Light Meson Structure from Basis Light-front Quantization

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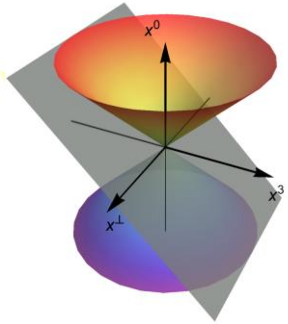


Jeju Island, Korea, November 29 - December 4, 2021

Outline

- Basis Light-front Quantization approach
- Application to Light Mesons
  - A review of BLFQ-NJL
  - Light mesons with one dynamical gluon

    (LFWF, EMFF, PDA, TFF, PDF, GPD & TMD)
- Summary & Outlook

Basis Light-front Quantization

- Nonperturbative eigenvalue problem [Vary et al, 2008]

$$P^{-}|\beta\rangle = P^{-}_{\beta}|\beta\rangle$$

  - P^{-}: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^{-}_{β}: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + \cdots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^{b}_{n,m}(\vec{p}_{\perp})$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

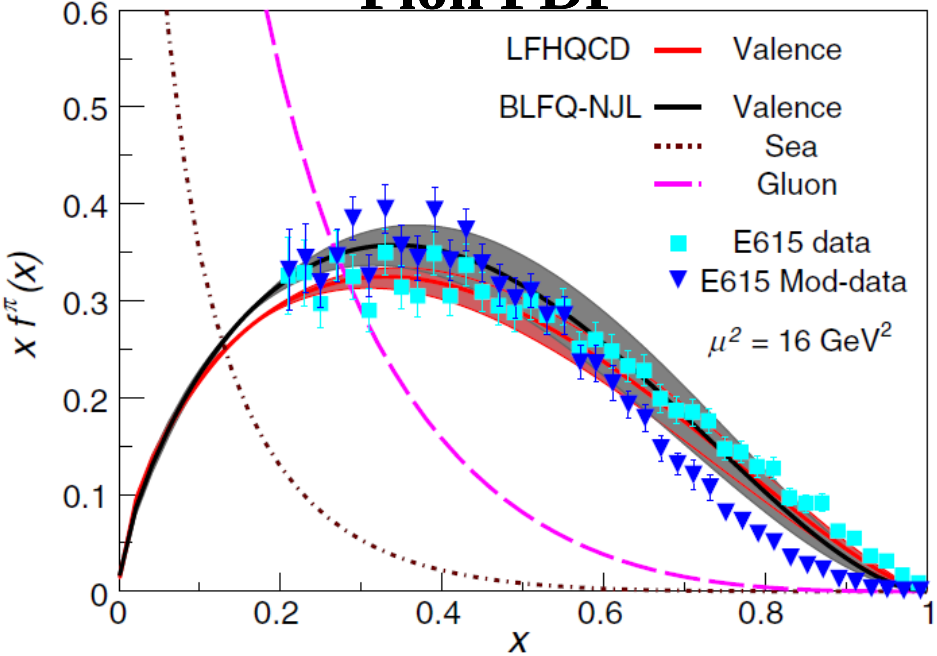
Large N_{max} and K : High UV cutoff & low IR cutoff

PDFs for Light Mesons

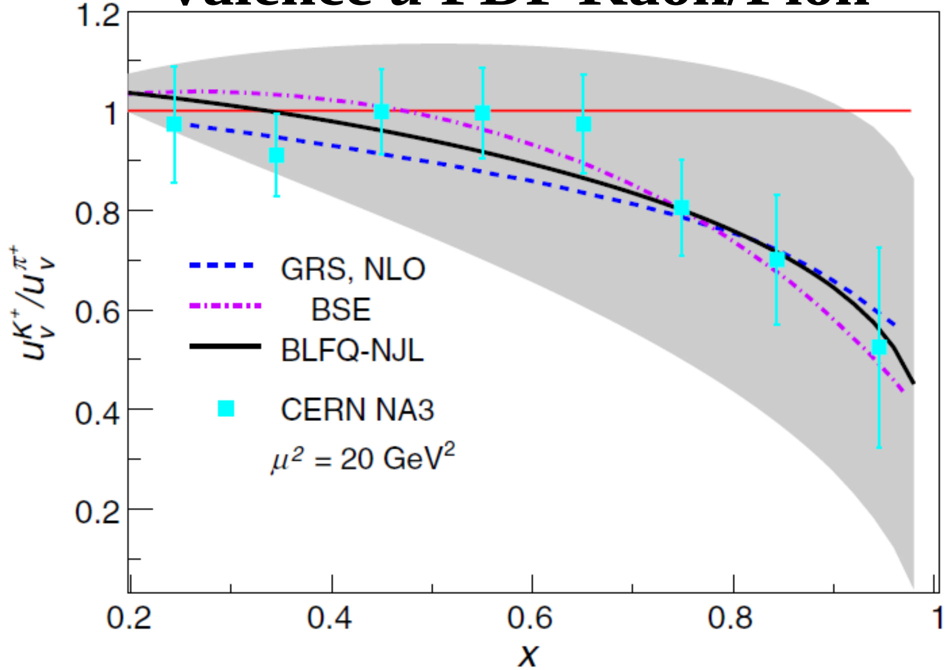
$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$

Diagonalizing H_{eff} → LF wave function → Initial PDFs → Scale evolution

Pion PDF



Valence *u* PDF Kaon/Pion

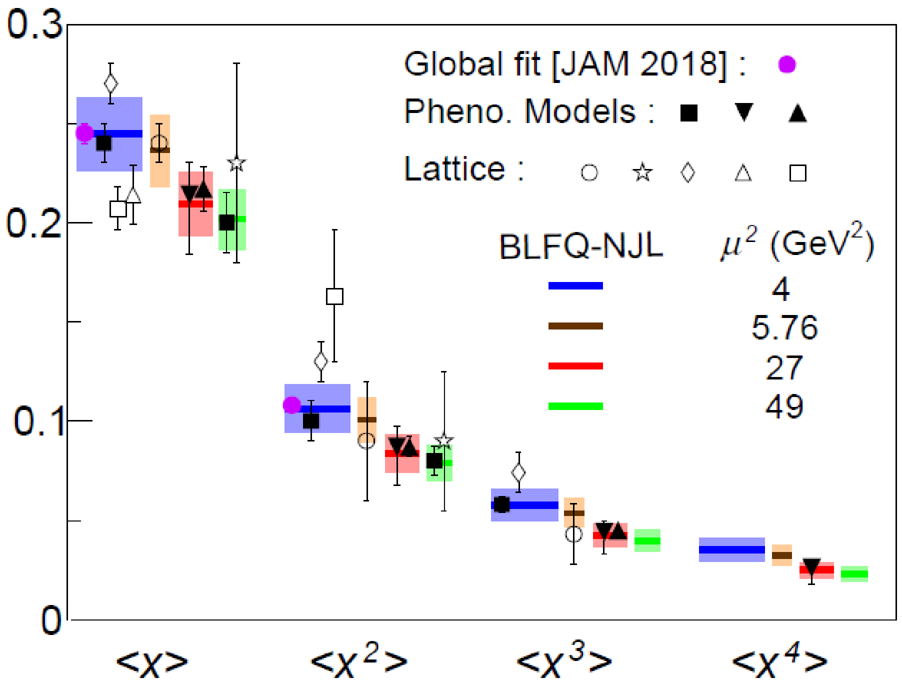


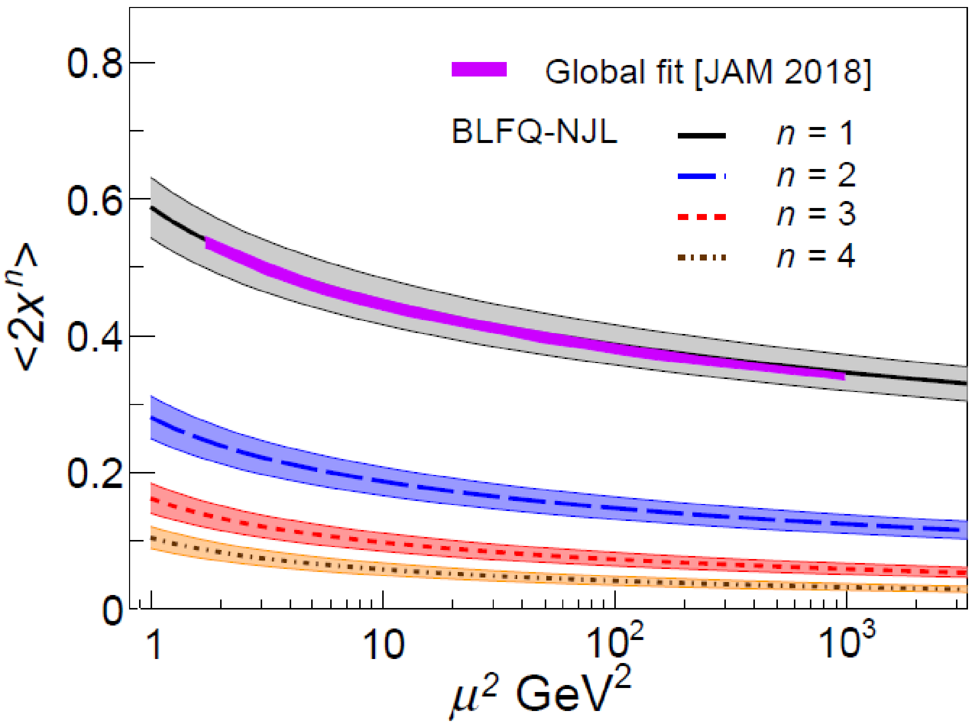
[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

Agree with experimental results

Moments of Pion PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f^{\pi}_{v,s,g}(x, \mu^2), \qquad n = 1, 2, 3, 4.$$





$\langle x \rangle$ @ 4 GeV²	Valence	Gluon	Sea
BLFQ-NJL	**0.489**	**0.398**	**0.113**
[Ding *et. al.*, BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

Agree with other results

[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020), arXiv: 2003.12249]

