Basis Light Front Quantization
Progress and Prospects

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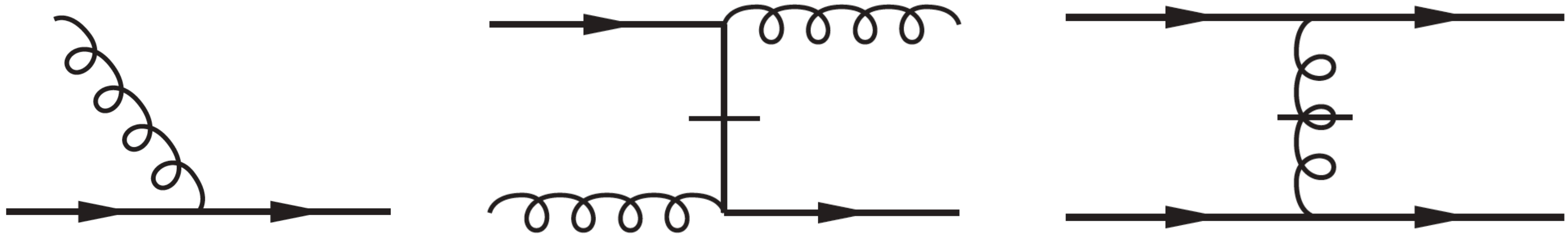
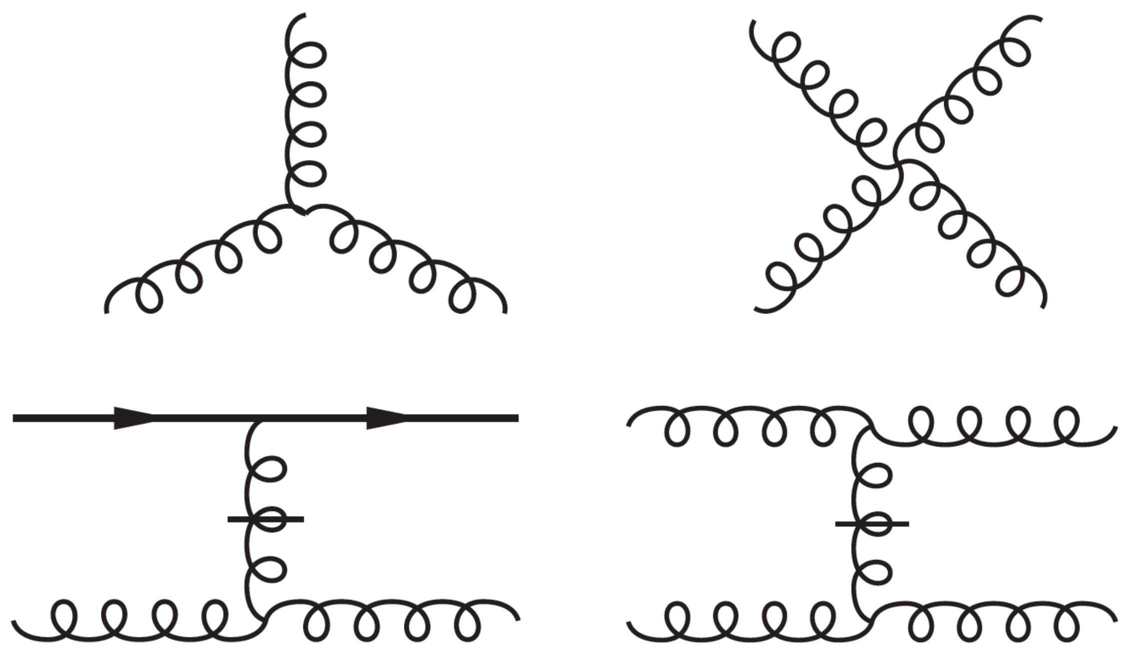
Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



QED & QCD



QCD

Discretized Light Cone Quantization

[H.C. Pauli & S.J. Brodsky, PRD32 (1985)]

⇩

Basis Light Front Quantization

[J.P. Vary, et al., PRC81 (2010)]

$$\phi\left(\vec{k}_\perp, x\right) = \sum_\alpha \left[f_\alpha\left(\vec{k}_\perp, x\right) a_\alpha + f_\alpha^*\left(\vec{k}_\perp, x\right) a_\alpha^\dagger \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $$\int f_\alpha\left(\vec{k}_\perp, x\right) f_{\alpha'}^*\left(\vec{k}_\perp, x\right) \frac{d^2 k_\perp dx}{(2\pi)^3 2x(1-x)} = \delta_{\alpha\alpha'}$$

Complete: $$\sum_\alpha f_\alpha\left(\vec{k}_\perp, x\right) f_\alpha^*\left(\vec{k}'_\perp, x'\right) = 16\pi^3 \sqrt{x(1-x)}\,\delta^2\left(\vec{k}_\perp - \vec{k}'_\perp\right)\delta\left(x - x'\right)$$

For mesons we adopt (later extended to baryons): [Y. Li, et al., PLB758 (2016)]

$$f_{\alpha=\{nml\}}\left(\vec{k}_\perp, x\right) = \phi_{nm}\left(\vec{k}_\perp / \sqrt{x(1-x)}\right)\chi_l(x)$$

ϕ_{nm} 2D-HO functions as in AdS/QCD

χ_l Jacobi polynomials times $x^a(1-x)^b$

BLFQ
Symmetries & Constraints

Baryon number $\sum_i b_i = B$

Charge $\sum_i q_i = Q$

Angular momentum projection (M-scheme) $\sum_i (m_i + s_i) = J_z$

Longitudinal momentum (Bjorken sum rule) $\sum_i x_i = \sum_i \frac{k_i}{K} = 1$

Longitudinal mode regulator (Jacobi) $\sum_i l_i \leq L$

Transverse mode regulator (2D HO) $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$

"Internal coordinates" $\vec{k}_{i\perp} = \vec{p}_{i\perp} - x_i \vec{P}_\perp \quad \Rightarrow \quad \sum_i \vec{k}_{i\perp} = 0$

$H \rightarrow H + \lambda H_{CM}$

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

All $J \geq J_z$ states in one calculation

Finite basis regulators

Preserve transverse boost invariance

Light-Front Wavefunctions (LFWFs)

$$|\psi_h(P,j,\lambda)\rangle = \sum_n \int [\mathrm{d}\mu_n]\, \psi_{n/h}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n)|\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

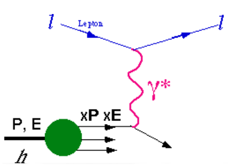
LFWFs are *frame-independent* (boost invariant) and depend only on the relative variables: $x_i \equiv p_i^+/P^+, \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i\vec{P}_\perp$

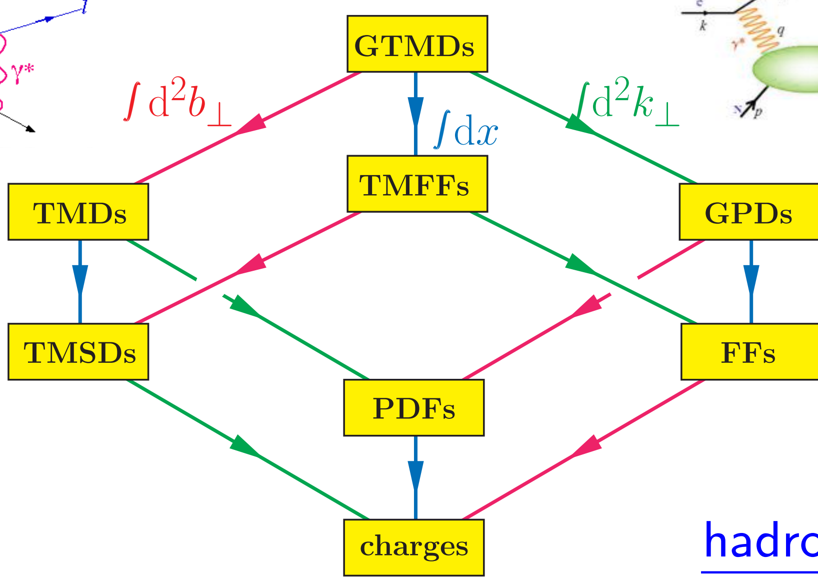
LFWFs provide intrinsic information of the structure of hadrons, and are indispensable for exclusive processes in DIS [Lepage '80]

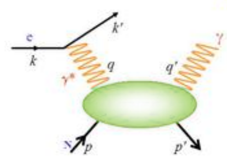
- Overlap of LFWFs: structure functions (e.g. PDFs), form factors, ...
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

"Hadron Physics without LFWFs is like Biology without DNA!"

— Stanley J. Brodsky

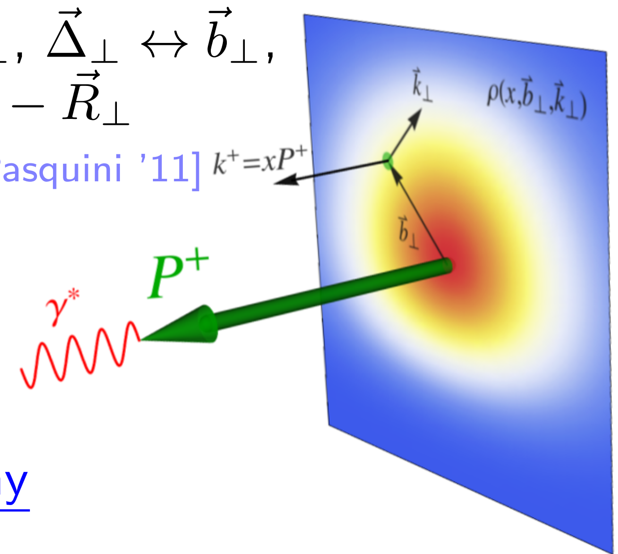






$\vec{k}_\perp \leftrightarrow \vec{r}_\perp,\ \vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp,$
$\vec{b}_\perp = \vec{r}_\perp - \vec{R}_\perp$

[Lorcé & Pasquini '11]



hadron tomography

Light-Front Regularization and Renormalization Schemes

1. Regulators in BLFQ (N_{max}, L or K)
2. Additional Fock space truncations (if any)
3. Counterterms identified/tested*
4. Sector-dependent renormalization**
5. RGPEP***
6. SRG & OLS in NCSM**** - adapted to BLFQ
   as a non-perturbative EFT approach (future)

* D. Chakrabarti, A. Harindranath and J.P. Vary, Phys. Rev. D **69**, 034502 (2004)
* P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

** R.J. Perry, A. Harindranath and K. G. Wilson, Phys. Rev. Lett. **65**, 2959 (1990)
** V. A. Karmanov, J.-F. Mathiot, and A. V. Smirnov,
Phys. Rev. D **77**, 085028 (2008); Phys. Rev. D **86**, 085006 (2012)
** Y. Li, V.A. Karmanov, P. Maris and J.P. Vary, Phys. Letts. B. 748, **278** (2015)
** **X. Zhao, plenary talk on Tuesday at 11:50**

*** **S. Glazek, plenary talk today at 14:30**; S. Glazek, Phys. Rev. D 103, 014021 (2021)

**** B.R. Barrett, P. Navratil and J.P. Vary, Prog. Part. Nucl. Phys. **69**, 131 (2013)

[Kaiyu Fu et al, in preparation]

Positronium Spectrum

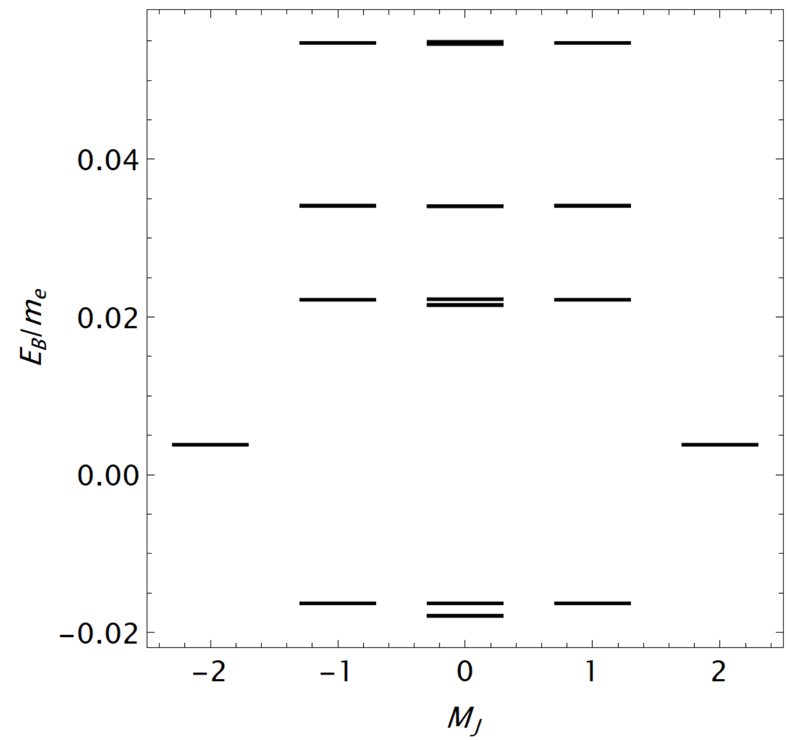
Fock space expansion:

$$|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle$$

$$E_B = M_P - 2M_e$$

- E_B: binding energy of positronium
- M_P: Invariant mass of positronium
- M_e: Invariant mass of free electron

$N_{\max} = 8, K = 9, \alpha = 0.3$



lowest 8 states of Mj=0 : parity and charge conjugation parity agree with hydrogen atom.

The value of Nmax, K and basis scale b is chosen by restoring the rotational symmetry of 1^3S_1 and maximizing the probanm of the ground state (which indicating the parameter set is the best match for the ground state scale).

