Lensing function relation in hadrons

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3DSPIN
MAPPING
THE PROTON IN 3D

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Motivation

Semi inclusive processes
exhibit single spin asymmetry (SSA) effects

Different contributions, one is from T-odd
Transverse Momentum Dependent parton distributions (TMDs)

T-odd TMDs are present
due to the final state interactions (FSIs)

Can FSIs be factorised, explaining the SSA as
the convolution of a spatially distorted distribution of partons
and a lensing function?

Pasquini, Lorcé PRD84(2011)

$$\mathcal{W}^{[\Gamma]}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$ Wigner distributions

$$\int d\boldsymbol{b}_\perp$$

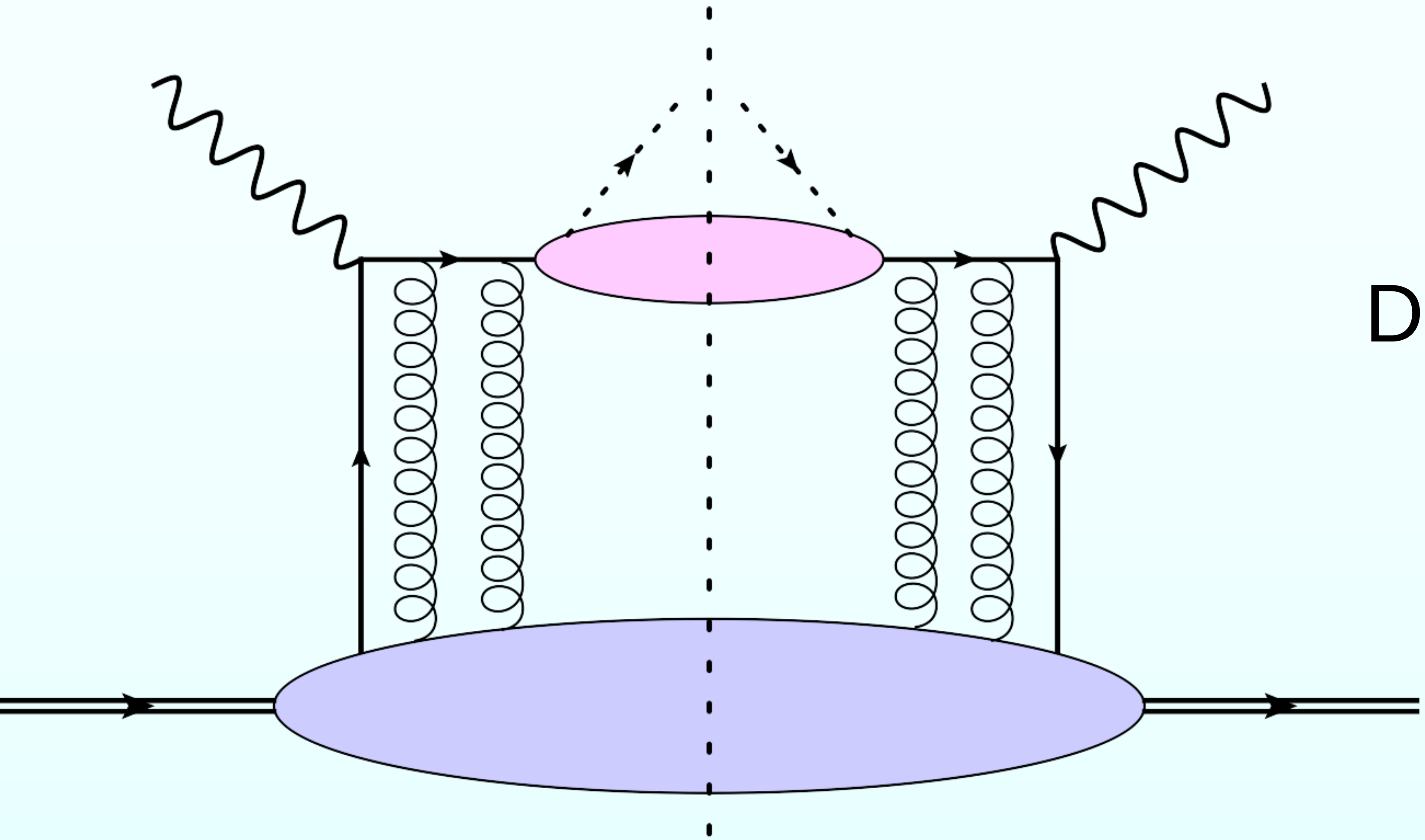
$$\int d\boldsymbol{k}_\perp$$

Transverse Momentum Dependent parton distributions (TMDs)

Impact Parameter parton Distributions (IPDs)

