Abel Tomography: Charge & EMT Densities of the Nucleon

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Introduction

Modern Understanding of Nucleon form factors

- GPDs

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', \sigma' | \overline{\psi}_q \left(-\frac{\lambda n}{2}\right) \not{n} \psi_q \left(\frac{\lambda n}{2}\right) | p, \sigma \rangle$$

$$= H^q(x,\xi,t) \overline{u}(p',\sigma') \not{n} u(p,\sigma) + E^q(x,\xi,t) \overline{u}(p',\sigma') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_B} u(p,\sigma)$$

- Mellin moments of the GPDs

- The first moments of the GPDs H & E yield the well-known EM form factors

$$\int_{-1}^{1} dx \sum_q H^q(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx \sum_q E^q(x,\xi,t) = F_2(t)$$

- The second moments of the GPDs H & E give the gravitational (EMT) FFs (Ji's sum rules).

$$\int_{-1}^{1} dx x \sum_q H^q(x,\xi,t) = A^Q(t) + D^Q(t)\xi^2,$$

$$\int_{-1}^{1} dx x \sum_q E^q(x,\xi,t) = 2J^Q(t) - A^Q(t) - D^Q(t)\xi^2$$

D. Müller et al. Fortschr. Phys. 42 (1994).

X. D. Ji, PRL 78, PRD 55 (1997).

A. V. Radyushkin, PLB 380 (1996)

Critical view on Nucleon form factors

- **Traditional interpretation of the nucleon form factors**

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \rho(\mathbf{r}) \rightarrow \quad \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Particle number fixed.

- This is valid for atoms and nuclei: $\frac{\delta r}{r} = \frac{m_e \alpha}{M} \sim 10^{-5}$

- **Crucial criticism on the traditional definition of the nucleon form factors.**

- It is not valid anymore for the nucleon: $r \sim 0.8\,\mathrm{fm} \quad \delta r \sim \frac{\hbar}{M_N c} \approx 0.2\,\mathrm{fm}$

$\delta r / r \sim 0.25$

The nucleon is a **relativistic** particle!

Particle creation and annihilation inside a nucleon

- Validity of the nucleon 3D distributions was put into question.
- View on the nucleon form factors has been modernized.

G.A. Miller, PRL **99** (2007)

C. Lorce, PRL **125** (2020)

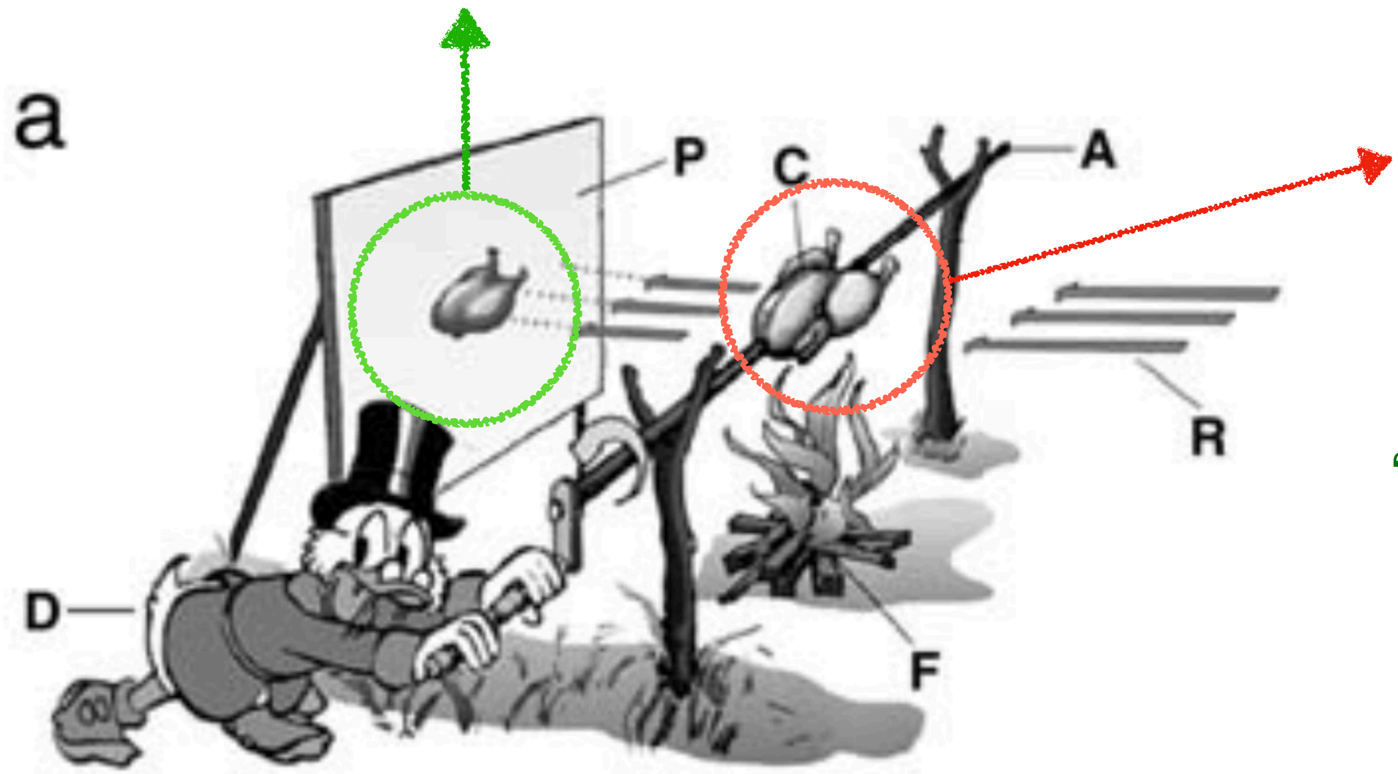
R. L. Jaffe, PRD **103** (2021)

M. Burkardt, PRD 62 (2000) [**66** (2002)]

Belitsky & Radyushkin, Phys.Rept. **418** (2005)

Abel & Radon transforms & Nucleon tomography

2D transverse distributions (QM probabilistic)



3D distributions
(Quasi-probabilistic)

"Electron tomography, edited by J. Frank"

- Abel transformation maps 3D distributions of a particle with spin 0 or 1/2 at rest onto 2D transverse plane in the IMF (spherically symmetric cases only). (Radon transform is required for that with higher spin.)

M. Burkardt, PRD 62 (2000) [66 (2002)]

G. A. Miller, PRL. 99 (2007).

Carlson & Vanderhaeghen, PRL 100 (2008)

C. Lorce, PRL 125 (2020).

Panteleeva & Polyakov, PRD 104, 014008(2021)

