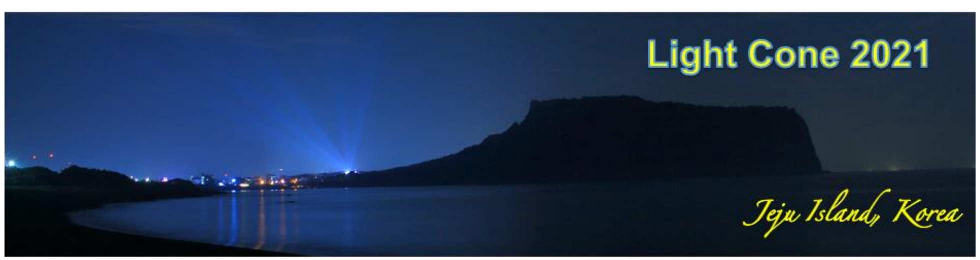
Positronium Structure from Light-front QED Hamiltonian

Xingbo Zhao[†], Kaiyu Fu[†], Hengfei Zhao[†],
Yang Li[‡], James P. Vary [*]

[†] **Institute of Modern Physics, CAS, Lanzhou, China**
[‡] **University of Science and Technology of China, China**
[*] **Iowa State University, Ames, US**



LIGHT CONE 2021, Jeju Island, Korea, Nov 30, 2021

Outline

- Motivation
- Solving Positronium in Basis Light-front Quantization
- Positronium Structure
- Summary and Outlook

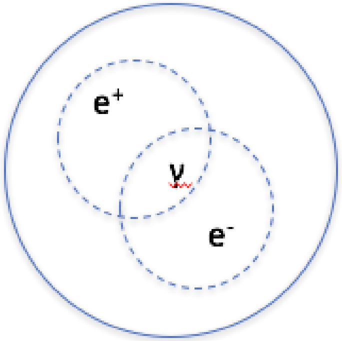
Questions for LF Hamiltonian Methods

- Picture of relativistic bound states beyond leading Fock sector (in gauge theories)?
  - Gluon and sea quark distribution in hadrons

- Consequences of Fock sector truncations? Can they be managed in a phenomenologically acceptable way? Rotational symmetry?

- First-principles (or effective interaction) calculation with Fock sector truncation possible?

Why Positronium?



- Simplest bound state in QED formed by e^+ and e^-

- Positronium has long been considered as testing ground for mesons

[Krautgärtner, et al, 1992] DLCQ
[Trittmann, et al, 1997] DLCQ
[Lamm, et al, 2014] TMSWIFT
[Wiecki, et al, 2015] BLFQ

- Precious works:
  1. Effective one-photon-exchange interaction in $|e^+e^-\rangle$ sector
     - Convergence is ok with additional counterterm (removing δ functionlike interaction), which shows negative impact on rotational symmetry
  2. Explicit $|e^+e^-\gamma\rangle$ sector [Kaluža, et al, 1992] DLCQ
     - Only ground state calculated, convergence is poor, rotational symmetry was not checked

- Time to revisit positronium with explicit $|e^+e^-\gamma\rangle$ sector in BLFQ after 30 years!

Basis Light-front Quantization

[Vary et al, 2008]

See James Vary's talk on Monday

- Nonperturbative eigenvalue problem

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

  - P^-: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^-_β: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + \mathrm{c}|\gamma\rangle + \mathrm{d}|e\bar{e}e\bar{e}\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

$\Lambda_{\mathrm{UV}} \propto \sqrt{N_{max}}b$

Large N_{max} and K : High UV cutoff & low IR cutoff

Light-front QED Hamiltonian

- QED Lagrangian $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$$

- Light-front QED Hamiltonian from standard Legendre transformation

$$P^- = \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \; F^{\mu +} \partial_+ A_\mu + i\bar{\Psi}\gamma^+ \partial_+ \Psi - \mathcal{L}$$

Light-cone gauge: $\left(A^+ = 0\right)$

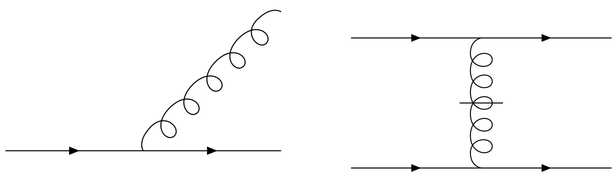
$$= \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \; \frac{1}{2}\bar{\Psi}\gamma^+ \frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2} A^j (i\partial^\perp)^2 A^j$$

kinetic energy terms

$$+ e j^\mu A_\mu + \frac{e^2}{2} j^+ \frac{1}{(i\partial^+)^2} j^+$$

vertex interaction

instantaneous photon interaction

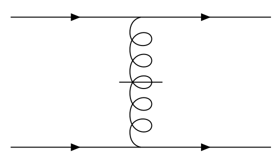
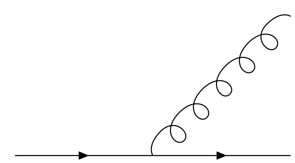
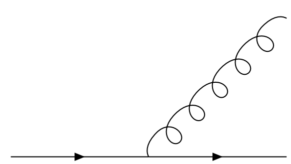


Interaction Part Of Hamiltonian

m_e=1.0MeV

$$|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + c|\gamma\rangle + d|e\bar{e}e\bar{e}\rangle + \ldots$$

$$\alpha = \frac{e^2}{4\pi} = 0.3$$

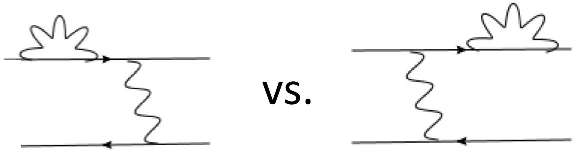
H_{int}	$\vert e\bar{e}\rangle$	$\vert e\bar{e}\gamma\rangle$
$\langle e\bar{e}\vert$		
$\langle e\bar{e}\gamma\vert$		0 excluded by *gauge principle* [Tang et al, 1991]

