Resonance in coupled channel reactions

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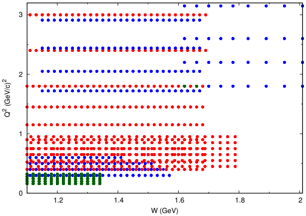
arXiv.2108.11605, W. Yamada, O. Morimatsu, T. Sato, K. Yazaki

"Near threshold spectrum from uniformized Mittag-Leffler Expansion - pole structure of Z(3900) -"

Motivation (N^* and Δ)

N^* and Δ from the amplitude meson production reaction

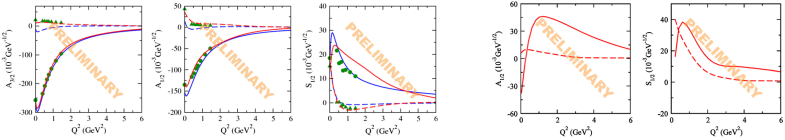
Analysis of pion, photon, electron induced meson production reaction ($\pi N, \eta N, K\Lambda, K\Sigma, \pi\pi N, \omega N..$) on nucleon and deuteron for various $W, Q^2, \theta..$



Data of single pion production(W, Q^2) S. X. Nakmura et al, PRD92 (2015)074024

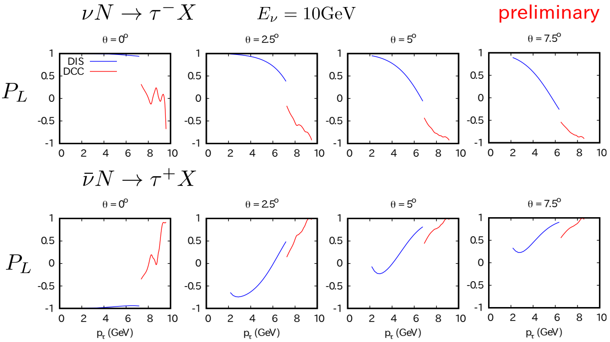
Motivation (N^* and Δ)

Electromagnetic transition form factor (ANL-Osaka DCC model H. Kamano, Few-Body Sys. (2018) 59:24)



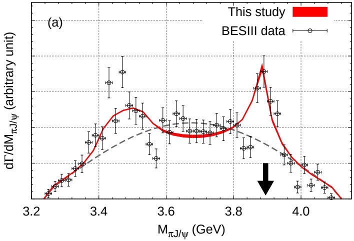
Δ_{33} $N^*(1/2^+)$

Application: τ polarization (ANL-Osaka Model(RES) and parton model(DIS) R. Alam, L. Alvarez Ruso, T. Sato)



Motivation (Z3900)

Enhancement/ peak in the invariant mass distribution



$Y \to \pi[\pi J/\Psi]$ Y. Ikeda for HAL QCD Collaboration J. Phys. G 45(2018)024002

New tools for less model dependent analysis

- Deep learning as a unified model-selection tool(Denny Sombillo, Parallel session 2-B)

  D. Sombillo, Y. Ikeda, T. Sato, A. Hosaka, Phys. Rev. D102(2020)016024

  D. Sombillo et al., Phys. Rev. D104(2021)036001

- Uniformied Mittag-Leffler expansion and near threshold spectrum

  W. A. Yamada, O. Morimatsu, Phys. Rev. C102 (2020)055201

  W.A. Yamada, O. Morimatsu Phys. Rev. C103 (2021) 045201

  W. A. Yamada, O. Morimatsu, T. Sato, K. Yazaki, arXiv:2018.11605

Mittag-Leffler expansion

analytic function that have only well separated poles

$$f(z) = c + \sum_{n=1}^{\infty} \frac{b_n}{z - a_n}$$

J. Humbelet, L. Rosenfeld(1961), W. Romo(1978)

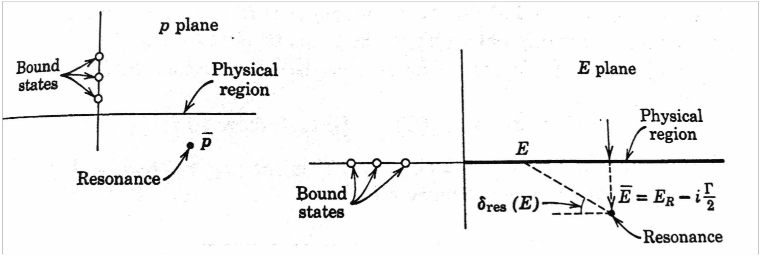
Uniformization

"introduce variable of which the S matrix is single valued"
"purpose: parametrization of the construction of phenomenological model"

J. R. Cox(1962), M. Kato(1965) ,H. Weidenmuller(1964)

- single channel
- two-channel
- pole near threshold
- summary and three-channel

From 'Scattering Theory', J. R. Taylor, Dover Pub.

