# Deeply virtual Compton scattering: the Compton form factors of the $^4$He nucleus[1]

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[1] This work was done in collaboration with Chueng-Ryong Ji

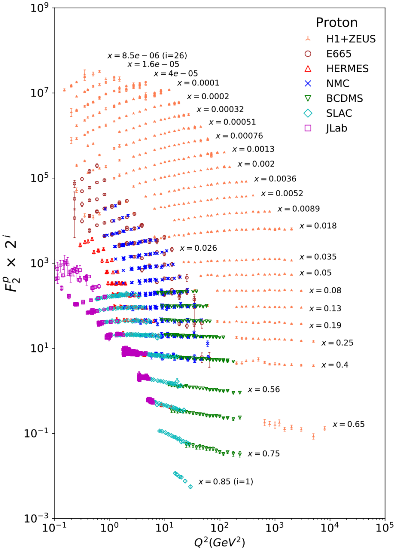
# Outline

# Motivation

The traditional motivation for the Parton Distribution approach to the study of hadronic structure is based on the ideas of factorization and scaling. These ideas have worked well in DIS, where the PDFs are determined, which are Lorentz scalars.

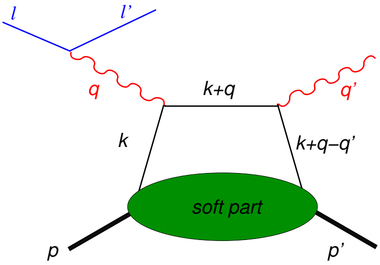
For large enough $Q$, scaling is seen as a weak dependence of the PDFs on $Q$ as illustrated by the compilation by the Particle Data Group.

*18. Structure Functions*




Source: PDG 2020

Deeply-virtual Compton scattering (DVCS) has been proposed to determine the generalized-parton distributions (GPDs) of hadrons. A hard, virtual photon with momentum $q$, $q^2 = -Q^2$, with $Q$ much larger than the characteristic hadronic scales, probes the quark content of the hadronic target. The detection of the outgoing, real photon provides information not contained in DIS.




Handbag diagram for VCS, including the leptonic part

It is usually assumed that to allow for the extraction of the GPDs, the experiments should be set-up in (approximately) collinear kinematics. Such kinematics may not always be realized in concrete experiments.

**We propose to first analyze the experimental data in terms of Lorentz-invariant amplitudes, Compton form factors (CFFs).**

By definition, the CFFs can be determined in any suitable kinematics. Once they are measured, theorists may use them to extract the GPDs.

Here, we present our work on VCS off the $^4$He nucleus, motivated by a considerable numbers of experiments about VCS on $^4$He, one of the most recent example is the work of R. Dupré et al., CLAS collaboration at Jefferson Lab[2]

We shall in particular discuss the importance of considering all CFFs to analyze the data.

[2] R. Dupré et al., Phys. Rev. C **104**, 025203 (2021))

# Formal Framework

In Compton scattering the physical amplitudes can be written in terms of a leptonic and a hadronic part

$$\mathcal{M}_{\rm VCS}(\lambda', \lambda, h') = \sum_h L^{\rho}_{\rm VCS}(\lambda', \lambda)\epsilon^*_\rho(q, h)\frac{1}{q^2}\epsilon^*_\mu(q', h')T^{\mu\nu}\epsilon_\nu(q, h)$$

The leptonic part is given by

$$L^{\rho}_{\rm VCS}(\lambda', \lambda) = \bar{u}(k', \lambda')\gamma^\rho u(k, \lambda)$$

The tensor $T^{\mu\nu}$ must be transverse to $q'_\mu$ and $q_\nu$.

In order not to introduce **unwarranted restrictions**, it is important to use the most general form of that tensor operator consistent with EM gauge invariance.

The quark-gluon structure of hadrons is supposed to manifest itself most transparently in processes where the hadrons are subjected to strongly virtual probes.