3D distributions ⟷ 2D distributions

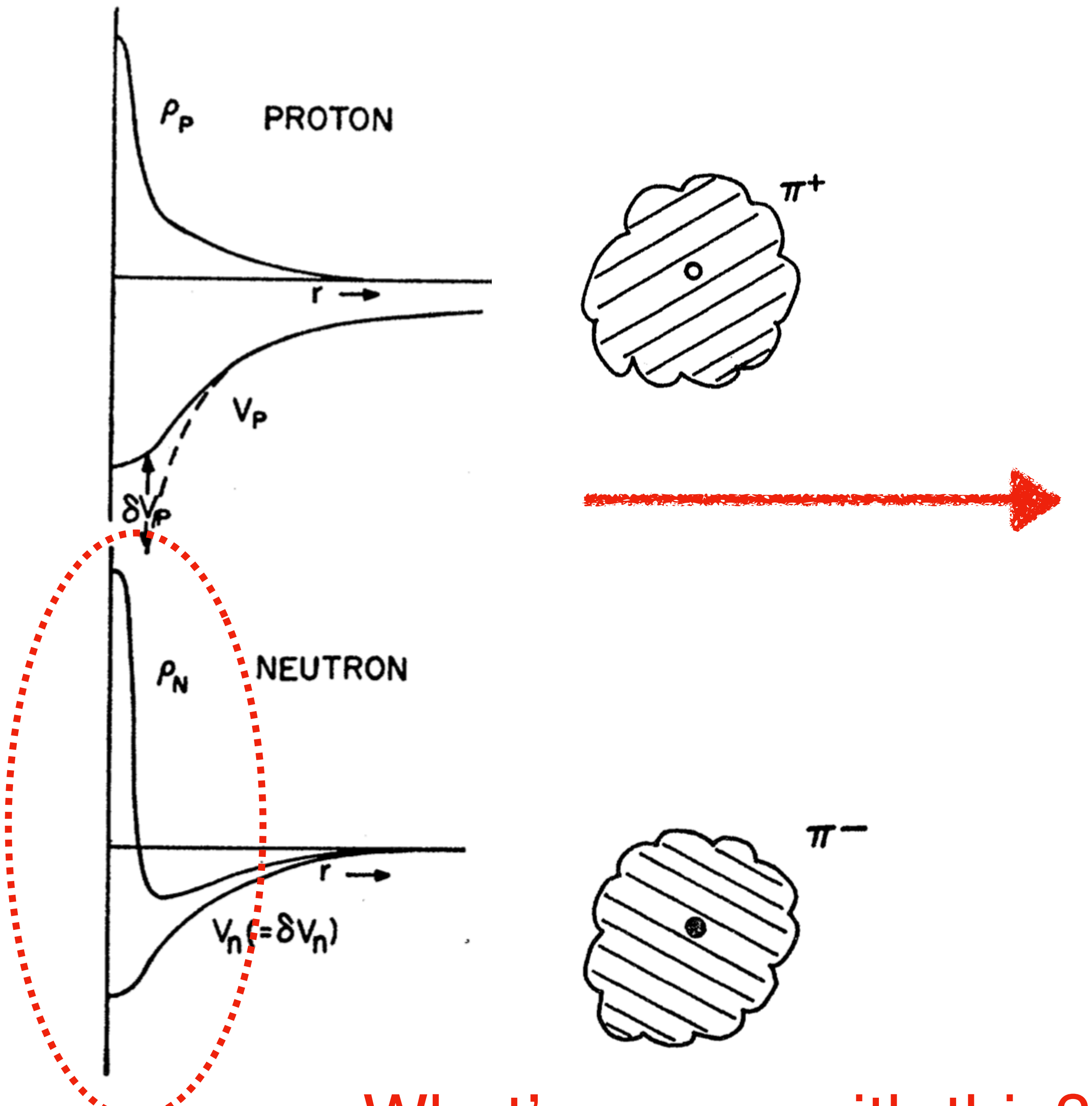
This is the subject of the present talk.

Charge densities
of
the nucleon

Charge distributions of the nucleon

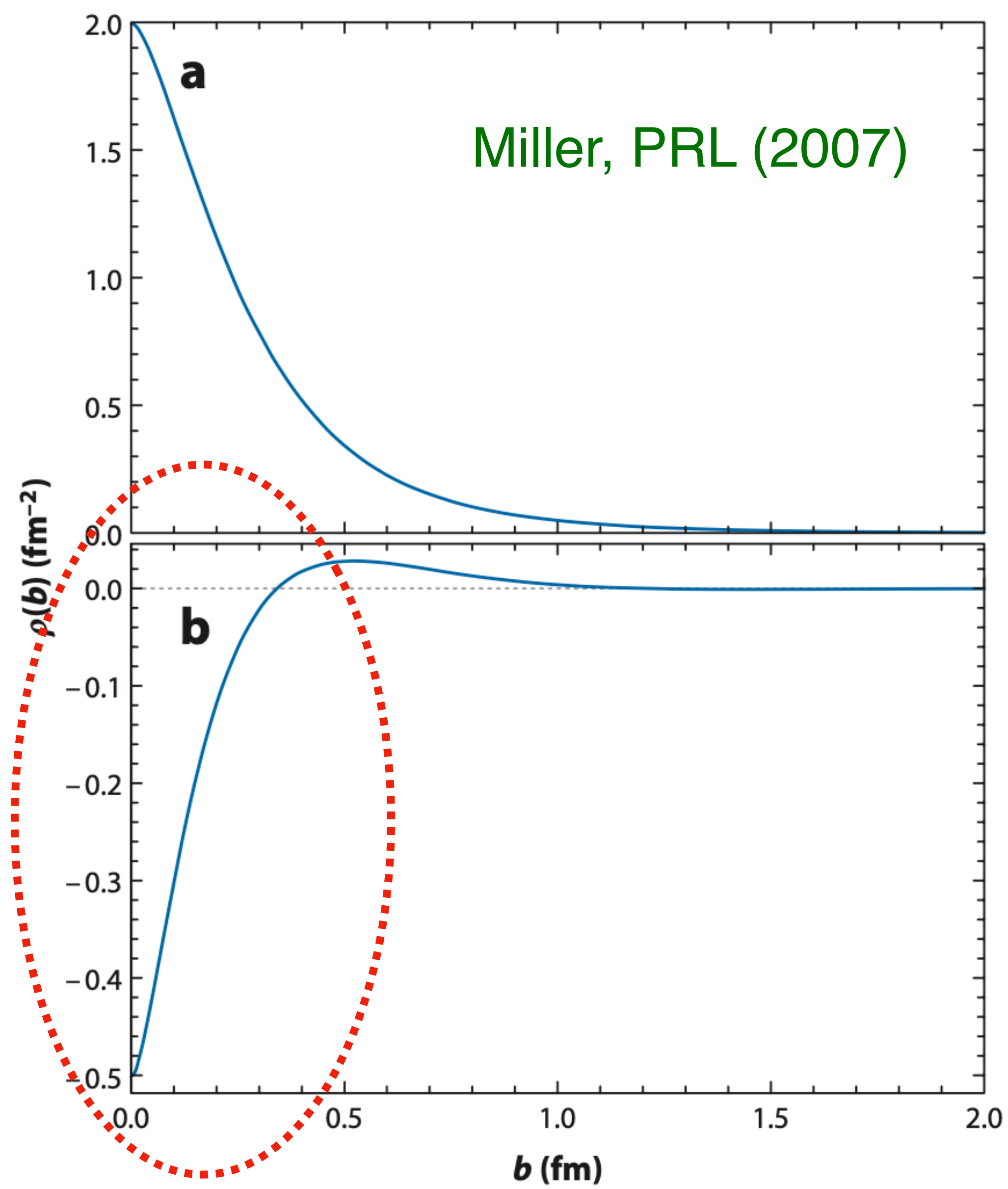
3D charge densities of the nucleon in the BF

Yennie et al., RMP (1957)



2D Transverse charge densities of the nucleon in the IMF

Miller, PRL (2007)



What's wrong with this?

(Actually, nothing wrong. Lorce's talk on Monday)

2D density exhibits correctly QM probabilistic meaning.

Charge distributions of the nucleon

- Quantum mechanical nucleon matrix element of the EM current

C. Lorce's talk on Monday

$$\langle \hat{J}^{\mu}(\boldsymbol{r})\rangle = \int \frac{d^3\boldsymbol{P}}{(2\pi)^3}\int d^3\boldsymbol{R}\, W(\boldsymbol{R},\boldsymbol{P})\langle \hat{J}^{\mu}(\boldsymbol{r})\rangle_{\boldsymbol{R},\boldsymbol{P}}$$

E. Wigner, PR 40, 749 (1932)
M. Hillery et al., Phys. Rep. 106, 121 (1984).

Wigner distribution (Quasi-probabilistic)

$$W(\boldsymbol{R},\boldsymbol{P}) = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{R}}\tilde{\psi}^*\left(\boldsymbol{P}+\frac{\boldsymbol{\Delta}}{2}\right)\tilde{\psi}\left(\boldsymbol{P}-\frac{\boldsymbol{\Delta}}{2}\right)$$
$$= \int d^3\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot\boldsymbol{P}}\psi^*\left(\boldsymbol{R}-\frac{\boldsymbol{z}}{2}\right)\psi\left(\boldsymbol{R}+\frac{\boldsymbol{z}}{2}\right)$$

$\boldsymbol{R}$: average position

$\boldsymbol{\Delta}$: momentum transfer

$\boldsymbol{P}$: average momentum

$\boldsymbol{z}$: spatial separation between the initial & final nucleons

$$\psi(\boldsymbol{r}) = \langle \boldsymbol{r}|\psi\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi^3)} e^{i\boldsymbol{p}\cdot\boldsymbol{r}}\tilde{\psi}(\boldsymbol{p}), \quad \tilde{\psi}(\boldsymbol{p}) = \frac{1}{\sqrt{2p_i^0}}\langle p|\psi\rangle,$$

$$|\boldsymbol{r}\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3\sqrt{2p^0}} e^{-i\boldsymbol{p}\cdot\boldsymbol{r}}|p\rangle$$

$$\int \frac{d^3\boldsymbol{P}}{(2\pi)^3} W_N(\boldsymbol{R},\boldsymbol{P}) = |\psi_N(\boldsymbol{R})|^2, \quad \int d^3\boldsymbol{R}\, W_N(\boldsymbol{R},\boldsymbol{P}) = |\tilde{\psi}_N(\boldsymbol{P})|^2$$