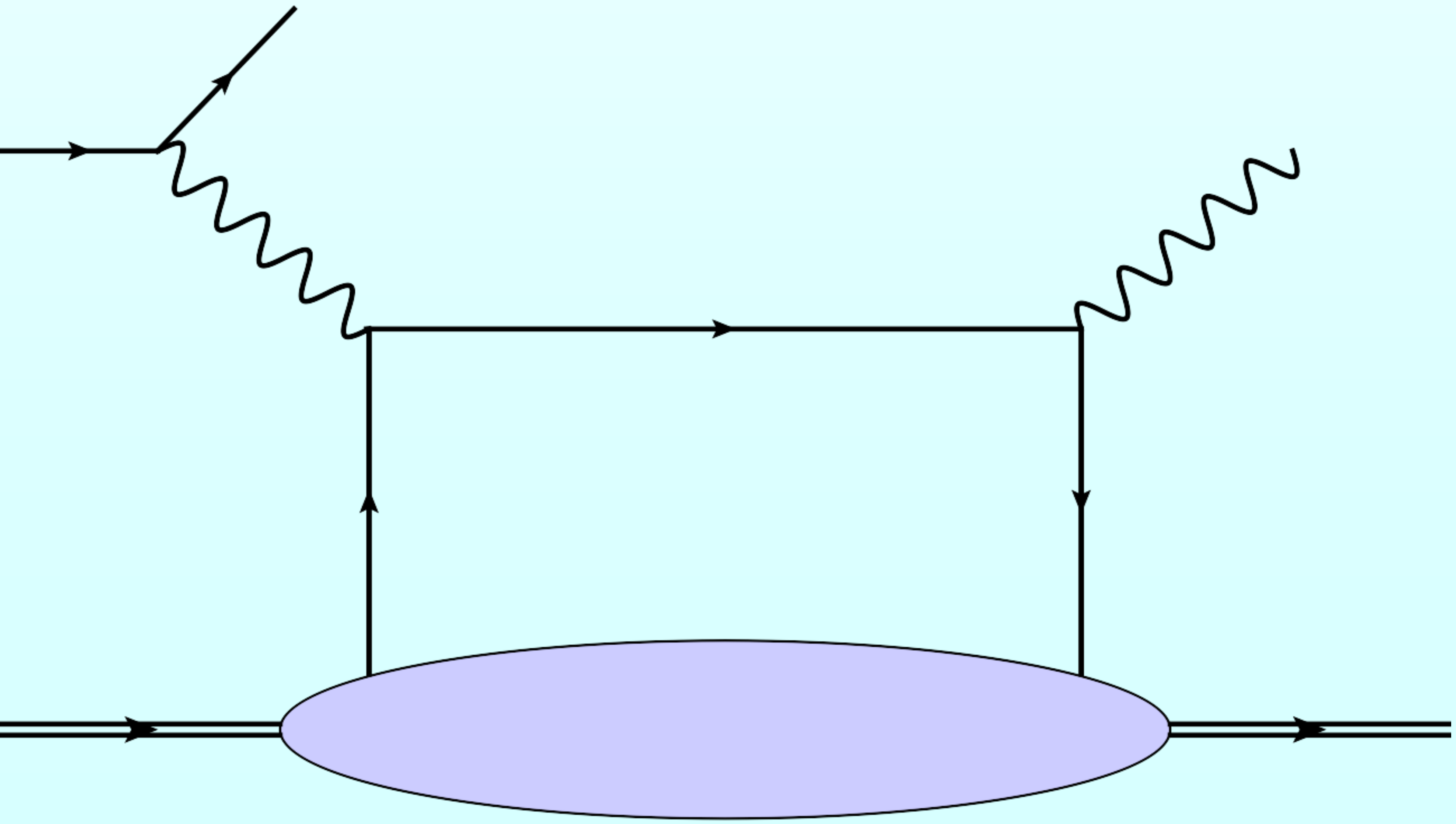
?

F.T. of Generalised Parton Distributions (GPDs)



Semi-inclusive
Deep inelastic scattering
(TMDs)

Light-Cone gauge
(with advanced
boundary condition)



Deeply Virtual
Compton Scattering
(GPDs)

Burkardt IJMPA18 (2003)

IPDs

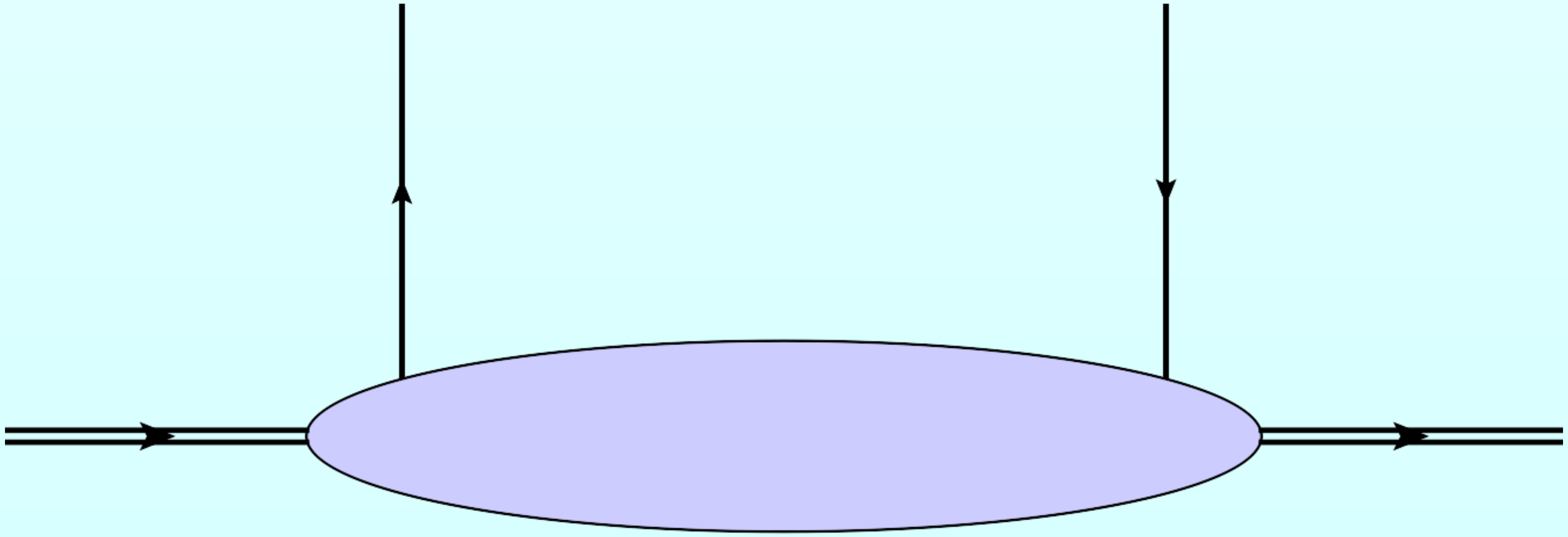
$$z_{1,2} = \left(0^+, \mp\frac{z^-}{2}, \boldsymbol{b}_\perp\right)$$

$$\mathcal{F}^{[\Gamma]}(x, \boldsymbol{b}_\perp, S) = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}$$
$$\times\langle p^+, \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, S|\overline{\psi}(z_1)\Gamma\psi(z_2)|p^+, \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, S\rangle$$

