et al. (The JLab F_π Collaboration), PRL 123, 2019
- Clear signal from backward regime of $ep \to e' p\omega$.

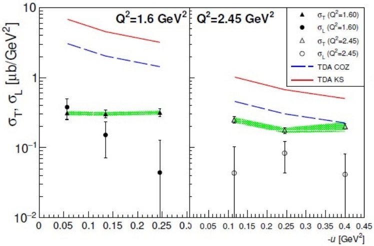


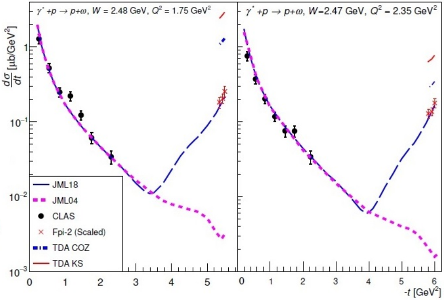
- Full Rosenbluth separation: $\sigma_{\rm T}$ and $\sigma_{\rm L}$ extracted to address $\sigma_{\rm T} \gg \sigma_{\rm L}$ issue.

$$2\pi \frac{d^2\sigma}{dt d\phi} = \frac{d\sigma_{\rm T}}{dt} + \epsilon \frac{d\sigma_{\rm L}}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{\rm LT}}{dt} \cos\phi + \epsilon \frac{d\sigma_{\rm TT}}{dt} \cos 2\phi$$

Backward ω-production at JLab Hall C II

- For $Q^2 = 2.45$ GeV^2: $\sigma_\mathrm{L}/\sigma_\mathrm{T} < \mu^2/Q^2$ and $\sigma_\mathrm{T} \gg \sigma_\mathrm{L}$;





- Experiment v.s. the predictions of the cross-channel nucleon exchange model for $p \to \omega$ TDAs.
- Combined (CLAS and F_π-2 data for $\gamma^* p \to \omega p$).
- TDA-based predictions v.s. the Regge-based J.M. Laget's JML'18 model.

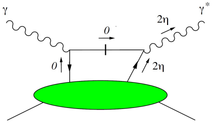
Time-like Compton scattering

$$\gamma(q) + N(p_1) \to \gamma^*(q') + N(p_2) \to \ell\bar{\ell} + N(p_2)$$

- Near-forward TCS E. Berger, M.Diehl, B.Pire'01:

$$\text{large} \quad q'^2 = Q'^2 \quad \text{and} \quad s; \quad \text{small} \quad -t.$$

- Fixed $\tau = \frac{Q'^2}{2p_1 \cdot q} = \frac{Q'^2}{s - m_N^2}$: analog of the Bjorken variable.



(b)

$q'^2 = +Q'^2 > 0$

- A complementary access to GPDs. Check of universality.

  at LO : $\mathcal{H}_{TCS} = \mathcal{H}^*_{DVCS}$; $\tilde{\mathcal{H}}_{TCS} = -\tilde{\mathcal{H}}^*_{DVCS}$

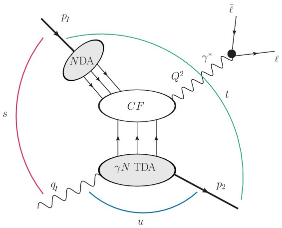
  at NLO $\mathcal{H}_{TCS} = \mathcal{H}^*_{DVCS} - i\pi Q^2 \frac{d}{dQ^2}\mathcal{H}^*_{DVCS}$; $\tilde{\mathcal{H}}_{TCS} = -\tilde{\mathcal{H}}^*_{DVCS} + i\pi Q^2 \frac{d}{dQ^2}\tilde{\mathcal{H}}^*_{DVCS}$

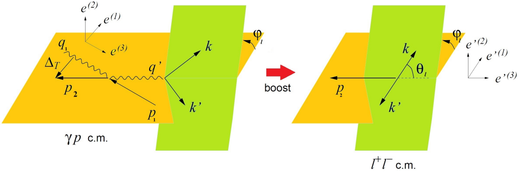
- First experimental data on TCS from CLAS12 arXiv:2108.11746.

Backward time-like Compton scattering

$$\gamma(q_1) + N(p_1) \to \gamma^*(q') + N(p_2) \to \ell\bar{\ell} + N(p_2)$$

large s and $q_2^2 \equiv Q^2$; fixed x_B; small $-u = -(p_2 - q_1)^2$.

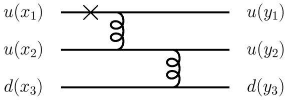




- γ_T^* dominance: $(1 + \cos^2\theta_\ell)$ angular dependence;
- large $-t$: small BH background?
- Crude cross section estimates: VMD + $\gamma^* N \to VN +$ crossing.