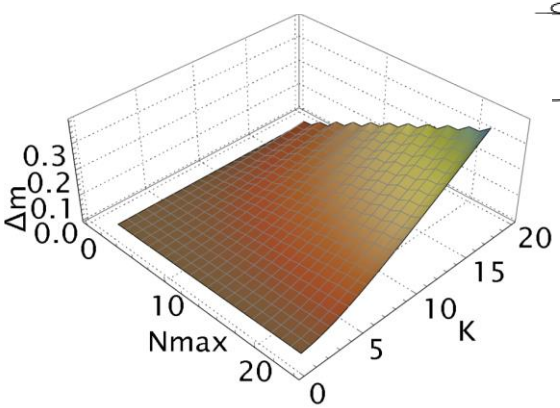
Mass Renormalization

- Mass counterterm $\Delta_m = m_{bare} - m_{phys}$ is needed for fermion self-energy correction

- Mass renormalization needs to be performed on single physical electron
  - Prediction power on positronium mass

- Mass counterterm is determined by fitting single electron mass [Karmanov et al, 2008]
  - Complication: Δ_m depends on UV cutoff and thus is basis state dependent. An extension of sector-dependent renormalization
  - $\Delta_m(N_{max}, K)$ needed



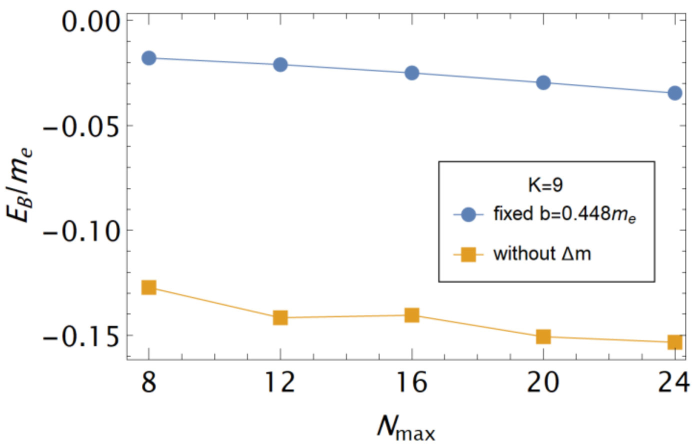


[Kaiyu Fu et al, in preparation]

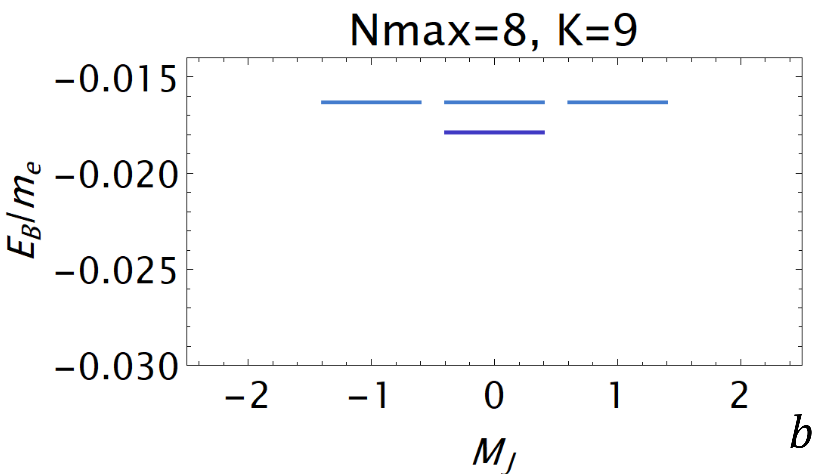
- Mass counterterm is on a larger order of magnitude $\Delta_m \propto \alpha m\, E_B \propto \alpha^2 m$

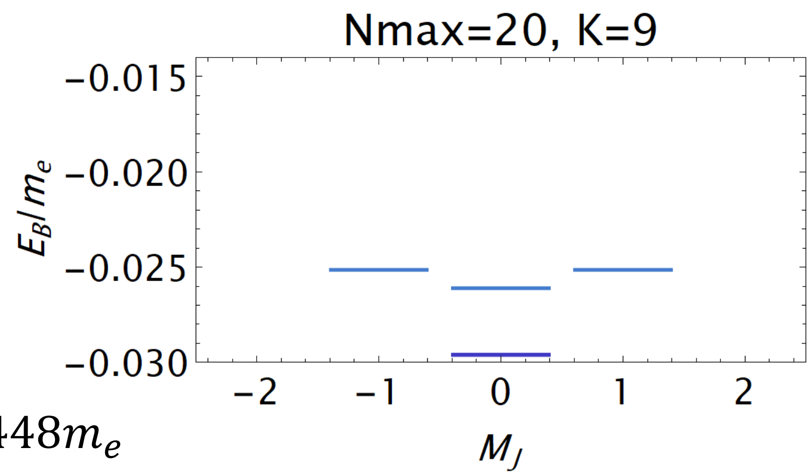
Mass Renormalization is not Enough

- After mass renormalization, positronium mass still diverges with N_{max}



- Rotational symmetry gets worse as N_{max} increase
  - Degeneracy of 1^3S_1 used as an indicator of rotational symmetry