Light-Cone gauge
(with advanced boundary condition)

$$A^+(\xi^+, \xi^-, \boldsymbol{\xi}_\perp) = 0,$$
$$\boldsymbol{A}_\perp(\xi^+, \xi^- = -\infty^-, \boldsymbol{\xi}_\perp) = \boldsymbol{0}_\perp$$

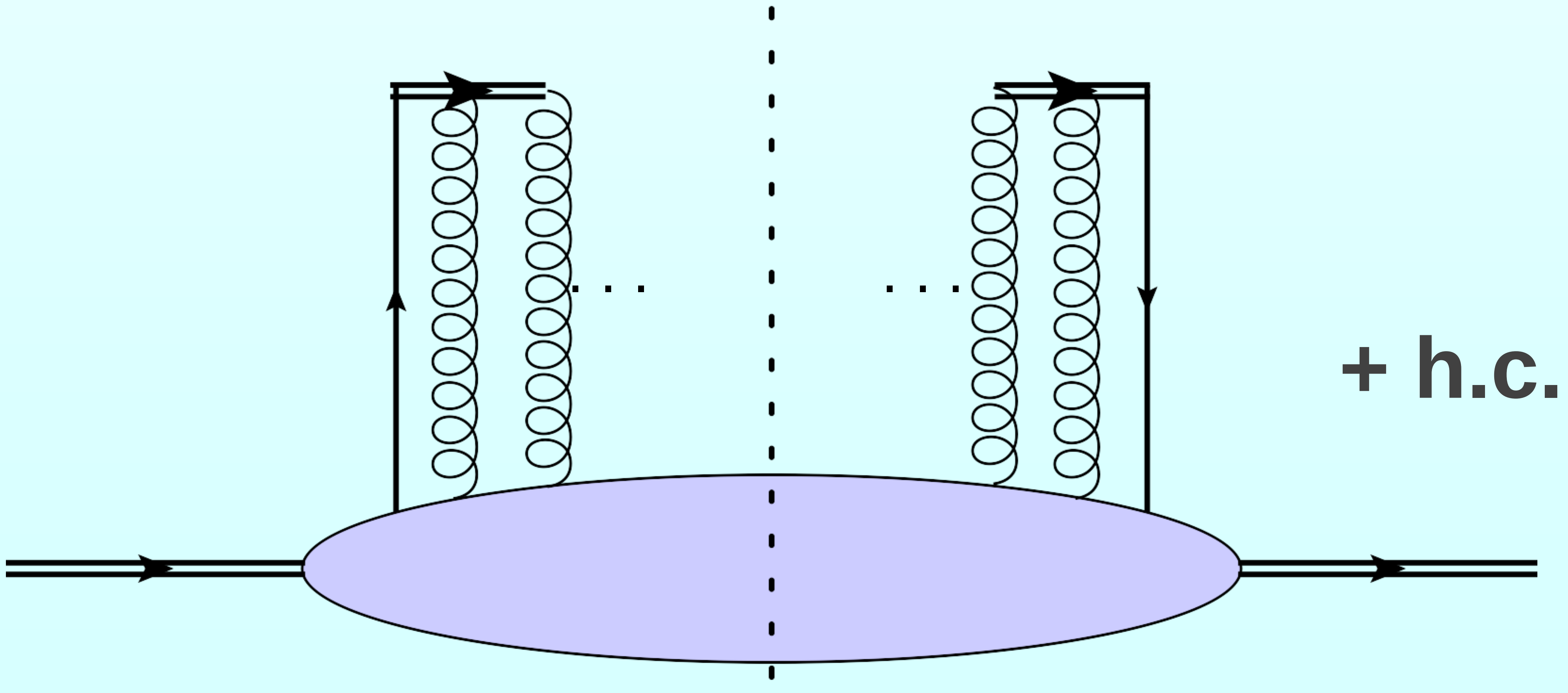
$$\int\frac{d\boldsymbol{\Delta}_\perp}{(2\pi)^2}e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}$$