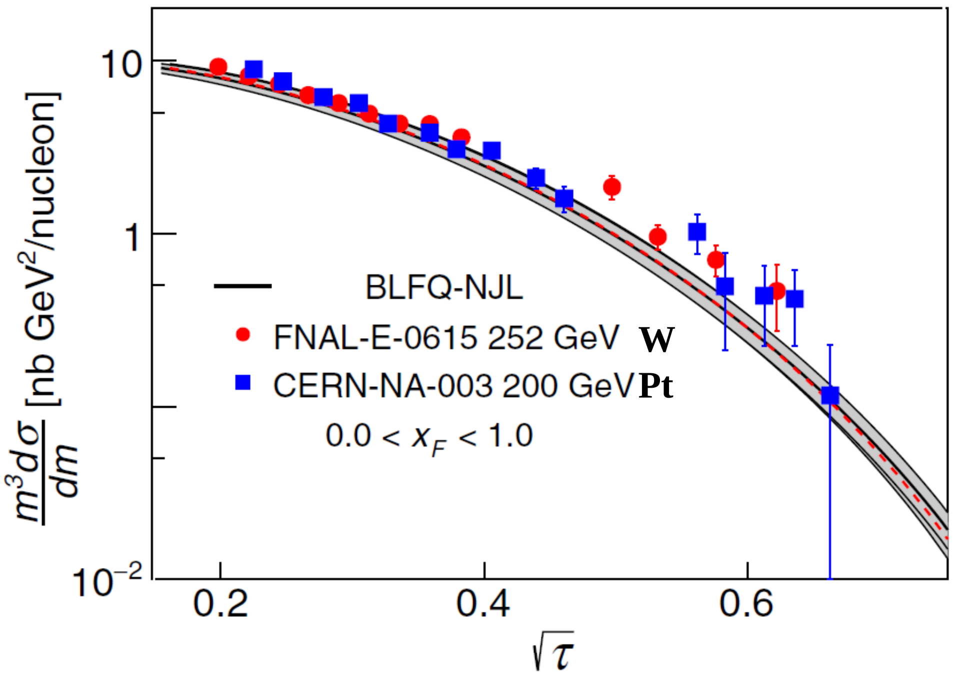
Drell-Yan Cross Section

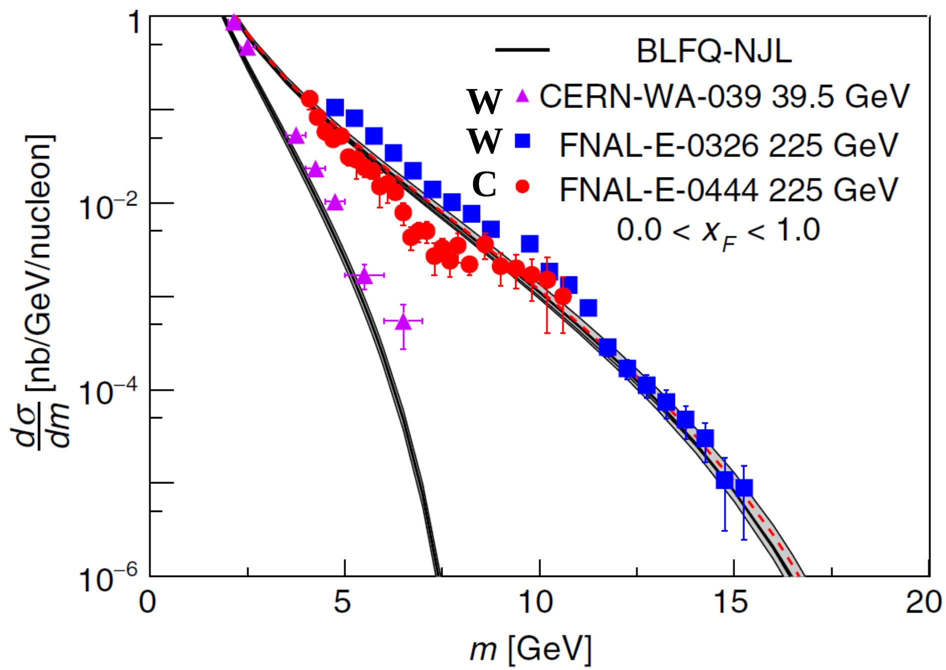
[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

$$\pi^- \text{ Nucleus} \rightarrow \mu^+ \mu^- X$$

[nCTEQ 2015]

$$\frac{m^3 d^2\sigma}{dmdY} = \frac{8\pi\alpha^2}{9}\frac{m^2}{s}\sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$





Agree with experimental data (**FNAL E615**, **326**, **444**, & **CERN NA3**, **WA-039**).

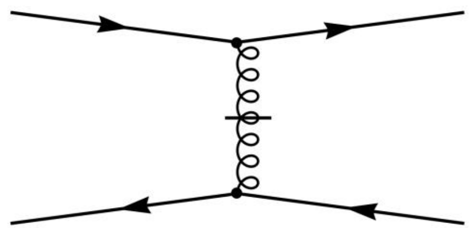
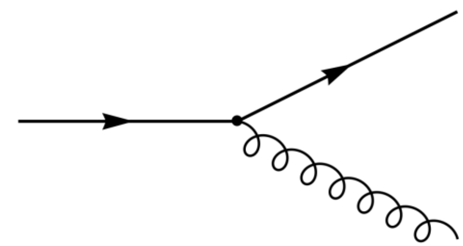
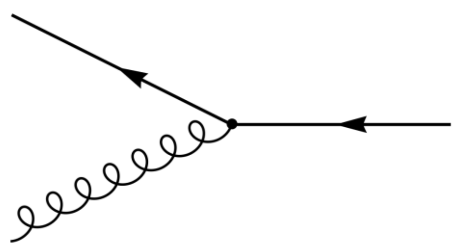
[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$$|\text{meson}\rangle = |q\bar{q}\rangle + \cdots$$

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Interaction Part of Hamiltonian

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

H_{int}	$\lvert q\bar{q}\rangle$	$\lvert q\bar{q}g\rangle$
$\langle q\bar{q}\rvert$		
$\langle q\bar{q}g\rvert$		0

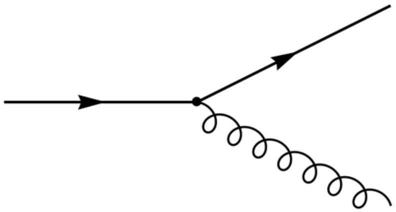
$$P^- = \frac{\overrightarrow{k_\perp^2} + m_q^2}{x} + \frac{\overrightarrow{k_\perp^2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^{\,2} - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right) + H_{\text{int}}$$

Light-front QCD Hamiltonian

[Brodsky et al, 1998]

$$P^-_{LFQCD} = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\,\psi - \frac{1}{2}\int d^3x\, A^i_a (i\partial^\perp)^2 A^i_a$$

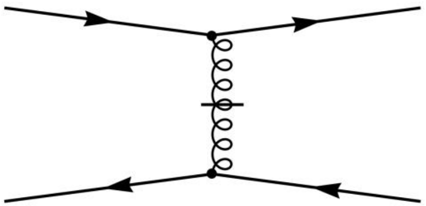
$$+g\int d^3x\, \bar{\psi}\gamma_\mu A^\mu\, \psi$$



$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+}\gamma_\nu A^\nu \psi$$

$$-ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^+ T^c\psi \frac{1}{(i\partial^+)^2}(i\partial^+ A^\mu_a A_{\mu b})$$

$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^+ T^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a\psi$$

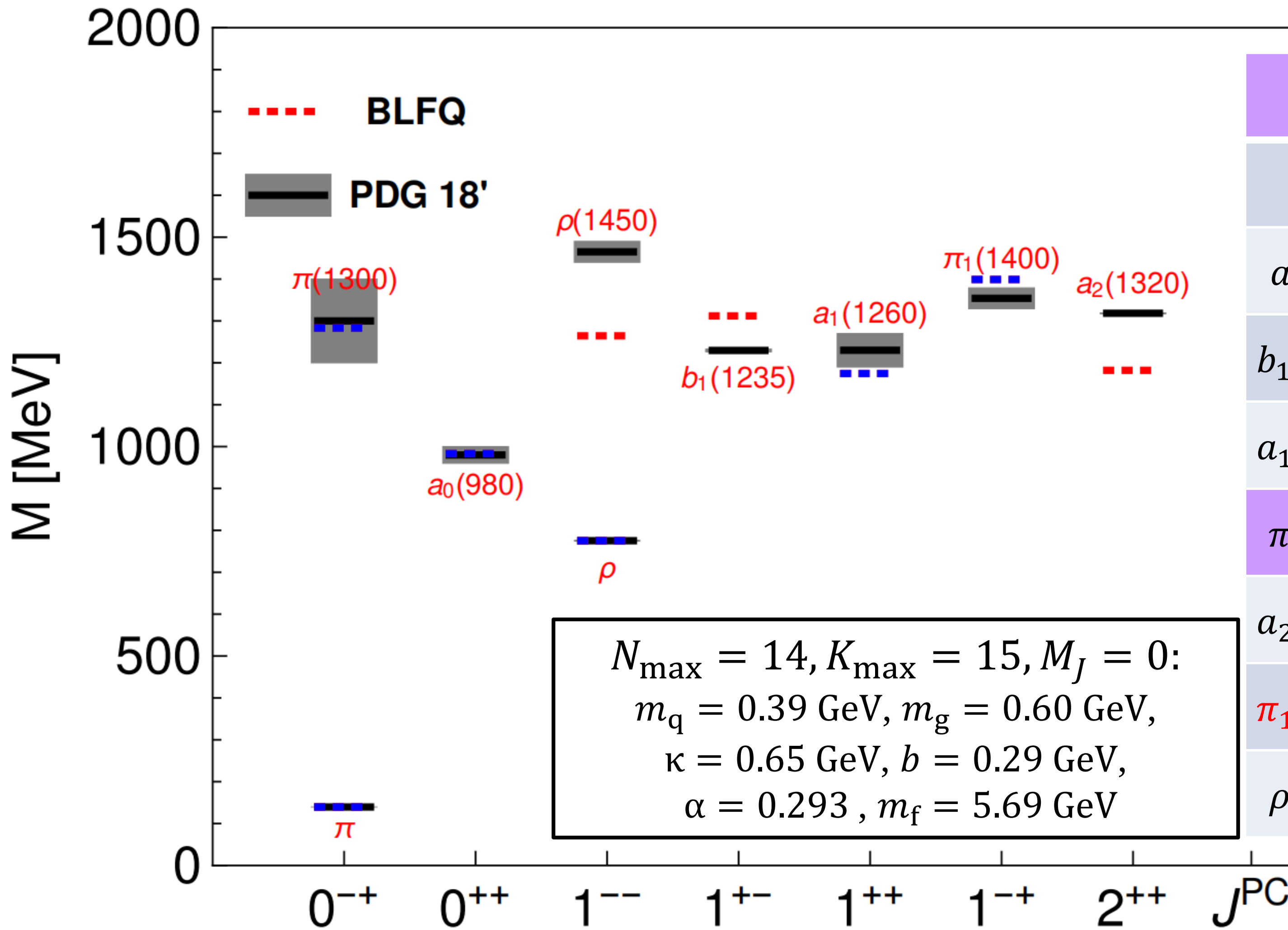


$$+ig\int d^3x\, f^{abc} i\partial^\mu A^{\nu a} A^b_\mu A^c_\nu$$

$$-\frac{1}{2}g^2\int d^3x\, f^{abc}\, f^{ade}\, i\partial^+ A^\mu_b A_{\mu c} \frac{1}{(i\partial^+)^2}(i\partial^+ A^+_d A_{\nu e})$$

$$+\frac{1}{4}g^2\int d^3x\, f^{abc}\, f^{ade}\, A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.$$