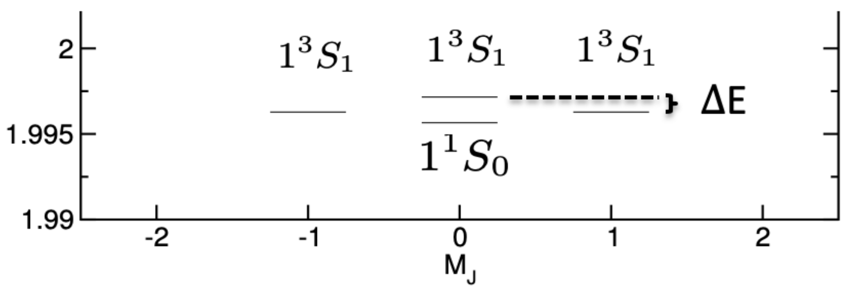


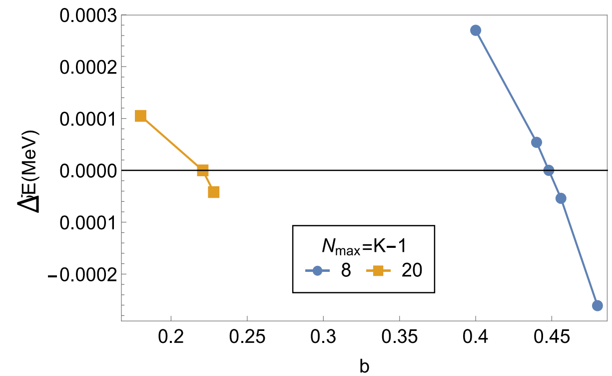


$b = 0.448 m_e$

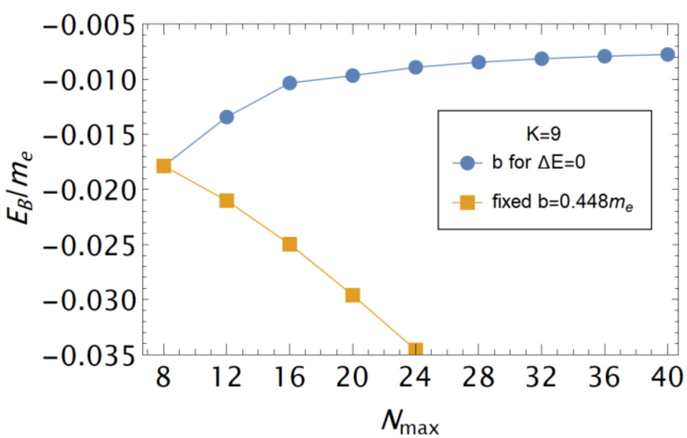
Basis Scale and Rotational Symmetry

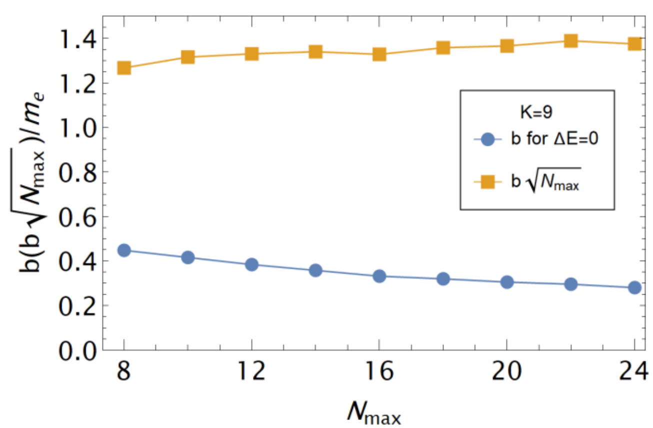
- Adjust the 2d harmonic oscillator basis scale parameter b to minimize the energy difference within the triplet 1^3S_1





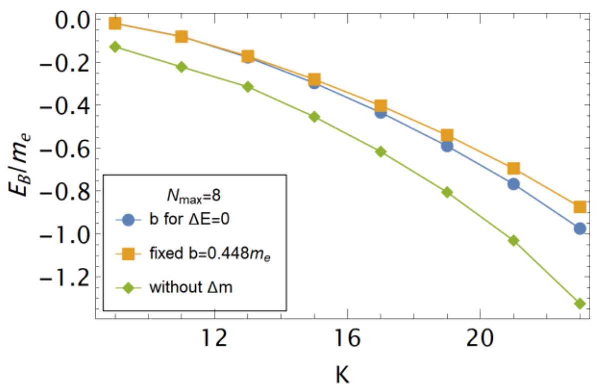
- Maintaining rotational symmetry leads to a corresponding UV cutoff

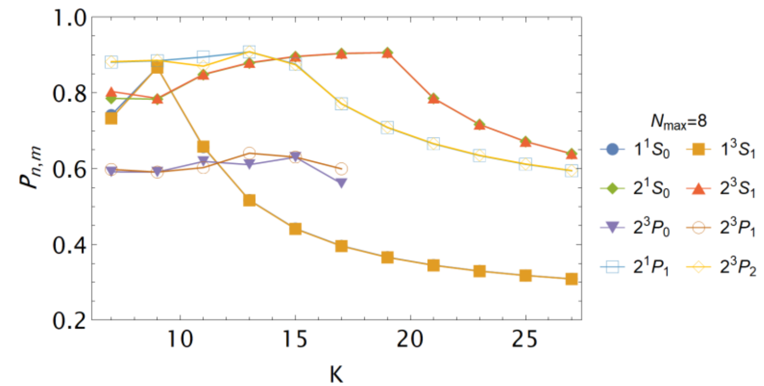




Longitudinal Behavior

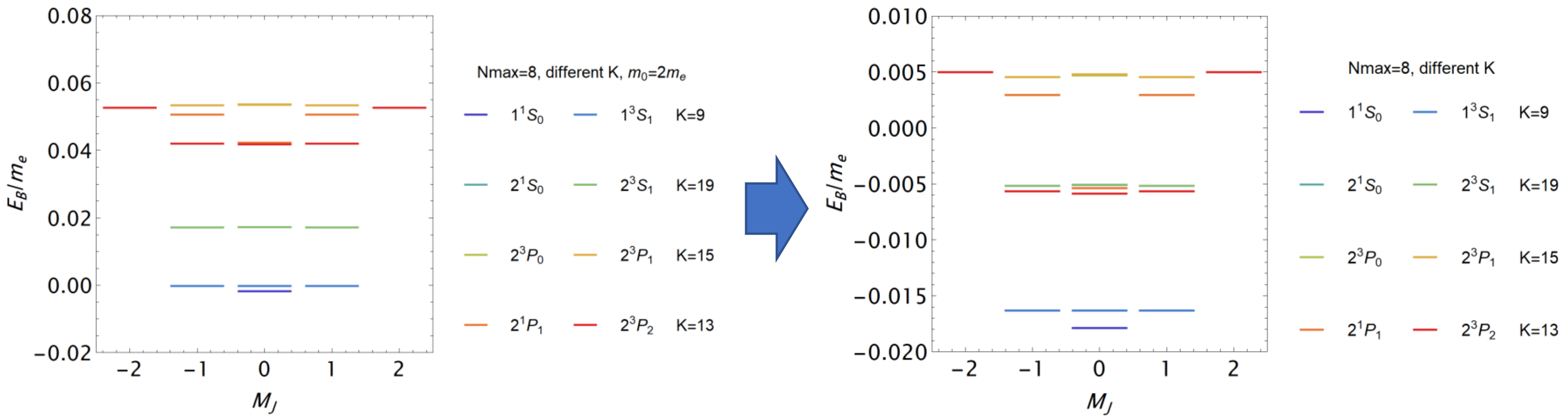
- No convergence with respect to K even after fixing transverse basis scale