Calculation of the LO amplitude

- LO amplitude for $\gamma^* + N \to V + N^p$ computed as in B. Pire, K.S. and L. Szymanowski'16;
- 21 diagrams contribute;



$$\mathcal{M}^{\gamma^* N \to \gamma N'} \approx \bar{u}\left(N'\right) \widehat{\mathcal{E}}(q) u(N) \int dx_i dy_i DA\left(y_i; Q^2\right) T_H\left(x_i, y_i, Q^2\right) \mathrm{TDA}\left(x_i, \xi, u; Q^2\right).$$

- $\mathrm{DA}(y_i; Q^2)$ = proton distribution amplitude;
- T_H: hard scattering amplitude, calculated in the collinear approximation.

$$\text{C.f.} \quad A(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x \pm \xi \mp i\epsilon} \int_0^1 dy \frac{\phi_M(y)}{y}$$

- At leading order, the amplitude for the time-like process is the complex conjugate of space-like (i.e. electroproduction) amplitude.
- Scaling law for the amplitude:

$$\mathcal{M}\left(Q^2, \xi\right) \sim \frac{\alpha_s \left(Q^2\right)^2}{Q^4}$$

Vector meson dominance I

- J. J. Sakurai'1960s: vector meson dominance model

$$\left\langle 0 \left| J_\mu^{\rm em}(0) \right| {\rm V} \right\rangle = \varepsilon_\mu m_{\rm V}^2 / f_{\rm V};$$

- Coupling constants:

$$\Gamma\left({\rm V} \to e^+ e^-\right) \approx \frac{1}{3}\alpha^2 m_{\rm V}\left(f_{\rm V}^2/4\pi\right)^{-1}, \quad {\rm V} = \rho, \omega, \phi.$$

- VMD for photoproduction reactions: A and B - hadron states

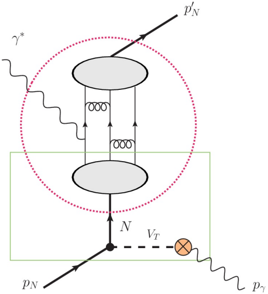
$$[\gamma A \to B] = e\frac{1}{f_\rho}\left[\rho^0 A \to B\right] + (\omega) + (\phi).$$

Vector meson dominance II

- VMD-based model for nucleon-to-photon TDAs

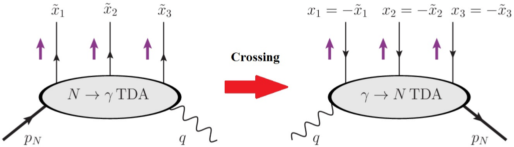
$$V_{\Upsilon}^{\gamma N} = \frac{e}{f_\rho} V_{\Upsilon}^{\rho_T N} + \frac{e}{f_\omega} V_{\Upsilon}^{\omega_T N} + \frac{e}{f_\phi} V_{\Upsilon}^{\phi_T N};$$

- Check of consistency: transverse polarization of V 16 out of 24 leading twist-3 VN TDAs;
- Cross-channel nucleon exchange model for $V_T N$ TDAs:



- Dumbrajs et al.'1982: $\frac{f_\rho^2}{4\pi} = 2.26$; $\frac{f_\omega^2}{4\pi} = 18.4$; $\frac{f_\phi^2}{4\pi} = 14.3$.

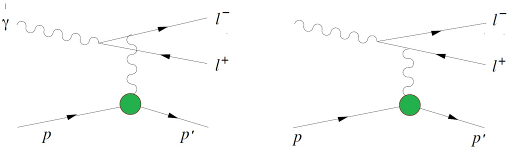
Crossing $\gamma \to N$ to $N \to \gamma$ TDAs



- Crossing relation established in B.Pire, K.S., L. Szymanowski, PRD'95 for $\pi \to N$ and $N \to \pi$ TDAs.

$$V_i^{N\gamma}(x_i, \xi, u) = V_i^{\gamma N}(-x_i, -\xi, u)\, ; A_i^{N\gamma}(x_i, \xi, u) = A_i^{\gamma N}(-x_i, -\xi, u)$$
$$T_i^{N\gamma}(x_i, \xi, u) = T_i^{\gamma N}(-x_i, -\xi, u)\, .$$

BH contribution in the near-backward regime I



$$\frac{d\sigma_{BH}}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}^3}{4\pi(s-M^2)^2}\,\frac{\beta}{-tL}\left[\left(F_1^2 - \frac{t}{4M^2}F_2^2\right)\frac{A}{-t} + (F_1+F_2)^2\,\frac{B}{2}\right]$$

$$\begin{aligned} A &= (s-M^2)^2\Delta_T^2 - t\,a(a+b) - M^2b^2 - t\,(4M^2-t)Q'^2 \\ &\quad + \frac{m_\ell^2}{L}\left[\left\{(Q'^2-t)(a+b) - (s-M^2)\,b\right\}^2 + t\,(4M^2-t)(Q'^2-t)^2\right]; \\ B &= (Q'^2+t)^2 + b^2 + 8m_\ell^2 Q'^2 - \frac{4m_\ell^2(t+2m_\ell^2)}{L}(Q'^2-t)^2; \end{aligned}$$