**The amplitudes must scale with the virtuality $Q$ to allow for a partonic interpretation.**

To obtain the **complete** amplitudes, one must add the ones associated with the **Bethe-Heitler (BH) process**. These amplitudes can be written as the convolution of the leptonic (QED) amplitude and a hadronic amplitude, which involves the electro-magnetic form factor of the $^4$He nucleus, which is well known. We use the parametrisation by Frosch *et al.*[3].

As the Bethe-Heitler amplitudes follow directly from QED, we concentrate here on the question what is the most general form of the Compton tensor $T^{\mu\nu}$ and can we estimate the effects on the analysis of the data by using a restricted form.

In particular we shall pay attention to the situation where the BH process does not contribute to the total amplitude. This situation occurs when the EM form factor of the $^4$He nucleus vanishes. The model of Frosch et al., that very well interpolates the data, gives us the clue.

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[3] R.F. Frosch, J.S. McCarthy, R.E. Rand, and M.R. Yearian, Phys. Rev. **180**, 874 (1967)

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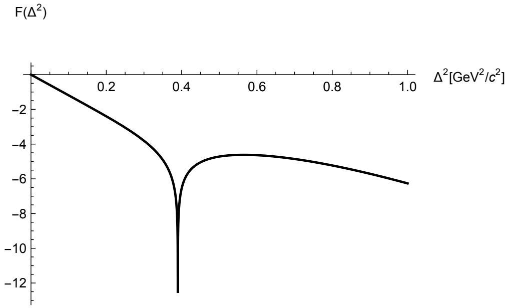



Note the node in the $^4$He form factor at $Q = 0.624$ GeV$/c$. This node is important, because it marks the point where the contribution of the BH process changes sign. At this point both the BH amplitude and its interference with the hadronic amplitude vanish.

# Tensor Formulations

In the thesis of Metz[4] the Compton tensor is denoted as $M^{\mu\nu}$ and the CFFs for a scalar particle, denoted as $B_1$, $B_2$, $B_3$, $B_4$, and $B_{19}$, are defined by the following equations:

$$
\begin{aligned}
M^{\mu\nu} &= B_1 M_1^{\mu\nu} + B_2 M_2^{\mu\nu} + B_3 M_3^{\mu\nu} + B_4 M_4^{\mu\nu} + B_{19} M_{19}^{\mu\nu}, \\
M_1^{\mu\nu} &= -q' \cdot q\, g^{\mu\nu} + q^\mu q'^\nu, \\
M_2^{\mu\nu} &= -(\overline{P} \cdot q)^2 g^{\mu\nu} - q' \cdot q\, \overline{P}^\mu \overline{P}^\nu + \overline{P} \cdot q\, (\overline{P}^\mu q'^\nu + q^\mu \overline{P}^\nu), \\
M_3^{\mu\nu} &= q'^2 q^2 g^{\mu\nu} + q' \cdot q\, q'^\mu q^\nu - q^2\, q'^\mu q'^\nu - q'^2\, q^\mu q^\nu, \\
M_4^{\mu\nu} &= \overline{P} \cdot q\, (q'^2 + q^2)\, g^{\mu\nu} - \overline{P} \cdot q\, (q'^\mu q'^\nu + q^\mu q^\nu) - q^2\, \overline{P}^\mu q'^\nu - q'^2\, q^\mu \overline{P}^\nu \\
&\quad + q' \cdot q\, (\overline{P}^\mu q^\nu + q'^\mu \overline{P}^\nu), \\
M_{19}^{\mu\nu} &= (\overline{P} \cdot q)^2\, q'^\mu q^\nu + q'^2 q^2\, \overline{P}^\mu \overline{P}^\nu - \overline{P} \cdot q\, q^2\, q'^\mu \overline{P}^\nu - \overline{P} \cdot q\, q'^2\, \overline{P}^\mu q^\nu,
\end{aligned}
$$

Note that for the case $q'^2 = 0$, the tensors $M_3^{\mu\nu}$ and $M_{19}^{\mu\nu}$ do not contribute to the hadronic amplitude.