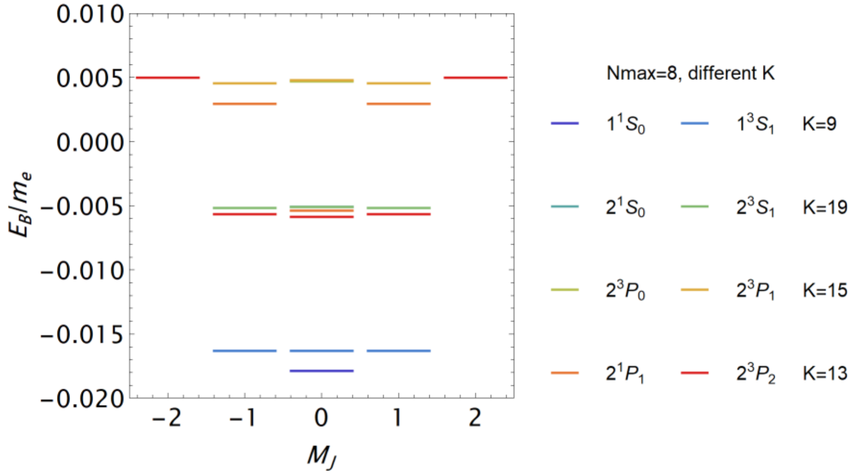
- Define P_{nm} to be the probability taken by lowest HO basis states
  - P_{nm} measures the agreement between the basis scale determined by rotational symmetry and that of the bound state
  - Smaller P_{nm} means the wave function is "stretched" to satisfy rotational symmetry
  - We choose the K which maximize P_{nm} as the optimal K for a given eigenstate
  - Fock sector truncation translates to UV cutoff $b\sqrt{N_{max}}$ and K_{opt}
  - Excited states have larger K_{opt} because of smaller transverse momentum

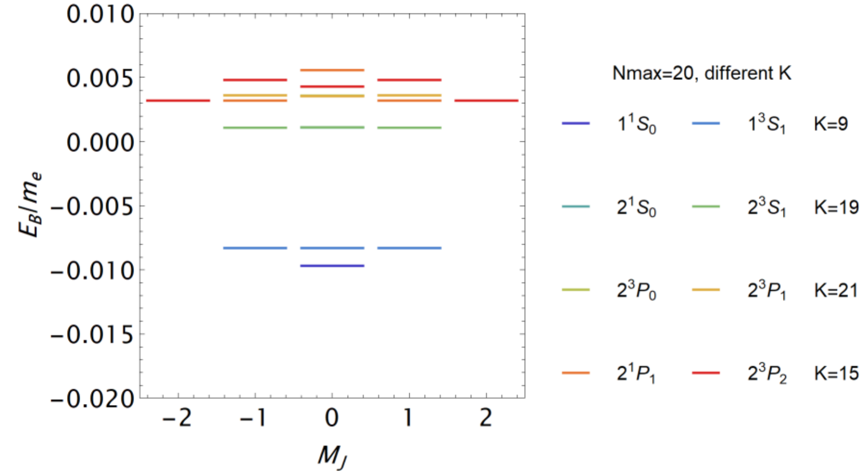
Correction for Finite Basis Effect



- In finite harmonic oscillator basis, IR cutoff > 0, so we use the ground state invariant mass without interaction as the reference for calculating E_B
- For p-wave states the ground state with M_j=2 is used

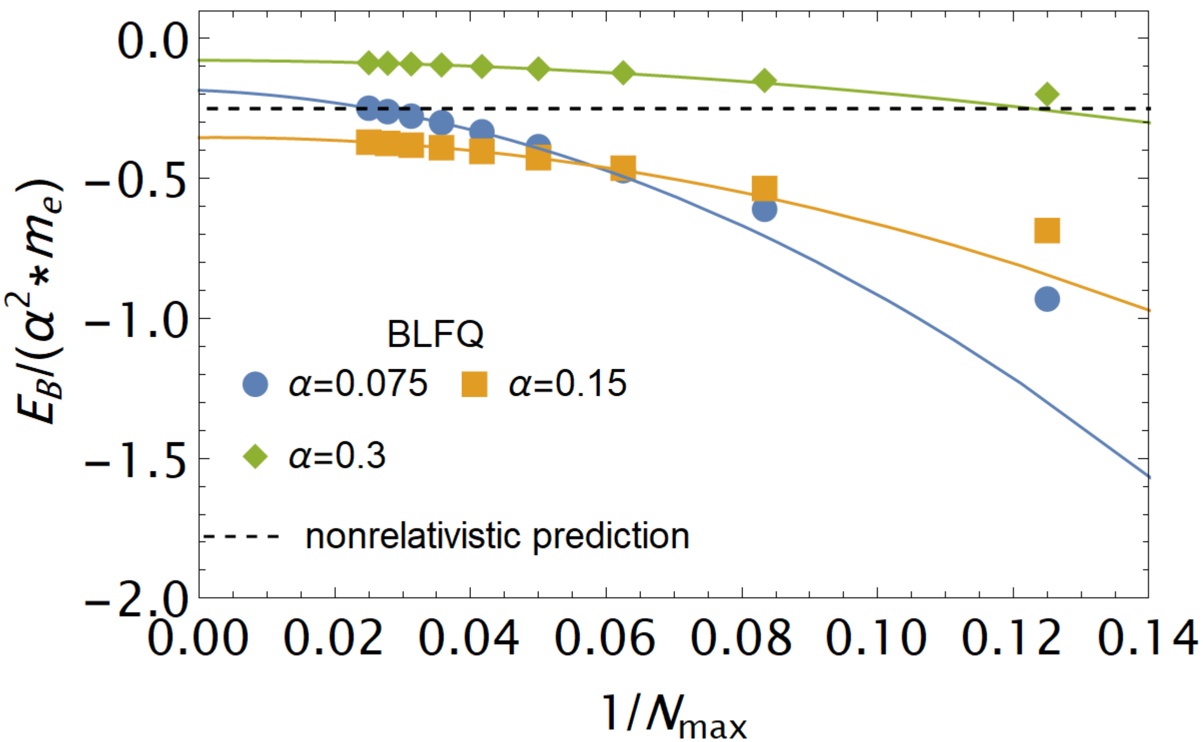
Positronium Mass Spectrum





- As N_{max} increases, rotational symmetry for excited states are restoring

Ground State Binding Energy at Different α



- Agreement with nonrelativistic quantum mechanics results improves as α decreases