Mass Spectrum



	Norm$q\bar{q}$	DC[MeV]
π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

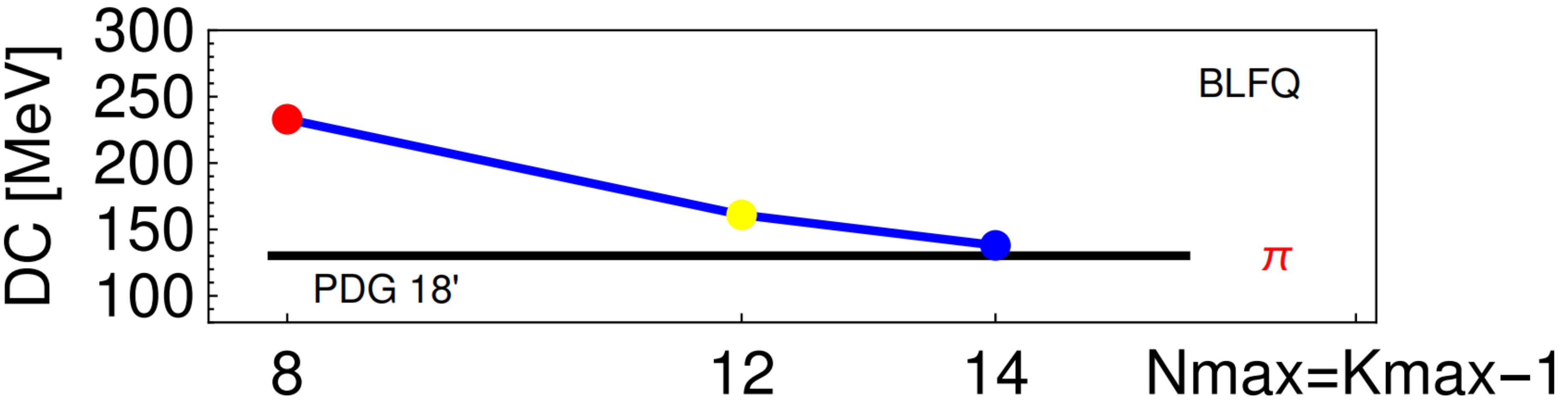
$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion Decay Constant

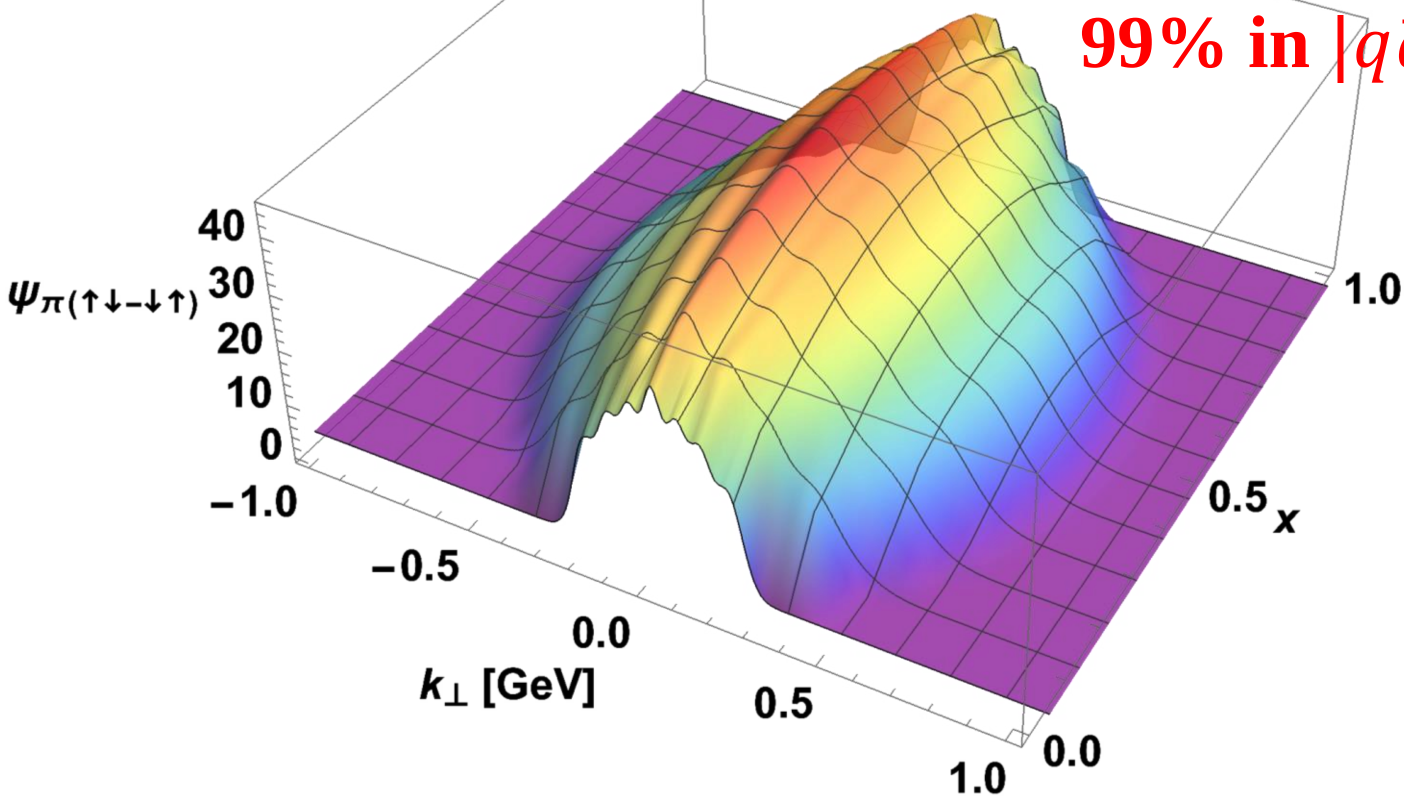


With increasing basis size, the DC approaches the experimental value

Wave Function

$$\int \frac{dx\, d^2k_\perp}{16\pi^3} |\psi_{q\bar{q}}|^2 = 1$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

