From Instant to Light-Front
Opening LC2021

November 29, 2021

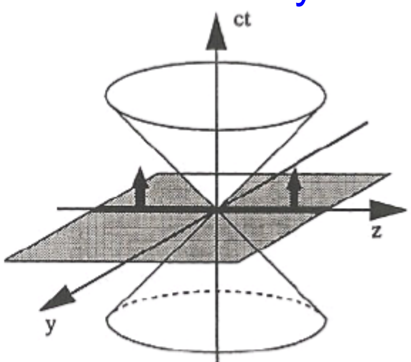
Chueng-Ryong Ji
North Carolina State University

Outline

- Dirac's proposition in 1949
- ILCAC WP in 2014
- EIC YR in 2021
- Interpolation between IFD and LFD
- QCD_{1+1} at large Nc limit
- Bridge between QCD and Quark Model

IFD

Instant Form Dynamics

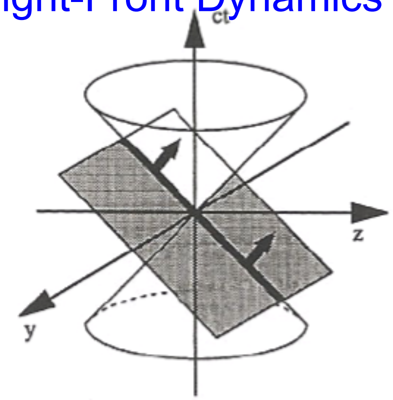


The instant form

1949

LFD

Light-Front Dynamics



The front form

Traditional approach
evolved from NR dynamics

Close contact with
Euclidean space

T-dept QFT, LQCD, IMF,
etc

Innovative approach
for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

Available online at www.sciencedirect.com

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Nuclear Physics B (Proc. Suppl.) 251–252 (2014) 165–174

NUCLEAR PHYSICS B PROCEEDINGS SUPPLEMENTS

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LIGHT-FRONT QUANTUM CHROMODYNAMICS
A framework for the analysis of hadron physics

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[a]*Member of ILCAC, Inc.*

Developing predictions for tests at the new and upgraded hadron experimental facilities

JLAB,

LHC,

J-PARC,

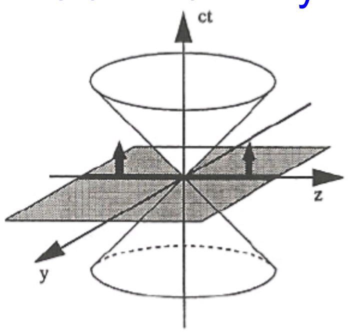
GSI-FAIR.

EIC CD-1 is Approved

June 25, 2021

IFD

Instant Form Dynamics

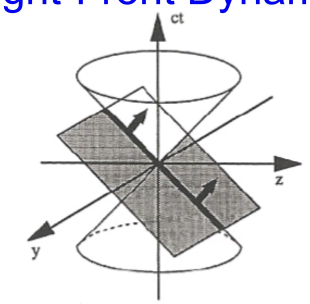


The instant form

1949

LFD

Light-Front Dynamics



The front form

Can IFD and LFD be linked?

K. Hornbostel, PRD45,3781(1992)

"Nontrivial vacua from equal time to the light cone"

.....

C.Ji, Z.Li,B.Ma & A.Suzuki, PRD98,036017(2018)

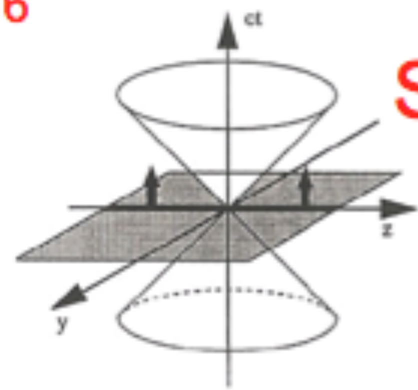
"Interpolating quantum electrodynamics between instant and front forms"

LC2018 JLAB Vacuum Session May, 5, 2018

Correspondence between IFD and LFD: Vacuum and related issues	*Chueng R. Ji*	
Auditorium, Jefferson Lab - CEBAF Center	08:30 - 08:55	
Contrasting light-front and canonical representations of quantum field theory	*W. Polyzou*	
Auditorium, Jefferson Lab - CEBAF Center	08:55 - 09:20	
Non-triviality of the vacuum in light-front quantization	*John Collins*	
Auditorium, Jefferson Lab - CEBAF Center	09:20 - 09:45	
Physics on the Light Front: The Light-Front Vacuum and Light-Front Holography	*Prof. Stanley Brodsky*	
Auditorium, Jefferson Lab - CEBAF Center	09:45 - 10:10	
coffee break		
Jefferson Lab - CEBAF Center	10:10 - 10:40	
Chirally constraining the proton light-cone wavefunction	*S. Beane*	
Auditorium, Jefferson Lab - CEBAF Center	10:40 - 11:05	
What ET thinks of the LF vacuum	*J. Hiller*	
Auditorium, Jefferson Lab - CEBAF Center	11:05 - 11:30	
discussion		
Auditorium, Jefferson Lab - CEBAF Center	11:30 - 12:20	

How many generators leave the time surface invariant?