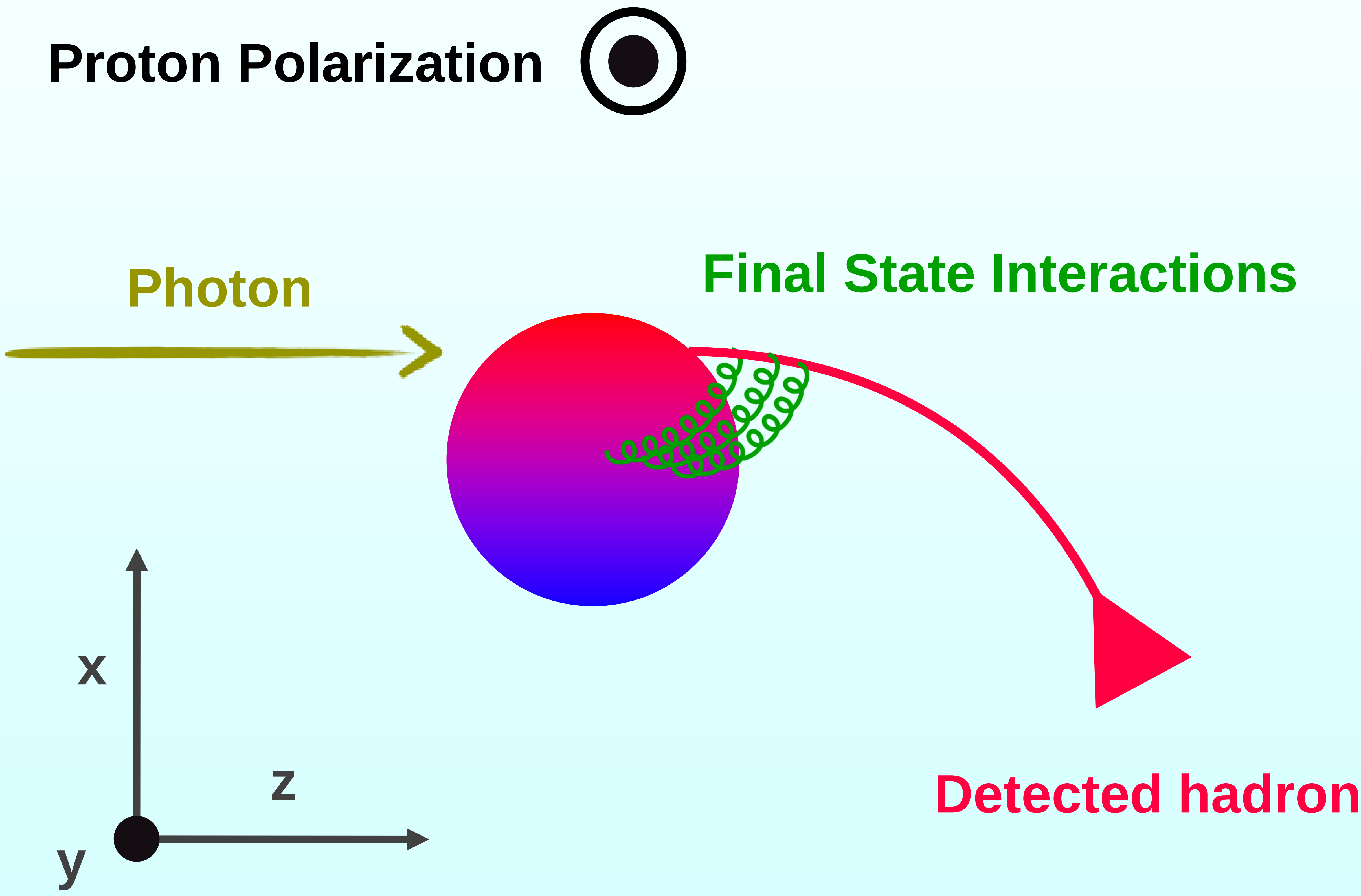


Meissner, Metz, Goeke PRD76 (2007)

TMDs

$$\Phi^{[\Gamma]}(x, \boldsymbol{k}_\perp, S) = \frac{1}{2}\int \frac{dz^- d\boldsymbol{z}_\perp}{(2\pi)^3} e^{ixp^+z^- - i\boldsymbol{k}_\perp \cdot \boldsymbol{z}_\perp}$$
$$\times \langle p, S|\overline{\psi}\left(-\frac{z}{2}\right)\Gamma \mathcal{W}\left(-\frac{z}{2}, \frac{z}{2}\right)\psi\left(\frac{z}{2}\right)|p, S\rangle|_{z^+=0}$$




Proton Polarization
Photon
Final State Interactions
x
z
y
Detected hadron

How IPDs and TMDs can be related?

$$\langle k_\perp^i(x)\rangle_{UT} = \int d\boldsymbol{k}_\perp k_\perp^i \, \Phi^{[\gamma^+]}(x, \boldsymbol{k}_\perp, \boldsymbol{S}_\perp)$$

$$\langle k_\perp^i(x)\rangle_{UT} \approx \int d\boldsymbol{b}_\perp \mathcal{L}^i\big(\boldsymbol{b}_\perp/(1-x)\big)\mathcal{F}^{[\gamma^+]}(x, \boldsymbol{b}_\perp, \boldsymbol{S}_\perp)$$

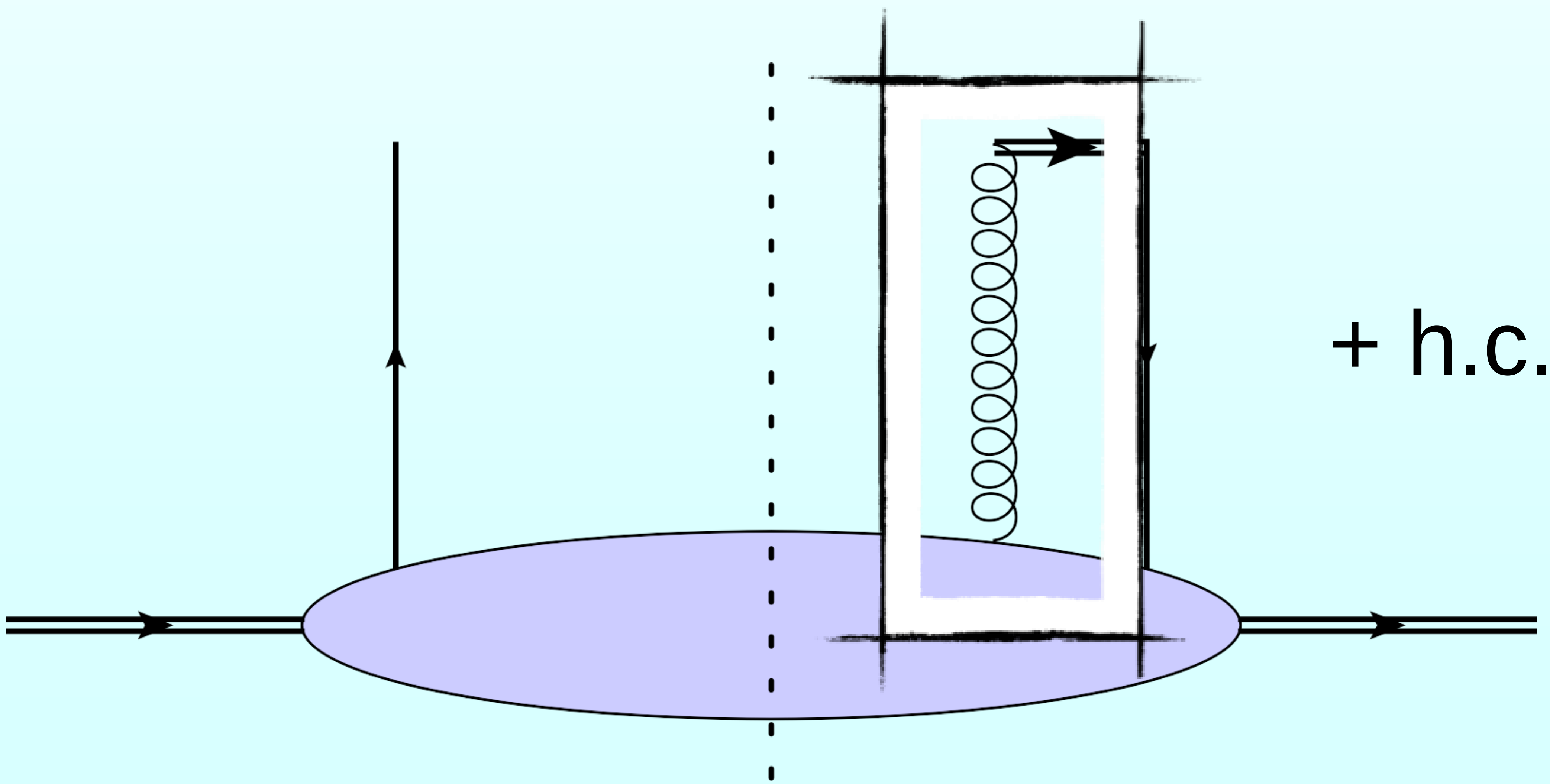
Burkardt PRD69 (2004)