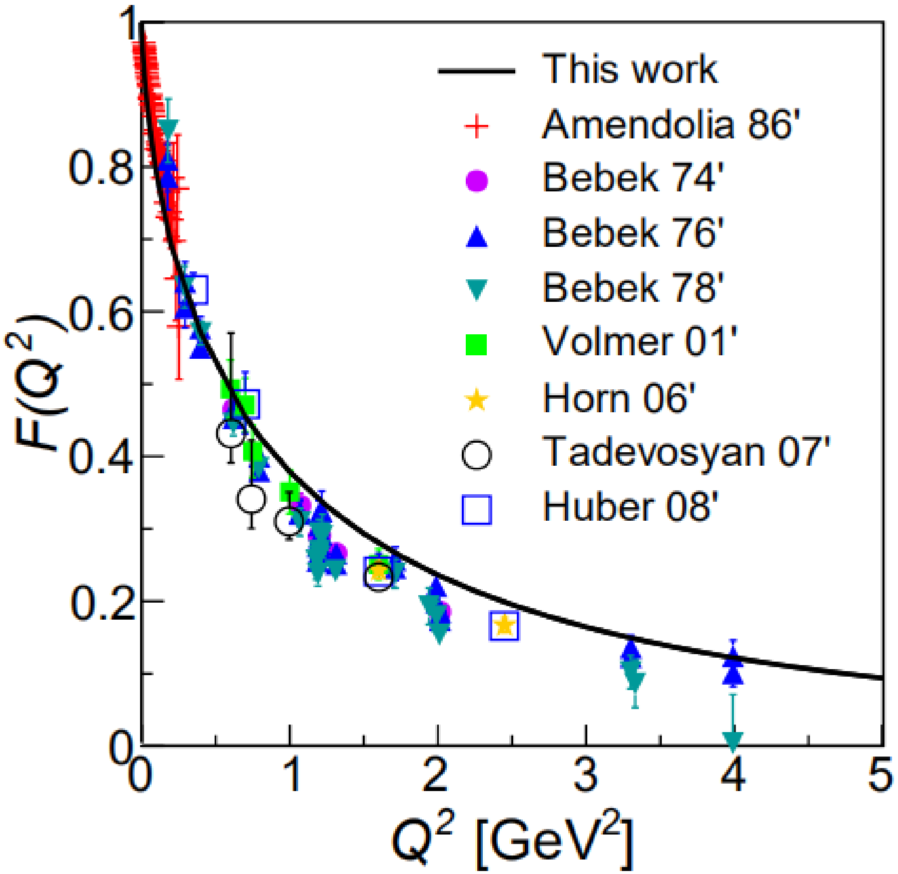


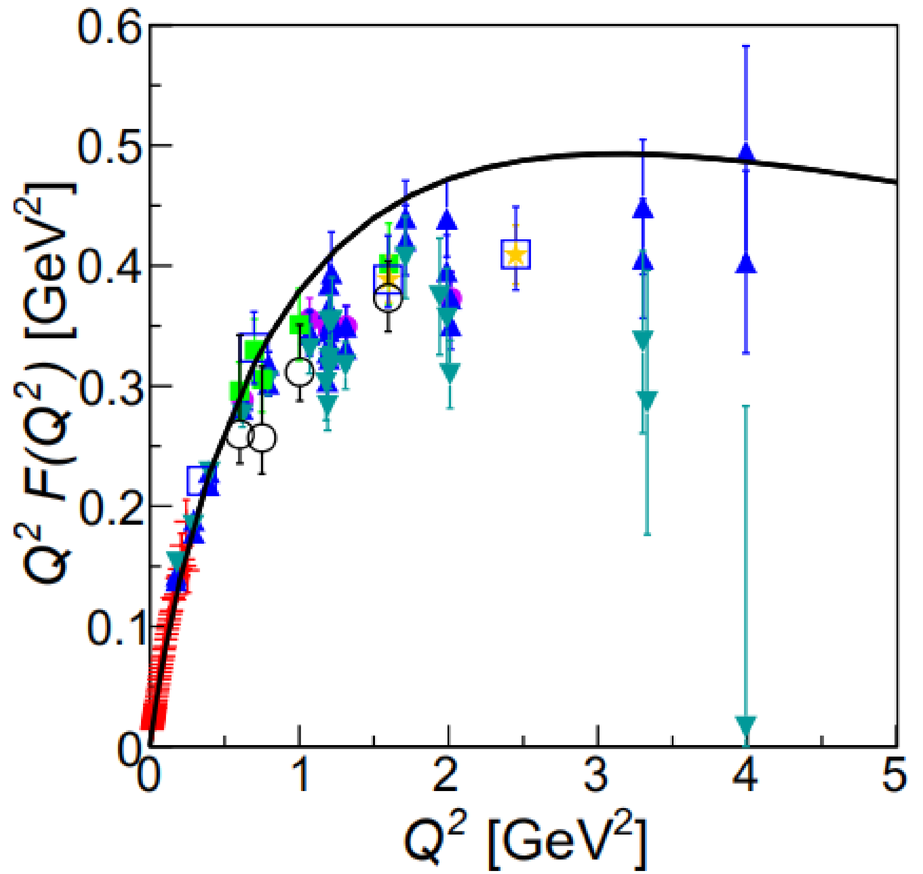
- At endpoint x, $\psi \sim k_\perp$: lightly narrow
- At middle x, $\psi \sim k_\perp$: a little bit wide

Preliminary

Pion Electromagnetic Form Factor

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$





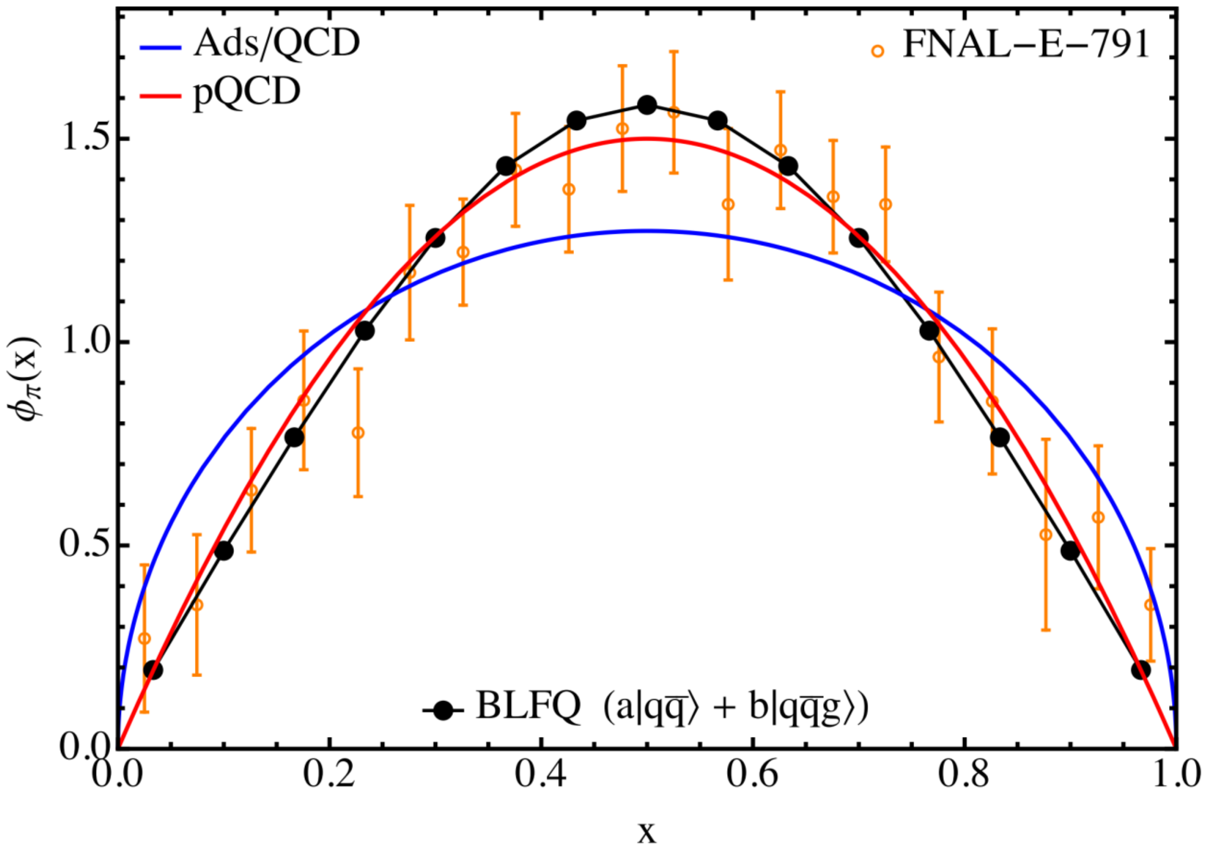
- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/\,Q^2$ for large Q^2

[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion PDA

$$f_\pi \phi_\pi^q(x) = \frac{1}{16\pi^3}\int d^2k_\perp\, \psi(x, k_\perp)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$



- Endpoint behavior almost agrees with pQCD
- Consistent with FNAL-E-791 experiment

Preliminary

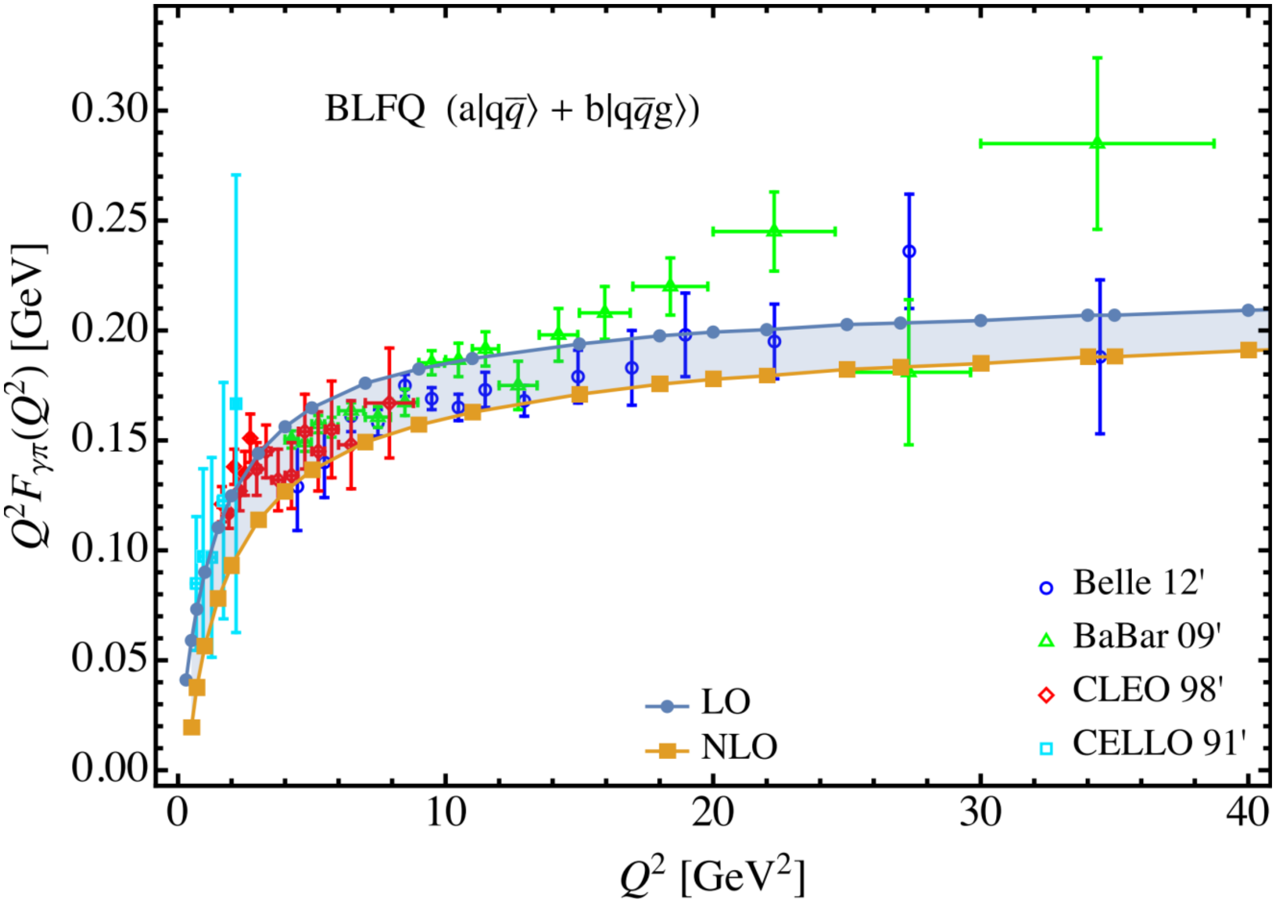
$\pi \to \gamma^*\gamma$ Transition Form Factor