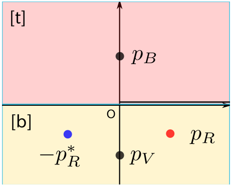


- two to one mapping from p to E
- E-plane : physical sheet [t] and second sheet(un-physical) [b]
- p-plane : upper ($Im(p) > 0$ physical), lower ($Im(p) < 0$ un-physical)

- Uniformization variable p: two E sheets $\to$ single p sheet
- Mittag-Leffler expansion of amplitude

$$\mathcal{F}(p) = \frac{S(p)-1}{2i} = \sum_B \frac{r_B}{p-p_B} + \sum_V \frac{r_V}{p-p_V} + \sum_R [\frac{r_R}{p-p_R} + \frac{r_R^*}{p+p_R^*}]$$

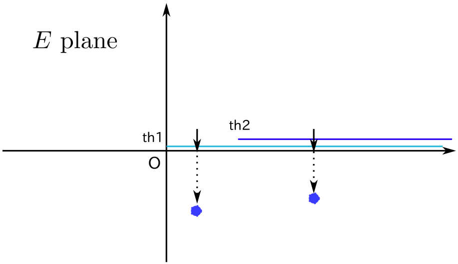
$(S(-p^*) = S^*(p))$



Breit-Wigner form : $$\frac{r_R}{p-p_R} + \frac{r_R^*}{p+p_R^*} = -\frac{p\gamma}{p^2-|p_R|^2+ip\gamma}$$

where $\gamma = -2Im(p_R) > 0$

Two Channel

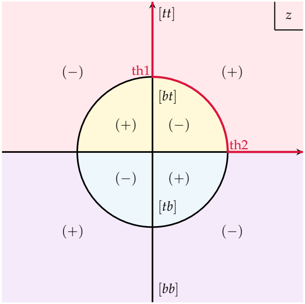


- S-matrix is not single valued function of p_1
- $Im[p_i] > 0$ $[t]$, $Im[p_i] < 0$ $[b]$, $i = 1, 2$
- 4-sheets: $[tt]$(physical), $[bt]$, $[bb]$, $[tb]$
- pole close to physical sheet : $[bt]$ between two thresholds, $[bb]$ above higher threshold

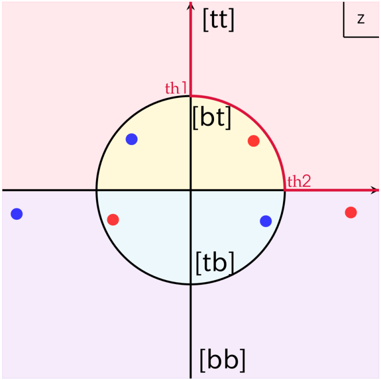
$$p_1 = \sqrt{s - \epsilon_1^2}, p_2 = \sqrt{s - \epsilon_2^2}, \quad \epsilon_2 = m_{a2} + m_{b2} > \epsilon_1 = m_{a1} + m_{b1}$$

Uniformization and amplitude

$$z = \frac{p_1 + p_2}{\sqrt{\epsilon_2^2 - \epsilon_1^2}}, \qquad \mathcal{A}(z) = -\frac{1}{\pi}\sum_n \left(\frac{r_n}{z - z_n} - \frac{r_n^*}{z + z_n^*}\right)$$



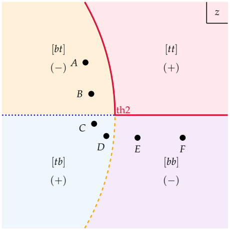
Two-channel

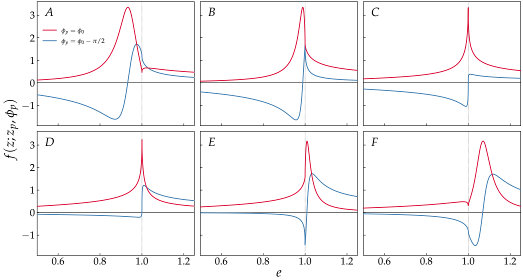


- 'conventional pole': $[bt]$ between th2 and th1, $[bb]$ above th2
- pole on $[tb]$ can be close to real s.

Two Channel

$$f(z; z_p, \phi_p) = -\frac{1}{\pi} Im[\frac{\exp(i\phi_p)}{z - z_p} - \frac{\exp(-i\phi_p)}{z + z_p^*}]$$



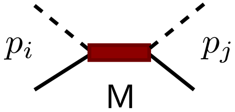


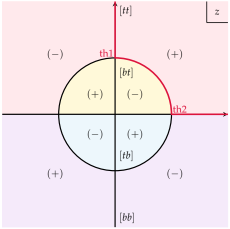
$e = (s - \epsilon_1^2)/(\epsilon_2^2 - \epsilon_1^2)$, $e = 1$ at th2

- A,B $[bt]$, E,F $[bb]$
- C,D $[tb]$ (peak at th2, peak energy $\neq Re(e_p)$, width of spectrum $\neq Im(e_p)$)

Two Channel

Flatte's formula (Two-channel Breit-Wigner)





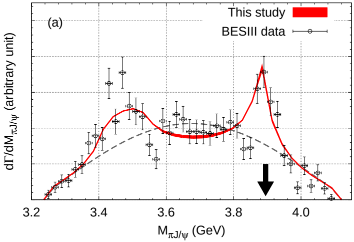
$$\mathcal{F}_{11} = -\frac{\gamma_1 p_1}{E - M + i\gamma_1 p_1 + i\gamma_2 p_2} = \sum_{j=1,2}[\frac{r_j}{z - z_j} + \frac{r_j^*}{z + z_j^*}]$$