C. Lorce´, PRL, 125, 232002 (2020)
J.-Y. Kim & HChK, PRD 104, 074003 (2021)

Charge distributions of the nucleon

- Given P and R, the matrix element conveys information on the internal structure of the target localized around the average position R and average momentum P.

$$\langle \hat{J}^\mu(\boldsymbol{r})\rangle = \int \frac{d^3\boldsymbol{P}}{(2\pi)^3} \int d^3\boldsymbol{R}\, W(\boldsymbol{R},\boldsymbol{P}) \langle \hat{J}^\mu(\boldsymbol{r})\rangle_{\boldsymbol{R},\boldsymbol{P}}$$

$\boldsymbol{x} = \boldsymbol{r} - \boldsymbol{R}$: shifted position

$$\langle \hat{J}^\mu(\boldsymbol{r})\rangle_{\boldsymbol{R},\boldsymbol{P}} = \langle \hat{J}^\mu(0)\rangle_{-\boldsymbol{x},\boldsymbol{P}} = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{x}\cdot\boldsymbol{\Delta}} \frac{1}{\sqrt{2p_i^0}\sqrt{2p_f^0}} \langle p_f,\lambda_f|\hat{J}^\mu(0)|p_i,\lambda_i\rangle$$

$$\hat{J}^\mu(0) = \bar{\psi}(0)\gamma_\mu \hat{Q}\psi(0)$$

$$\langle p_f,\lambda_f|\hat{J}^\mu(0)|p_i,\lambda_i\rangle = \overline{u}_{\lambda_f}(p_f)\left[\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_N}F_2(t)\right]u_{\lambda_i}(p_i)$$

$$= \overline{u}_{\lambda_f}(p_f)\left[\frac{M_N P^\mu}{P^2}G_E(t) + \frac{i\epsilon^{\mu\nu\alpha\beta}\Delta_\nu P_\alpha \gamma_\beta\gamma_5}{2P^2}G_M(t)\right]u_{\lambda_i}(p_i)$$

C. Lorce´, PRL, 125, 232002 (2020)
J.-Y. Kim & HChK, PRD 104, 074003 (2021)

Charge distributions of the nucleon

$$\int dx^- \langle \hat{J}^\mu \rangle_{\boldsymbol{R},\boldsymbol{P}} = \int \frac{d^2\Delta_\perp}{(2\pi^3)} e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}_\perp} \frac{\langle p_f, \lambda_f | \hat{J}^\mu | p_i, \lambda_i \rangle}{2P^+} \Bigg|_{\Delta^+=0, x^+=0}$$

- 2D charge & magnetization densities on the light front

$$\rho_{\mathrm{ch}}(b) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}_\perp} F_1(-\Delta_\perp^2),$$

$$\rho_{\mathrm{M}}(b) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}_\perp} F_2(-\Delta_\perp^2)$$

(Unpolarized nucleon)

$(P_z \to \infty)$

Abel transformation
+ Lorentz Boost

$$\rho_{\mathrm{ch}}^{\mathrm{BF}}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{x}\cdot\boldsymbol{\Delta}} \frac{m}{P^0} G_E(t),$$

$$\rho_{\mathrm{M}}^{\mathrm{BF}}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{x}\cdot\boldsymbol{\Delta}} \frac{m}{P^0} G_M(t)$$

NR limit

$$\rho_{\mathrm{ch}}^{\mathrm{NR}}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{x}\cdot\boldsymbol{\Delta}} G_E(t),$$

$$\rho_{\mathrm{M}}^{\mathrm{NR}}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{x}\cdot\boldsymbol{\Delta}} G_M(t)$$

$$t = -\boldsymbol{\Delta}^2$$

J.-Y. Kim & HChK, PRD 104, 074003 (2021)

Abel transformation

- Abel transform in the present work

$$A[g](b) = \mathcal{G}(b) = \int_b^\infty \frac{dr}{r} \frac{g(r)}{\sqrt{r^2 - b^2}}$$

(More convenient for the EMT densities)

- Inverse Abel transform

$$g(r) = -\frac{2}{\pi} r^2 \int_r^\infty db \frac{d\mathcal{G}(b)}{db} \frac{1}{\sqrt{b^2 - r^2}}$$

M. V. Polyakov, PLB **659** (2008) 542

A. M. Moiseeva & M. Polyakov NPB 832 (2010) 241

- Standard Abel transform

$$A[g](b) = \mathcal{G}(b) = \int_b^\infty dr \frac{g(r)}{\sqrt{r - b}}, \quad b > 0$$

$$A[g](b) = 2 \int_b^\infty r dr \frac{g(r)}{\sqrt{r^2 - b^2}},$$

- Usual inverse Abel transform

$$g(r) = -\frac{1}{\pi} \int_r^\infty db \frac{d\mathcal{G}(b)}{db} \frac{1}{\sqrt{b^2 - r^2}}$$

Cormack's inversion formula, J. App. Phys, 34 (1963) 2722

(1979 Nobel laureate in medicine)

- Abel transform: a special case of the Radon transform

Spherical symmetry is essential: Abel transform applies only to the 2D images of spin 0 & 1/2 hadrons.

Abel, J. Reine und Angew. Math. 1 (1826)

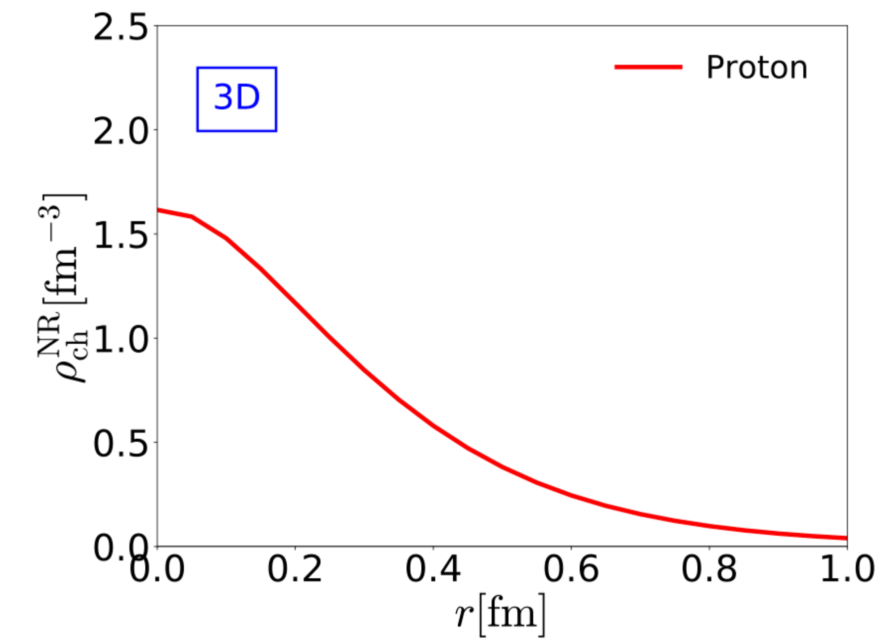
Abel transformations of the 3D densities

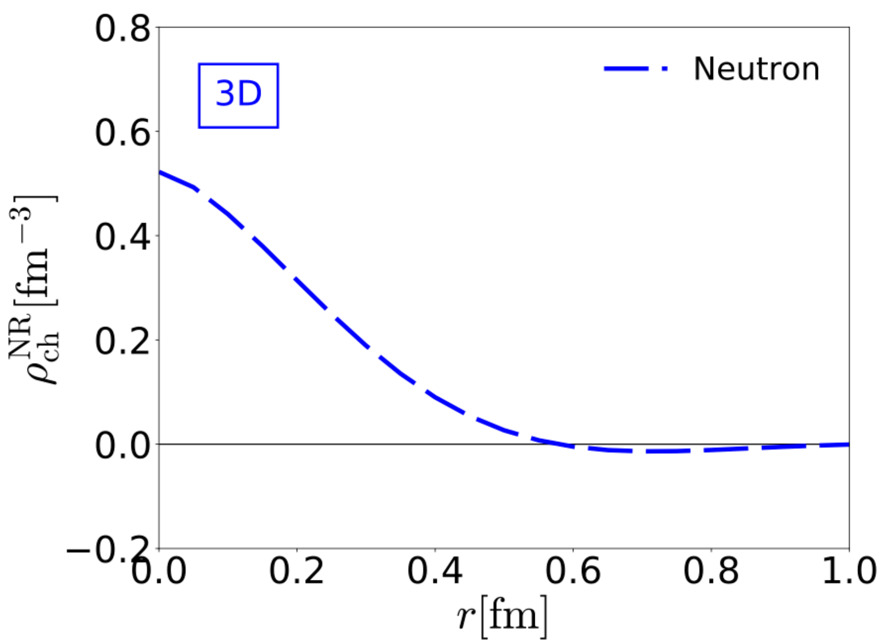
J.-Y. Kim & HChK, PRD 104, 074003 (2021)

$$\rho_{\rm ch}(b) + \frac{1}{4M_N^2}\partial_\perp^2 \rho_{\rm M}(b) = \int_b^\infty \frac{2r\,dr}{\sqrt{r^2-b^2}}\rho_{\rm ch}^{\rm NR}(r),$$