$$\langle k_\perp^i(x)\rangle_{UT} = \frac{1}{2}\int d\boldsymbol{b}_\perp \int \frac{dz^-}{2\pi} e^{ixp^+z^-}$$

$$\times \langle p^+, \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, \boldsymbol{S}_\perp | \bar{\psi}(z_1)\mathcal{W}(z_1; z \quad \mathcal{I}^i(z_2)\gamma \quad \psi(z_2) | p^+, \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, \boldsymbol{S}_\perp\rangle$$

In light-cone gauge

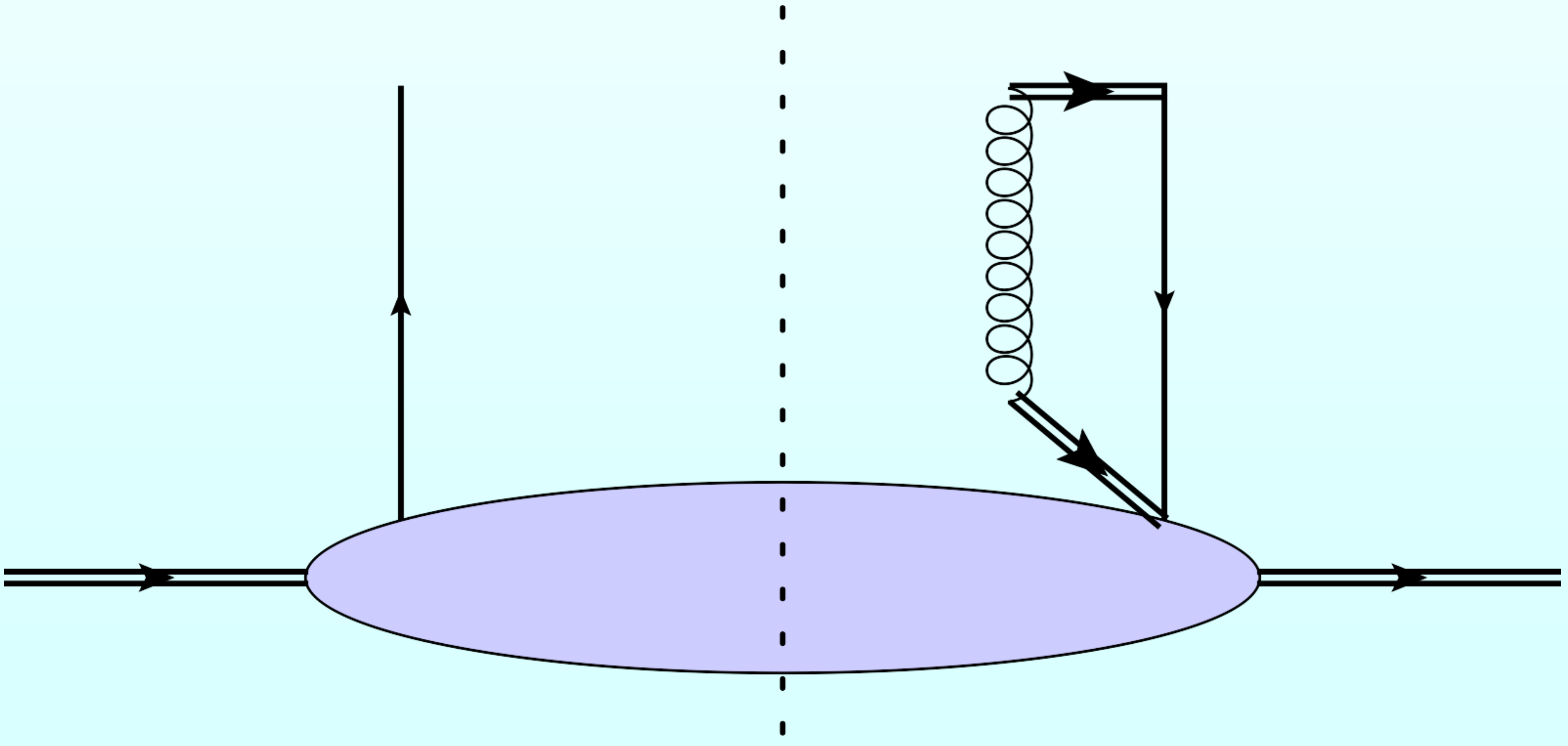
$$\mathcal{I}^i(z_2) = \frac{g_s}{2} A_\perp^i(\infty^-, 0^+, \boldsymbol{b}_\perp)$$



+ h.c.