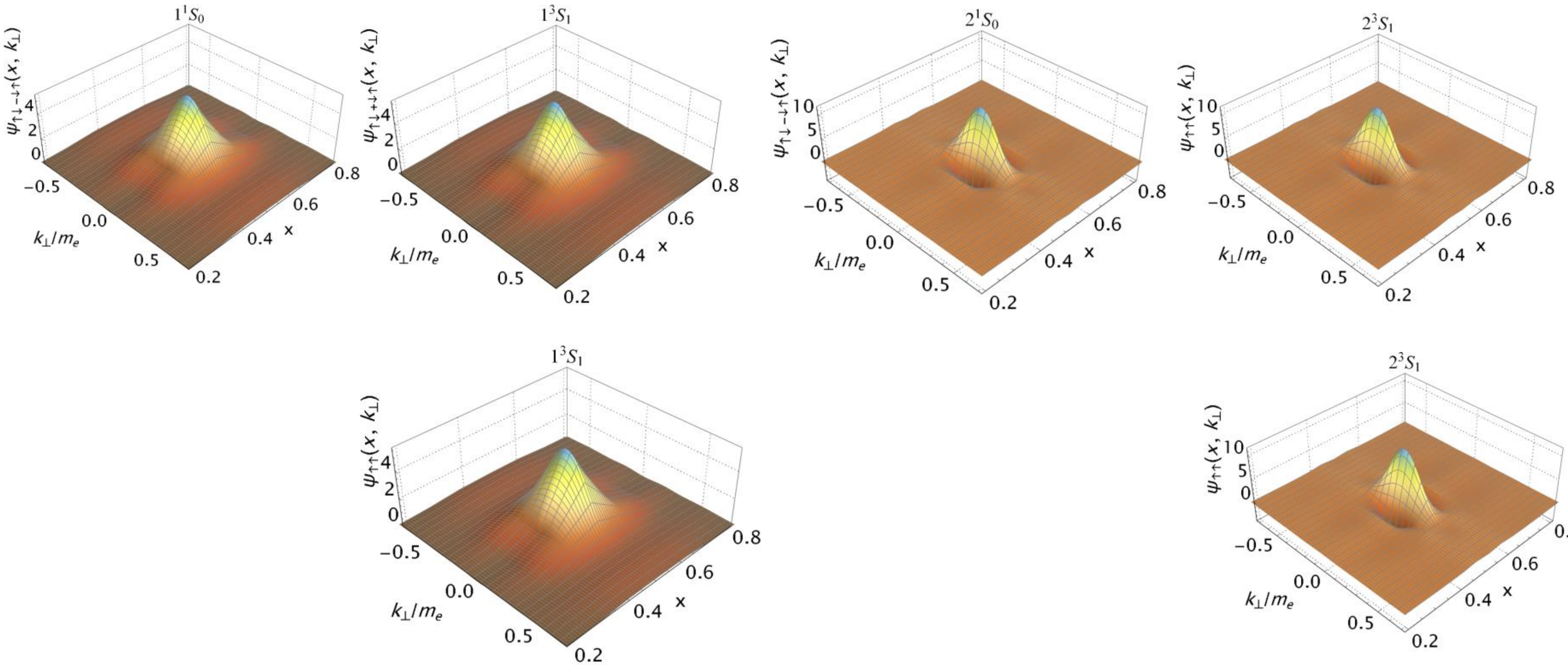
[Kaiyu Fu et al, in preparation]

Wave Functions for S-Wave States

N_{max}=8

M_J=0

M_J=1



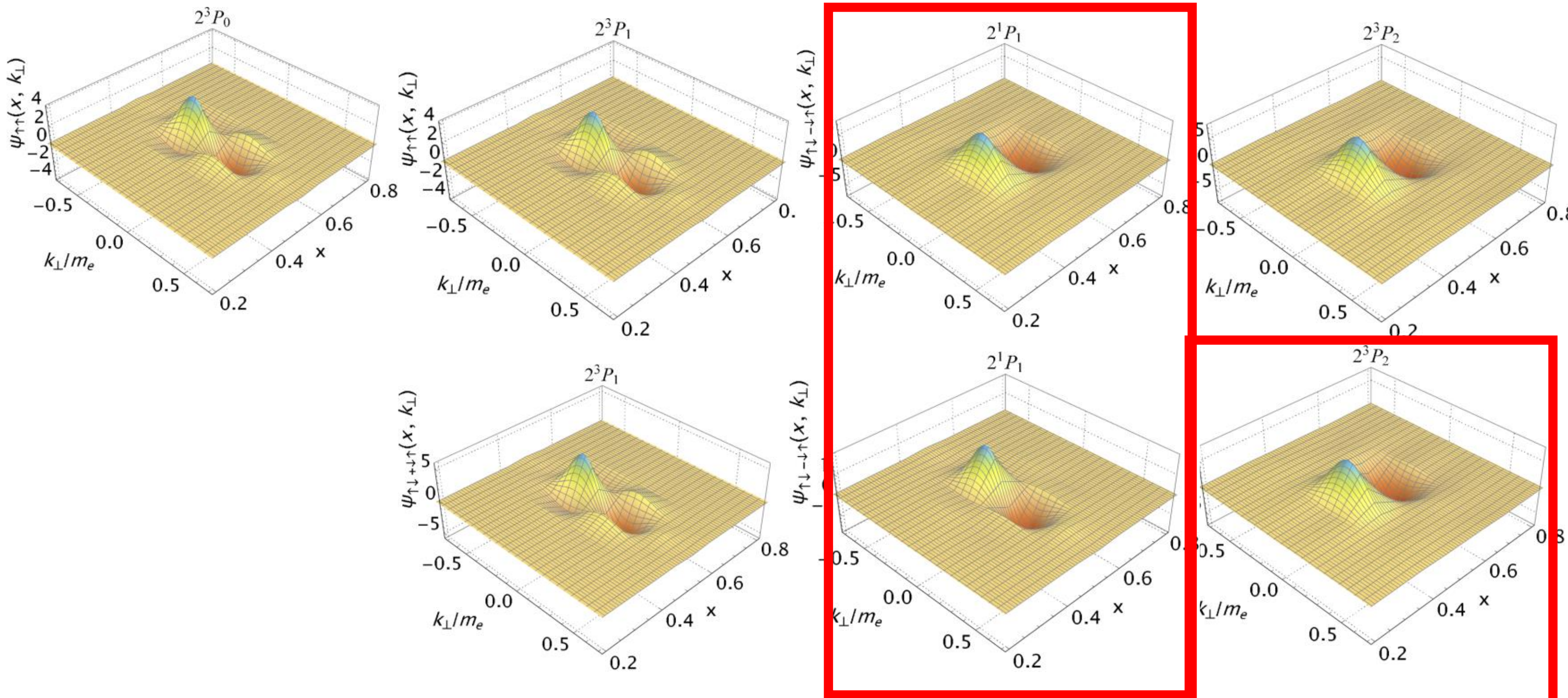
- Wave functions in $|e^+e^-\rangle$ Fock sector, dominant helicity component
- Nodal structure visible in radially excited states