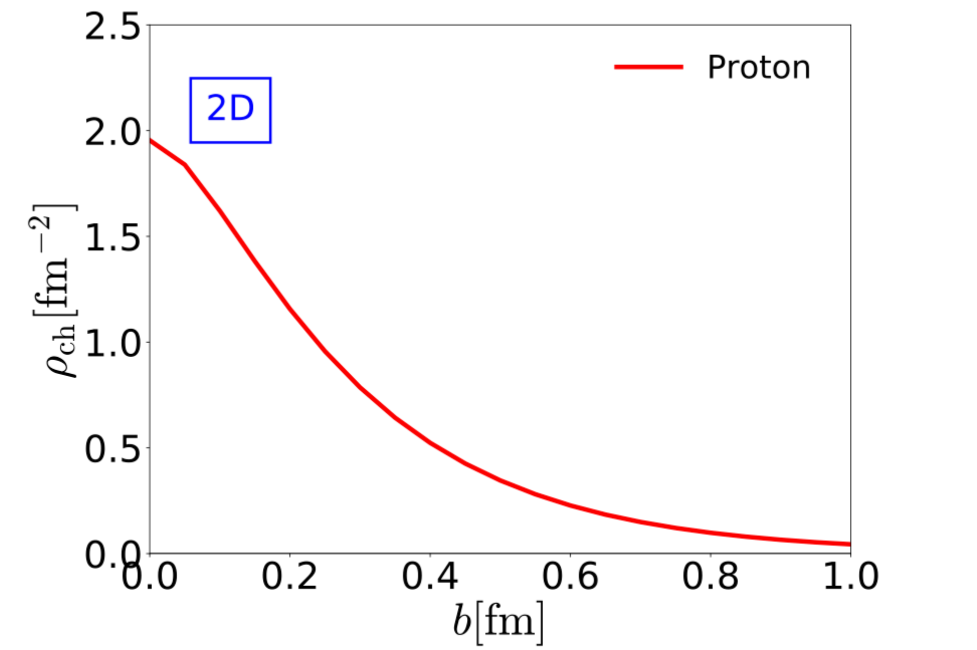
$$\rho_{\rm ch}(b) + \rho_M(b) = \int_b^\infty \frac{2r\,dr}{\sqrt{r^2-b^2}}\rho_{\rm M}^{\rm NR}(r)$$

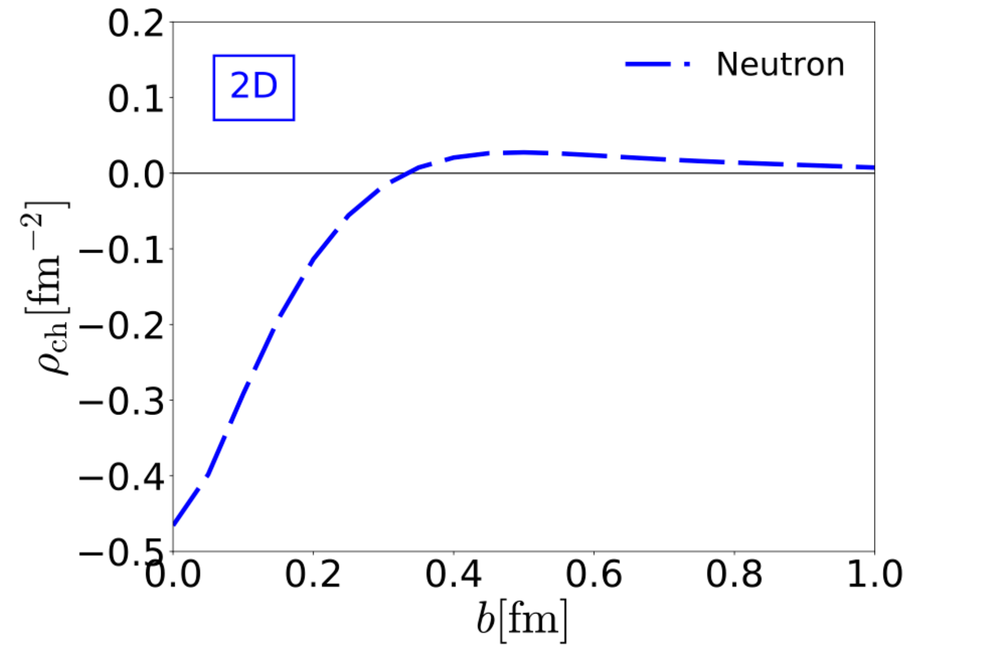
for an unpolarized nucleon





By the Abel transform





Exactly the same as those by Miller!

EMT densities
of
the nucleon

Gravitational form factors

- EMT current in QCD & GFFs

Kobzarev et al. 1962; Pagels, 1966

$$T_q^{\mu\nu} = \frac{1}{4}\overline{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\right)\psi_q - g^{\mu\nu}\overline{\psi}_q\left(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\right)\psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta}\,F^{a,}{}_\eta{}^\nu + \frac{1}{4}g^{\mu\nu}F^{a,\kappa\eta}\,F^{a,}{}_{\kappa\eta}.$$

D(Druck)-term

Weiss & Polyakov, 1999

$$\langle p'|T^{\mu\nu}(0)|p\rangle = \bar{u}(p')\left[A^a(t)\frac{P^\mu P^\nu}{M_N} + J^a(t)\frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M_N} + D^a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + M_N\bar{c}^a(t)g^{\mu\nu}\right]u(p)$$

δg^{00} → Mass: $\sum_a A^a(0) = 1$

δg^{0i} → Spin: $\sum_a J^a(0) = \frac{1}{2}$

δg^{ij} → Deformation of space = mechanical properties of the nucleon

Non-conservation piece of EMT FFs

Pressure & Shear-force distributions (pressure anisotropy)

Pressure & Shear-force distributions

$$T^a_{ij}(\boldsymbol{r},\sigma',\sigma) = p^a(r)\delta^{ij}\delta_{\sigma'\sigma} + s^a(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)\delta_{\sigma'\sigma}$$

- 3D Shear-force density in the BF

$$s^a(r) = -\frac{1}{4M_B} r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{D}^a(r)$$

- 3D Pressure density in the BF

M.V. Polyakov, PLB555 (2003)

$$p^a(r) = \frac{1}{6M_B}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\tilde{D}^a(r) - M_B\int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\bar{c}^a(t)$$

$$\tilde{D}^a(r) = \int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}D^a(t)$$

- This term is related to forces between quark and gluon subsystems (Polyakov & Son, 2018).
- It contributes to gluon and quark parts of energy density (mass decomposition). (Lorce, 2018)
- It vanishes for Goldstone bosons (P. Schweitzer & M.V. Polyakov, 2019).

Abel transforms

- Abel transform from 3D in the BF to 2D in the IMF (Also invertible)

$$\mathcal{E}(x_\perp) = 2\int_{x_\perp}^{\infty}\left(\varepsilon(r) + \frac{3}{2}p(r) + \frac{1}{4m}\partial^2\left[\tilde{A}(r) - 2\tilde{J}(r)\right]\right)\frac{rdr}{\sqrt{r^2 - x_\perp^2}}$$

$$\rho_J^{(2D)}(x_\perp) = 3\int_{x_\perp}^{\infty}\frac{\rho_J(r)}{r}\frac{x_\perp^2\,dr}{\sqrt{r^2 - x_\perp^2}}$$

$$\mathcal{S}(x_\perp) = \int_{x_\perp}^{\infty}\frac{s(r)}{r}\frac{x_\perp^2\,dr}{\sqrt{r^2 - x_\perp^2}}$$

$$\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) = \frac{1}{2}\int_{x_\perp}^{\infty}\left(\frac{2}{3}s(r) + p(r)\right)\frac{rdr}{\sqrt{r^2 - x_\perp^2}}$$

Abel, J. Reine und Angew. Math. 1 (1826)

- Abel transform is used for tomography of spherically symmetric systems (spin 0 & 1/2 hadrons).
- For non-spherical objects (spin > 1/2), the Radon transform comes into play.