“Eikonalization” of the Wilson gluon

Is it enough to factorise the Final State Interactions?



No…

The Final State Interactions operator should:

1) connect Fock states with the same number of constituents and the same parton, helicity and color content;

2) transfer the total transverse momentum $\mathbf{l}_\perp/(1-x)$ to the whole spectator system;

3) NOT transfer momentum in the light-cone direction to the spectator system;

4) **transfer a fraction $x_i = w_i^+/p^+$ of the total transverse momentum to each constituent of the spectator system.**

$$
\begin{aligned}
&\langle\{q_i^+, \boldsymbol{q}_{\perp,i}\}_n, \beta'|I^i(l)|\{w_i^+, \boldsymbol{w}_{\perp,i}\}_m, \beta\rangle \\
&= 2\pi L^i\left(\frac{\boldsymbol{l}_\perp}{1-x}\right)\delta_{n,m}\delta_{\beta\beta'}\delta(l^+) \\
&\times \prod_{i=1}^{n}(2\pi)^3 2q_i^+\delta(q_i^+ - w_i^+)\delta\left(\boldsymbol{q}_{\perp,i} - \boldsymbol{w}_{\perp,i} - x_i\frac{\boldsymbol{l}_\perp}{1-x}\right)
\end{aligned}
$$

Models and the lensing relaition

When does it work?

Two-body system:
Pion (q-antiq pair)
Scalar diquark model for the proton

Gamberg, Schlegel PLB685 (2010)

Burkardt NPA735 (2004)

When does it NOT work?

Many-body system:
Three-quark model for the proton

Pasquini, Yuan PRD81 (2010)

Massive remnant with spin > 1/2:
Axial-vector diquark model for the proton

Bacchetta, Conti, Radici PRD78 (2008)

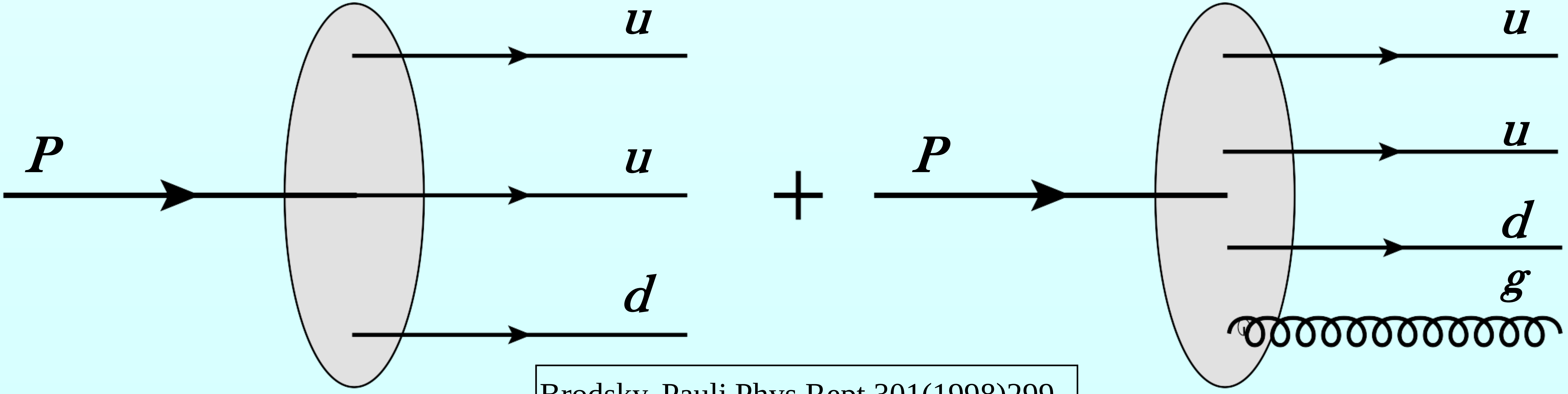
How do we study the models?

Light-Front Wave Functions

$$|P,\Lambda\rangle = \Psi^{\Lambda}_{3q}|3q\rangle + \Psi^{\Lambda}_{3q+g}|3q+g\rangle + \cdots$$

Probability amplitudes of the corresponding Fock state

All parton distributions can be written as
an overlap of Light-Front Wave Functions



Brodsky, Pauli Phys.Rept.301(1998)299

Gamberg, Schlegel PLB685 (2010)

Pion

Instead of unpolarized quark in a transversely polarised hadron transversely polarised quark in an unpolarized hadron:

Boer-Mulders function

$$\frac{\Delta_\perp^k}{2M_\pi}\widetilde{H}_{T,\pi}(x,0,-\boldsymbol{\Delta}_\perp^2) = \frac{T_\pi^2}{2(2\pi)^3}\int d\boldsymbol{k}_\perp G^k\left(x,\boldsymbol{k}_\perp\middle|\middle|x,\boldsymbol{k}_\perp+(1-x)\boldsymbol{\Delta}_\perp\right)$$

$$\langle k_\perp^i\rangle_{TU}^j = -\frac{2\alpha_s}{(2\pi)^4}\frac{4}{3}T_\pi^2\int\frac{d\boldsymbol{q}_\perp}{\boldsymbol{q}_\perp^2}\int d\boldsymbol{k}_\perp k_\perp^i\epsilon_\perp^{kj}G^k\left(x,\boldsymbol{k}_\perp\middle|\middle|x,\boldsymbol{k}_\perp-\boldsymbol{q}_\perp\right)$$

$$\langle k_\perp^i\rangle_{TU}^j = \int d\boldsymbol{b}_\perp\frac{\epsilon_\perp^{kj}b_\perp^k}{M_\pi}\mathcal{L}^i\left(\frac{\boldsymbol{b}_\perp}{1-x}\right)\left(\widetilde{\mathcal{H}}_{T,\pi}(x,\boldsymbol{b}_\perp^2)\right)'$$