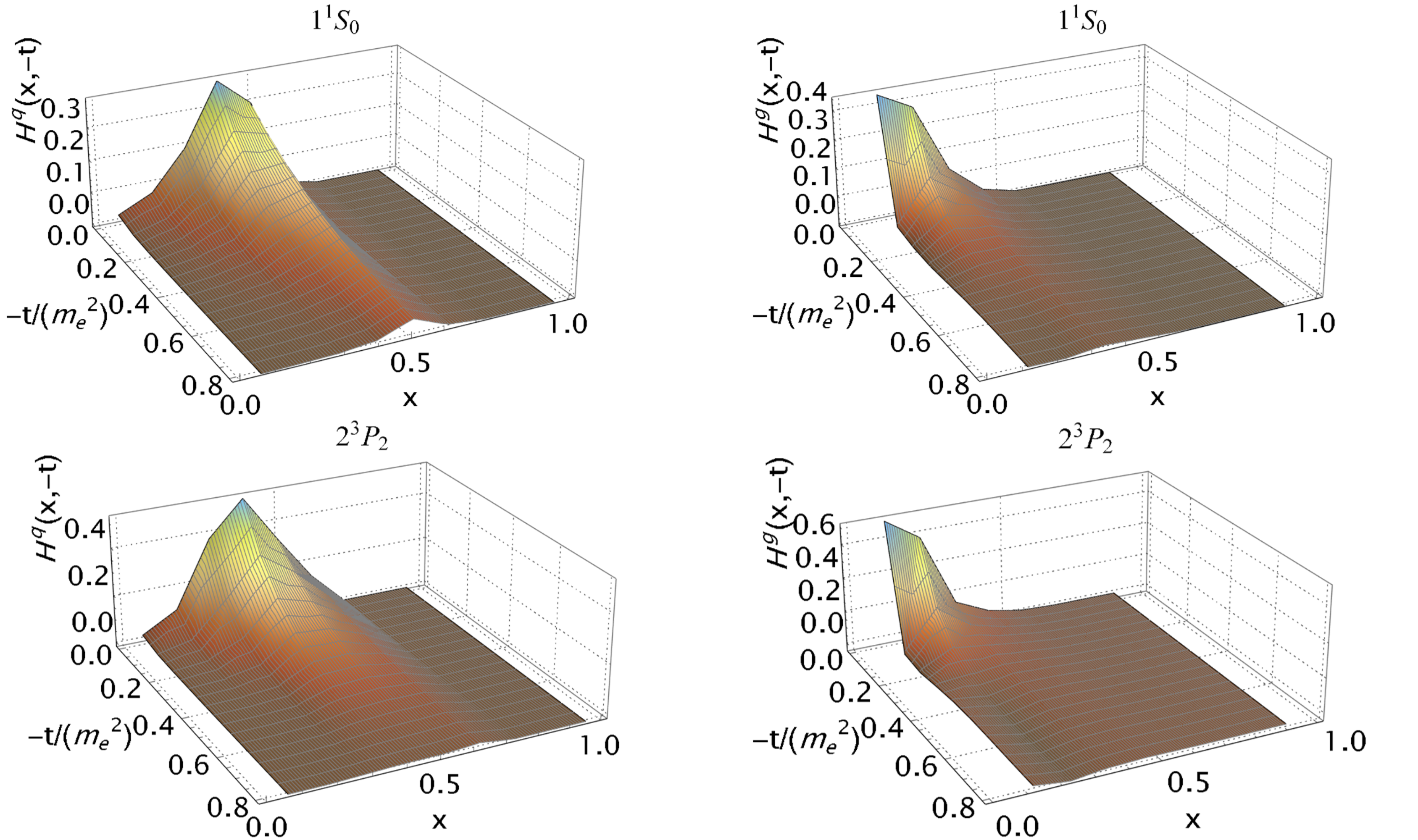
Xingbo Zhao, Plenary Session 6, Tuesday, Nov. 30 at 11:50

Positronium GPD

$N_{\max} = 8, K = 9, \alpha = 0.3$

For electron

For photon



Xingbo Zhao, Plenary Session 6, Tuesday, Nov. 30 at 11:50

Overview of BLFQ/tBLFQ applications to mesons and baryons at LC2021

Common features

Transverse confinement from 2D HO (in common with LF Holography)
Longitudinal confinement (Y. Li, et al, PLB 2016, PRD 2017)
Basis states from exact solutions of this reference Hamiltonian
Compare results with experiment, lattice, DSE/BSE, . . .

Distinct features

For Veff
1) perturbative one-gluon exchange (Y. Li, et al, PLB 2016, PRD 2017)
2) NJL model for light meson applications (S. Jia, et al, PRC 2019)

For Fock space truncation
1) Valence sector
2) Valence sector plus dynamical gluon

For observables
1) Single state properties and decays
2) Transitions between states
3) Non-perturbative probes (tBLFQ)

Complementary Methods

BLFQ on Quantum Computers

Heavy Quarkonia [Y.Li,PLB758,2016; PRD96,2017]

- Effective Hamiltonian in the $q\bar{q}$ sector

$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$

where $x = p_q^+ / P^+,\ \vec{k}_\perp = \vec{k}_{q\perp} = \vec{p}_{q\perp} - x\vec{P}_\perp = -\vec{k}_{\bar{q}\perp} = -\left(\vec{p}_{\bar{q}\perp} - (1-x)\vec{P}_\perp\right),\ \vec{r}_\perp = \vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp}.$

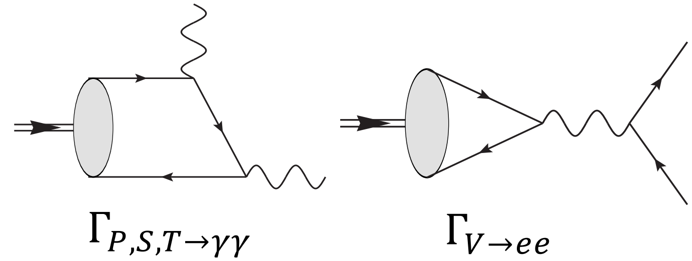
  - Confinement
    transverse holographic confinement [S.J.Brodsky,PR584,2015]
    longitudinal confinement [Y.Li,PLB758,2016]
  - One-gluon exchange with running coupling
    $V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$

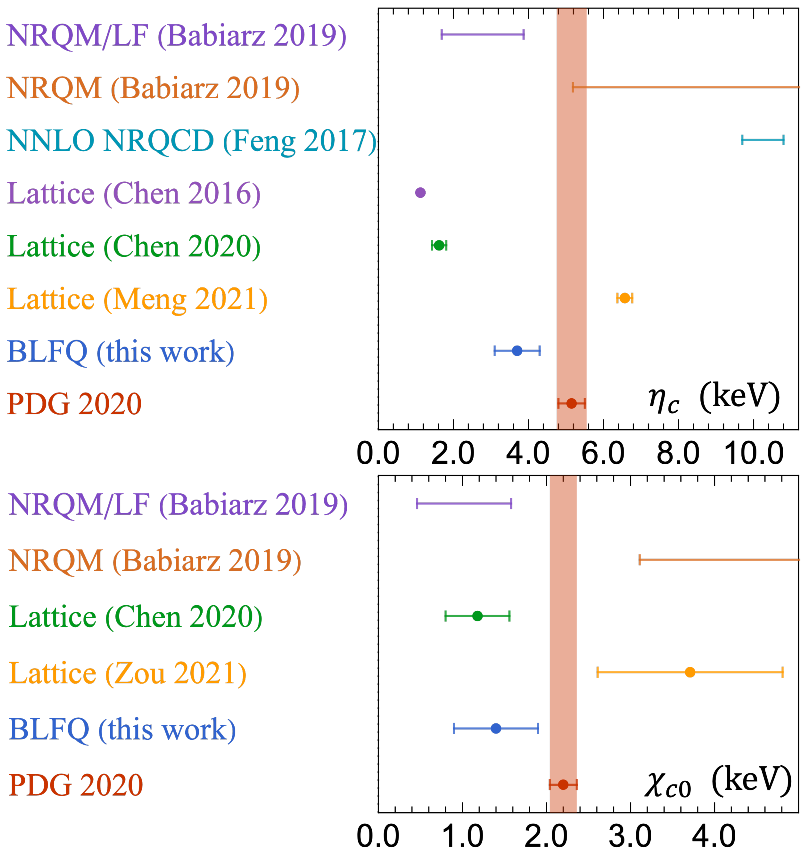
- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of H_0 (LF kinetic energy+ confinement)

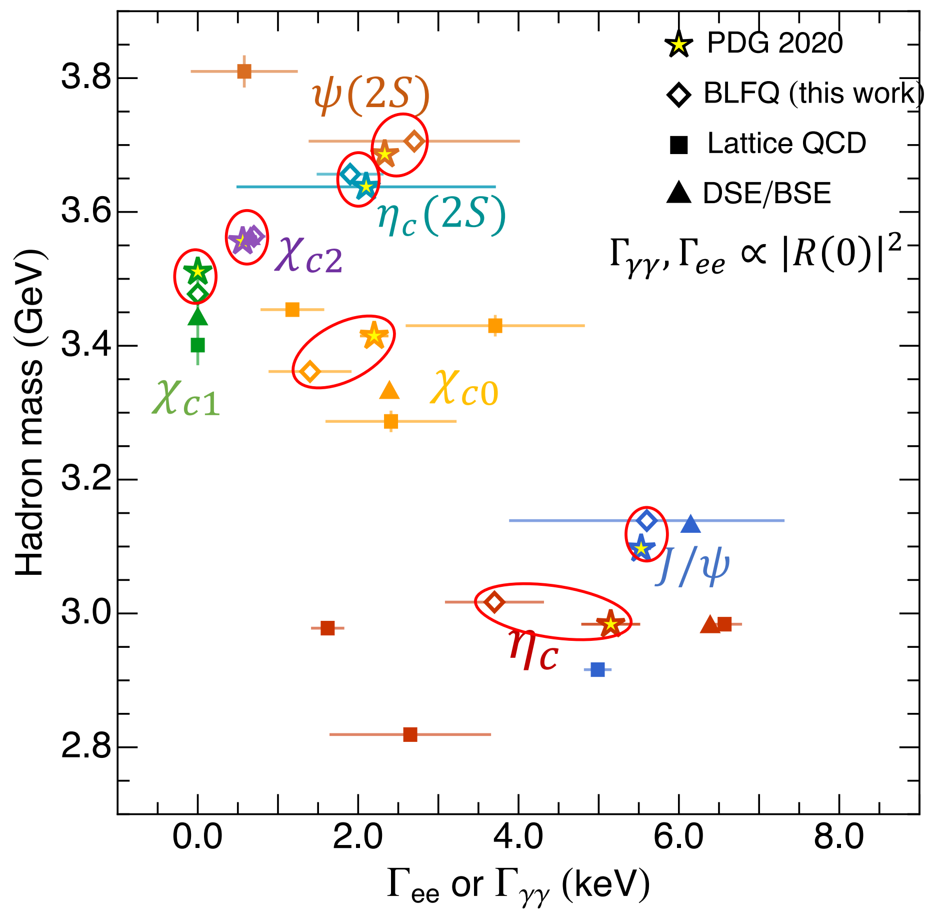
Yang Li, Parallel Session 1-B, Wednesday, Dec. 1 at 14:10

Diphoton width $\Gamma_{\gamma\gamma}$ of charmonia in BLFQ

- ✓ Notoriously challenging
- ✓ BLFQ predictions are very competetive!
  - ✓ No parameters were adjusted!



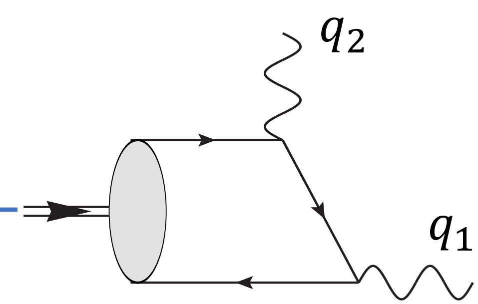




Lattice: Dudek '06, Chen '16, Chen '20, Meng '21, Zou '21;
DSE: Chen '17
NRQCD: Feng '15 & '17
NRQM: Babiarz '19 & '20

Comparison of theoretical prediction of masses and dilepton/diphoton widths combined

Transition form factor: η_c

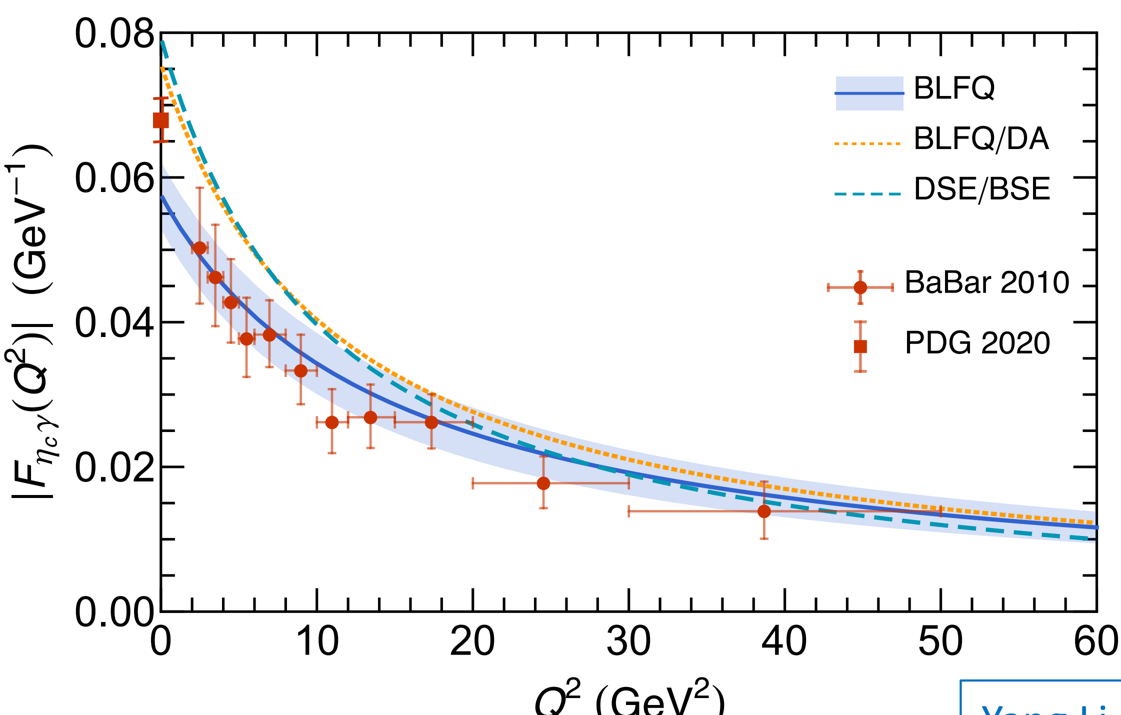


$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2)$$

- ✓ Diphoton width $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha_{em}^2 M_P^3 \left|F_{P\gamma\gamma}(0,0)\right|^2$
- ✓ Single-tag TFF $F_{P\gamma}(Q^2) = F_{P\gamma\gamma}(Q^2, 0) = F_{P\gamma\gamma}(0, Q^2)$

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_c}\int \frac{dx}{2\sqrt{x(1-x)}}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{\psi_{\uparrow\downarrow-\downarrow\uparrow}(x,\vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2}$$

Lepage '80, Feldman '97, Babiarz, '19



DSE/BSE: Chen PRD 2017

- BABAR data: described by monopole form with pole mass $\Lambda^2 = 8.5 \pm 0.6 \pm 0.7\ \mathrm{GeV}^2$, and width 5.12 (53) keV
- BLFQ: using N_{max}=8 wave function corresponding to $\mu \approx 2m_c$. Basis sensitivity band is taken as the difference between the N_{max}=8,16 results.
- BLFQ/DA: prediction using the LCDA obtained from the LFWF
- Theoretical prediction in good agreement with both the width and the form factor.