Panteleeva & Polyakov, PRD **104** (2021)

J. Y. Kim & HChK, PRD **104** (2021) 7

Equivalence of the 3D BF & 2D LF distributions

- Von Laue Conditions

A. Freese and G. A. Miller, PRD **103** (2021)

$$\int_0^\infty dr\ r^2 p(r) = 0 \quad \Longleftrightarrow \quad \int d^2x_\perp \mathcal{P}(x_\perp) = 0$$

- Local stability Conditions

$$\frac{2}{3}s(r) + p(r) > 0 \quad \Longleftrightarrow \quad \frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) > 0$$

Geometric factor

$$\Omega_d = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d+2}{2}\right)}$$

- Mechanical radius

$$\langle x_\perp^2 \rangle_{\rm mech} = \frac{\int d^2x_\perp x_\perp^2 \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)}{\int d^2x_\perp \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)} = \frac{4D(0)}{\int_{-\infty}^0 dt D(t)} = \frac{2}{3}\langle r^2 \rangle_{\rm mech}$$

- D(Druck)-terms

$$D(0) = -\frac{4M_N}{15}\int d^3r r^2 s(r) = m\int d^3r r^2 p(r) \quad \Longleftrightarrow \quad D(0) = -m\int d^2x_\perp x_\perp^2 \mathcal{S}(x_\perp) = 4m\int d^2x_\perp x_\perp^2 \mathcal{P}(x_\perp)$$

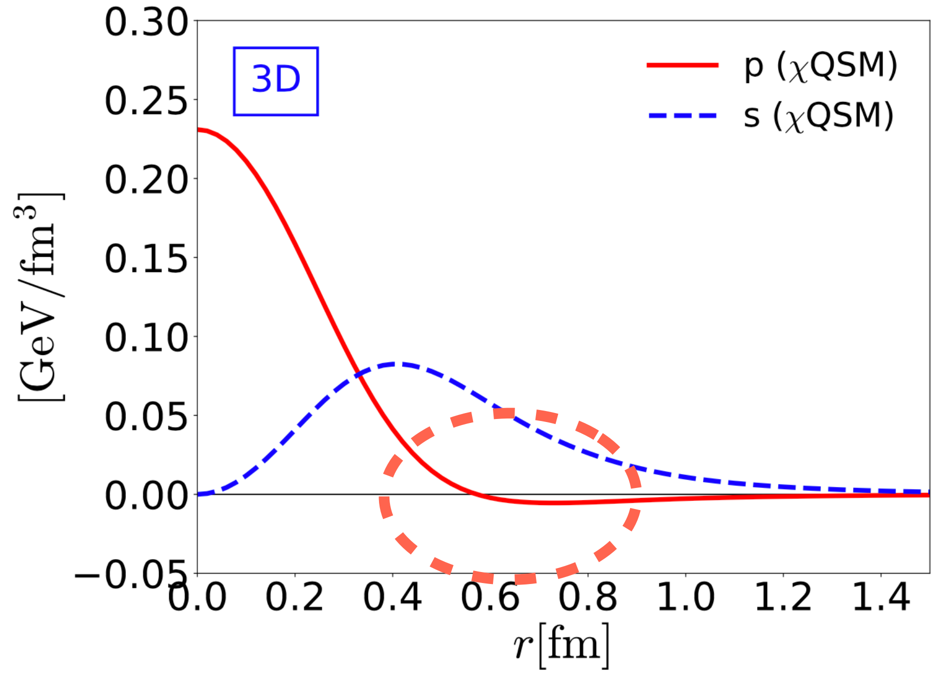
Panteleeva & Polyakov, PRD **104** (2021)

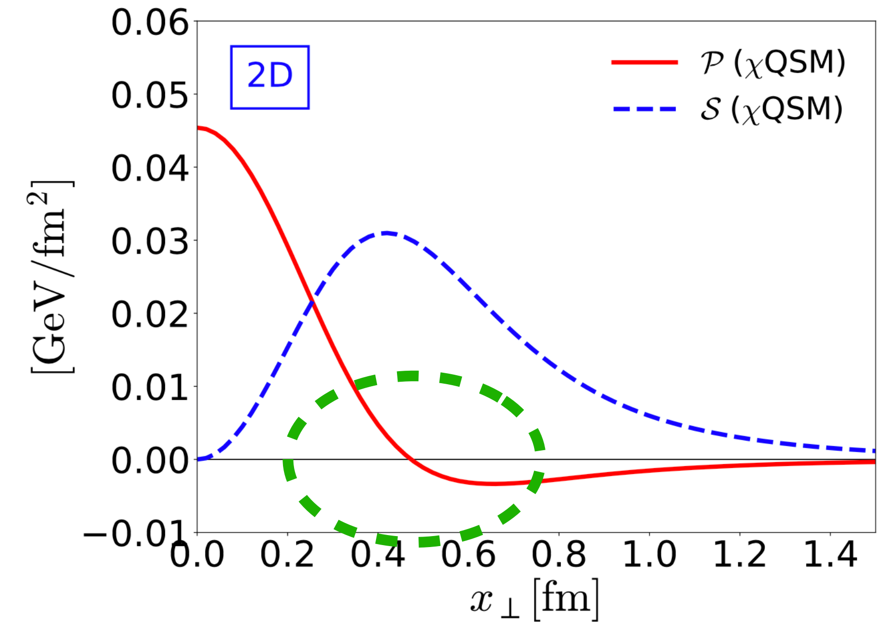
The 3D & 2D pressure & shear-force densities