[Kaiyu Fu et al, in preparation]

Wave Functions for p-Wave States

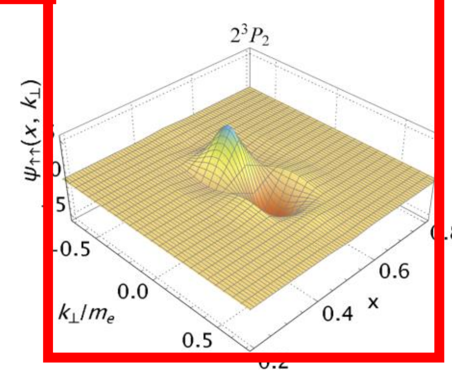
M_J=0

M_J=1



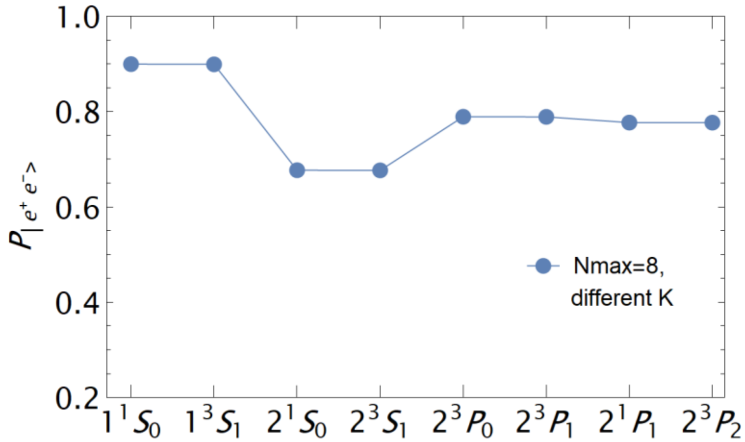
M_J=2

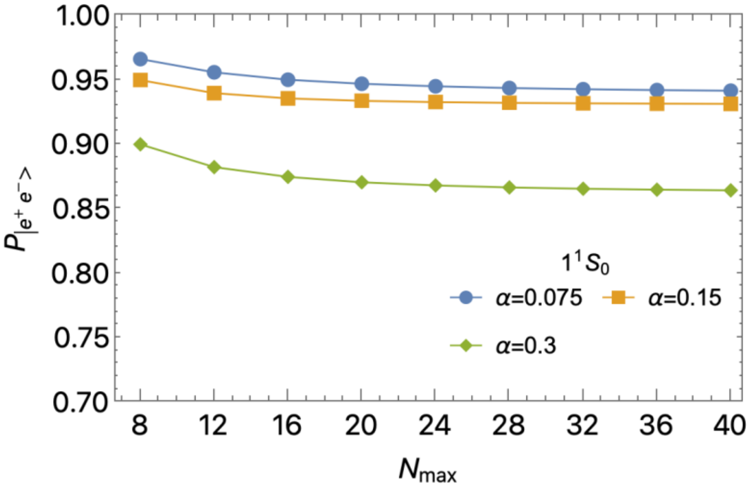
- Nodal structure visible in azimuthal directions
- Different orientation for different M_J states



[Kaiyu Fu et al, in preparation]