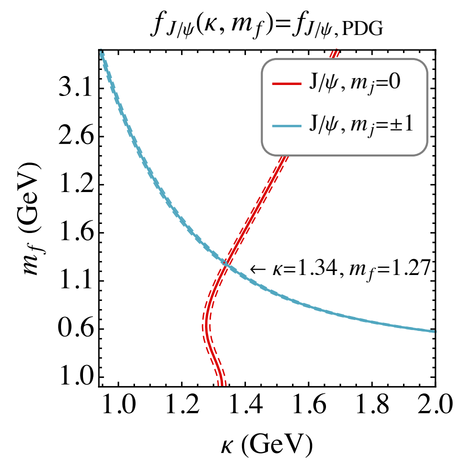
Yang Li, Parallel Session 1-B, Wednesday, Dec. 1 at 14:10

Charmonium LFWFs on a small-basis

- η_c, J/ψ, ψ' and $\psi(3770)$ LFWFs on a small BLFQ Hamiltonian basis: LF-1S, LF-2S, LF-1P, LF-1D.

$$H_0 = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) r_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x)\partial_x) \,,$$

- m_c and κ determined by J/ψ decay constant.



arXiv: 2111.07087, Guangyao Chen at 11:30 on Dec 2nd, Parallel Session 3-A

Charmonium LFWFs on a small-basis

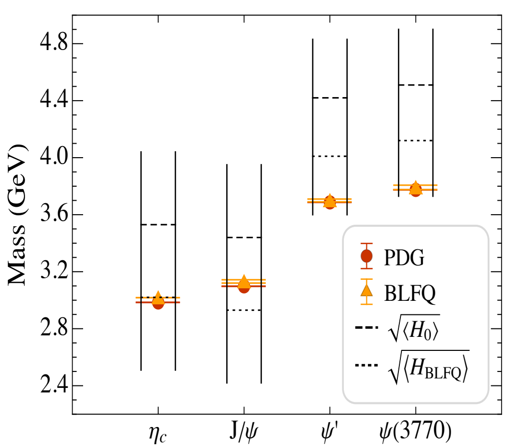
- Physical observables calculated:

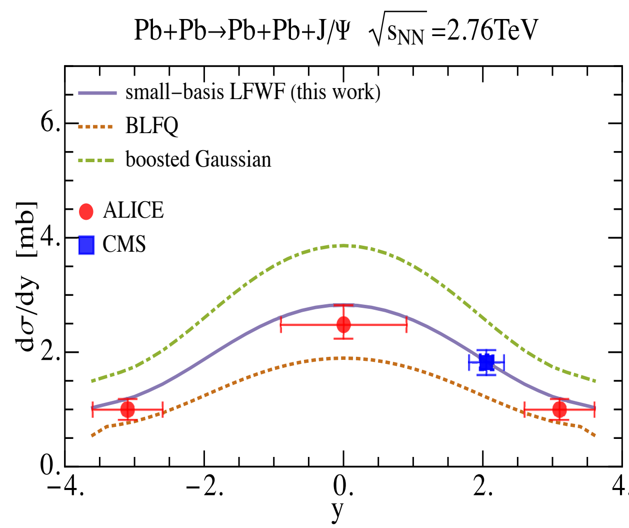
i. Mass and charge radii.

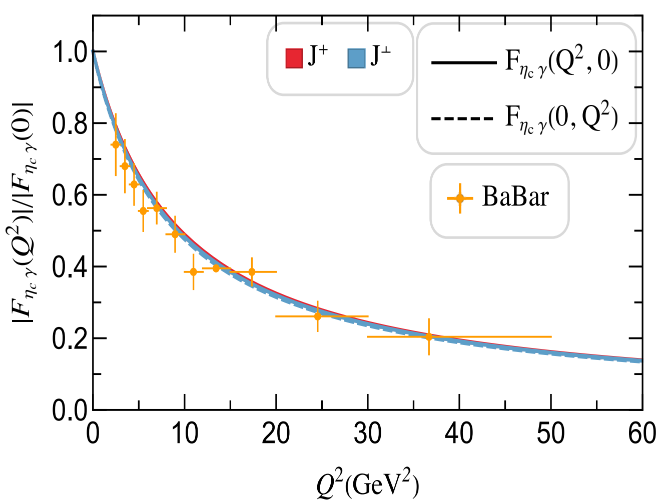
ii. PDFs.

iii. J/ψ production at HERA and LHC.

iv. η_c diphoton transition form factor.

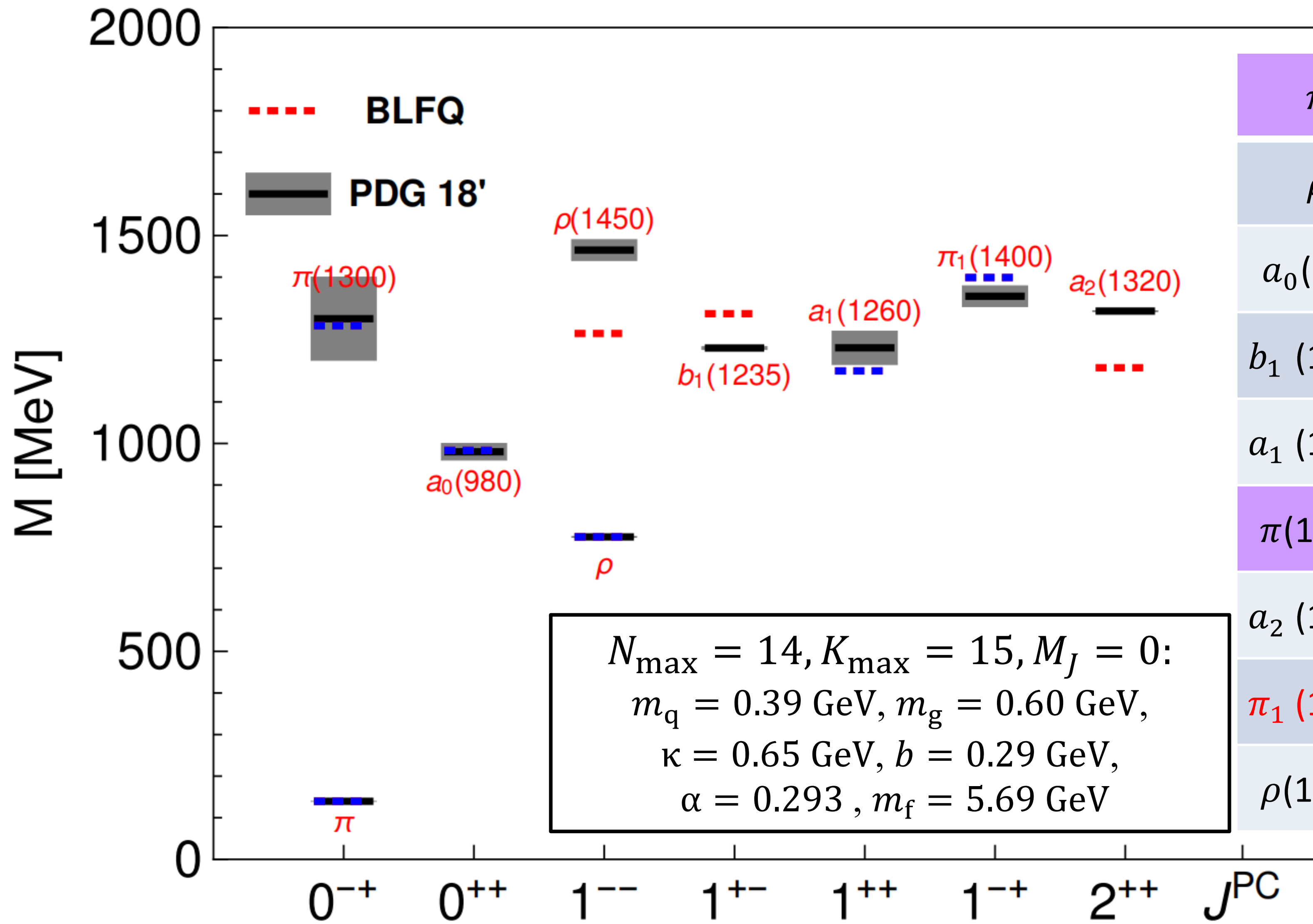






arXiv: 2111.07087, Guangyao Chen at 11:30 on Dec 2nd , Parallel Session 3-A

Light Meson Mass Spectrum Including One Dynamical Gluon



	Norm $q\bar{q}$	DC[MeV]
π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

[Lan, Fu, et al., arXiv 2106.04954]

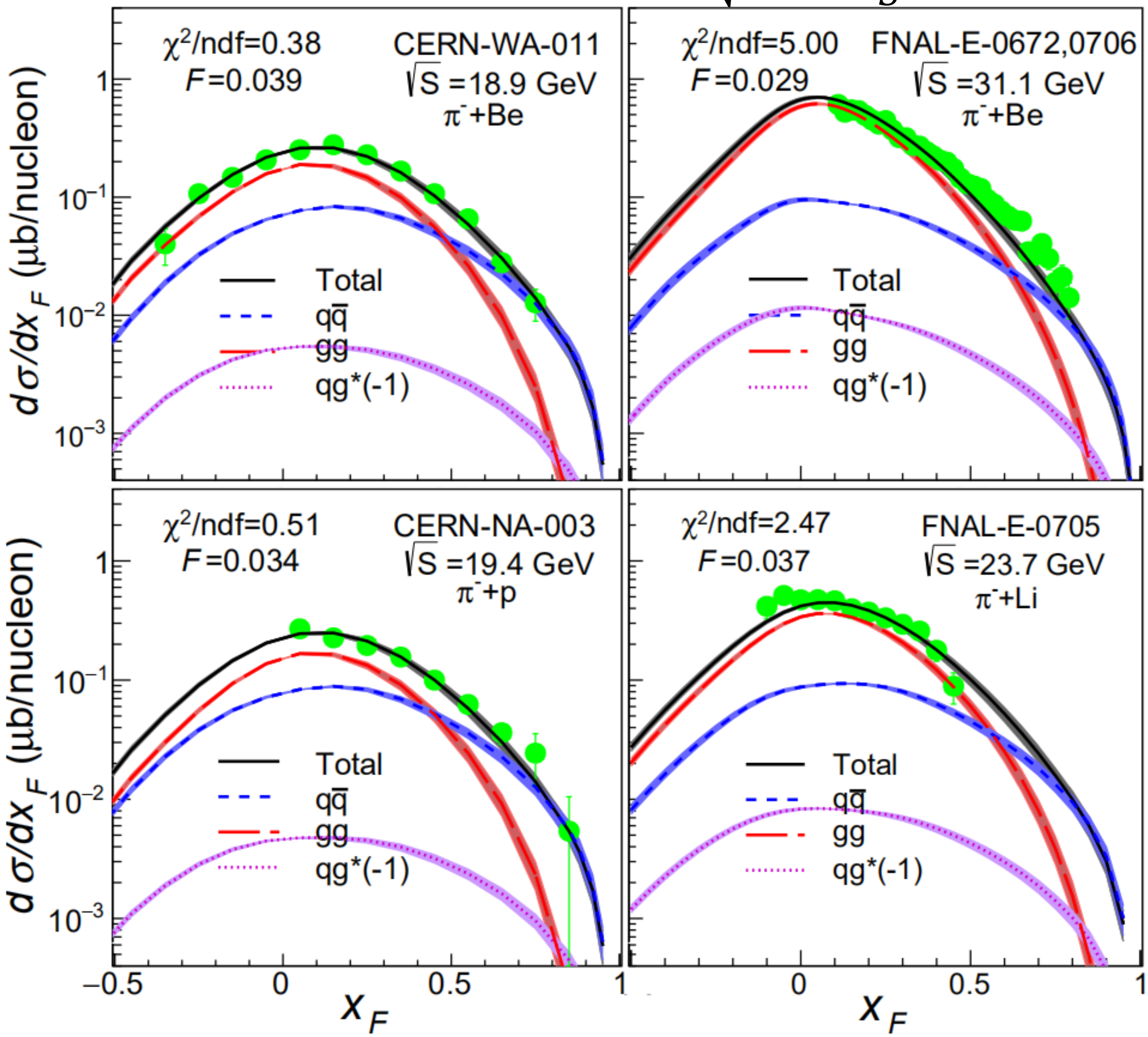
Jiangshan Lan, McCartor Award Session, Thursday, Dec. 2 at 9:05

J/ψ production cross section

$\pi^{\pm} N \rightarrow J/\psi\ X$

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^{\pm}}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM

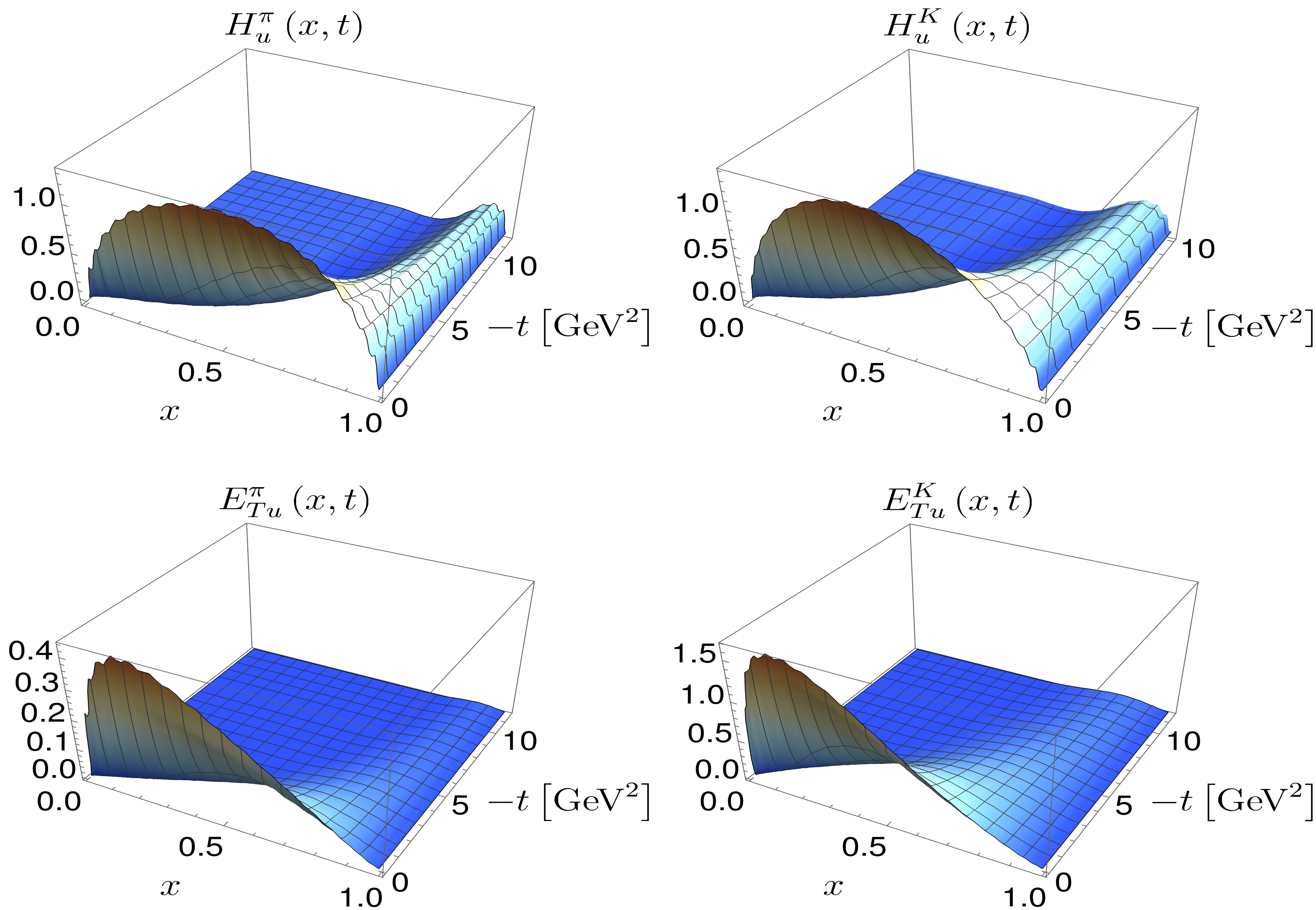
[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

Jiangshan Lan, McCartor Award Session, Thursday, Dec. 2 at 9:05

Agree with experimental data (FNAL E672, E706, E705, CERN NA3,WA11).

[Lan, et al., arXiv 2106.04954]

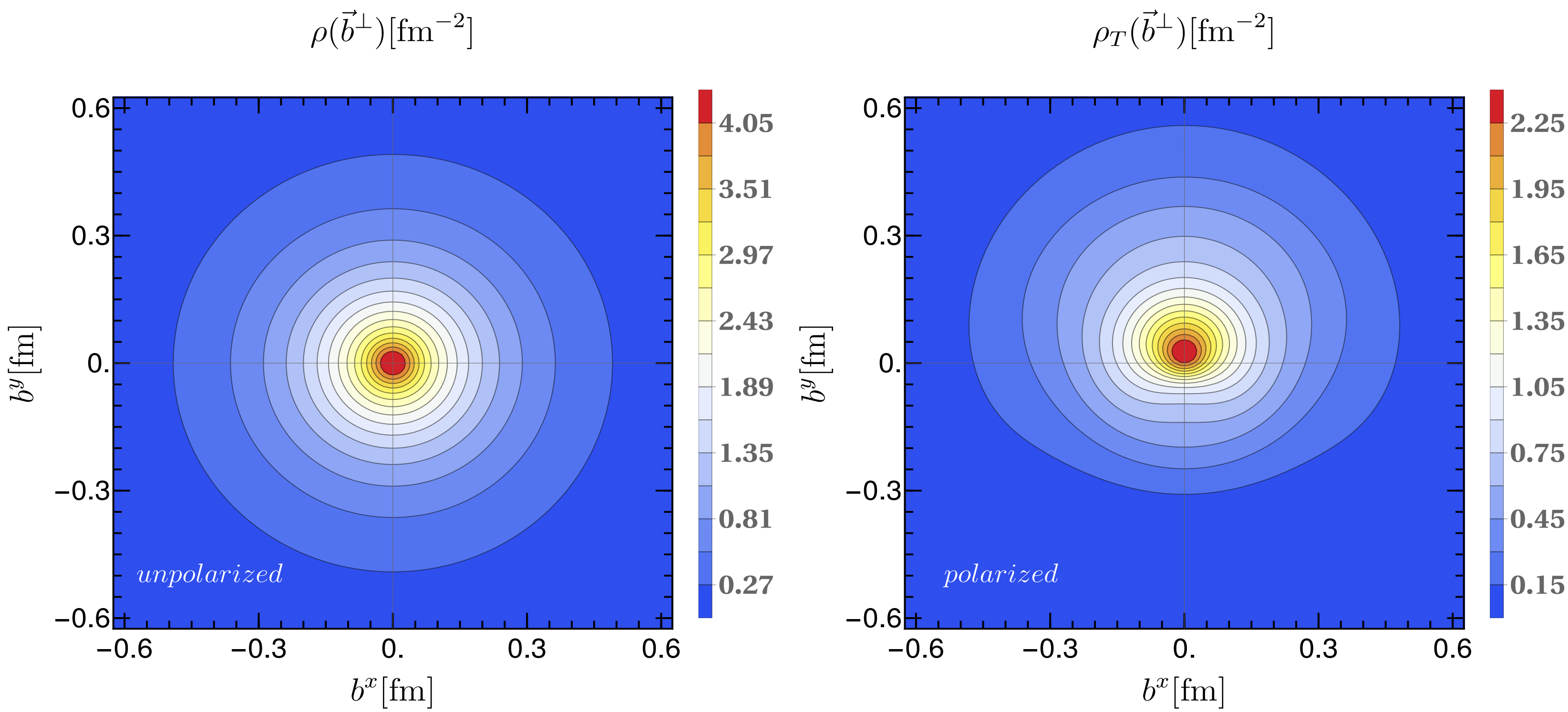
Generalized PDFs for the Pion and Kaon from BLFQ-NJL (Jia 2019)



Sreeraj Nair, Parallel Session 1-B, Tuesday, Nov. 30 at 14:30

Spin Densities in Impact Parameter Space

$$\rho(x,\vec{b}^{\perp},\vec{s}^{\perp}) = \frac{1}{2}\left[q(x,\vec{b}^{\perp}) - \frac{\vec{s}_i^{\perp}\epsilon_{ij}^{\perp}\vec{b}_j^{\perp}}{M_{\mathcal{P}}}\,q_T'(x,\vec{b}^{\perp})\right]$$



Sreeraj Nair, Parallel Session 1-B, Tuesday, Nov. 30 at 14:30

Distribution Amplitude from BLFQ-NJL

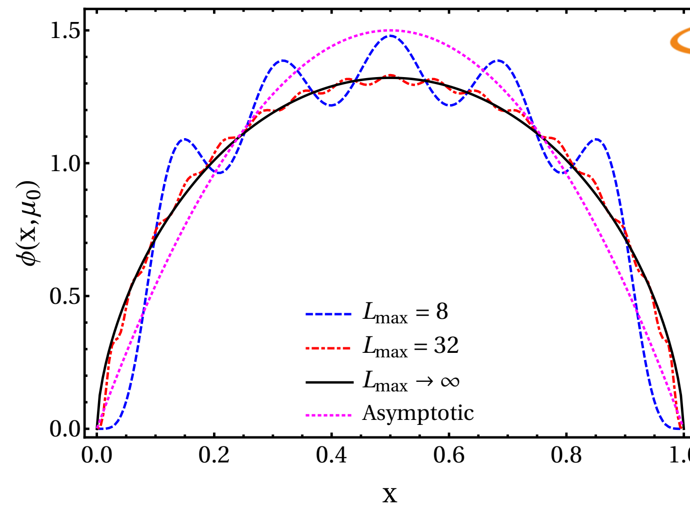
DAs of pseudoscalar states

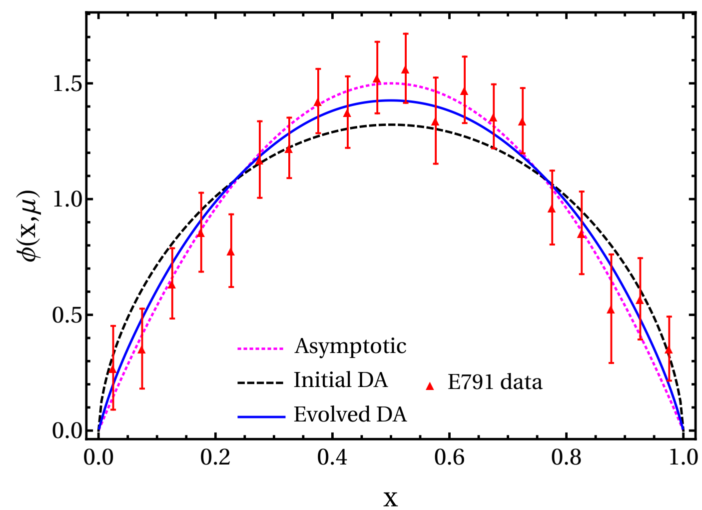
$$\phi(x,\mu_0) \sim \frac{1}{\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2\vec{k}_\perp}{2(2\pi)^3} \frac{(\psi_{\uparrow\downarrow} - \psi_{\downarrow\uparrow})}{\sqrt{2}}$$

- DA evolution: ERBL equations (Gegenbauer basis)

  E. R. Arriola, et al., PRD 66 (2002)

- Oscillations → Basis artifacts
- With increasing $L_{\max}$ the DA tends toward a smooth function
- Evolved DA (10 GeV^2) : Asymptotic DA





- Consistent with the FNAL-E-791

Decay constant f_π:

BLFQ (Basis [8, 32]): 145.3 MeV

Experimental data: 130.2 ± 1.7 MeV

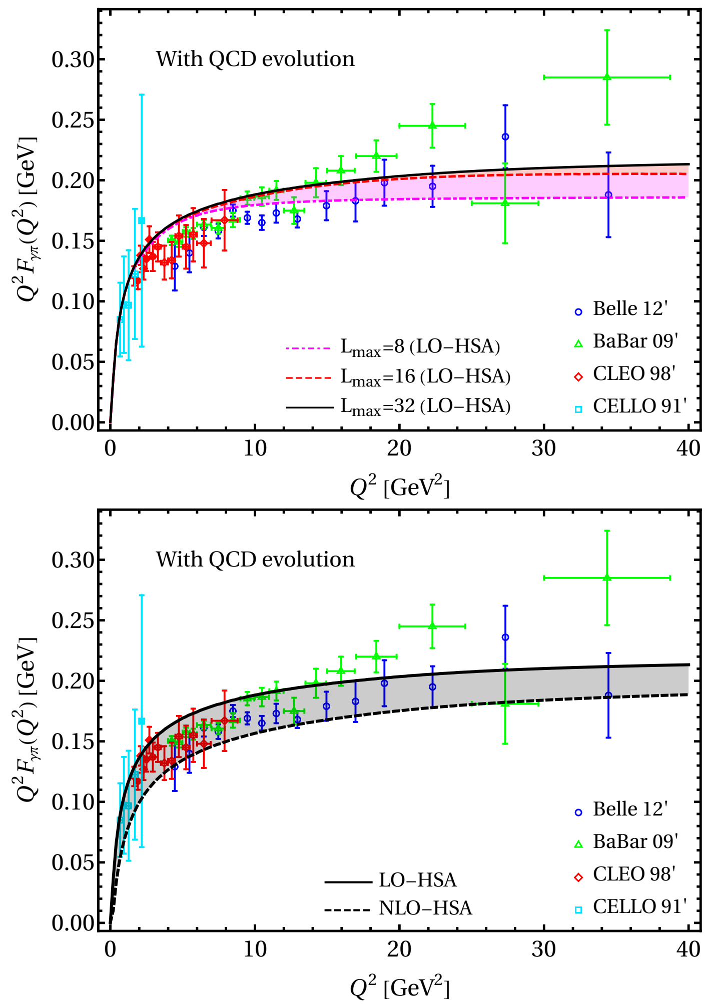
[1] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

Chandan Mondal, Parallel Session 2-B, Wednesday, Dec. 1 at 15:10

$\pi \to \gamma^* \gamma$ Transition Form Factor

$$\langle \gamma(P-q)|J^\mu|M(P)\rangle = -ie^2 F_{M\gamma}(Q^2)\epsilon^{\mu\nu\rho\sigma}P_\nu \epsilon_\rho q_\sigma \, ,$$

- Results for $\{N_{\max}, L_{\max}\} \equiv \{8, 8\}$, $\{8, 16\}$, and $\{8, 32\}$ (upper panel)
- The results show a good convergence trend over the range of Q^2
- Consistent with data reported by Belle Collaboration.
- Deviates from the rapid growth of the large Q^2 data reported by BaBar Collaboration.
- ERBL evolution effects & α_s order correction in the hard scattering amplitude (HSA) have been considered (lower panel).