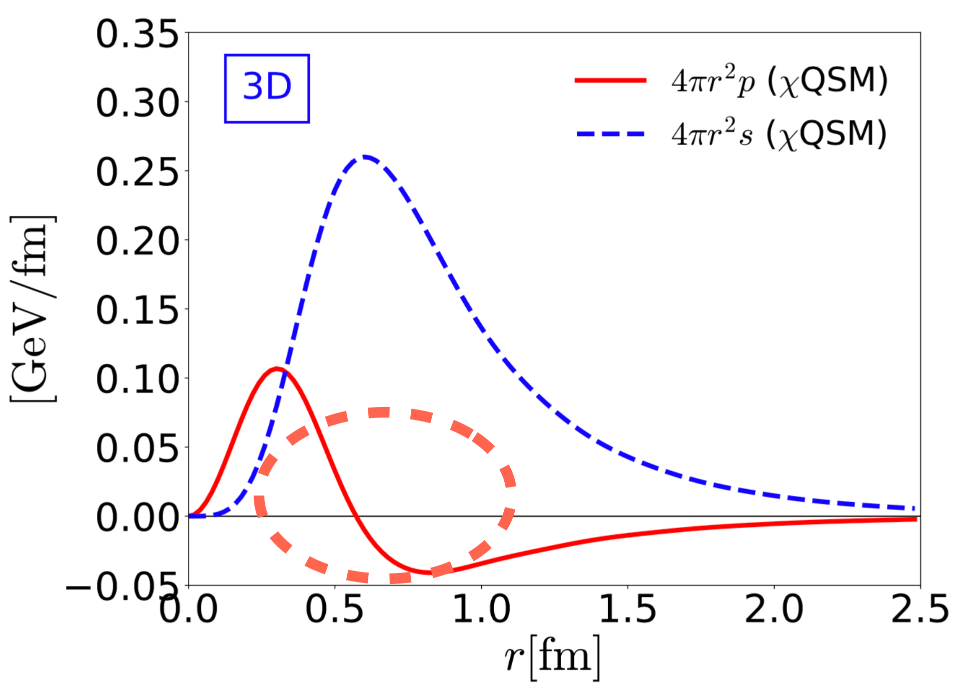
3D in BF　　　　2D in IMF　　　　χQSM

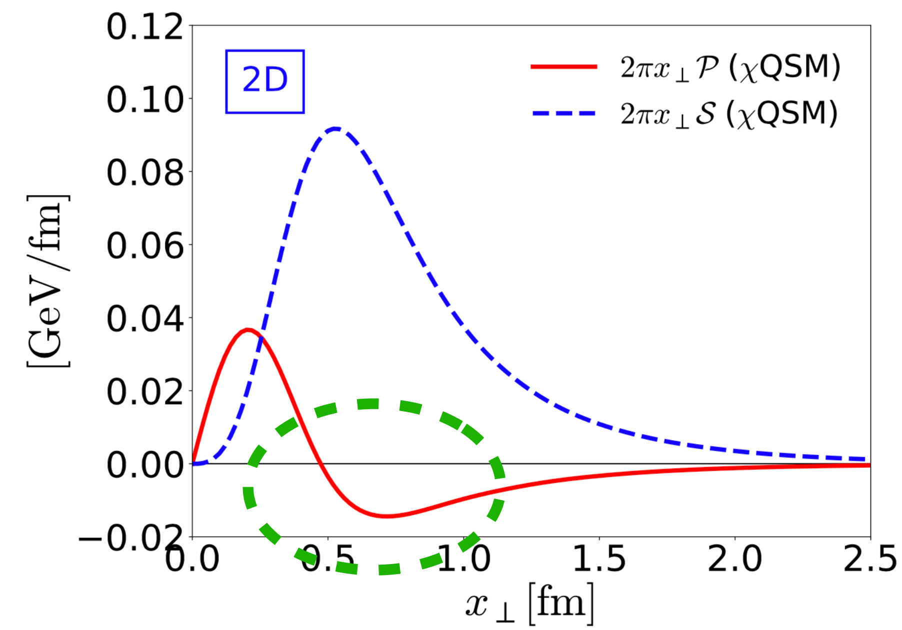
Abel transforms

$$\int_0^\infty dr\ r^2 p(r) = 0 \qquad \int d^2x_\perp \mathcal{P}(x_\perp) = 0$$









J. Y. Kim & HChK, PRD **104** (2021) 7

Radii of the proton

$$\langle x_\perp^2\rangle_{\rm mass} < \langle x_\perp^2\rangle_{\rm mech} < \langle x_\perp^2\rangle_{\rm charge} < \langle x_\perp^2\rangle_J \quad \text{(2D } \chi\text{QSM)}$$

$$\langle r^2\rangle_{\rm mech} < \langle r^2\rangle_{\rm mass} < \langle r^2\rangle_{\rm charge} < \langle r^2\rangle_J \quad \text{(3D } \chi\text{QSM)}$$

$$\langle x_\perp^2\rangle_{\rm mass} = \frac{1}{m}\int d^2x_\perp x_\perp^2 \mathcal{E}(x_\perp) = \frac{2}{3}\langle r^2\rangle_{\rm mass} + \frac{D(0)}{m^2} \qquad (D(0) < 0)$$

Note that 2D mass radius is smaller than the 3D one.

$\langle x_\perp^2\rangle_{\rm mass}$ (fm^2)	$\langle x_\perp^2\rangle_J$ (fm^2)	$\langle x_\perp^2\rangle_{\rm mech}$ (fm^2)	$\langle x_\perp^2\rangle_{\rm charge}$ (fm^2)
0.39	1.19	0.42	0.58
$\langle r^2\rangle_{\rm mass}$ (fm^2)	$\langle r^2\rangle_J$ (fm^2)	$\langle r^2\rangle_{\rm mech}$ (fm^2)	$\langle r^2\rangle_{\rm charge}$ (fm^2)
0.66	1.49	0.63	0.86

J. Y. Kim & HChK, PRD **104** (2021) 7

Stability conditions

- Conservation of the statice EMT current → Global & local stability conditions

$$\partial^i T_{ij} = \frac{r_j}{r}\left[\frac{2}{3}\frac{\partial s(r)}{\partial r} + \frac{2s(r)}{r} + \frac{\partial p(r)}{\partial r}\right] = 0$$

- Von Laue condition: Global stability condition

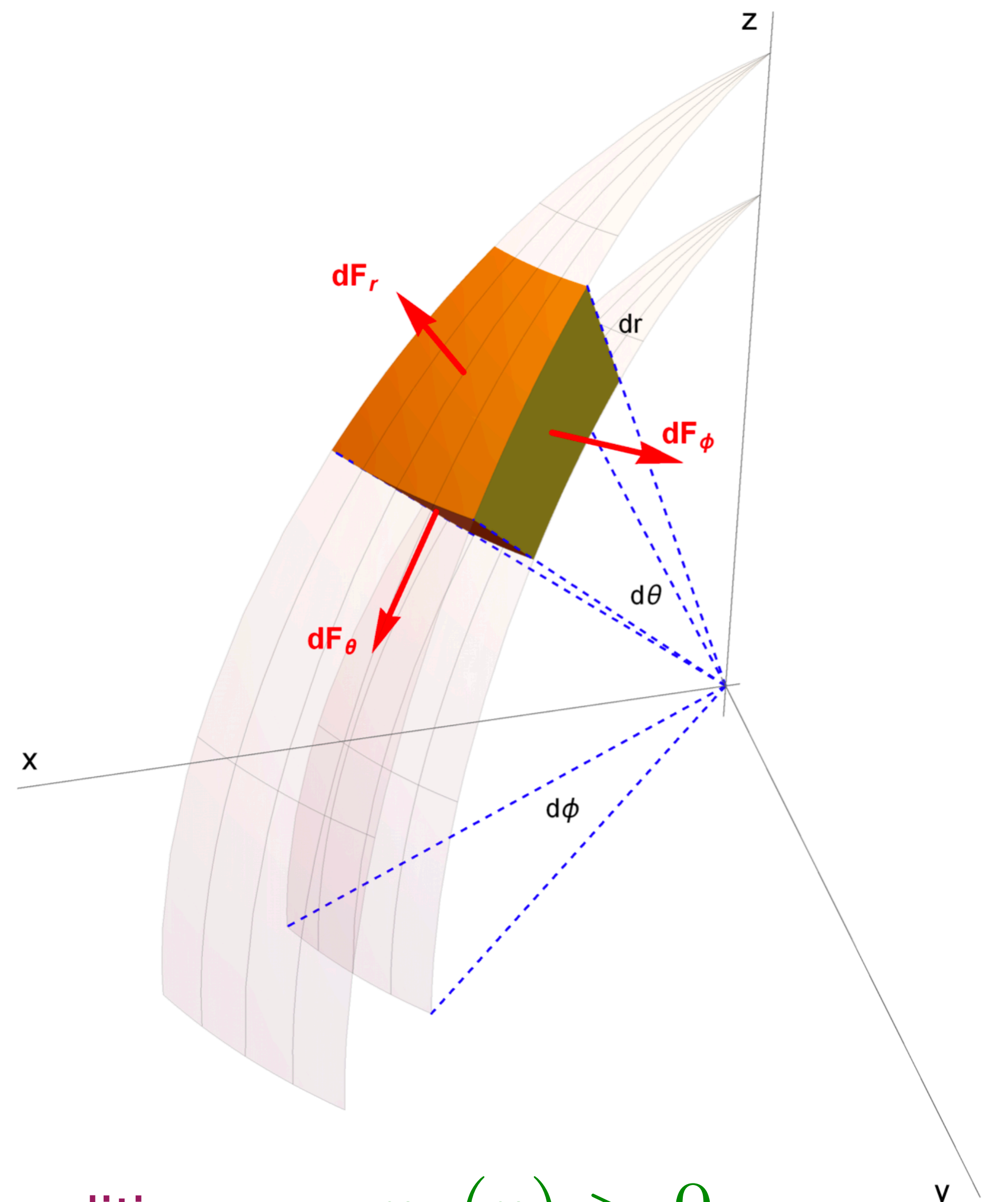
$$\int_0^\infty dr\ r^2 p(r) = 0$$

$$dF^i_{(r,\theta,\phi)} = T^{ij} dS_{(r,\theta,\phi)} \boldsymbol{e}^j_{(r,\theta,\phi)}$$