[Brodsky et al, PRD 84, 033001 (2011)]

Preliminary

$$Q^2 F_{\gamma\pi}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 T_H(x,Q)\, dx \int_0^{\bar{x}Q} \frac{d^2\boldsymbol{k}_\perp}{16\pi^3}\ \psi_{q\bar{q}}(x,\boldsymbol{k}_\perp^2)$$

$$T_H(x,Q) = \frac{1}{\bar{x}} + \frac{\alpha_s(\mu_R)}{4\pi}\ C_F \frac{1}{\bar{x}}\left[-9 - \frac{\bar{x}}{x}\ln(\bar{x}) + \ln^2\bar{x} + (3 + 2\ln\bar{x})\ln\left(\frac{Q^2}{\mu_R^2}\right)\right]$$



- Reasonably agrees with experimental results
- Flat at large Q^2

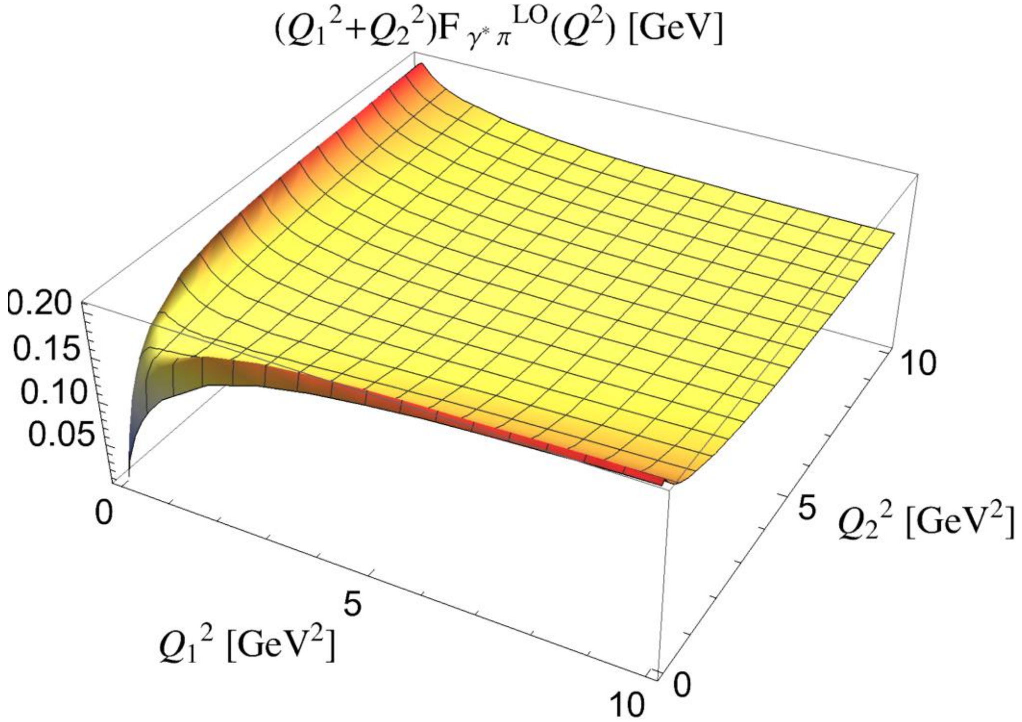
$\pi \to \gamma^* \gamma^*$ Transition Form Factor

[Brodsky et al, PRD 84, 033001 (2011)]
[CM et al, Phys. Rev. D 104, 094034 (2021)]

Preliminary

$$(Q_1^2 + Q_2^2) F_{\gamma^* \pi}(Q_1^2, Q_2^2) = \frac{4}{\sqrt{3}} \int_0^1 \frac{dx}{\bar{x} Q_1^2 + x Q_2^2} \int_0^{\bar{x} Q_1 + x Q_2} \frac{d^2 \boldsymbol{k}_\perp}{16\pi^3} \, \psi_{q\bar{q}}(x, \boldsymbol{k}_\perp^2)$$

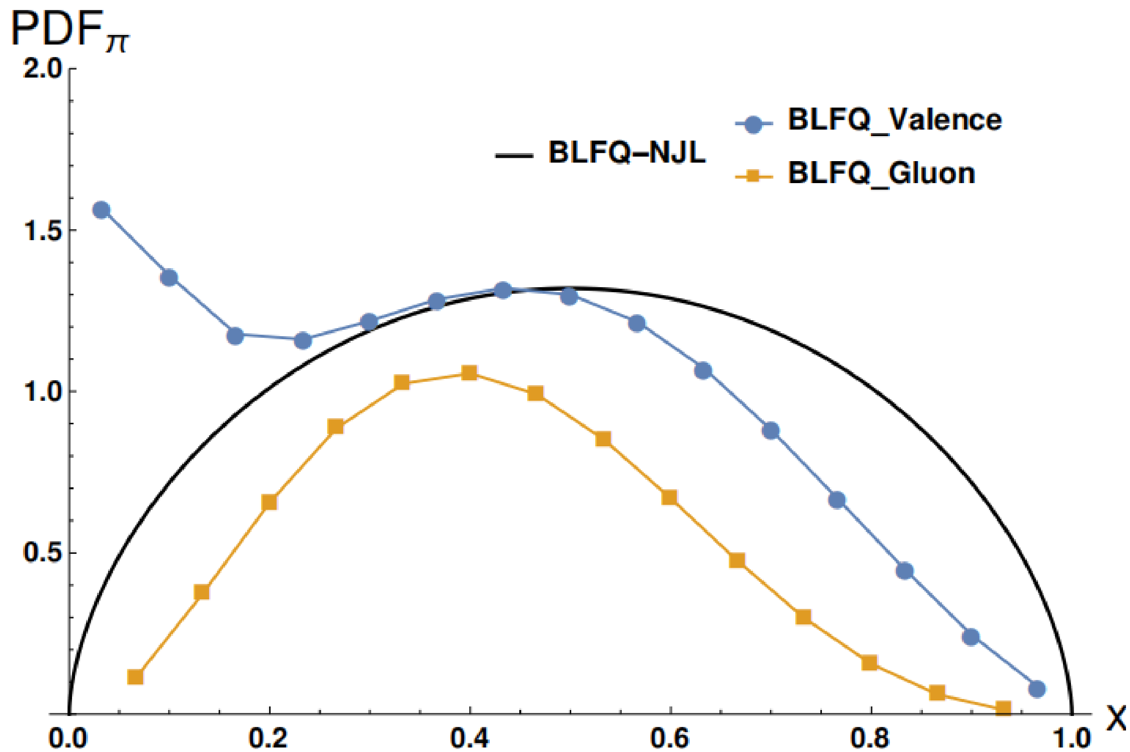
- $F_{\pi\gamma^*}(Q_1^2, Q_2^2) \sim 1/(Q_1^2 + Q_2^2)$ when $(Q_1^2 + Q_2^2) \to \infty$
- Reasonably agrees with pQCD prediction
- Symmetric



Pion initial PDF

$$f_{q,g}(x) = \frac{1}{16\pi^3} \sum_{\mathcal{N},\lambda} \int d^2k_\perp \left|\psi^{\mathcal{N}}(x,k_\perp)\right|^2 \delta(x - x_i)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