Probability Of $|e^+e^-\rangle$ Fock Sector



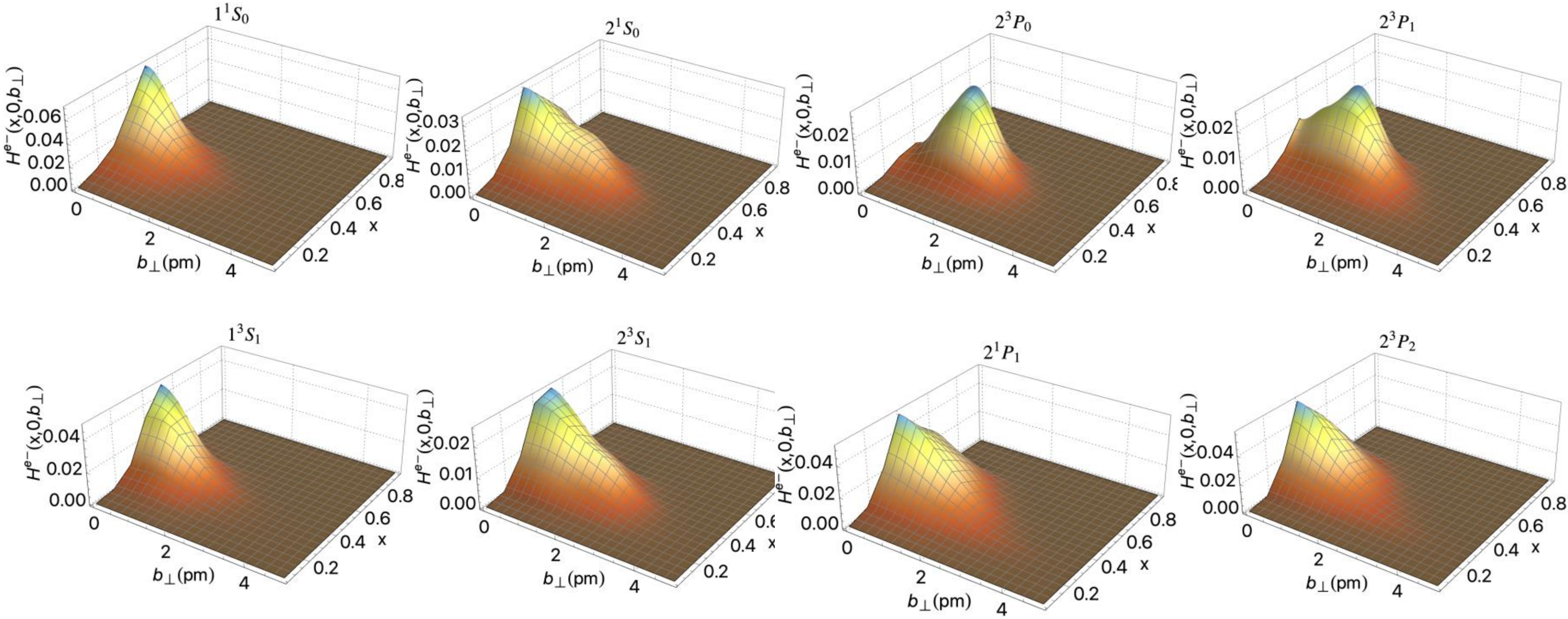


- Smaller probability of $|e^+e^-\rangle$ Fock sector for excited states
- Probability of $|e^+e^-\rangle$ Fock sector decreases as α increases

[Kaiyu Fu et al, in preparation]

Generalized Parton Distribution for Electron

M_J=0

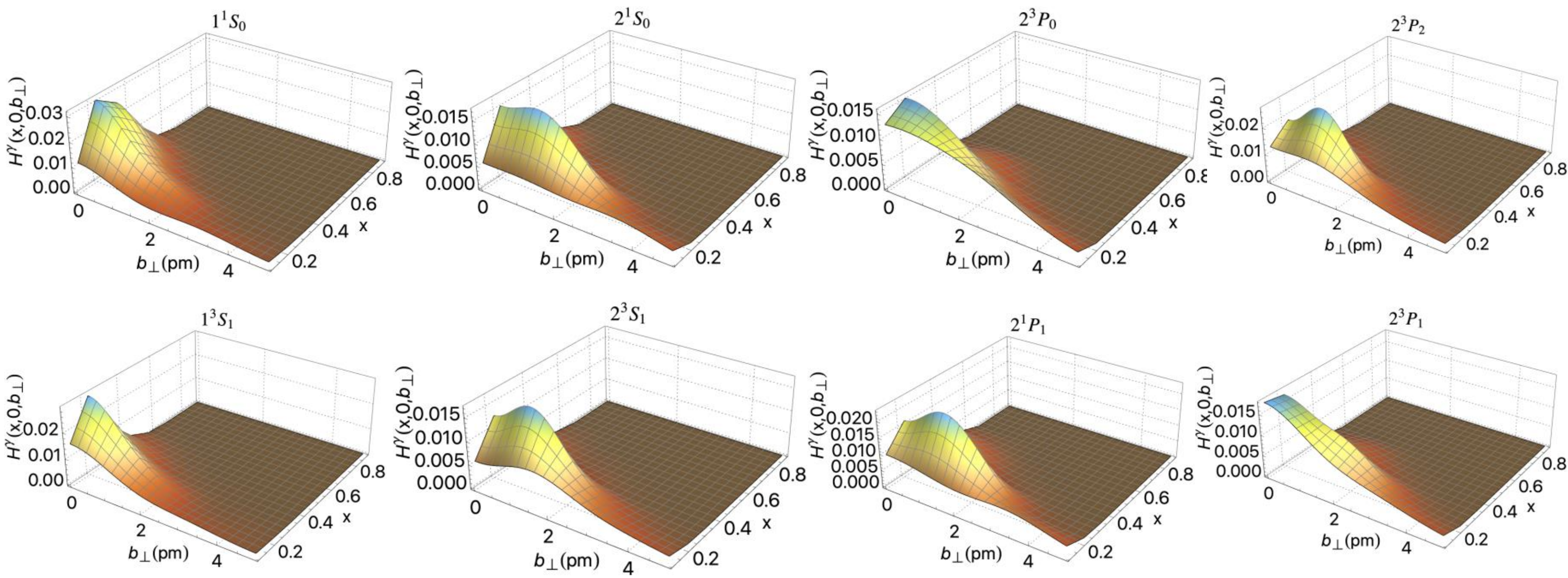


- GPD in impact parameter space measures the parton distribution in transverse coordinate space
- Both $|e^+e^-\rangle$ and $|e^+e^-\gamma\rangle$ Fock sectors contribute

[Kaiyu Fu et al, in preparation]

Generalized Parton Distribution for Photon

M_J=0



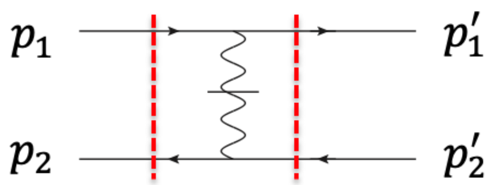
- “Ridge” structure in small-x region, which is absent in physical electron

[Kaiyu Fu et al, in preparation]

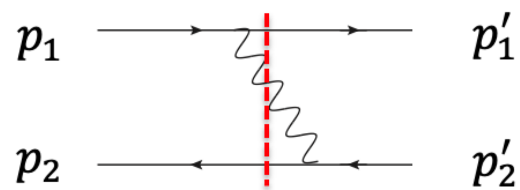
UV Cutoff for Instantaneous Photon b_{inst}