[4] A. Metz, *Virtuelle Comptonstreuung un die Polarisierbarkeiten des Nukleons* (in German), PhD thesis, Universität Mainz, 1997.

# A novel projection method

We have proposed a method[5] that is free of poles *ab initio*. The back bone of the Compton tensor is

$$d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\beta} g^{\nu\alpha}.$$

We note that $d^{\mu\nu\alpha\beta}$ is symmetric under the simultaneous interchange $\mu \leftrightarrow \nu$, $\alpha \leftrightarrow \beta$ and changes sign by the interchanges $\mu \leftrightarrow \alpha$, and $\nu \leftrightarrow \beta$. Using this back bone we construct pieces of "DNA" by contracting it with the three basis four vectors. With an obvious notation we write them as follows:

$$\begin{aligned}
G^{\mu\nu}(q'q) &= q'_\alpha d^{\mu\nu\alpha\beta} q_\beta = q' \cdot q\, g^{\mu\nu} - q^\mu q'^\nu, \\
G^{\mu\nu}(qq) &= q_\alpha d^{\mu\nu\alpha\beta} q_\beta = q^2 g^{\mu\nu} - q^\mu q^\nu, \\
G^{\mu\nu}(q'q') &= q'_\alpha d^{\mu\nu\alpha\beta} q'_\beta = q'^2 g^{\mu\nu} - q'^\mu q'^\nu, \\
G^{\mu\nu}(\overline{P}q) &= \overline{P}_\alpha d^{\mu\nu\alpha\beta} q_\beta = \overline{P} \cdot q\, g^{\mu\nu} - q^\mu \overline{P}^\nu, \\
G^{\mu\nu}(q'\overline{P}) &= q'_\alpha d^{\mu\nu\alpha\beta} \overline{P}_\beta = \overline{P} \cdot q'\, g^{\mu\nu} - \overline{P}^\mu q'^\nu.
\end{aligned}$$

The momentum $\overline{P}$ is the sum of the hadron momenta: $\overline{P} = p' + p$.

[5] B.L.G. Bakker and C.-R. Ji, Few-Body Syst., **58**, 1 (2017)

Given these building blocks we write the transverse tensor as

$$T^{\mu\nu}_{\text{DNA}} := \sum_{i=1}^{5} \mathcal{S}_i T^{(i)\,\mu\nu}_{\text{DNA}} = \mathcal{S}_1 G^{\mu\nu}(q', q) + \mathcal{S}_2 G^{\mu\lambda}(q', q') G_{\lambda}^{\ \nu}(q, q)$$

$$+\mathcal{S}_3 G^{\mu\lambda}(q', \bar{P}) G_{\lambda}^{\ \nu}(\bar{P}, q) + \mathcal{S}_4 \left( G^{\mu\lambda}(q', \bar{P}) G_{\lambda}^{\ \nu}(q, q) + G^{\mu\lambda}(q', q')) G_{\lambda}^{\ \nu}(\bar{P}, q) \right)$$

$$+\mathcal{S}_5 G^{\mu\lambda}(q', q') \bar{P}_{\lambda} \bar{P}_{\lambda'} G^{\lambda'\nu}(q, q).$$

Where the $\mathcal{S}_i$ are the CFFs in the DNA construction. By direct computation one may check that the DNA representation is simply related to Metz's .

$$T^{(1)}_{\mathrm{DNA}} = -M_1, \quad T^{(2)}_{\mathrm{DNA}} = M_3, \quad T^{(3)}_{\mathrm{DNA}} = -M_2, \quad T^{(4)}_{\mathrm{DNA}} = M_4, \quad T^{(5)}_{\mathrm{DNA}} = M_{19}.$$

Note that for the case $q'^2 = 0$, the CFFs $\mathcal{S}_2$ and $\mathcal{S}_5$ do not contribute to the hadronic amplitude.