- two poles : $(z_1, -z_1^*)$ and $(z_2, -z_2^*)$
- $|z_1 z_2| = 1$: $[bt] + [bb]$ or $[tb] + [bb]$

HAL QCD Collaboration: $\pi J/\Psi - \rho\eta_c - \bar{D}D^*$ coupled-channel interaction

Poles of scattering amplitude(Case I)

	$W_{pole} - m_D - m_{\bar{D}^*}$ in MeV	sheet
1	-167(94)(27) - i 83(46)(19)	[bbb]
2	-146(112)(108) - i 38(148)(32)	[ttb]
3	-93(55)(21) - i 9(25)(7)	[ttb]
4	-177(116)(62) - i 175(30)(22)	[tbb]
5	-369(129)(102) - i 207(61)(20)	[btb]

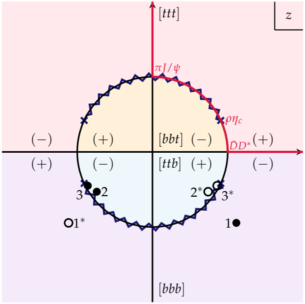


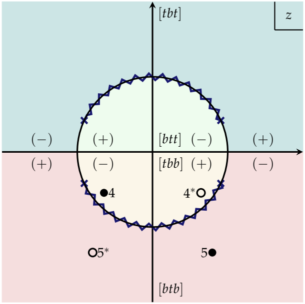
Y. Ikeda et al., J. Phys. G. 45 (2018)024002

Uniformization for two-channel $\pi J/\Psi - \bar{D}D^*$

- poles of HAL QCD results and their conjugate poles

	$1, 1^*$	$2, 2^*$	$3, 3^*$	$4, 4^*$	$5, 5^*$
z_p	$\pm 1.11 - 0.95i$	$\mp 0.74 - 0.53i$	$\mp 0.86 - 0.45i$	$\mp 0.65 - 0.54i$	$\pm 0.79 - 1.34i$
e_p	$0.60 \mp 0.41i$	$0.66 \mp 0.09i$	$0.79 \mp 0.02i$	$0.60 \mp 0.17i$	$0.16 \mp 0.44i$
sheet	$[bbb]$	$[ttb]$	$[ttb]$	$[tbb]$	$[btb]$

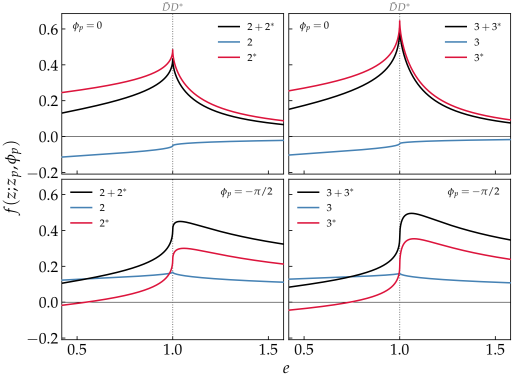




Two-Channel Z(3900) Uniformized-Mitag-Leffler expansion

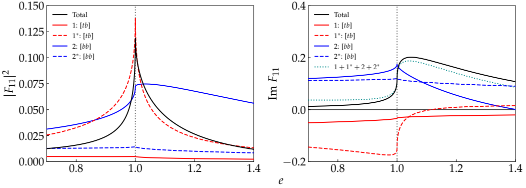
Uniformization for $\pi J/\Psi - \bar{D}D^*$ two-channel

Normalized spectrum $f(z; z_p, \phi_p)$ for each pole



- 3^* nearest to physical region
- $2^*, 3^*$ peak at DD^* threshold
- suggests contrbution of near threshold poles are the source of narrow structure

Separable potential model for $\pi J/\Psi(ch1) - \bar{D}D^*(ch2)$ two-channel



- monopole form factor, only $v_{12} \neq 0$
- amplitude can be explicitly expressed in terms of uniformization variable z

$$\mathcal{F}_{11}(z) = \sum_{j=1,2}\left[\frac{r_j}{z - z_j} + \frac{r_j^*}{z + z_j^*}\right] + \sum_{j=1,4}\frac{r'_j}{z - ia_j} + \sum_{j=1,2}\left[\frac{r_{\beta j}}{z - ia_{\beta j}} + \frac{ir'_{\beta j}}{(z - ia_{\beta j})^2}\right]$$

- poles simulating those of HAL-QCD

	z_p	sheet
$1, 1^*$	$\mp 0.754 - 0.229i$	$[tb]$
$2, 2^*$	$\pm 1.198 - 1.076i$	$[bb]$

Summary

- We have shown how S-matrix poles manifest themselves as physical spectrum near upper threshold from unformized Mittag-Leffler expansion
- Z(3900) can be naturally understood as a contribution of a set of poles in the domian near the $\bar{D}D^*$ threshold