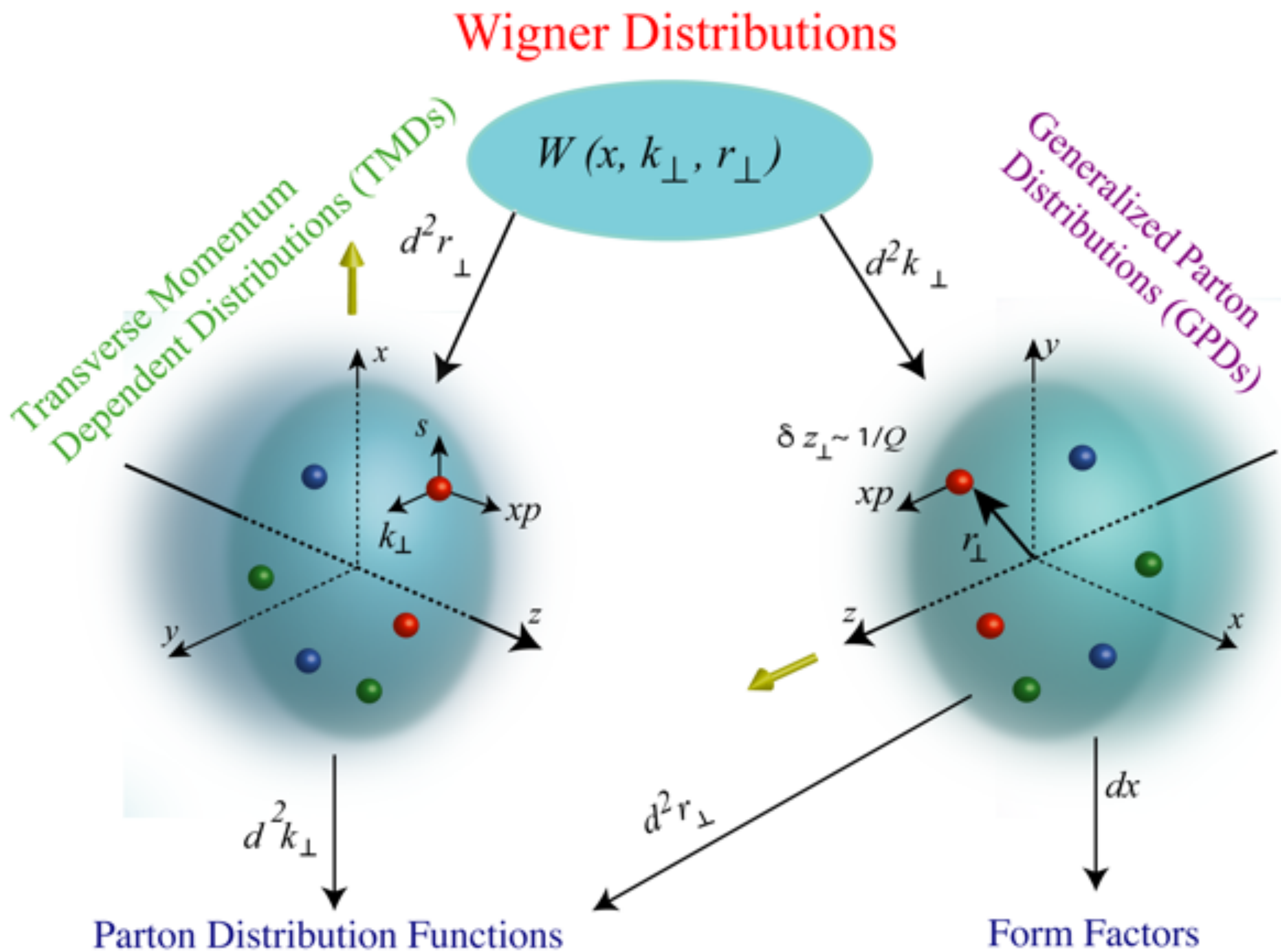


[1] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

Chandan Mondal, Parallel Session 2-B, Wednesday, Dec. 1 at 15:10

TMDs And Their Physical Significances

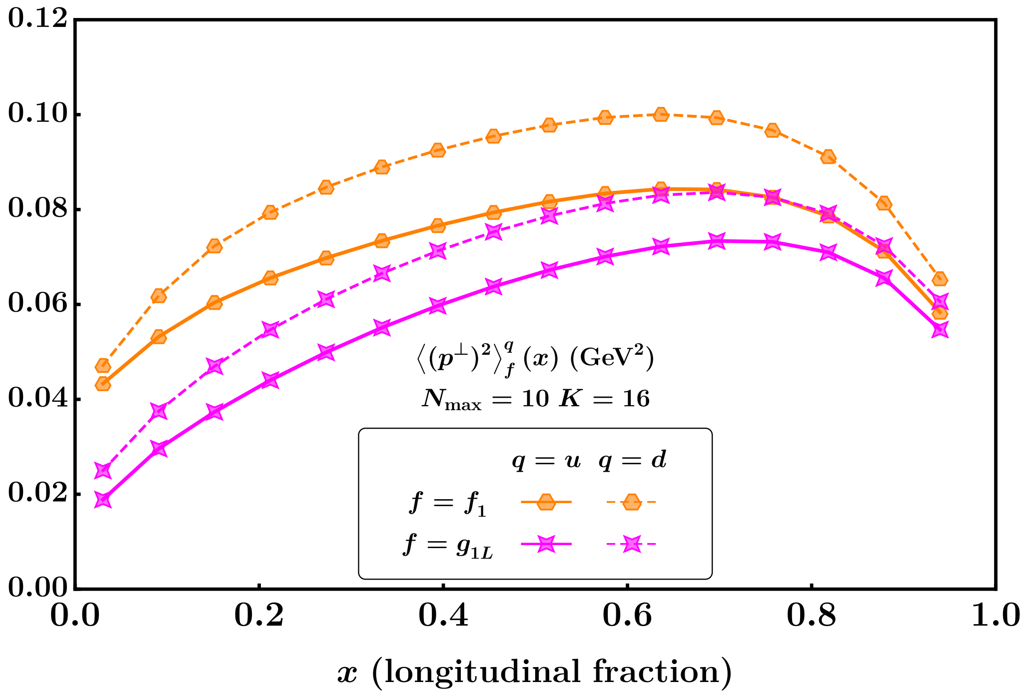
- They provide a 3D momentum space picture inside the hadron

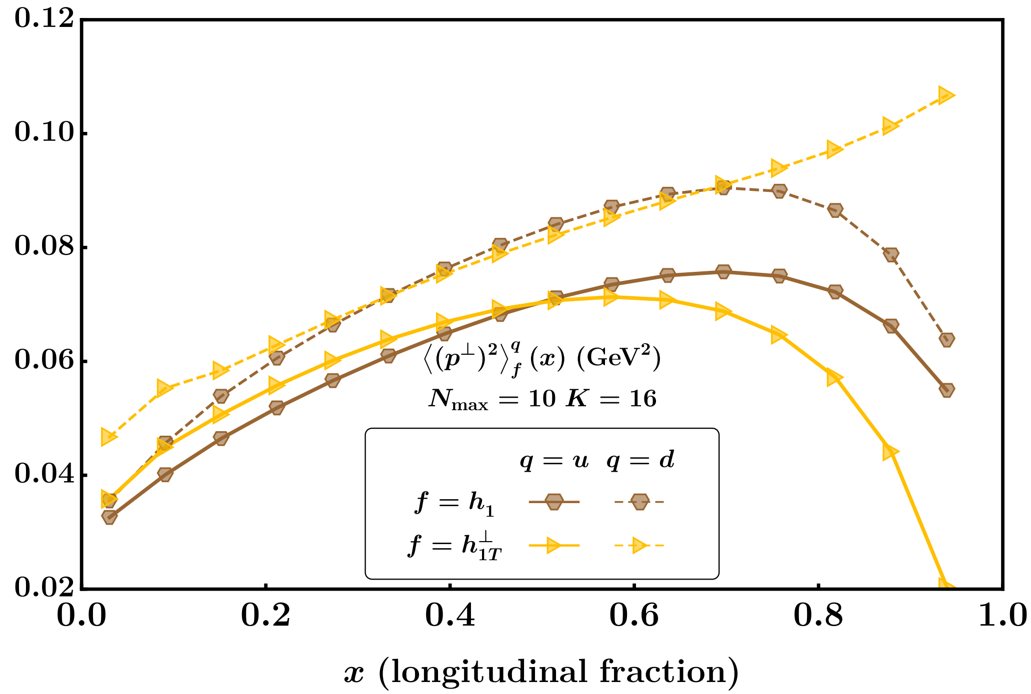


Zhi Hu, Parallel Session 1-A, Tuesday, Nov. 30 at 15:25

Proton TMDs (Zhi Hu et al. in preparation)

- x and flavour dependence of $\langle (p^{\perp})^2 \rangle$





$$\langle (p^{\perp})^2 \rangle_f^q (x) = \frac{\int d^2 p^{\perp} \, (p^{\perp})^2 f^q_{\mathrm{BLFQ}}(x, (p^{\perp})^2)}{\int d^2 p^{\perp} \, f^q_{\mathrm{BLFQ}}(x, (p^{\perp})^2)}$$

In the BLFQ calculation, we find a strong x and flavour dependence of $\langle (p^{\perp})^2 \rangle$

Zhi Hu, Parallel Session 1-A, Tuesday, Nov. 30 at 15:25

Light-Front Hamiltonian for Baryons

Siqi Xu, Parallel Session 4-A, Thursday, Dec. 2 at 14:30

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_{trans} \sim \kappa_T^4 r^2$$ -- Brodsky, Teramond arXiv: 1203.4025

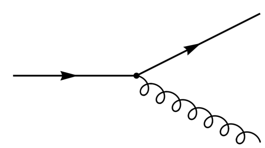
$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

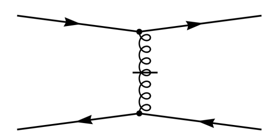
$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

- Include the first and second Fock sector

$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\, A_\mu^a + \frac{g^2 C_F}{2}\, j^+ \frac{1}{(i\partial^+)^2} j^+$$

$$N_{\max} = 9, K = 16.5$$

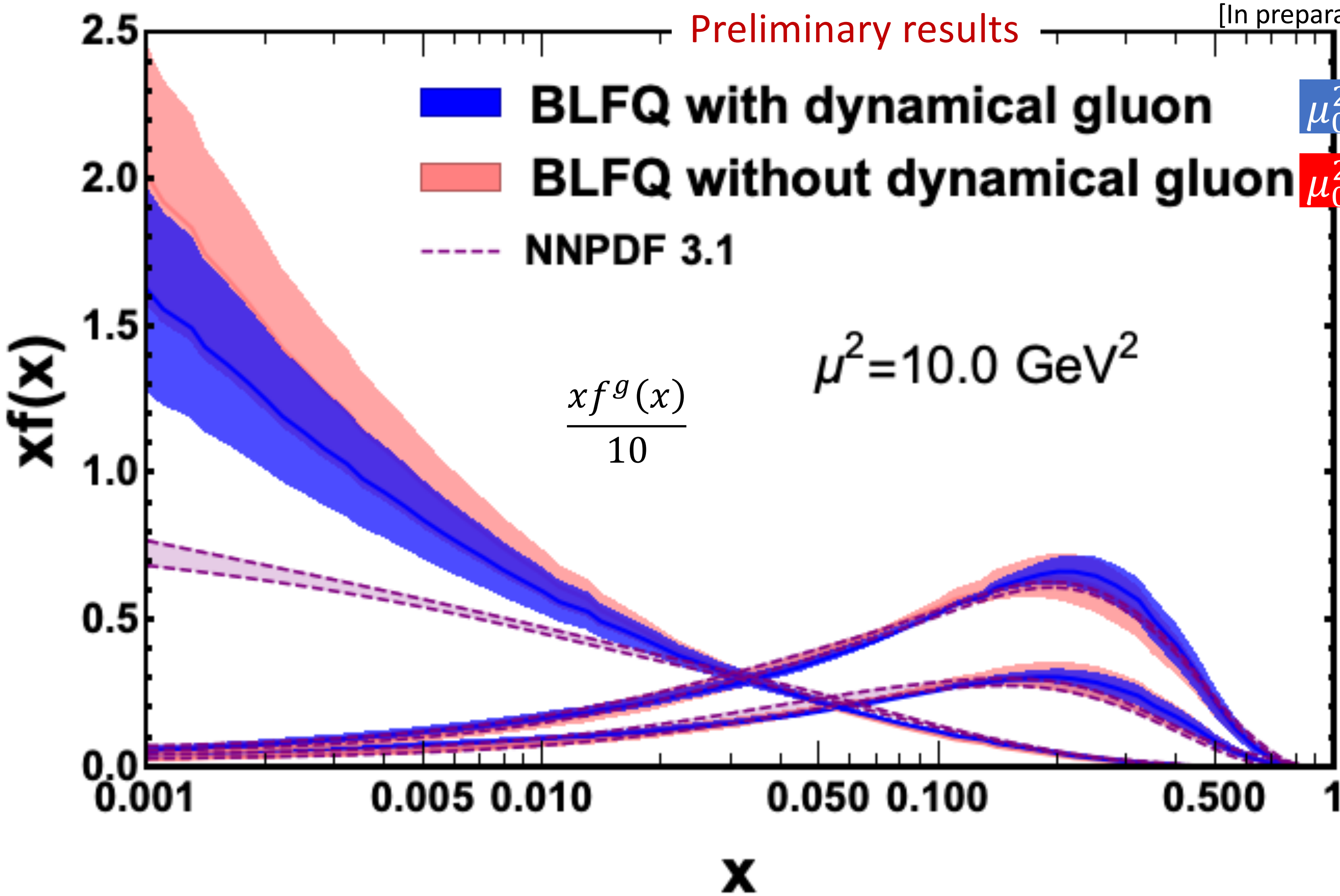




m_u	m_d	κ	m_g	m_{int}	b_{inst}	b	g
0.35 GeV	0.3 GeV	0.54 GeV	0.5 GeV	1.9 GeV	3 GeV	0.65 GeV	2.2

Unpolarized Parton Distribution Functions

[In preparation, Siqi Xu, C. Mondal *et.al*]



$\mu_0^2 = 0.25 \pm 0.2\ \mathrm{GeV}^2$

$\mu_0^2 = 0.19 \pm 0.2\ \mathrm{GeV}^2$

Siqi Xu
Parallel Session 4-A
Thursday, Dec. 2
at 14:30

Without second Fock sector $|qqqg\rangle$, the gluon is generated dynamically from the DGLAP evolution。

Including the One Dynamical Gluon Fock Sector, the gluon distribution is closer to the global fit.