# Kinematics

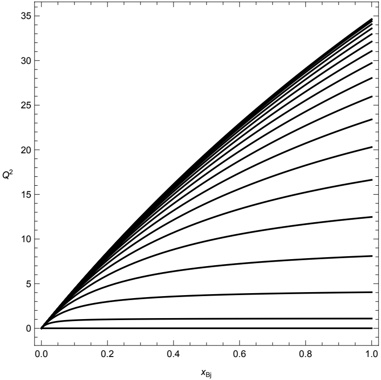
We shall I work in the target rest frame (TRF) with the $z$-axis along the three momentum $\boldsymbol{q}$ of the virtual photon. The amplitudes can be expressed in terms of three invariants and the azimuthal angles $\phi$, which is the angle between the *leptonic plane*, defined by the momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$ and the *hadronic plane* defined by $\boldsymbol{p}$ and $\boldsymbol{p}'$. The momentum $\overline{\boldsymbol{P}} = \boldsymbol{p}' + \boldsymbol{p}$ as well as the momentum $\boldsymbol{\Delta} = \boldsymbol{p}' - \boldsymbol{p}$ are in the hadronic plane, while $\boldsymbol{q}$ is in the intersection line of the two planes.

The relevant invariants are the mass $M$ of the hadronic target and

$$
\begin{aligned}
Q^2 &= -q^2, \quad x_{\mathrm{Bj}} = \frac{Q^2}{(2p\cdot q)}, \quad y = \frac{p\cdot q}{p\cdot k} = \frac{Q^2}{2E_b M x_{\mathrm{Bj}}}, \\
s_{\mathrm{had}} &= (p+q)^2 = M^2 + \frac{1-x_{\mathrm{Bj}}}{x_{\mathrm{Bj}}}\, Q^2, \\
t_{\mathrm{had}} &= (p-p')^2, \quad u_{\mathrm{had}} = (p-q')^2.
\end{aligned}
$$

$E_b$ is the energy of the incoming electron; it determines the overall energy and momentum scales. The invariants $t_{\mathrm{had}}$ and $u_{\mathrm{had}}$ depend on the azimuthal angle $\phi$. The invariants $x_{\mathrm{Bj}}$ and $y$ are both limited to the interval $[0,1]$.

The kinematical domain for fixed $M$ and $E_b$ is parametrized by the scattering angle $\theta_{\rm e}$ of the electron. $Q^2$ in GeV$^2/c^2$. The plots below are for $M = 3.7373$ GeV$/c^2$ and $E_{\rm b} = 6.064$ GeV$/c^2$.




The curves are lines of constant $\theta_{\rm e}$. This angle runs from $\theta_{\rm e} = 0$, the lowest curve, to $\theta_{\rm e} = \pi$, the highest, in steps of $\frac{\pi}{18}$. $Q^2$ is largest for $x_{\rm Bj} \to 1$.

This plot demonstrates that for small values of $Q^2$, say $Q^2 \sim 1 - 2\ \text{GeV}^2/c^2$, the curves for constant $\theta_e$ are flat for 0.2 - 0.3 $< x_{Bj} < 1$.
The Mandelstam variables $t_{had}$ and $u_{had}$ for large $Q$ are:

$$t_{had} \to -\frac{1-\cos\vartheta}{2x_{Bj}}\, Q^2 + \mathcal{O}(M^2), \quad u_{had} \to -\frac{1+\cos\vartheta}{2x_{Bj}}\, Q^2 + \mathcal{O}(M^2).$$

The quantity $\vartheta = \theta'_C - \theta_C$ is the photon scattering angle in the hadronic CMF. For small values of $\vartheta$, which are relevant here, it is close to the scattering angle in the TRF.