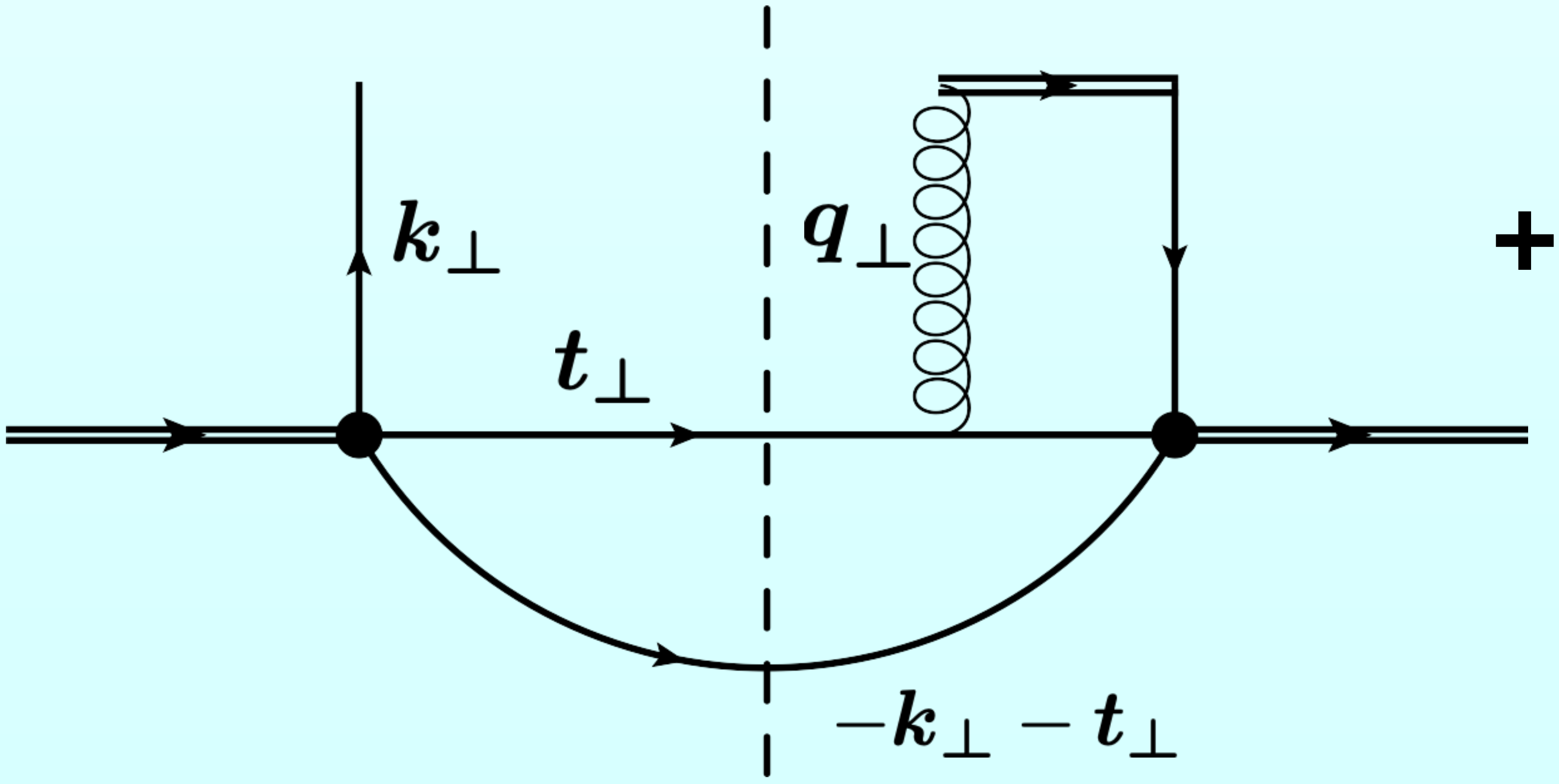
$$\mathcal{L}^i\left(\frac{\boldsymbol{b}_\perp}{1-x}\right) = -\frac{8}{3}\alpha_s 4\pi^2\frac{b_\perp^i}{\boldsymbol{b}_\perp^2}(1-x)$$

Proton

Three-quark bound system

$$\frac{i\epsilon_\perp^{ij}\Delta_\perp^j S_T^i}{M}E(x,\xi=0,-\boldsymbol{\Delta}_\perp^2)=\frac{1}{4(2\pi)^6}$$
$$\times\int d\boldsymbol{k}_\perp\int_0^x dy\int d\boldsymbol{t}_\perp G_T\left(x,\boldsymbol{k}_\perp;y,\boldsymbol{t}_\perp\right|\left|x,\boldsymbol{k}_\perp+(1-x)\boldsymbol{\Delta}_\perp;y,\boldsymbol{t}_\perp-y\boldsymbol{\Delta}_\perp\right)$$

$$\frac{\epsilon_\perp^{ij}k_\perp^j S_T^i}{M}f_{1T}^\perp\left(x,\boldsymbol{k}_\perp^2\right)=-\frac{\alpha_s}{3(2\pi)^7}$$
$$\times\int\frac{d\boldsymbol{q}_\perp}{\boldsymbol{q}_\perp^2}\int_0^x dy\int d\boldsymbol{t}_\perp G_T\left(x,\boldsymbol{k}_\perp;y,\boldsymbol{t}_\perp\right|\left|x,\boldsymbol{k}_\perp-\boldsymbol{q}_\perp;y,\boldsymbol{t}_\perp+\boldsymbol{q}_\perp\right)$$



+ h.c.