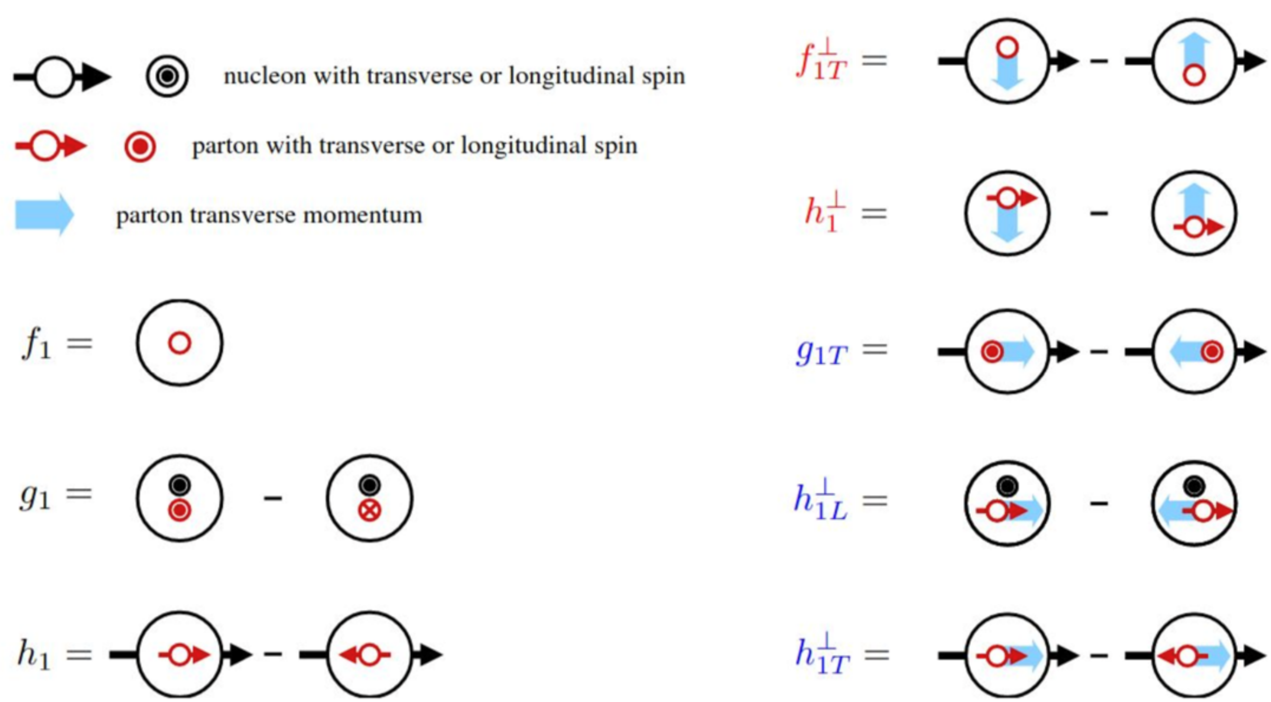
[Work in progress, C. Mondal, Siqi Xu, *et.al*]

TMDs and the spin-density in Λ^0 and Λ_c (ZhiMing Zhu et al. in preparation)

Spin-density definition: Pasquini B, Lorcé C. arXiv:1203.5006, 2012.

$$\rho\left(x, \boldsymbol{k}_T, (\lambda, \boldsymbol{s}_\perp), (\Lambda, \boldsymbol{S}_\perp)\right) = \frac{1}{2}\Big[f_1 + S_\perp^i \epsilon^{ij} k_T^j \frac{1}{M} f_{1T}^\perp + \lambda\Lambda g_{1L} + \lambda S_\perp^i k_T^i \frac{1}{M} g_{1T} + s_\perp^i \epsilon^{ij} k_T^j \frac{1}{M} h_1^\perp + \Lambda s_\perp^i k_T^i \frac{1}{M} h_{1L}^\perp + s_\perp^i S_\perp^i h_1 + s_\perp^i \left(2k_T^i k_T^j - \vec{k}_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{2M^2} h_{1T}^\perp\Big]$$

Red term is zero because of ignoring gauge link

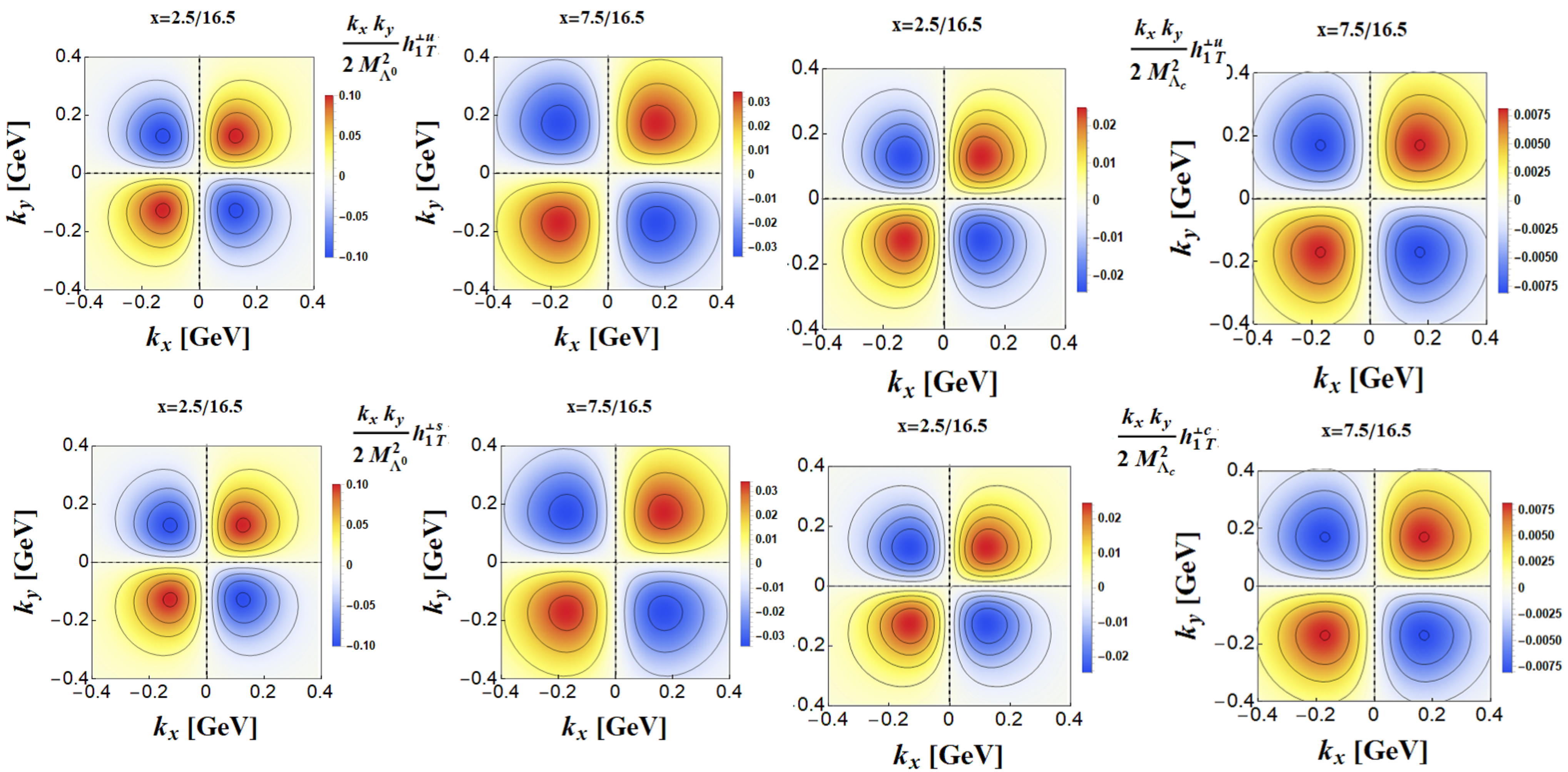


Zhi-Ming Zhu, Parallel Session 4-B, Thursday, Dec. 2 at 14:00

From Bacchetta's "Trento" Lecture notes

The spin-density for Λ^0 and Λ_c (ZhiMing Zhu et al. in preparation)

$$|\Lambda^0\rangle = |uds\rangle \qquad |\Lambda_c\rangle = |udc\rangle$$

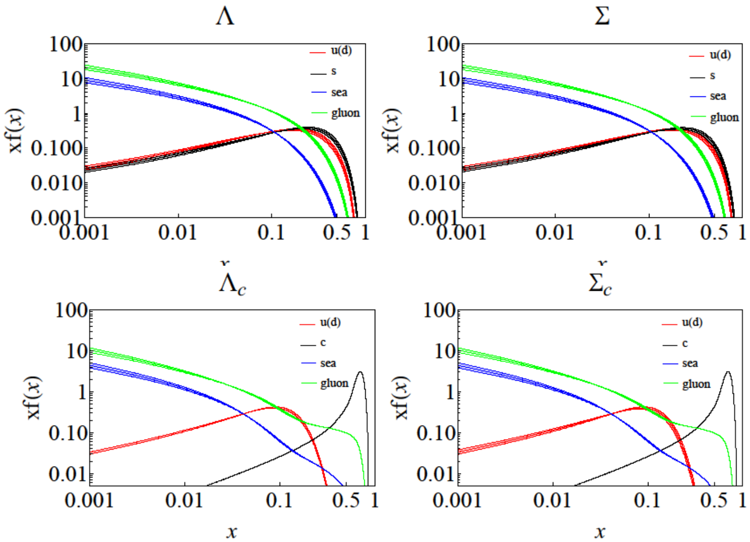


The distortion term is small

Zhi-Ming Zhu, Parallel Session 4-B, Thursday, Dec. 2 at 14:00

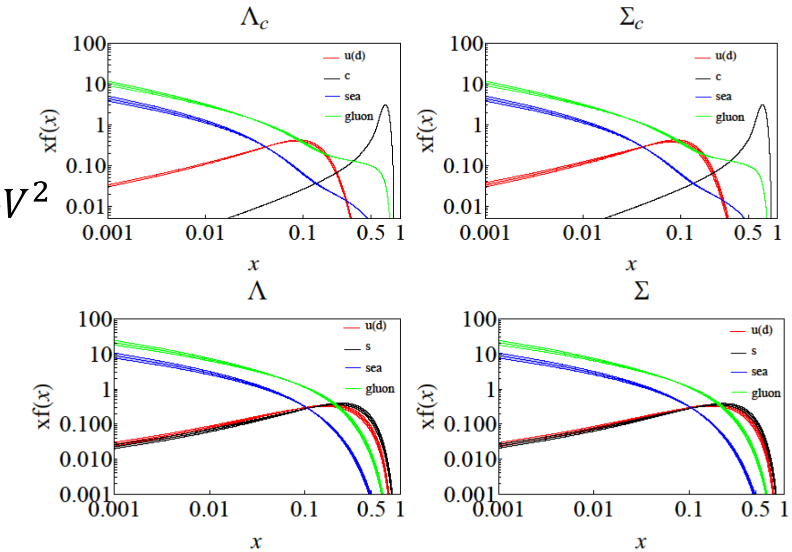
Basis light-front quantization approach to Λ(Σ0, Σ+, Σ−) and Λc (Σc+, Σc++, Σc0)

Unpolarized PDFs f(x) after QCD scale evolution



Initial scale
$\mu_0^2 = 0.195 \pm 0.020\ \mathrm{GeV}^2$

Final scale
$\mu^2 = 10\ \mathrm{GeV}^2$



Electromagnetic properties of these baryons – some sample results

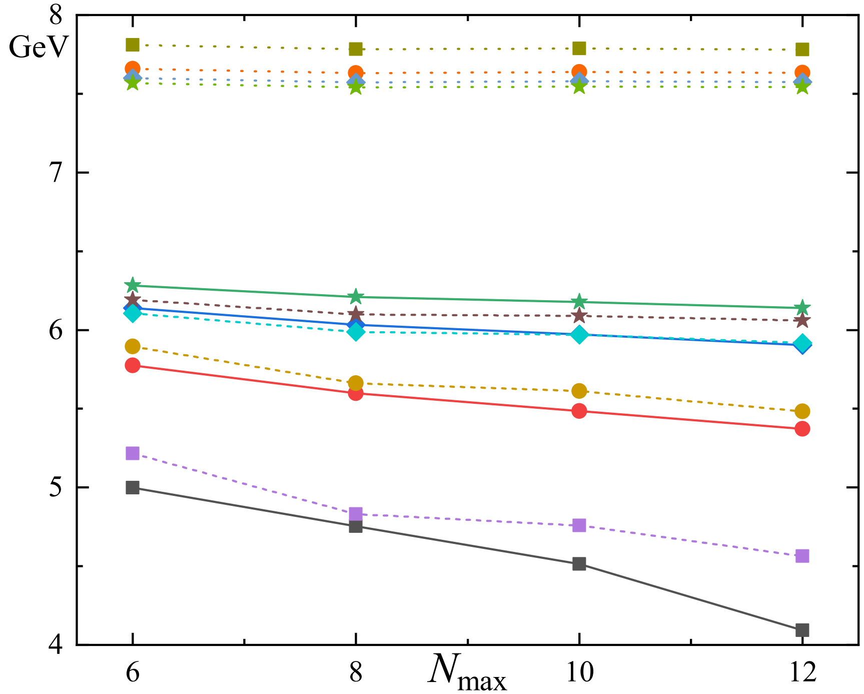
	$\mu_{\mathrm{BLFQ}}/\mu_N$	μ_{exp}/μ_N	$< r_E^2 >_{BLFQ}$ $/[fm^2]$	$< r_E^2 >_{exp}$ $/[fm^2]$	$< r_M^2 >_{BLFQ}$ $/[fm^2]$	$< r_M^2 >_{exp}$ $/[fm^2]$
Λ	$-0.494^{+0.028}_{-0.010}$	-0.613 ± 0.004	$-0.07^{+0.01}_{-0..01}$	-	$0.52^{+0.01}_{-0.01}$	-
Σ^0	$0.610^{+0.032}_{-0.051}$	-	$-0.07^{+0.00}_{-0..01}$	-	$0.82^{+0.00}_{-0.01}$	-
Σ^+	$2.323^{+0.067}_{-0.111}$	2.458 ± 0.010	$-0.07^{+0.05}_{-0..05}$	-	$0.79^{+0.00}_{-0.00}$	-
Σ^-	$-1.124^{+0.011}_{-0.007}$	-1.160 ± 0.025	$-0.07^{+0.02}_{-0..02}$	$0.60 \pm 0.08 \pm 0.08$	$0.70^{+0.02}_{-0.02}$	-