If $\vartheta \to 0$, $t_{had}$ goes to zero up to corrections of $\mathcal{O}(M^2)$, thus $t_{had}$ does not strictly vanish in the forward limit. This shows that to neglect $t_{had}$ in the analysis of the data is not precise.
For large $Q$ and small $\vartheta_{lim}$ one finds $|t| > \frac{\vartheta^2_{lim}}{4x_{Bj}}\, Q^2$.

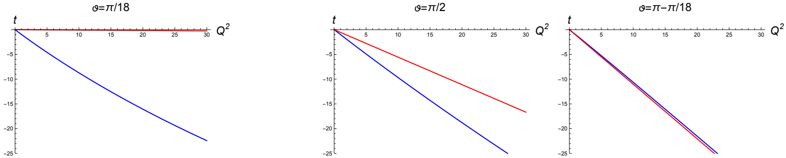
For any value of the scattering angle greater than 0, $t$ remains of order $Q^2$.

Moreover, if the target mass $M$ is not small compared to $Q$, which is the case in e.g. the VCS-on-$^4$He experiment done at Jlab, one must go to almost completely forward kinematics and $Q^2$ very large to make $t_{had}$ small compared to $Q^2$, which implies $x_{Bj} \approx 1$.

Because the Mandelstam variable $t$ plays a special role, we consider its behaviour at large $Q^2$ in more detail. Its expression in terms of the invariants is

$$t = -Q^2 \frac{Q^2(1-x_{\mathrm{Bj}}) + 2M^2 x_{\mathrm{Bj}}^2 - Q(1-x_{\mathrm{Bj}})\sqrt{Q^2 + 4M^2 x_{\mathrm{Bj}}^2}\cos\vartheta}{2x_{\mathrm{Bj}}(Q^2(1-x_{\mathrm{Bj}}) + M^2 x_{\mathrm{Bj}})}$$

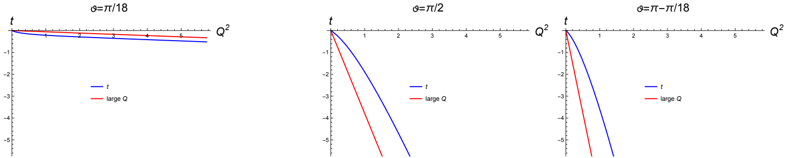
where the angle $\vartheta$ is the polar angle of the emitted photon momentum in the CMF. For a value of $x_{\mathrm{Bj}} = 0.9$ and the electron scattering angle $\theta_e = \pi$ we find the behaviour:




$t$, large $Q$ limit

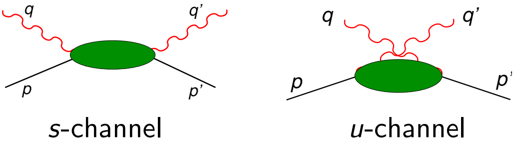
It is clear that the asymptotic limit is bad for small $\vartheta$, but becomes more accurate for $\vartheta \to \pi$. Note that in the whole domain $t \sim Q^2$. (Units: $\mathrm{GeV}^2/c^2$.)

For values of $x_{\rm Bj}$ in the range 0.1 - 0.3 the details are different. For $x_{\rm Bj} = 0.132$ and $\theta_e = \pi/18$ we find




Although the accuracy of of the large-$Q$ limit is different, the behaviour $t \sim Q^2$ is the same. The choice of $\theta_e$ corresponds to the CLAS collaboration kinematics.

# Model Calculation




As a **benchmark model** one may consider the tree-level case, which of course describes completely structureless particles. Any deviation of the cross sections from the predictions of this model implies that the hadron has structure.

The tree-level DVCS amplitude corresponds to the CFFs

$$\mathcal{S}_1^{\text{tree}} = -\left(\frac{1}{s_{\text{had}} - M^2} + \frac{1}{u_{\text{had}} - M^2}\right), \quad \mathcal{S}_3^{\text{tree}} = \frac{2}{(s_{\text{had}} - M^2)(u_{\text{had}} - M^2)}.$$

Thus, only 2 out of 5 CFFs contribute. We note that at large $Q$, $\mathcal{S}_3$ is of relative order $1/Q^2$ compared to $\mathcal{S}_1$.