$$p_r(r) := \frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r),$$

$$p_\theta(r) := \frac{dF_\theta}{dS_\theta} = -\frac{1}{3}s(r) + p(r),$$

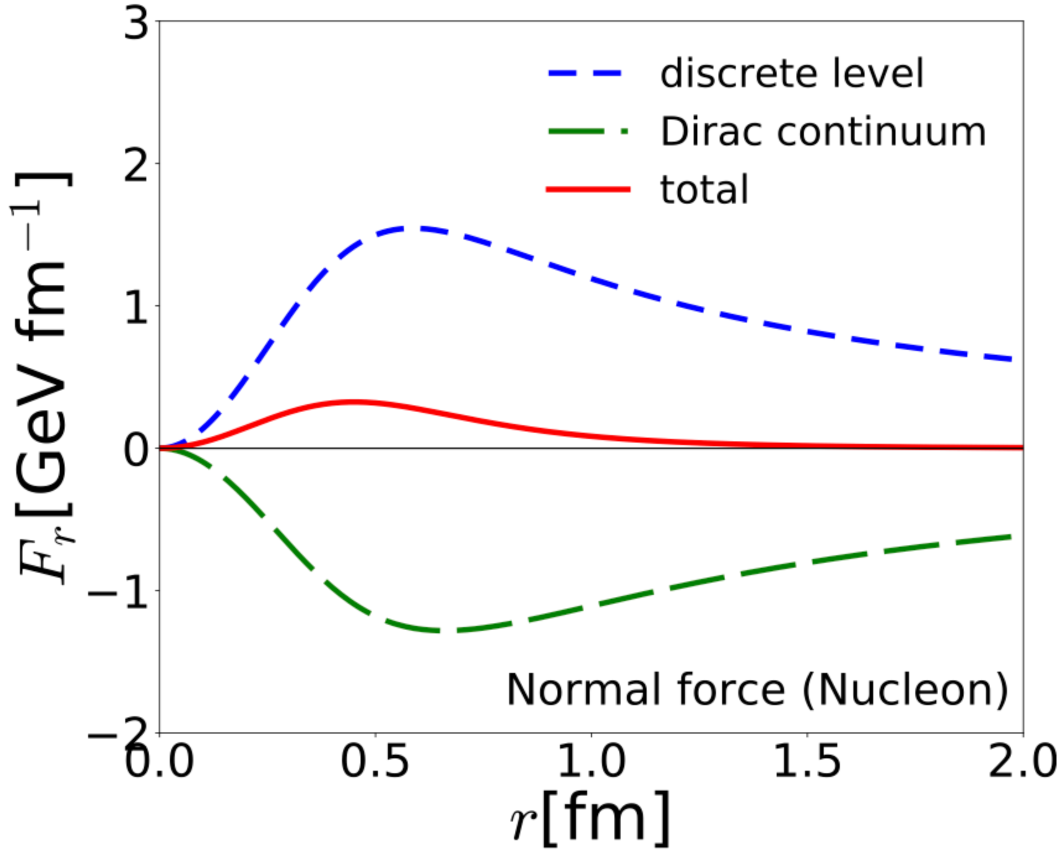
$$p_\phi(r) := \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r)$$

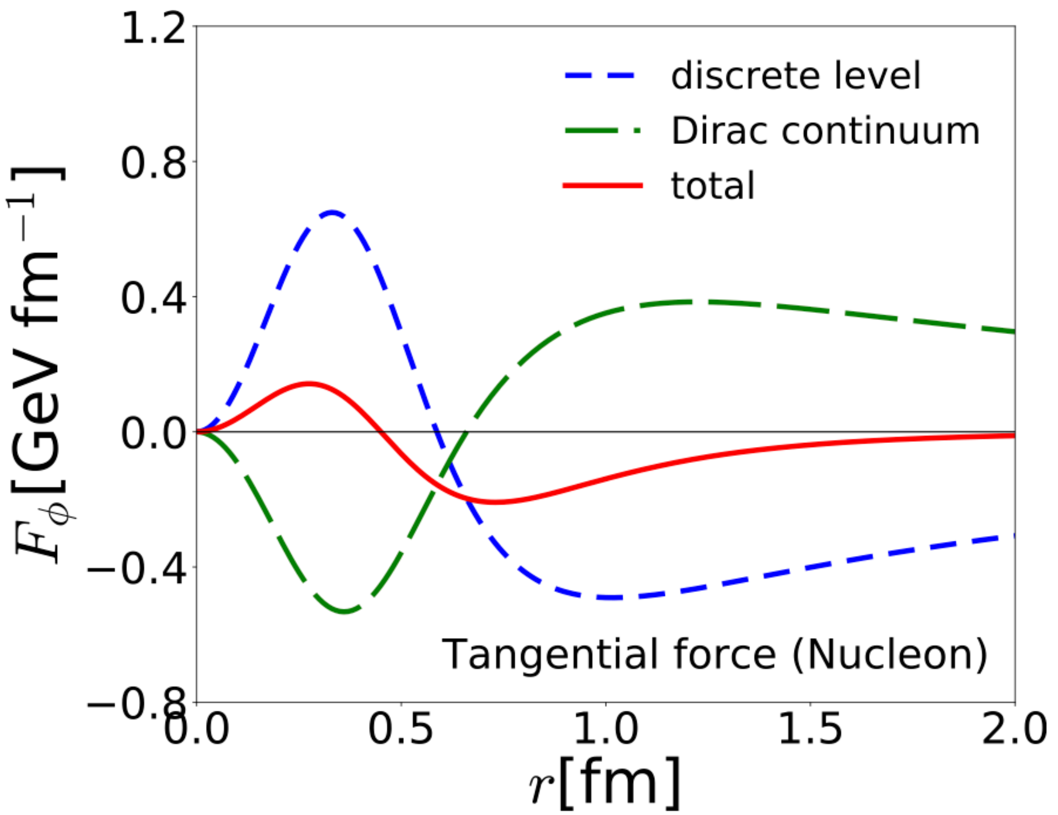


Local stability condition $p_r(r) > 0$

A. Perevalova, M. V. Polyakov and P. Schweitzer, PRD 94 (2016)

3D force fields & local stability





- Normal force is always positive:

$$p_r(r) > 0 \longrightarrow F_r(r) > 0$$

The discrete level overcomes the Dirac continuum.

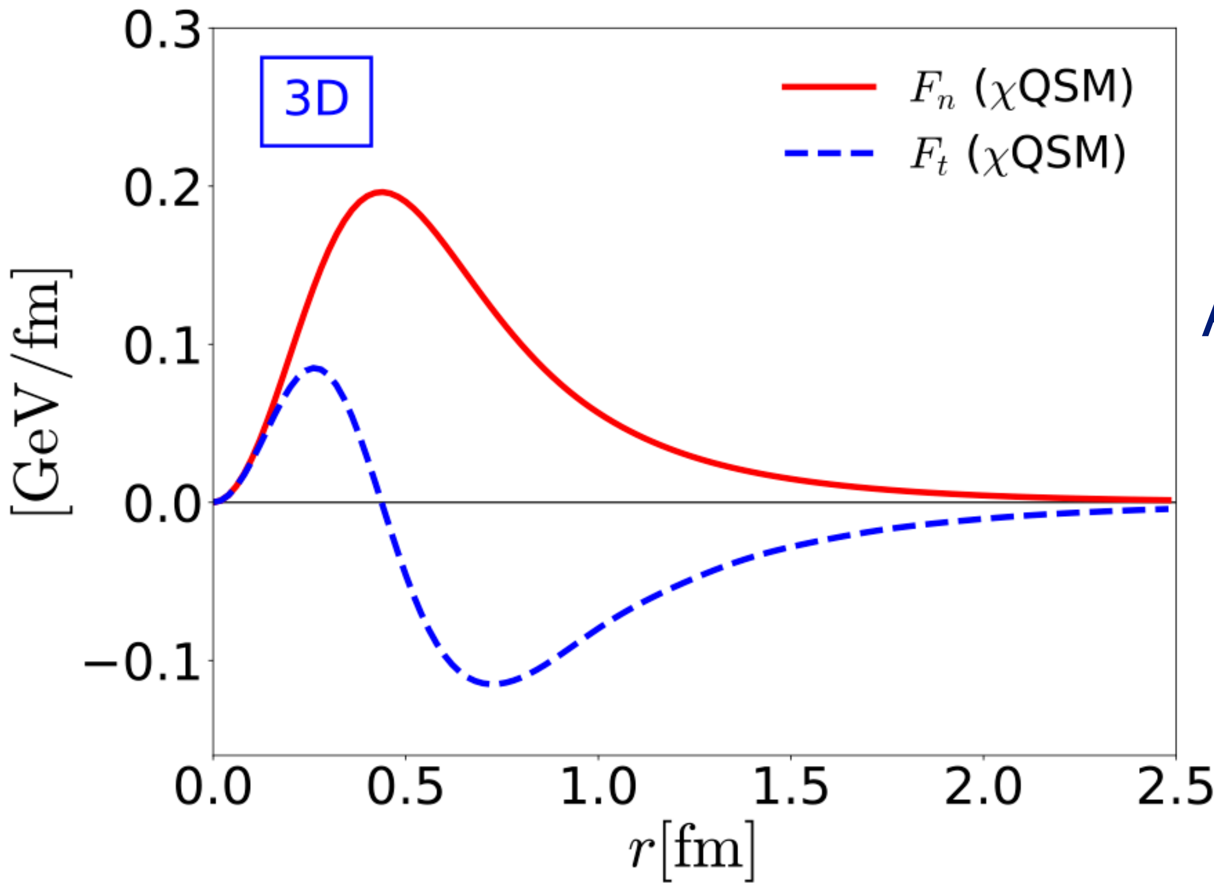
- Tangential force should at least have one nodal point.