Tiancai Peng, Parallel Session 2-B, Wednesday, Dec. 1 at 15:30

All-charm tetraquark using BLFQ

- New issues compared with mesons and baryons
  - Cluster decomposition principle for interactions
  - Identical particles issue
  - More than one color singlet
- Hamiltonian
  - Transverse confining potential like in AdS/QCD
  - Longitudinal confining potential (Głazek et al. PLB773, 172-178 (2017), different than $BLFQ_0$)
  - One-gluon-exchange spin-dependent potential (Wiecki et al.)
- Problem with negative M^2 solved by ad hoc modification of the Hamiltonian (which breaks cluster decomposition principle)

Kamil Serafin, Parallel Session 1-B, Tuesday, Nov. 30 at 14:50



$K = 6$ ■, 10 ●, 14 ♦, 18 ★

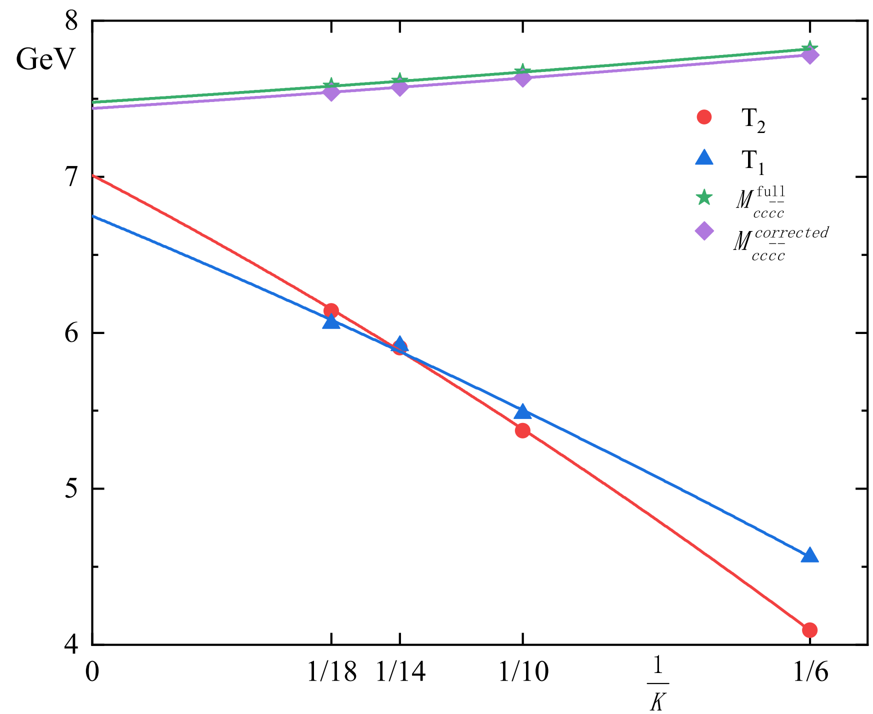
Dotted – $M^{\text{corrected}}_{cc\bar{c}\bar{c}}$

Solid – T_2

Dashed – T_1

Extrapolations $K \to \infty$

$N_{\max} = 12$



Kamil Serafin, Parallel Session 1-B, Tuesday, Nov. 30 at 14:50

Forward quark jet-nucleus scattering in a light-front Hamiltonian approach

(Meijian Li, et al, 2 Dec 2021, 16:25, Parallel Session 5-B)

Time-dependent Basis Light-Front Quantization (tBLFQ)

❖ First-principles:

In the light-front Hamiltonian formalism, the state obeys the time-evolution equation, and the Hamiltonian is derived from the QCD Lagrangian

$$\frac{1}{2}P^-(x^+)|\psi(x^+)\rangle = i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle$$

❖ Nonperturbative treatment:

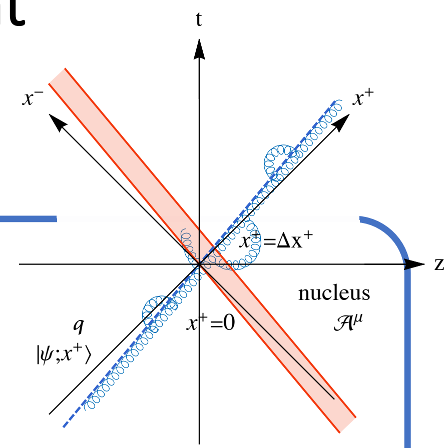
The time evolution operator is divided into many small timesteps, each timestep is evaluated numerically and intermediate states are accessible,

$$|\psi(x^+)\rangle = \mathcal{T}_+ \exp\left[-\frac{i}{2}\int_0^{x^+} dz^+ P^-(z^+)\right]|\psi(0)\rangle$$

$$= \lim_{n\to\infty}\prod_{k=1}^{n}\mathcal{T}_+ \exp\left[-\frac{i}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+)\right]|\psi(0)\rangle$$

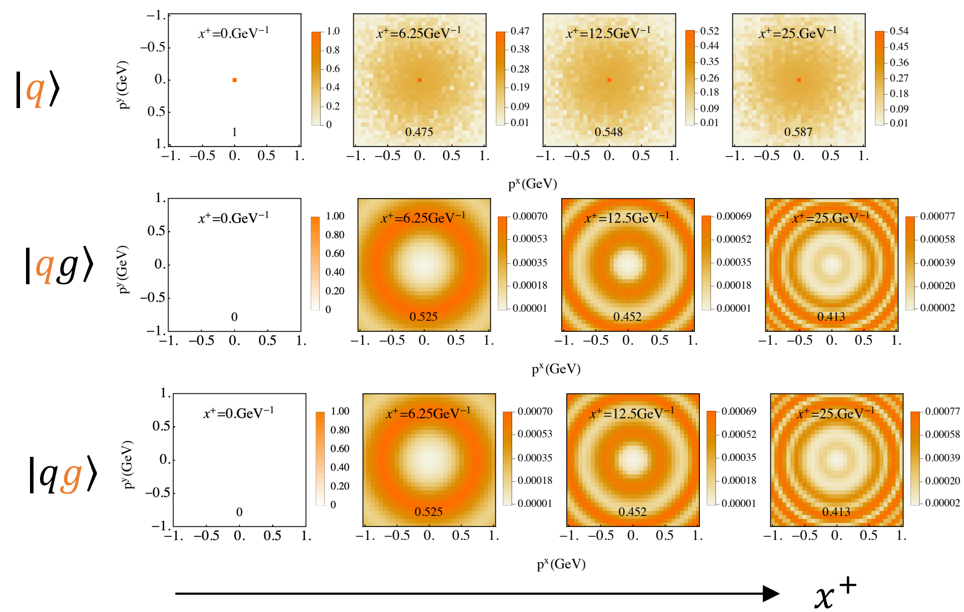
❖ Basis representation:

Optimal basis has the same symmetries of the system, and it is the key to numerical efficiency



We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.

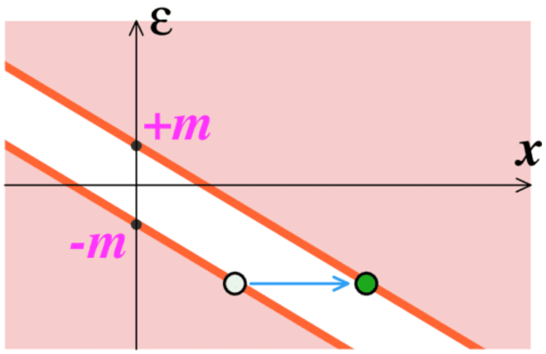
Time evolution of a quark state in the $|q\rangle + |qg\rangle$ Fock space observed from the transverse momentum space



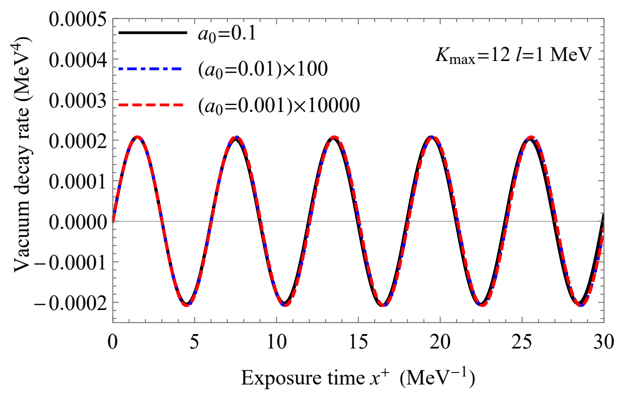
M. Li, T. Lappi and X. Zhao, Phys. Rev. D 104, 056104 (2021)

Pair production in strong electric fields using tBLFQ (Bolun Hu, 30 Nov, 15:30, Parallel Sesion 1-B)

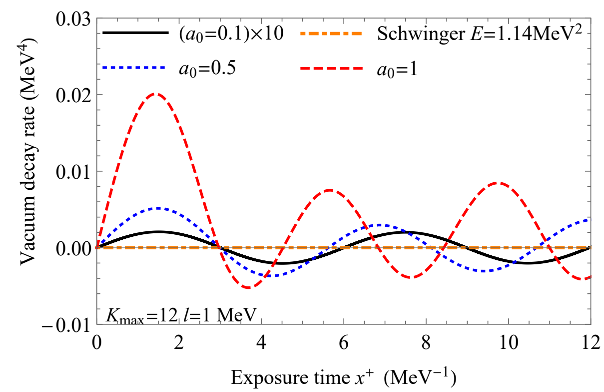
We investigate the e^+e^- pair production in strong electric fields.



We find a critical E around the Schwinger limit m_e^2/e above which the pairs are created efficiently

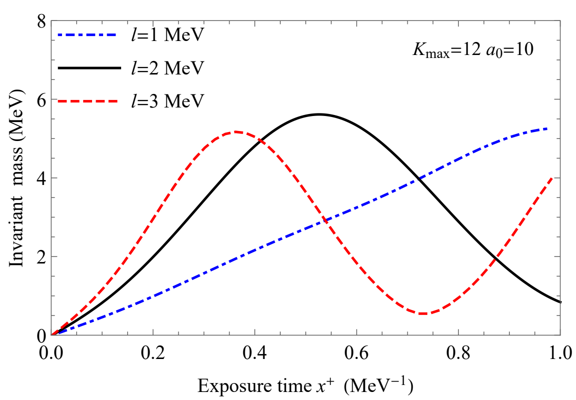


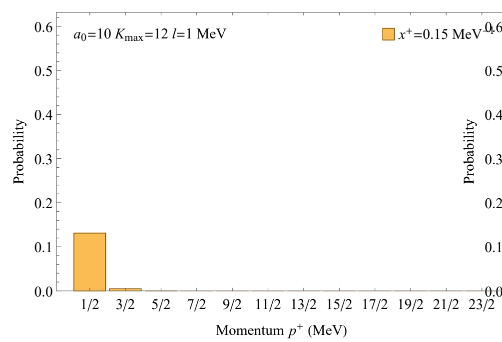
weaker fields

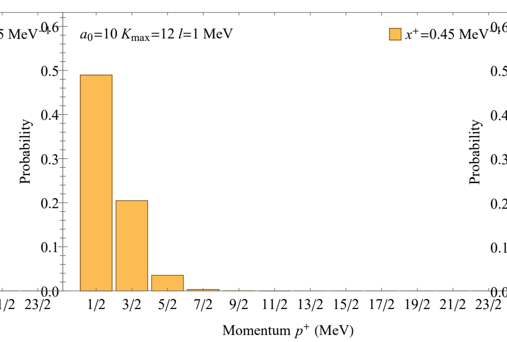


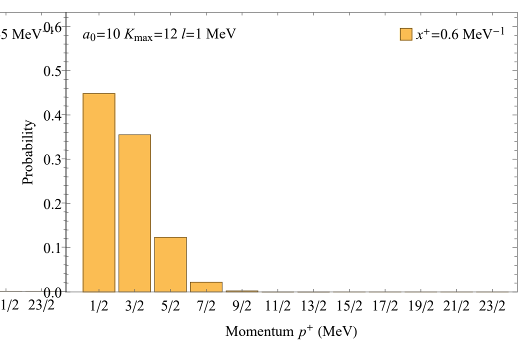
stronger fields

We can also calculate observables such as the invariant mass and the momentum distribution of the produced e^+e^- pairs as functions of time