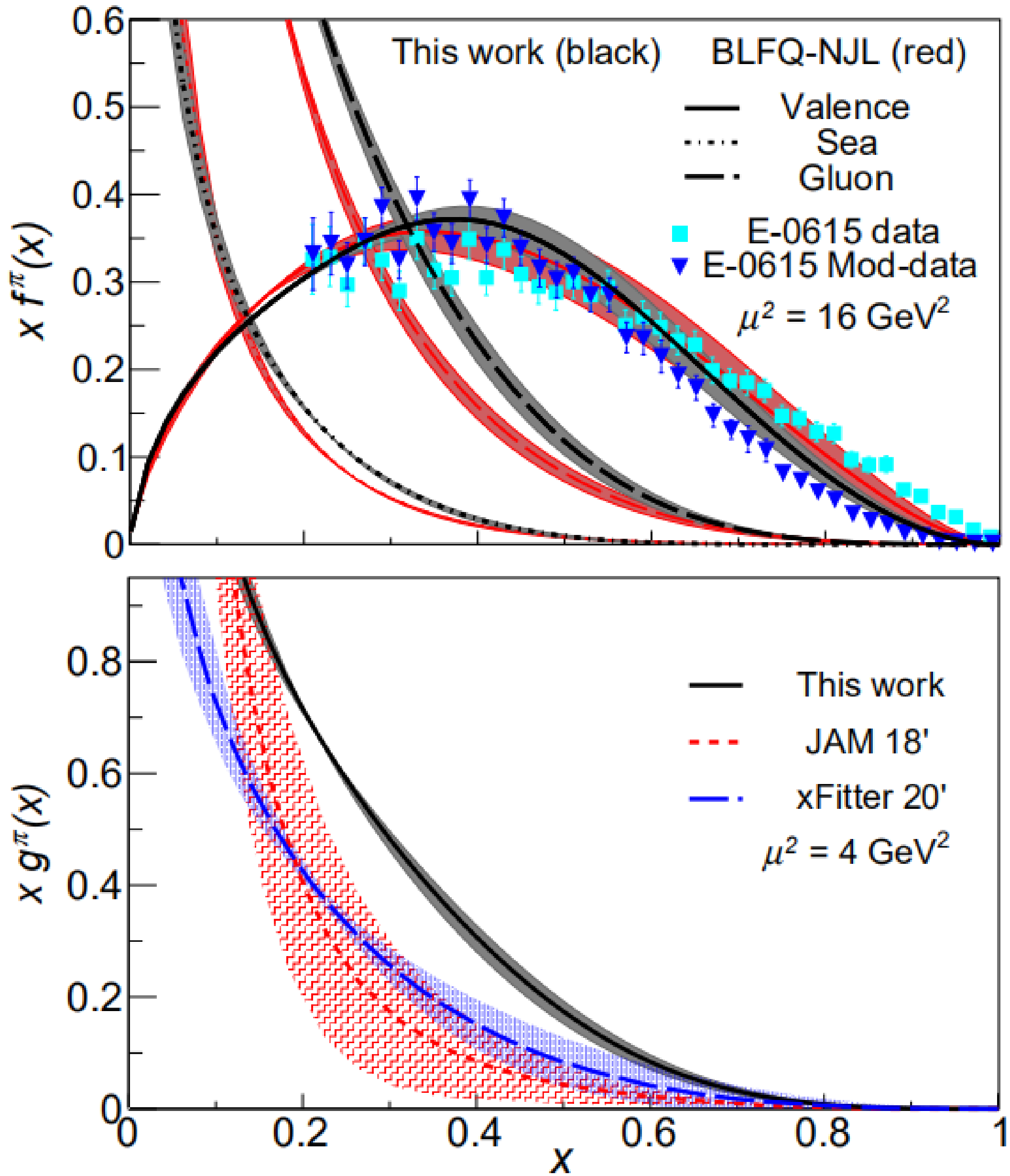


large x

$\mu^2_{0\text{BLFQ}-\text{NJL}} = 0.240\ \text{GeV}^2$	$\langle \boldsymbol{x} \rangle_{\textbf{gluon}} = \mathbf{0}$;	$\langle \boldsymbol{x} \rangle_{\textbf{valence}\ \boldsymbol{u}} = \mathbf{0.5}$	$(1-x)^{0.596}$
$\mu^2_{0\text{BLFQ}} = 0.34\ \text{GeV}^2$	$\langle \boldsymbol{x} \rangle_{\textbf{gluon}} = \mathbf{0.216}$;	$\langle \boldsymbol{x} \rangle_{\textbf{valence}\ \boldsymbol{u}} = \mathbf{0.392}$	$(1-x)^{1.4}$

[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion PDF



$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

- Large-x behavior $(1-x)^{1.77}$ closer to pQCD
- The gluon distribution significantly increases

$\langle x \rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ	**0.483**	**0.421**	**0.096**
BLFQ-NJL	0.489	0.398	0.113
[BSE 2019']	0.48(3)	0.41(2)	0.11(2)

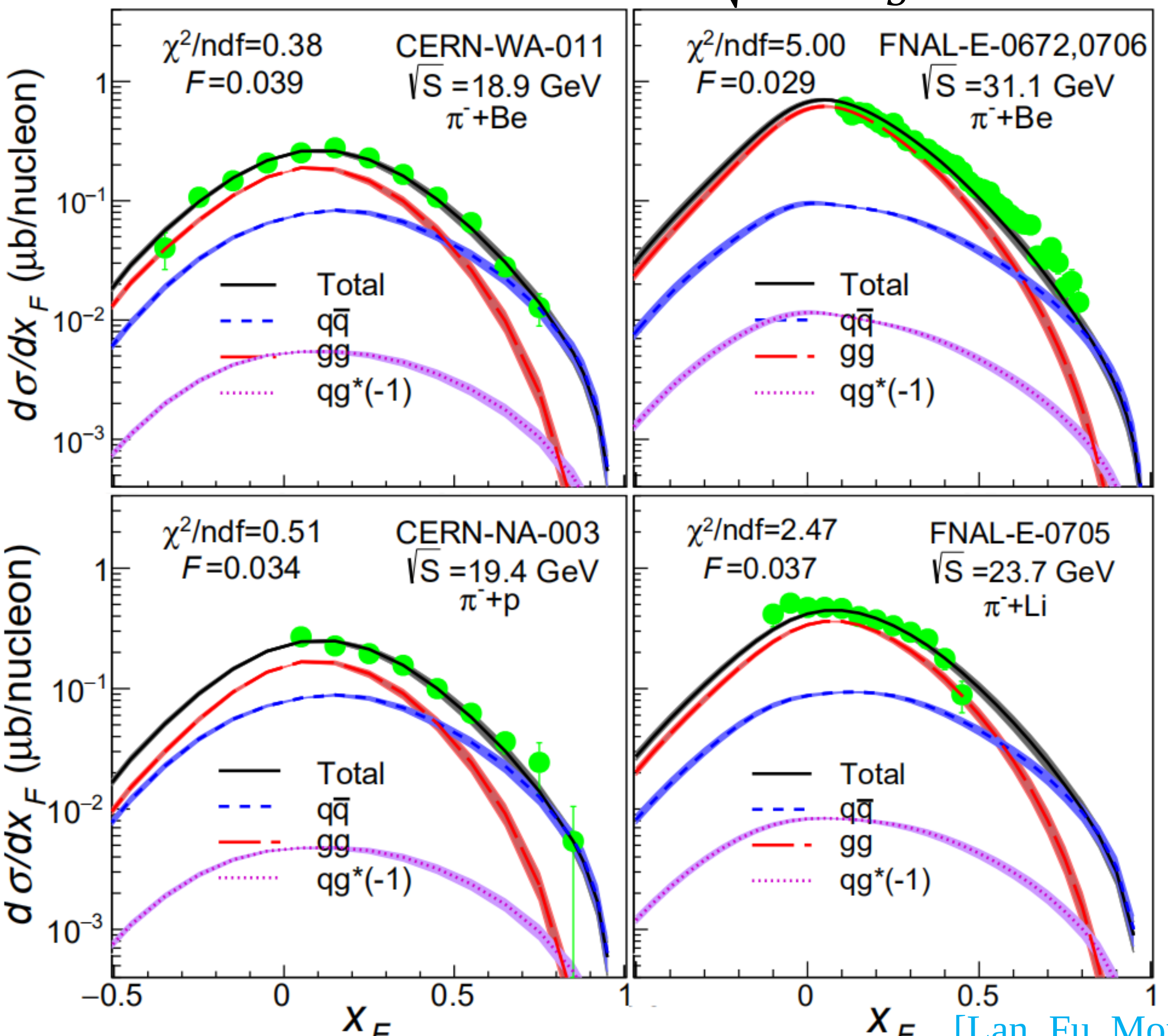
[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

J/ψ production cross section

$\pi^{\pm} N \rightarrow J/\psi\ X$

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^{\pm}}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM

[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

- significantly gg contribution
- various energies of pions
- different target

Agree with experimental data (FNAL E672, E706, E705, CERN NA3,WA11).

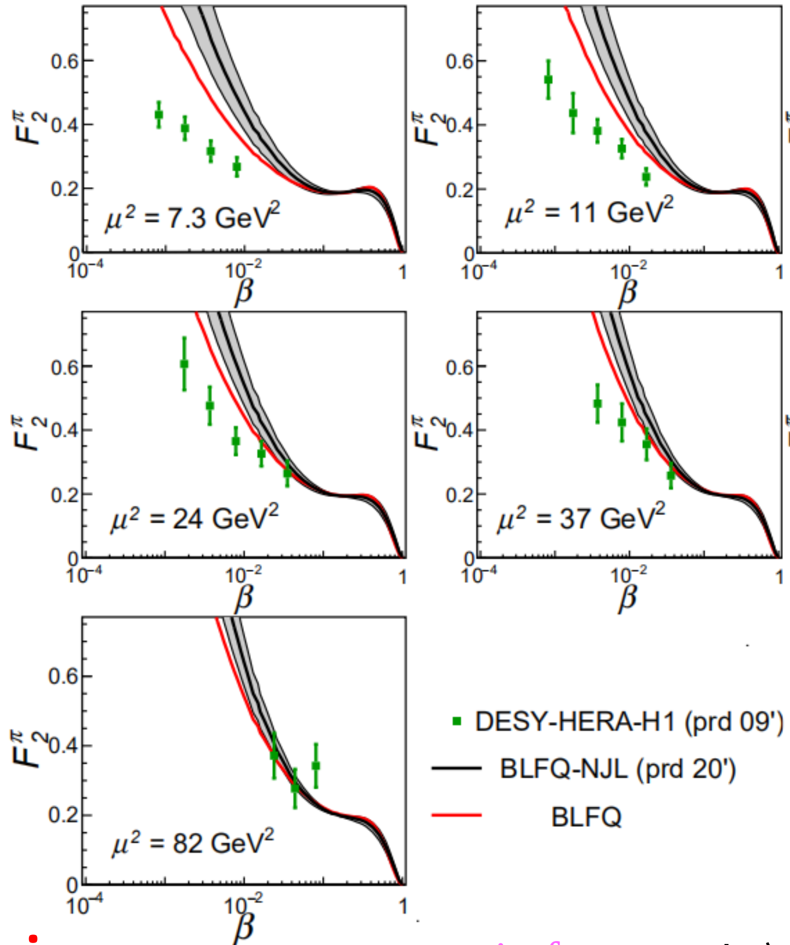
[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion Structure function

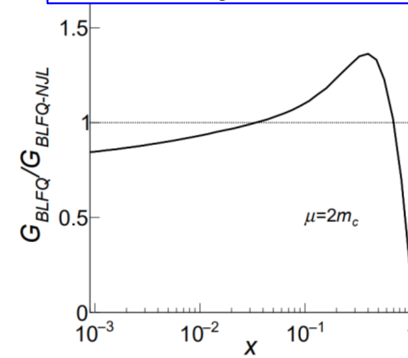
[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

$$F_2^\pi(\beta,\mu^2) = \sum_{q,g} e_q^2 \beta\{f_q^\pi(\beta,\mu^2) + f_{\bar{q}}^\pi(\beta,\mu^2) + \frac{\alpha_s(\mu^2)}{2\pi}[C_{q,2}\otimes(f_q^\pi + f_{\bar{q}}^\pi) + 2C_{g,2}\otimes f_g^\pi]\}$$