Because we study the relative importance of the CFFs, we do not include the factors $-e$ and $2e$ for the charges of the elektron and the $^4$He nucleus, respectively.

# VCS Amplitudes squared

Because the Bethe-Heitler and the *coherent* VCS processes are coherent, their amplitudes must be added when the cross section for the process $e + {}^4\mathrm{He} \to e' + {}^4\mathrm{He} + \gamma$ is calculated. Then the complete squared amplitudes can be split into a Bethe-Heitler part, a VCS part and a part that is obtained by the interference of the two amplitudes:

$$\begin{aligned} |A_{\mathrm{tot}}|^2 &= |A_{\mathrm{BH}} + A_{\mathrm{VCS}}|^2 \\ &= |A_{\mathrm{BH}}|^2 + |A_{\mathrm{VCS}}|^2 + A^*_{\mathrm{BH}} A_{\mathrm{VCS}} + A_{\mathrm{BH}} A^*_{\mathrm{VCS}}. \end{aligned}$$

As the BH amplitude does not depend on the CFFs, we shall not discuss it here, but rather concentrate on the VCS squared amplitude.

The values of the quantities $Q^2$, $x_{\mathrm{Bj}}$, and $t_{\mathrm{had}}$ for which we show the results are taken from the paper by Dupré et al., namely

$$x_{\mathrm{Bj}} = 0.132,\ Q^2 = 1.143,\ x_{\mathrm{Bj}} = 0.170,\ Q^2 = 1.423,\ x_{\mathrm{Bj}} = 0.255,\ Q^2 = 1.902$$

# VCS cross section and CFFs

Including the leptonic part of the VCS amplitudes, we calculate the physical squared VCS amplitude including the leptonic part.

$x_{\mathrm{Bj}} = 0.132,\ Q^2 = 1.143,\ t/Q^2 = -0.172058,\ \vartheta = 0$

| $Amp_{11}^2$ | $Amp_{13}^2$ | $Amp_{33}^2$ | Total |
|---|---|---|---|
| 0.00328065 | $-1.87056$ | 266.638 | 264.771 |

$x_{\mathrm{Bj}} = 0.170,\ Q^2 = 1.423,\ t/Q^2 = -0.217369,\ \vartheta = 0$

| $Amp_{11}^2$ | $Amp_{13}^2$ | $Amp_{33}^2$ | Total |
|---|---|---|---|
| 0.00393712 | $-1.44797$ | 129.874 | 128.448 |

$x_{\mathrm{Bj}} = 0.255,\ Q^2 = 1.902,\ t/Q^2 = -0.321459,\ \vartheta = 0$

| $Amp_{11}^2$ | $Amp_{13}^2$ | $Amp_{33}^2$ | Total |
|---|---|---|---|
| 0.00609192 | $-1.12551$ | 94.0326 | 50.8667 |

The units are GeV for $Q$ and $1/\mathrm{GeV}^2$ for the (partial) amplitudes squared. ($1/\mathrm{GeV}^2 \approx 0.4$ mbarn)

Comments

(i) The partial contribution (13) is negative, the contributions that are diagonal in the CFF label are positive.

(ii) Although the CFF $\mathcal{S}_3$ is of order $1/Q^2$ compared to $\mathcal{S}_1$, it dominates in the squared amplitude.

It is clear from these results that, when extracting the CFFs from the data, it is dangerous to rely on what has been considered the dominant CFF, in this case $\mathcal{S}_1$. The two CFFs we have included are not realistic. To begin with, they are both *real*, while there is no reason for all CFFs to be real.