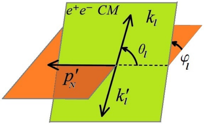
$$a = 2(k-k')\cdot p', \qquad b = 2(k-k')\cdot(p-p');$$

$$L = \left[(q-k)^2 - m_\ell^2\right]\left[(q-k')^2 - m_\ell^2\right] = \frac{(Q'^2-t)^2 - b^2}{4}; \quad \beta = \sqrt{1 - 4m_\ell^2/Q'^2}.$$

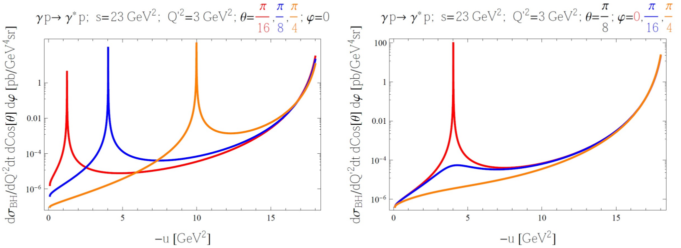
- BH contribution dominates in the near-forward regime: $\frac{F_1(t)}{t} \sim \frac{1}{t}$.

BH contribution in the near-backward regime II

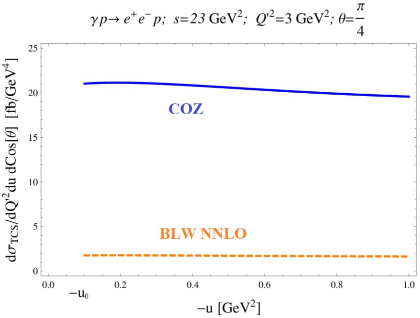
- The BH cross section peaks once ℓ goes "on-shell": L -small.
- Effect of the cut in the lepton polar angle θ: keep the BH peak out of the near-backward kinematics.

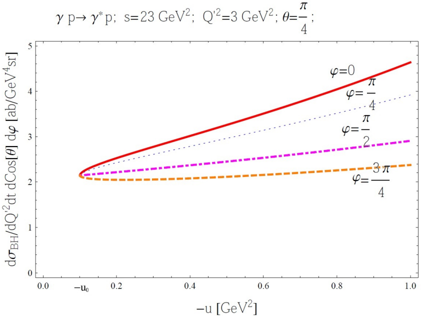


- The left peak is very narrow.



Cross section estimates: BH v.s. near-backward TCS





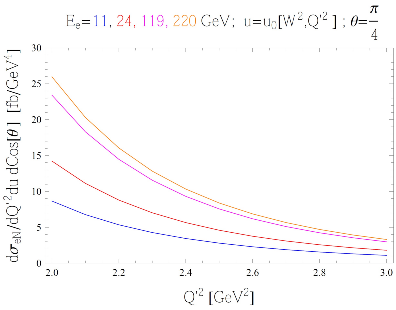
Cross section estimates for JLab and EicC

- Quasi-real photoproduction

$$\sigma_{eN} = \int dx \sigma_{\gamma N}(x) f(x); \quad x = \frac{s_{\gamma N} - m_N^2}{s_{eN} - m_N^2}.$$

- Weizsacker-Williams distribution

$$f(x) = \frac{\alpha_{\rm em}}{2\pi} \left\{ 2m_e^2 x \left(\frac{1}{Q^2_{\rm max}} - \frac{1-x}{m_e^2 x^2} \right) + \frac{\left((1-x)^2+1\right) \ln \frac{Q^2_{\rm max}(1-x)}{m_e^2 x^2}}{x} \right\}.$$



- JLab Hall C: assuming luminosity $10^{38} cm^{-2} s^{-1}$ - plenty of events!
- EicC luminosity (reported in ArXiv:2110.094) is $50 fb^-1$/year - several tens of events.

Conclusions & Outlook

1. Nucleon-to-meson TDAs provide new information about correlation of partons inside hadrons. A consistent picture for the integrated TDAs emerges in the impact parameter representation.
2. We strongly encourage to detect near forward and backward signals for various mesons (π, η, ω, ρ): there is interesting physics around!
3. First evidences for the onset of the factorization regime in backward $\gamma^* N \to N' \omega$ from JLab Hall C analysis and BSA measurements in $\gamma^* p \to \pi^+ n$ from CLAS.
4. PAC 48 decision is a challenge both for the experiment and for theory. New models for advanced feasibility studies. Regge-behavior for small x_B?
5. Backward time-like Compton scattering and backward DVCS may provide access to nucleon-to-photon TDAs. Seems to be sizable enough to be studied with JLab Hall C and A and future EicC. BH contribution is small in the near-backward regime.
6. Dispersion relation and the subtraction constant.
7. May be an addition to the ultraperipheral physics program at hadron colliders.

Thank you for your attention!