$\mu = 2m_c = 3$ GeV



Different BLFQ-NJL results

- Higher initial scale
- less gluon/sea quark at small x

Preliminary

in future $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + \cdots$

Pion GPD

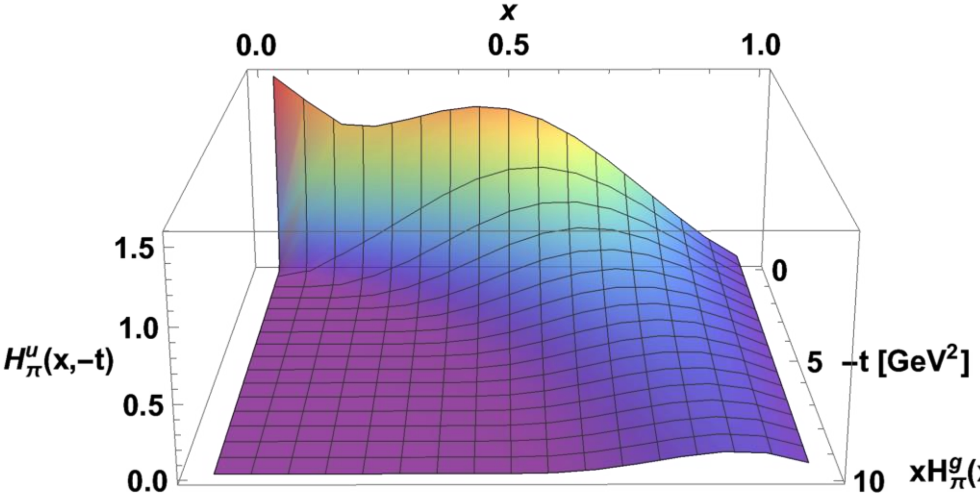
Preliminary

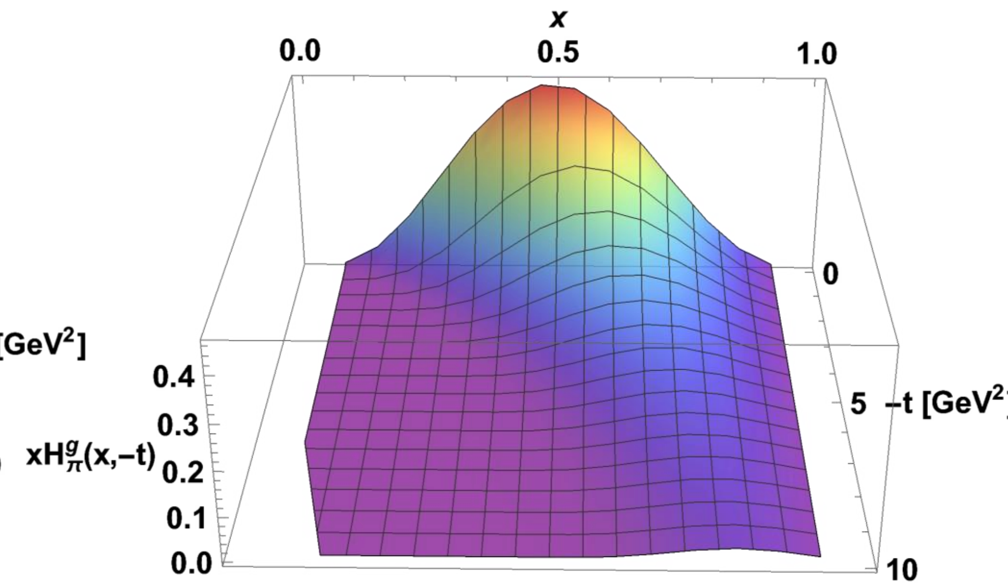
[M. Diehl, Phys. Rep. 388 (2003) 41-277]

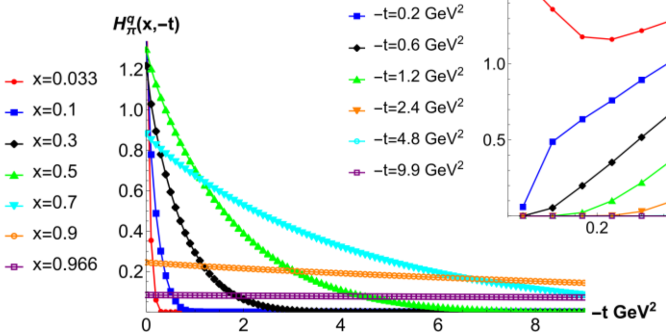
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

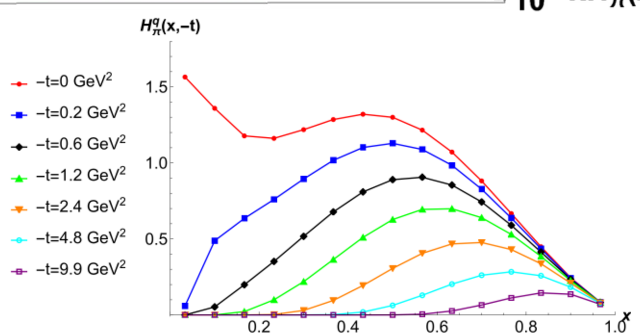
$$H_\pi^q(x,\xi=0,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\left|\pi, P-\frac{\Delta}{2}\right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

$$H_\pi^g(x,\xi=0,t) = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2}\right| G^{+\mu}\left(-\frac{z}{2}\right) G_\mu^+\left(\frac{z}{2}\right)\left|\pi, P-\frac{\Delta}{2}\right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$









- Quark content enhanced at small x with $|q\bar{q}g\rangle$
- Falls slowly at larger x
- Emerge at larger x range for larger $-t$

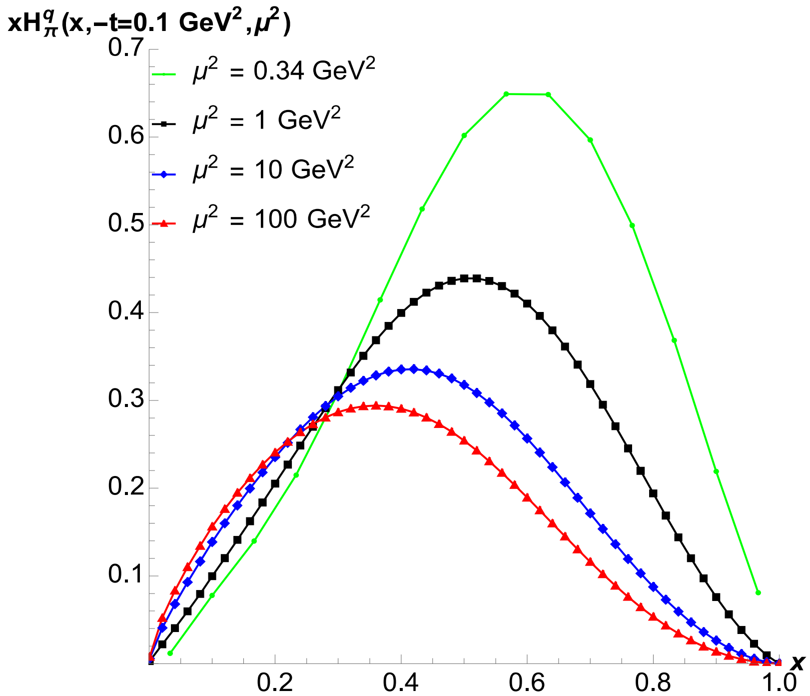
Pion GPD with Scale Evolution

Preliminary

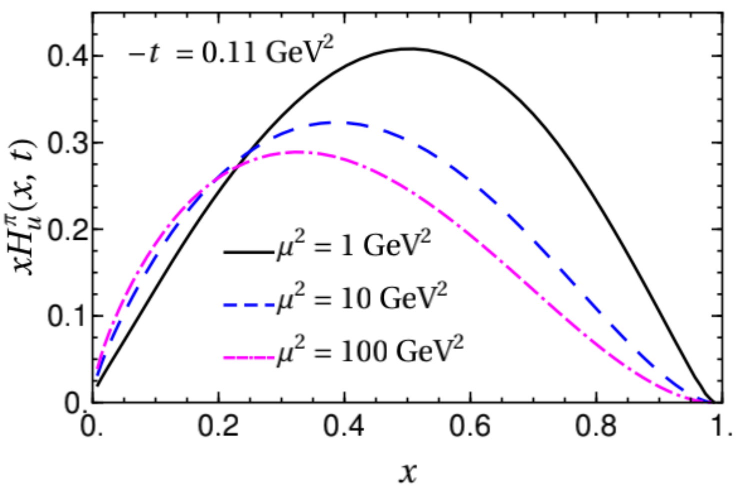
[M. Diehl, Phys. Rep. 388 (2003) 41-277]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$H_\pi^q(x, \xi = 0, t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$



BLFQ-NJL $\mu_0^2 = 0.24\ \mathrm{GeV}^2$



[Adhikari et al, arXiv: 2110.05048]

- Including valence quark only
- Scale evolution performed using DGLAP equation (HOPPET)