- Mismatch between explicit and instantaneous photon interactions:

for instantaneous photon:
$p_{\text{rel}} = p_1 - p_2$ not limited



for explicit photon:
$p_{\text{rel}} = p_1 - p_2$ subject to N_{max} truncation

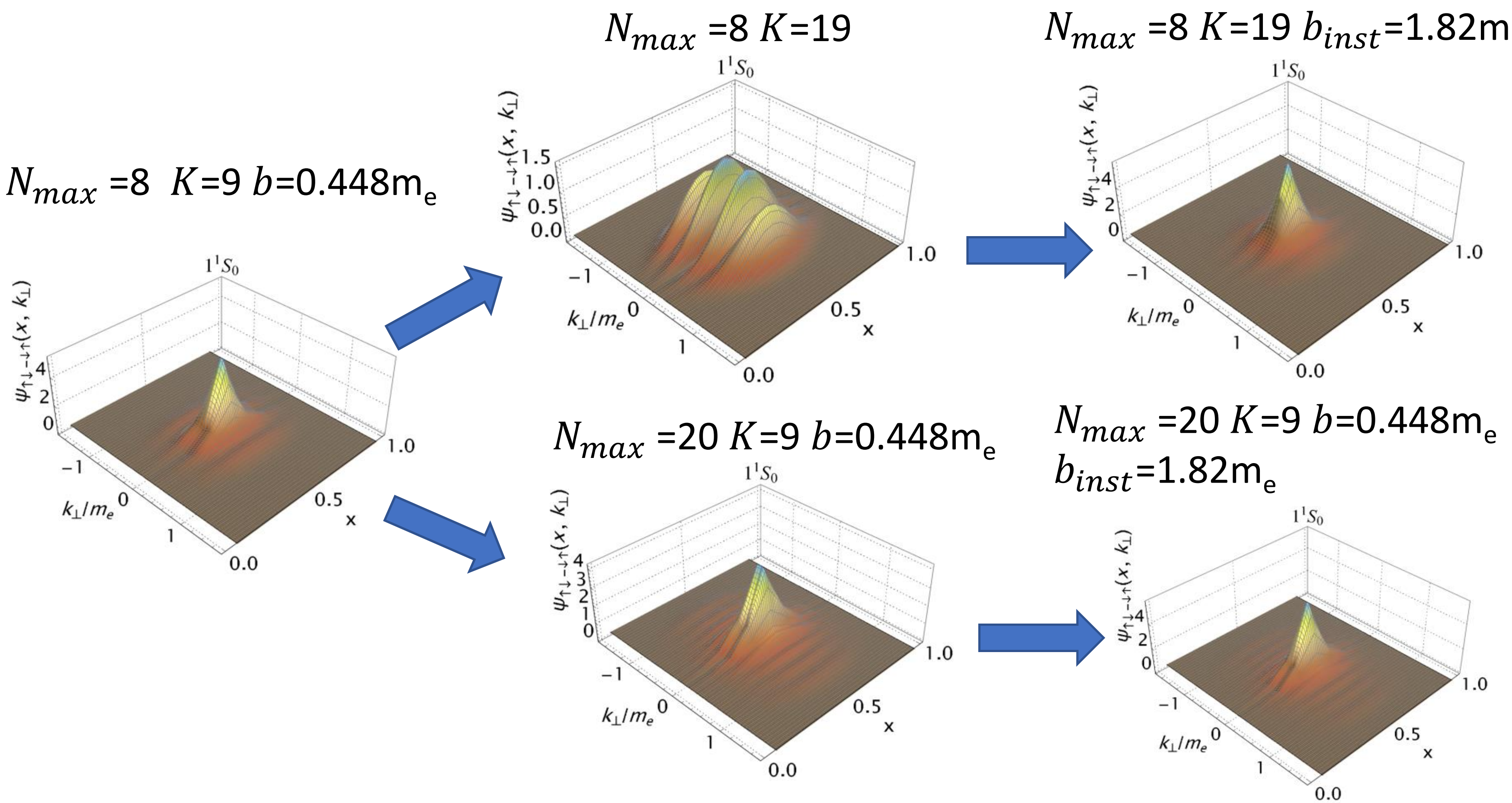


- Introduce cutoff parameter b_{inst} for instantaneous photon interaction:

$$V_{inst} \equiv \int \mathrm{d}^2 x^{\perp} \mathrm{d}x^- \, j^+ \frac{1}{(i\partial^+)^2} j^+ \quad \Longrightarrow \quad V_{inst} \times \exp\left(-\frac{p_\perp^2}{b_{\text{inst}}^2}\right)$$

“Extrapolate” Wave Functions

- By adjusting b_{inst} to match E_B at “optimal K”, one can “extrapolate” the wave functions beyond optimal K and transverse UV cutoff at the price of sacrifice the rotational symmetry a bit.



Summary of the Procedure

1. Solve a series of single electron system and generate mass counterterm table (from lowest N_{max} and K to those of the positronium system)

2. Solve positronium system in truncated bases

3. Fix basis scale parameter b for given N_{max} by examining rotational symmetry

4. Determine “optimal K” by requiring good match between the scale of bound state and basis

5. (Optional) Finite basis effect may need to be considered for mass spectrum

6. (Optional) Extrapolate the resulting wave function to required K and UV cutoff by using b_{inst}

Conclusions

1. Fock sector truncation translates to finite UV cutoff in transverse directions and finite resolution in longitudinal directions

2. Rotational symmetry plays key role in determining the scale associated with Fock sector truncation

3. Nonperturbative renormalization seems working

4. Basis states as eigenstates of J_z help a lot

Outlook

- Application in hadron systems
  - Jiangshan Lan, "Light meson structure with basis light-front quantization," Thursday, Dec. 2 at 9:05, McCartor Award Session
  - Siqi Xu, "Nucleon structure with dynamical gluon in light-front frame," Thursday, Dec. 2 at 14:30, Parallel Session 4-A

- Include $|e^+e^-e^+e^-\rangle$ Fock sector and study the scale obtained by rotational symmetry

- Include $|\gamma\rangle$ Fock sector and study the structure of virtual photon

Thank You!