→ Inner part of the tangential force is opposite to its outer part.

$$\int_0^\infty dr\ r\ p_\phi = 0$$ (2D von Laue condition)

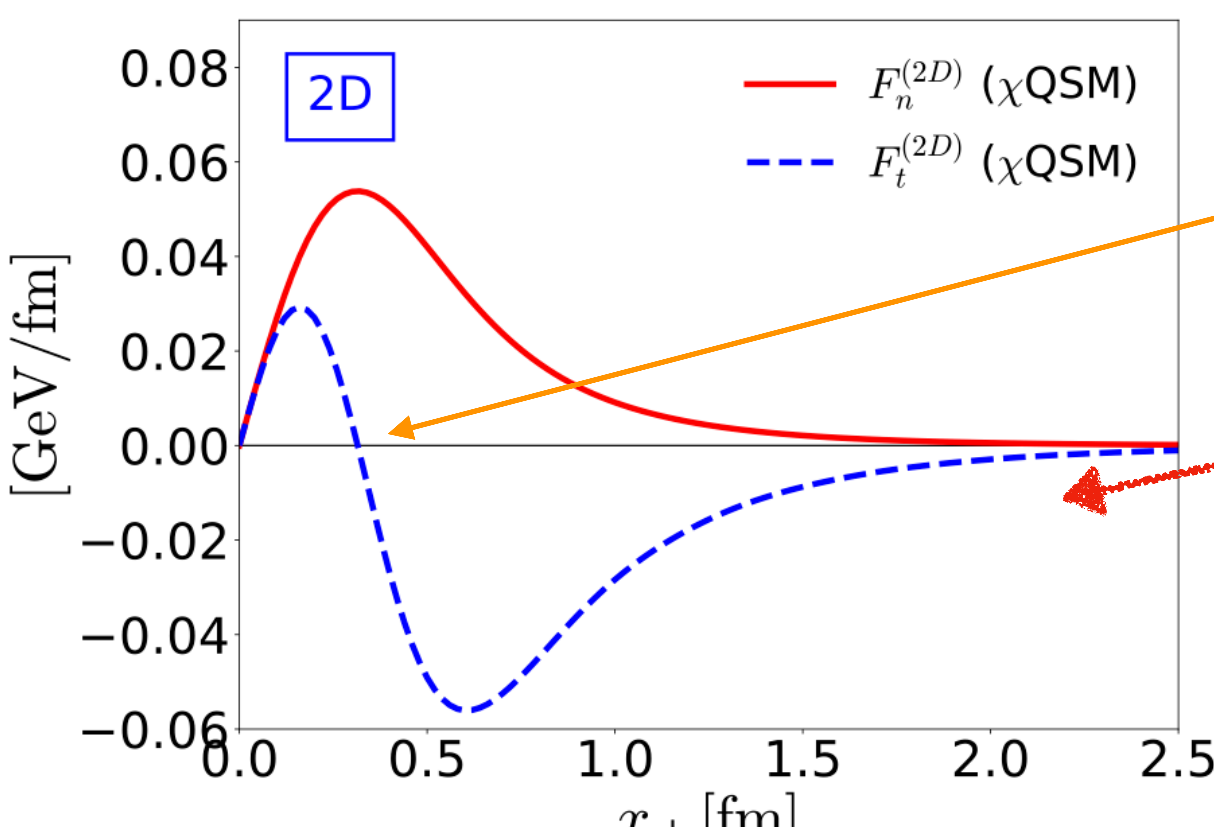
Kim, HChK, H. Son, M. Polyakov PRD 103 (2021)

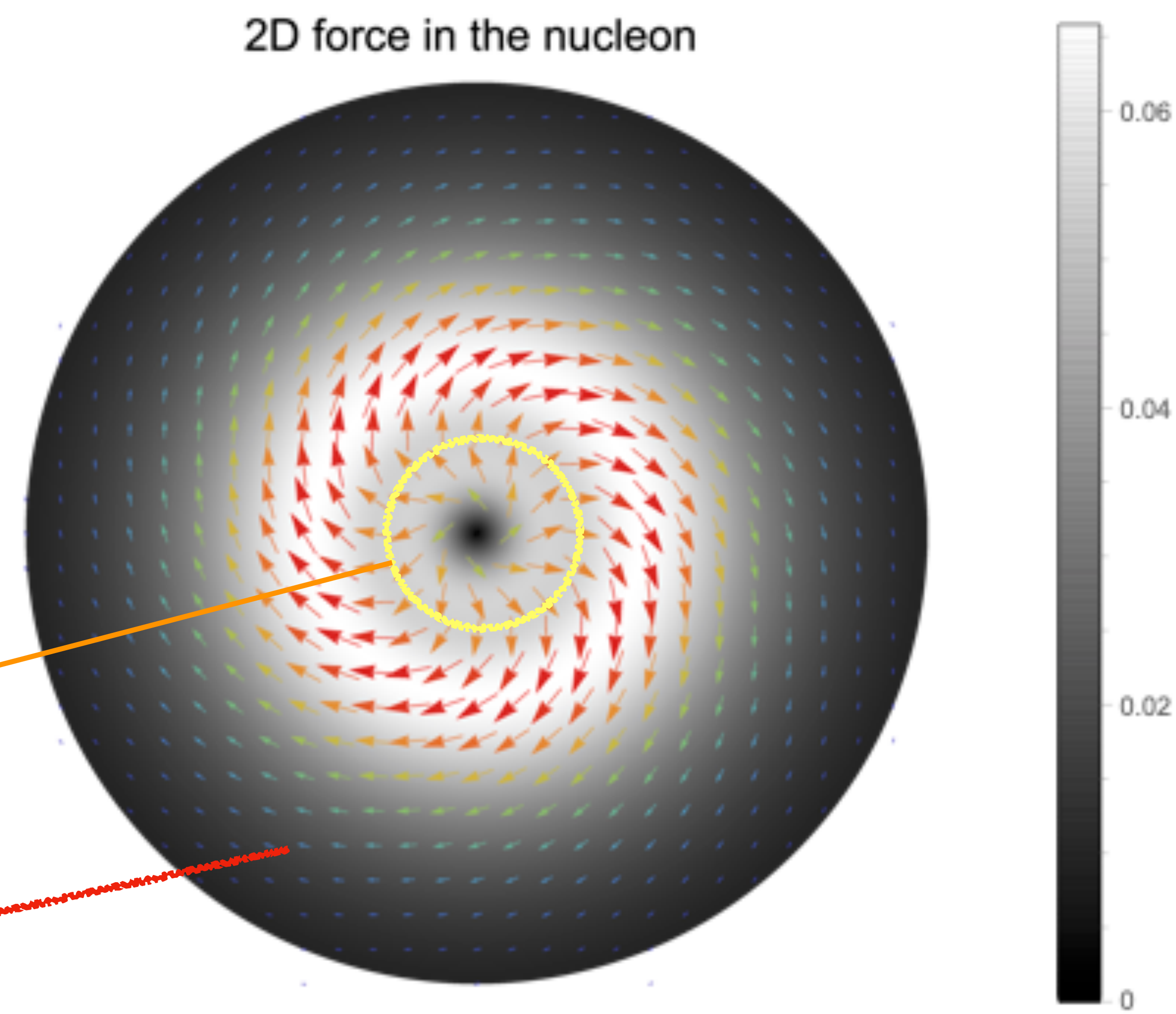
2D force fields & local stability



J. Y. Kim & HChK, PRD **104** (2021) 7

Abel transformation





Outer part is governed by
the tangential force. (Stability is acquired).

Summary & Conclusions

2D transverse structure of the Nucleon

- The nucleon is *per se* a relativistic particle.
- The 3D BF distributions have only a quasi-probabilistic meaning in a Wigner sense.
- Abel transform makes 3D NR densities equivalent to 2D LF ones.
  In 2D, we restore quantum mechanically probabilistic meaning of the densities.
- **The 3D global & local stability conditions are all conveyed to the 2D ones!**
- **3D distributions in BF still provide physical intuitions, even though they have only a quasi-probabilistic meaning.**
- As for the 2D densities of the polarized nucleon, we need the Radon transform.
- Higher-spin baryons are under investigation by using the Radon transform.

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Thank you very much for the attention!