Pion Polarized GPD

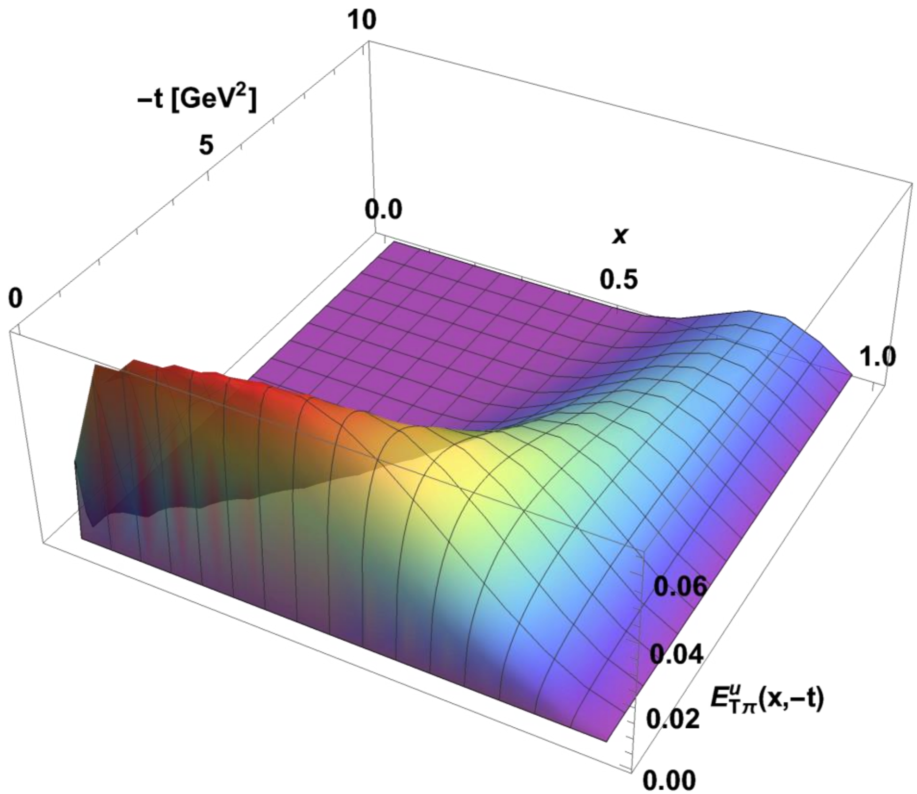
Preliminary

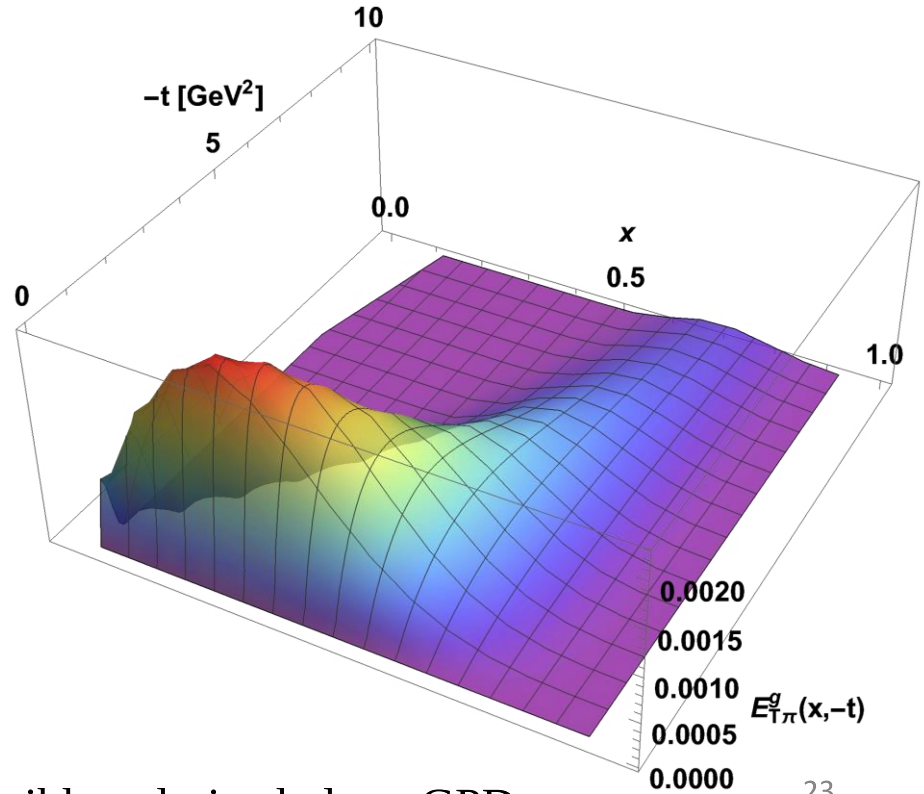
[Adhikari et al, arXiv: 2110.05048]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$E^q_{T\pi}(x,\xi=0,t) \sim \frac{1}{2}\sum_s \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}\left[(-i)^j \psi^*_{\uparrow s}(x,\vec{k}'_\perp)\psi_{\downarrow s}(x,\vec{k}_\perp) + (i)^j \psi^*_{\downarrow s}(x,\vec{k}'_\perp)\psi_{\uparrow s}(x,\vec{k}_\perp)\right]$$

$$E^g_{T\pi}(x,\xi=0,t) \sim \frac{1}{2}\sum_s \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}\left[(-i)^j \psi^*_{\uparrow s}(x,\vec{k}'_\perp)\psi_{\downarrow s}(x,\vec{k}_\perp) + (i)^j \psi^*_{\downarrow s}(x,\vec{k}'_\perp)\psi_{\uparrow s}(x,\vec{k}_\perp)\right]$$





- accessible polarized gluon GPD

Pion TMD

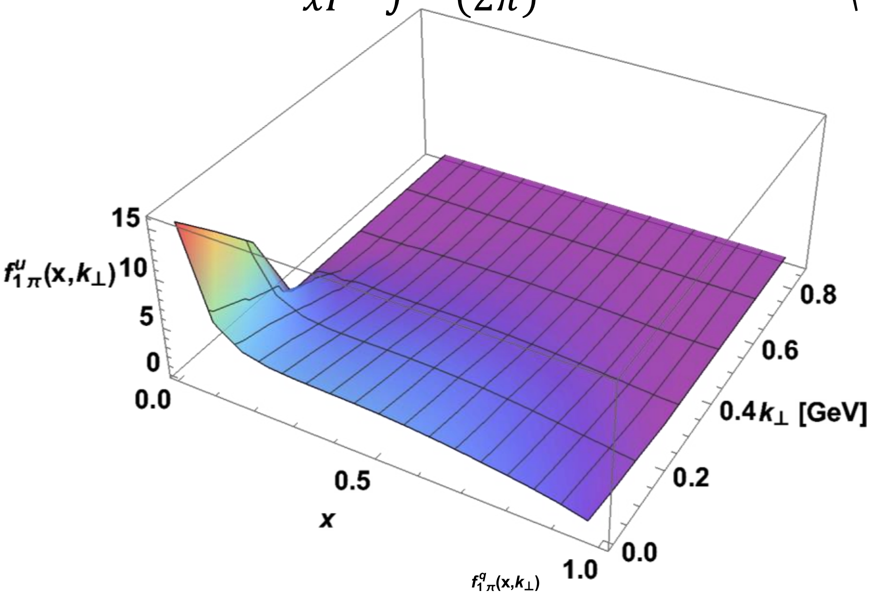
Preliminary

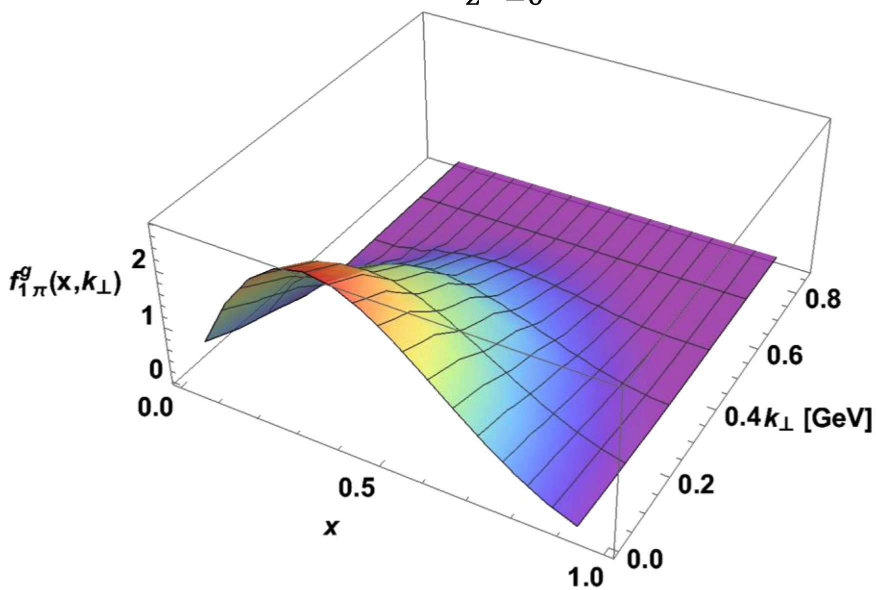
[Boer & Mulders PRD 57 (1998) 5780]
[Pasquini et al, PRD 90 (2014) 014050]

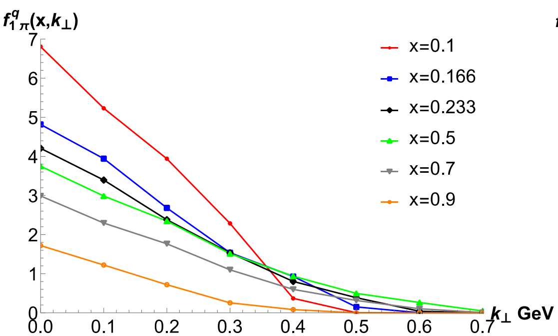
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

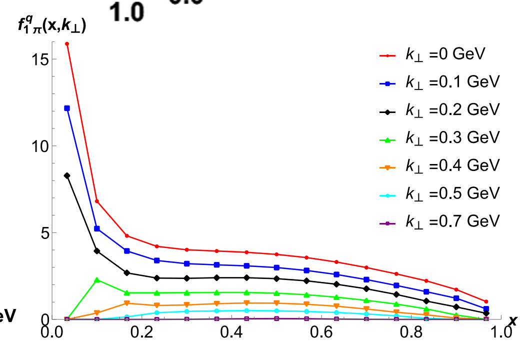
$$f_1^q(x, k_\perp) = \frac{1}{2}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(z^- k^+ - z_\perp k_\perp)} \left\langle \pi, P \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P \right\rangle_{z^+=0}$$

$$f_1^g(x, k_\perp) = \frac{1}{xP^+}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(z^- k^+ - z_\perp k_\perp)} \left\langle \pi, P \middle| G^{+\mu}\left(-\frac{z}{2}\right) G_\mu^+\left(\frac{z}{2}\right) \middle| \pi, P \right\rangle_{z^+=0}$$









- The TMD decreases with $k_\perp$
- Vanishes after $k_\perp \sim 0.7$ GeV

Conclusion & Outlook

- Light-front Hamiltonian approach: mass spectrum ⟺ structure
- Compared to NJL interaction, dynamical gluon in light meson:
  - ✓ Explains the properties of excited/exotic states such as $\pi(1300)$, $\pi_1(1400)$
  - ✓ Describes EMFF, $F(Q^2) \propto 1/Q^2$ for large Q^2
  - ✓ Improves endpoint behavior in PDF/PDA
  - ✓ Reasonable TFF
  - ✓ Generates more gluon at moderate x/less gluon at small x
  - ✓ Improves agreement on J/ψ production cross section with experimental data
  - ✓ Improves π structure function at small x
- Preliminary results on (polarized) **gluon** GPDs and TMDs of light mesons

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- Systematically expandable by including higher Fock sectors

$$|\text{Meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + \cdots$$

Thank you !

4A: Siqi Xu "Nucleon structure with dynamic gluon"