Burkardt NPA735 (2004)

Bacchetta, Conti, Radici PRD78 (2008)

Proton

quark-diquark
bound system

Scalar diquark: OK

Vector diquark

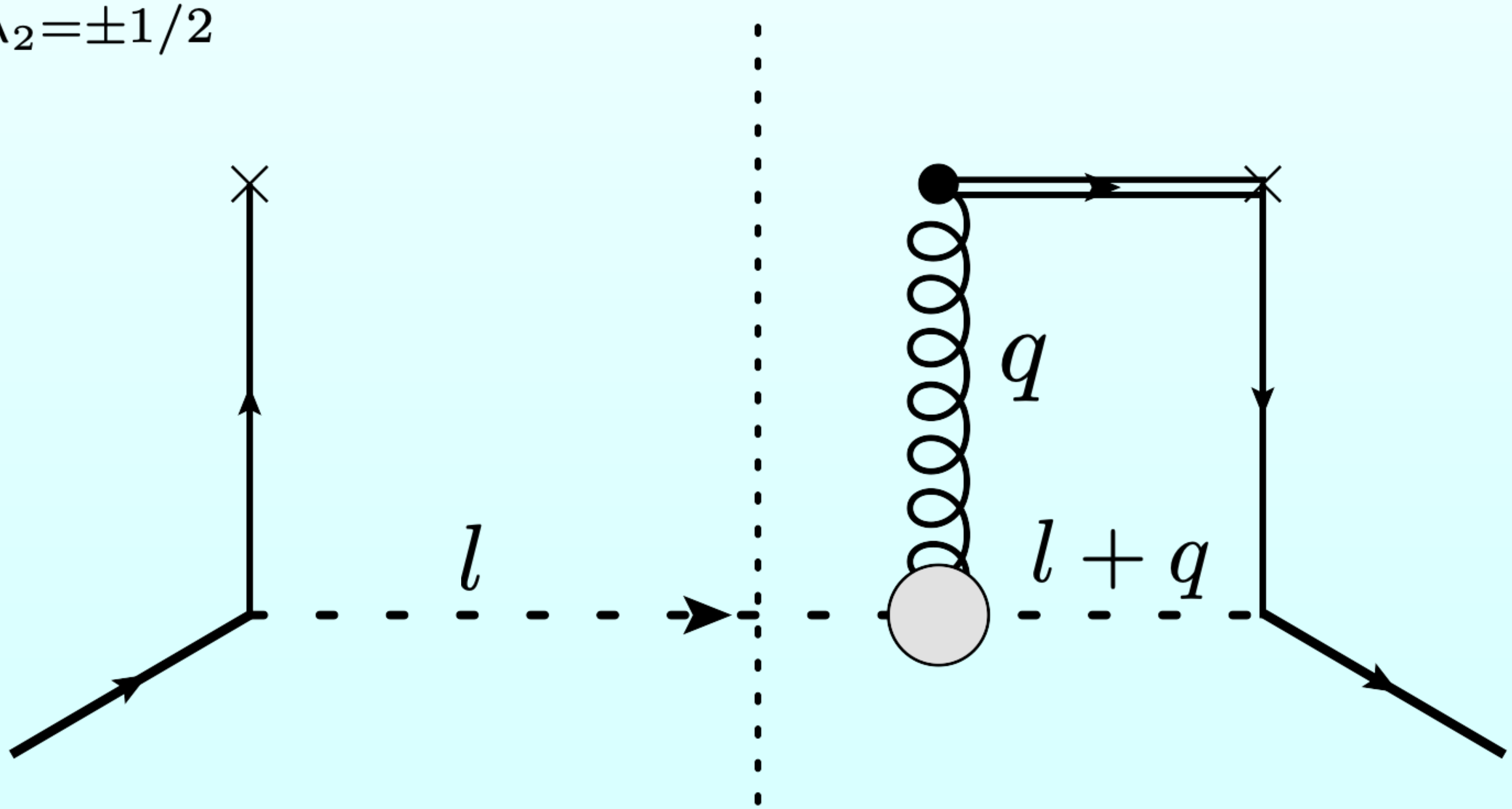
Inclusion of longitudinally
polarized diquark

Only transverse
polarisation allowed

$$\frac{i}{e_c}\Gamma^{\rho}_{\nu\sigma} = (2l+q)^{\rho} g_{\nu\sigma} - (l + (1+\kappa_a)q)_{\sigma}\delta^{\rho}_{\nu} - (l - \kappa_a q)_{\nu}\delta^{\rho}_{\sigma}$$

$$\mathcal{R}^{\rho} = \sum_{\lambda_1,\lambda_2=\pm 1,0} \varepsilon^{\nu *}_{\lambda_1}(l+q)\varepsilon^{\sigma}_{\lambda_2}(l)\Gamma^{\rho}_{\nu\sigma}$$ **With longitudinal polarisation**

$$\mathcal{R}^{\rho} = \sum_{\lambda_1,\lambda_2=\pm 1/2} \bar{v}_{\lambda_1}(l+q)\gamma^{\rho} v_{\lambda_2}(l)$$ **Pion for comparison**



$$\mathcal{R}^{+} \simeq \mathcal{O}(p^{+}) \quad \mathcal{R}^{i}_{\perp} \simeq \mathcal{O}(1) \quad \mathcal{R}^{-} \simeq \mathcal{O}(1/p^{+})$$

Conclusions

Starting point: T-odd TMD contribution to SSA
as convolution of lensing function and IPD?

full-QCD answer: NO
T-odd TMDs are independent from IPDs, they are both integrals of
Wigner Distributions (“mother distributions”)

Model’s answer: YES and NO
It depends on the model

Our answer:
a set of necessary and sufficient conditions
for the FSIs to be factorised

Take home message:
models are incredibly useful tools for phenomenological investigations,
but great care is needed extending model-induced relations.

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