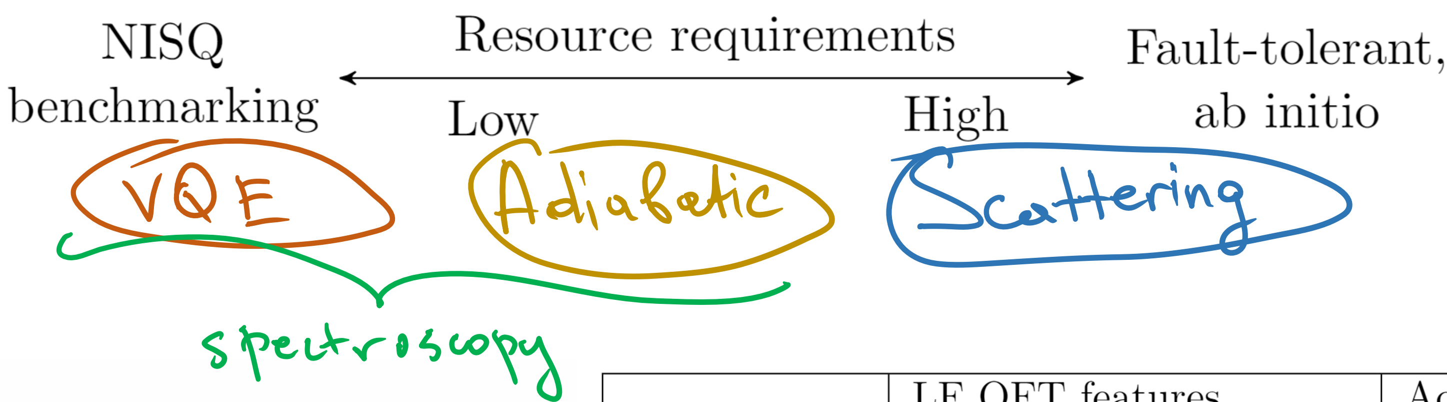




Quantum Simulation of QFT in the Front Form

Michael Kreshchuk, Thursday Dec 2 at 11:30 AM; Parallel Session 3-B

2002.04016, 2105.10941, 2011.13443, 2009.07885



$$\mathfrak{Q}_{\text{Direct}} = O(K \log K)$$

$$\mathfrak{Q}_{\text{Compact}} = O(\sqrt{K} \log K)$$

	Trotter	Oracle
Direct	✓	✓
Compact	✗	✓

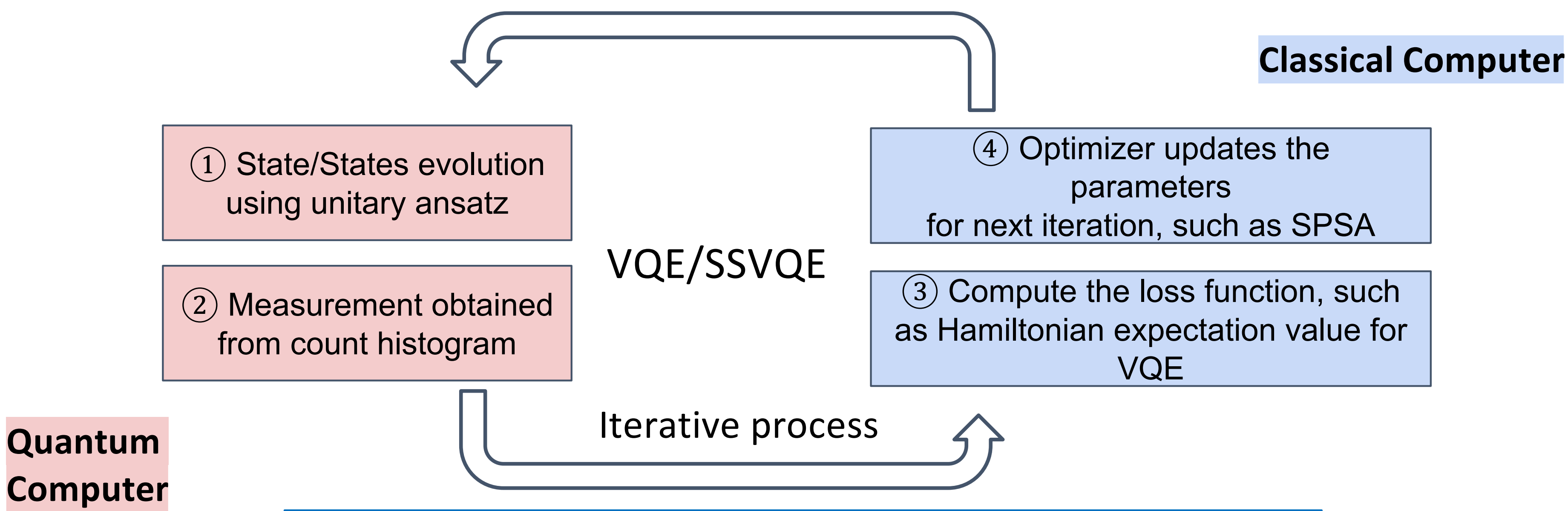
$$O_F\,|x, i\rangle = |x, y_i\rangle\ ,$$

$$O_H\,|x, y, 0\rangle = |x, y, H_{xy}\rangle$$

	LF QFT features	Advantages for QC
Resources	No ghost fields Linear EoM	Low qubit count
	LF momentum > 0	Efficient encoding
Evolution	Sparse Hamiltonians	Using sparsity-based methods
Measurement	LF wavefunction $\to$ $\to$ static quantities; Simple form of operators in the second-quantized formalism	Simple form of measurement operators
Other	Trivial vacuum, fewer cut-offs, no fermion doubling, form invariance of H	

Light front approach to hadrons on quantum computers

- Quantum computers: New tool to simulate many-body quantum system. (quantum mechanical nature and high scalability)
- In the Noisy Intermediate-Scale Quantum (NISQ) era, the Variational Quantum Eigensolver (VQE) and Subspace-search VQE (SSVQE) approaches are promising tools to solve nuclear physics problems. [Nakanishi, 1810.09434]
- Advantages of light front Hamiltonian formalism are directly applicable
- We first formulate the problem on the light front and then map the Hamiltonian to qubits (quantum bits)



Wenyang Qian, Parallel Session 1-A, Tuesday, Nov. 30 at 14:30

Formulating the problem on qubits

- We adopt the Hamiltonian used in a previous work: [Qian, 2005.13806]

$$H_{\text{eff},\gamma_5} = \overbrace{\underbrace{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\mathbf{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_{\text{eff}}} + V_g + H_{\gamma_5}$$

[Vary, 0905.1411]

- Basis representation (BLFQ) is key to represent the Hamiltonian on qubits.
- Small-size Hamiltonians (4-by-4 and 16-by-16) are used.
- Direct encoding and compact encoding are compared.

[Seeley, 1208.5986]
[Kreshchuk, 2002.04016]

	N_{f}	$\alpha_{\text{s}}(0)$	κ (MeV)	m_q (MeV)	$N_{\max}$	$L_{\max}$	Matrix dimension
$H_{\text{eff}}^{(1,1)}$	3	0.89	560 ± 10	300 ± 10	1	1	4 by 4
$H_{\text{eff}}^{(4,1)}$					4	1	16 by 16

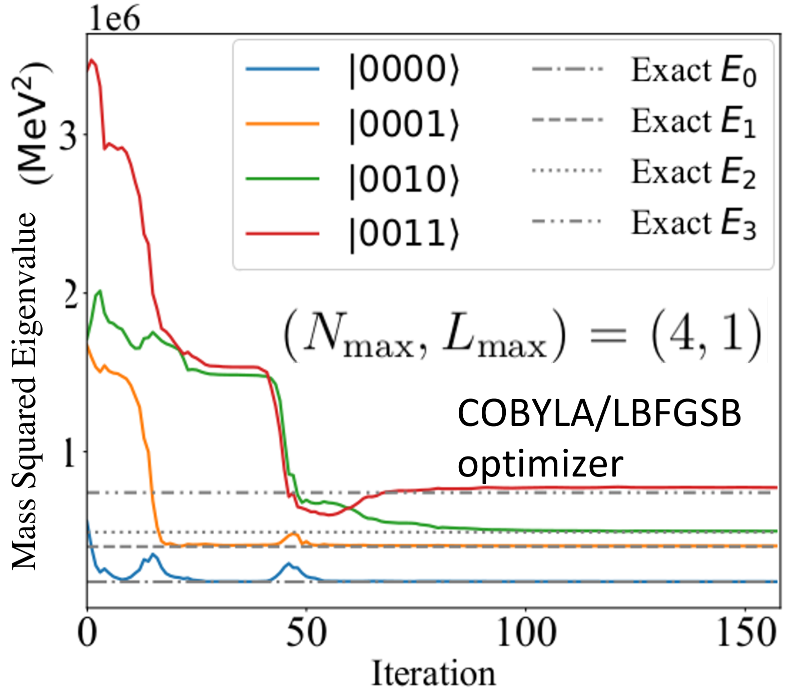
$$\begin{aligned} H_{\text{direct}}^{(1,1)} &= 2269462\,\text{IIII} - 284243\,(\text{ZIII} + \text{IIZI}) \\ &- 850488\,(\text{IZII} + \text{IIIZ}) + 12714\,(\text{XZXI} + \text{YZYI}) \\ &- 7883\,(\text{IXZX} + \text{IYZY}), \end{aligned}$$

$$\begin{aligned} H_{\text{compact}}^{(1,1)} &= 1134731\,\text{II} - 566245\,\text{IZ} \\ &+ 4831\,\text{XI} + 20598\,\text{XZ} \end{aligned}$$

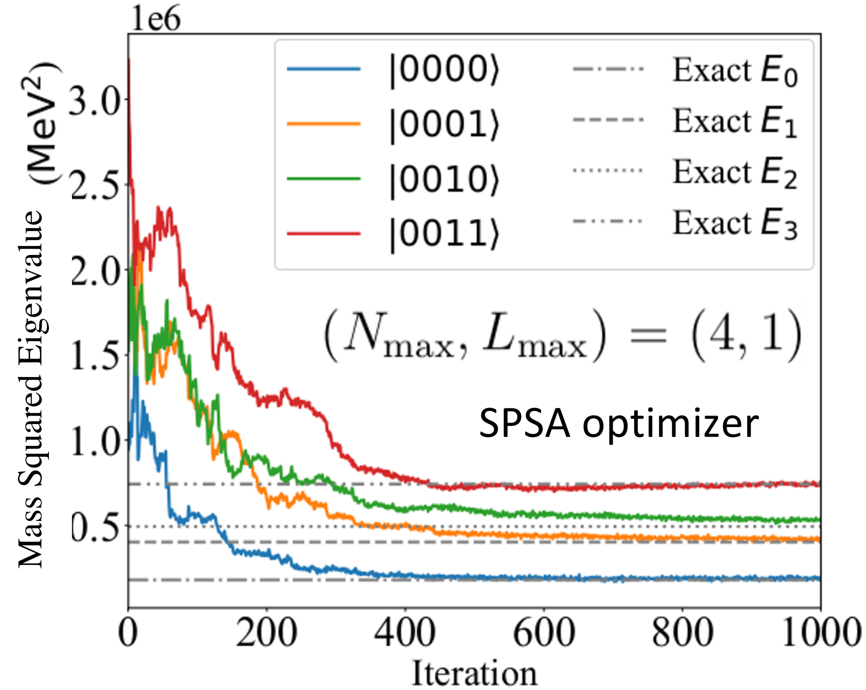
Wenyang Qian, Parallel Session 1-A, Tuesday, Nov. 30 at 14:30

Results on quantum simulators

- Low-lying spectroscopy (4 states) obtained from SSVQE for 16-by-16 Hamiltonian



(a) SSVQE with statevector simulator



(b) SSVQE with qasm simulator

- Both use Hardware efficient ansatz (6 repetition layer, 53 params) [Kandala, Nature 549, 242]
- In SSVQE: Each initial orthogonal quantum state evolves to the expected hadrons
- Loss function: $\mathcal{L} = 1.0\, E_{|0000\rangle} + 0.5\, E_{|0001\rangle} + 0.25\, E_{|0010\rangle} + 0.125\, E_{|0011\rangle}$
- With evolved states, other physical quantities such as decay constants, distribution functions, transition amplitudes can also be computed

Wenyang Qian, Parallel Session 1-A, Tuesday, Nov. 30 at 14:30

Summary and Outlook

Basis Light Front Quantization approach to mesons and baryons yields competitive descriptions and predictions

- Bound states and transitions of hadrons are described
- Time-dependent scattering applications are showing progress
- Plan: expand the Fock spaces (add gluons & sea quarks)
- Plan: renormalization, counterterms and regulators
- Efficient utilization of supercomputing resources
- Well positioned to exploit advances in quantum computing

Thank you for your attention

Applications of BLFQ to QED and QCD at LC2021

Invited

Xingbo Zhao, "Positronium structure from light-front QED Hamiltonian,"
Tuesday, Nov. 30 at 11:50, Plenary Session 6

Jiangshan Lan, "Light meson structure with basis light-front quantization,"
Thursday, Dec. 2 at 9:05, McCartor Award Session

Contributed

Wenyang Qian, "Light front approach to hadrons on quantum computers,"
Tuesday, Nov. 30 at 14:30, Parallel Session 1-A

Zhi Hu, "Proton and pion momentum-space 3D structure within basis light-front quantization,"
Tuesday, Nov. 30 at 15:25, Parallel Session 1-A

Sreeraj Nair, "Generalized parton distributions and spin structures of light mesons from basis light-front quantization," Tuesday, Nov. 30 at 14:30, Parallel Session 1-B

Kamil Serafin, "All-charm tetraquark using basis light-front quantization,"
Tuesday, Nov. 30 at 14:50, Parallel Session 1-B

Bolun Hu, "Pair production in strong inhomogeneous electric fields,"
Tuesday, Nov. 30 at 15:30, Parallel Session 1-B

Applications of BLFQ to QED and QCD at LC2021
Contributed (Continued)

Yang Li, “Two-photon transitions of charmonia on the light front,”
Wednesday, Dec. 1 at 14:10, Parallel Session 2-B

Chandan Mondal, “Pion to photon transition form factors with basis light-front quantization,”
Wednesday, Dec. 1 at 15:10, Parallel Session 2-B

Tiancai Peng, “Basis light-front quantization approach to Lambda and Lambda_c,”
Wednesday, Dec. 1 at 15:30, Parallel Session 2-B

Guangyao Chen, “Designing charmonium light-front wavefunctions on a small basis,”
Thursday, Dec. 2 at 11:30, Parallel Session 3-A

Michael Kreshchuk, “Simulation of nuclear physics on near-term quantum computers usig basis light-front quantization,” Thursday, Dec. 2 at 11:30, Parallel Session 3-B

Zhi-Min Zhu, “Transverse structure of heavy baryons in momentum space: a light-front Hamiltonian approach,” Thursday, Dec. 2 at 14:00, Parallel Session 4-B

Siqi Xu, “Nucleon structure with dynamical gluon in light-front frame,”
Thursday, Dec. 2 at 14:30, Parallel Session 4-A

Meijian Li, “Forward quark jet – nucleus scattering in a light-front Hamiltonian approach,”
Thursday, Dec. 2 at 16:25, Parallel Session 5-B