When the CFFs are complex, a beam spin asymmetry may show up in the VCS cross section. The common understanding is that the beam spin asymmetry is due to the interference part of the cross section

$$A^*_{\mathrm{BH}} A_{\mathrm{VCS}} + A_{\mathrm{BH}} A^*_{\mathrm{VCS}}$$

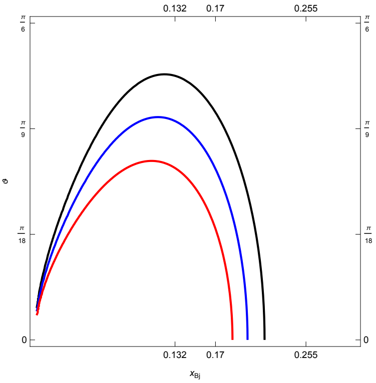
However, since $A_{\mathrm{BH}}$ is proportional to the $^4$He form factor, which has a node at $Q = 0.624$ GeV$/c$, which in the low-$Q$ part of the kinematic domain, one may perform a **crucial experiment** by measuring the beam spin asymmetry checking the minimum number of CFFs.

**If no beam spin asymmetry is measured, the minimal number of CFFs may be 1. If the beam spin asymmetry does not vanish, it is proof that at least two CFFs are involved and one of them must be complex.**

# Kinematics for the node in the $^4$He form factor

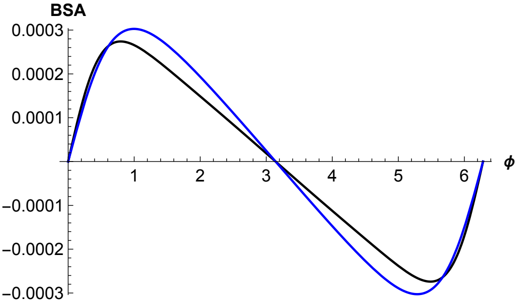
For the values of $x_{\mathrm{Bj}}$ and $Q^2$quoted in Dupré's paper we find the following nodal loci in the $x_{\mathrm{Bj}} - \vartheta$-plane.

$x_{\mathrm{Bj}} = 0.132$, $x_{\mathrm{Bj}} = 0.170$ , $x_{\mathrm{Bj}} = 0.255$ .




The nodal position is $\Delta^2 = 0.389941 \mathrm{GeV}^2/c^2$; the angle $\vartheta$ is the polar angle of the emitted photon in the CMF.

## Beam Spin Asymmetry




$x_{\mathrm{Bj}} = 0.132$, $x_{\mathrm{Bj}} = 0.170$. (For $x_{\mathrm{Bj}} = 0.255$ the node cannot be reached).

For the two kinematics from the CLAS experiment the BSA is tiny. Remarkably, the form of the BSA is not a pure sine, because the coefficient of this sine depends on $\cos\phi$ and $\cos 2\phi$.

# Summary and conclusions

- Our treatment of Virtual Compton Scattering is entirely phenomenological.
- We have discussed the number of Compton Form factors for a scalar target. This number is **three**.
- **We have presented a model-independent form of the Compton tensor, containing all three CFFs**.
- We have demonstrated that the partial tensors $T_i^{\mu\nu}$, $(i = 1, 2, 3)$ have different asymptotic behaviour as functions of $Q^2$. This behaviour is expected to compensate for the behaviour of the CFFs for large $Q^2$.

- For illustration, we have used the tree-level Compton tensor and modifications.
- We found that for the kinematics in the CLAS experiment at $E_b = 6$ GeV, the contribution of the part $\mathcal{S}_1 T_1^{\mu\nu}$ is much smaller than the contribution of the part $\mathcal{S}_3 T_3^{\mu\nu}$.
- Even without interference of the Bethe-Heitler process, there may occur a single-spin symmetry in VCS. This result is obtained because the VCS amplitude is the coherent sum of two parts, one related to the CFF $\mathcal{S}_1$, the other to $\mathcal{S}_3$.