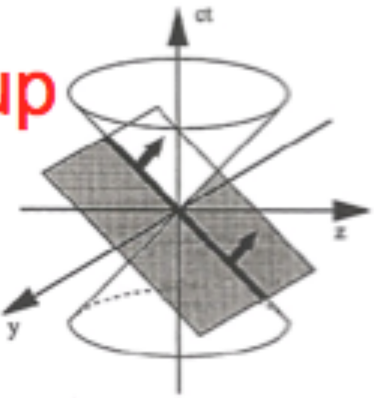
6



The instant form

Stability Group

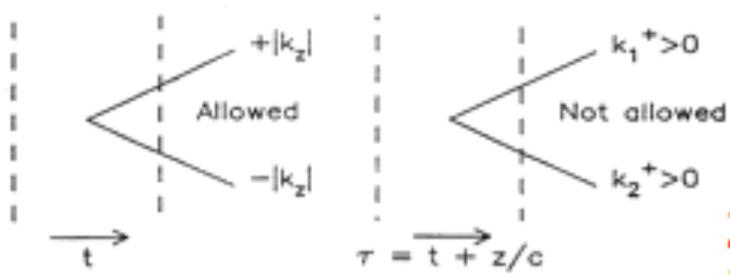
7



The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2}$$



(a) (b)

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$

Zero-modes

$k_1^+ = 0$, $k_2^+ = 0$

LC2021, Wednesday, Dec. 1, Plenary Session 8

8:30 - 9:00	M. Burkardt
9:00 - 9:30	W. Polyzou
9:30 - 10:00	J.-R. Hiller
10:00 - 10:30	P. Mannheim

B.Ma,C.Ji,PRD104,036004(2021)
"Interpolating 'tHooft model between instant and front forms"

Large Nc QCD in 1+1 dimensions

$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu} a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

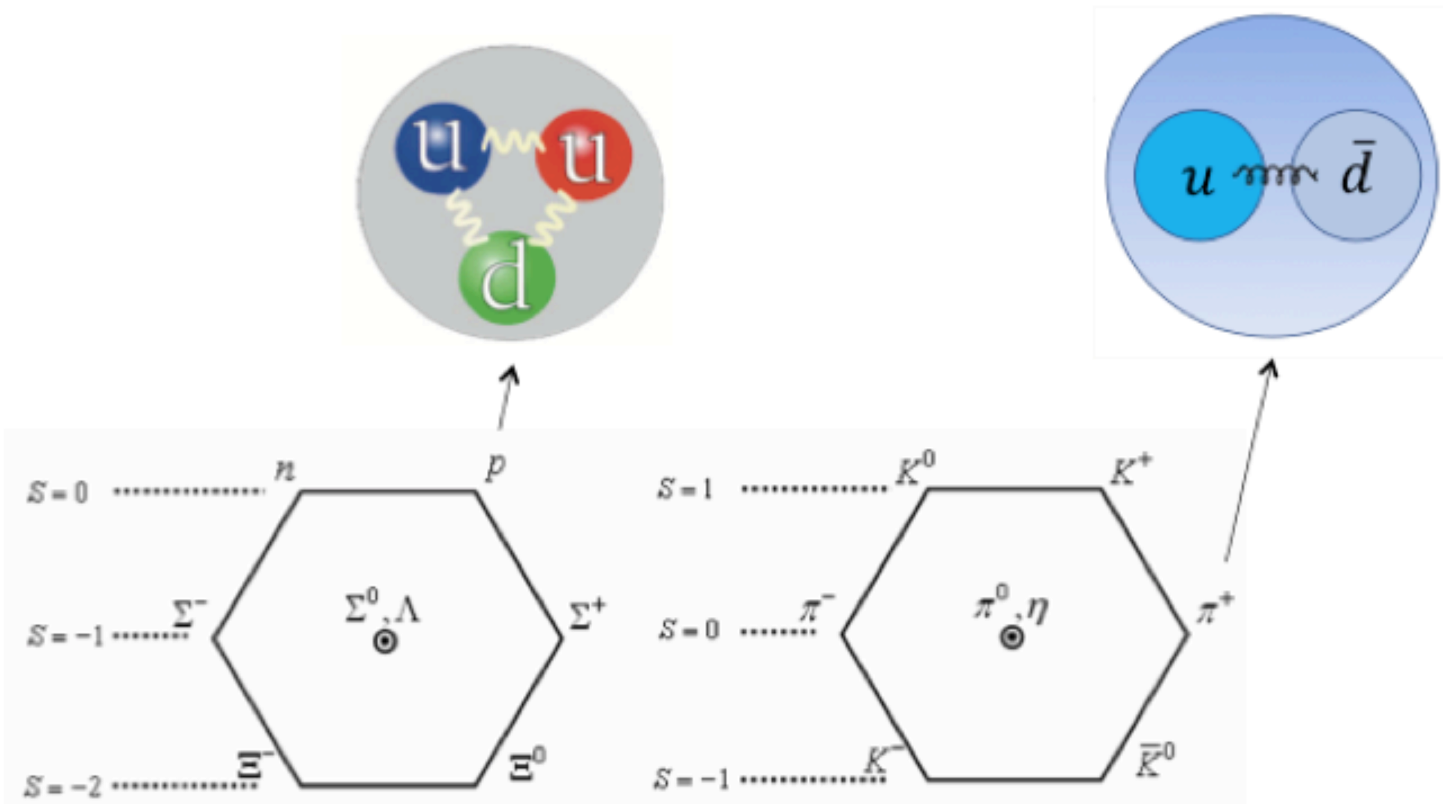
$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + gf^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ and mass m

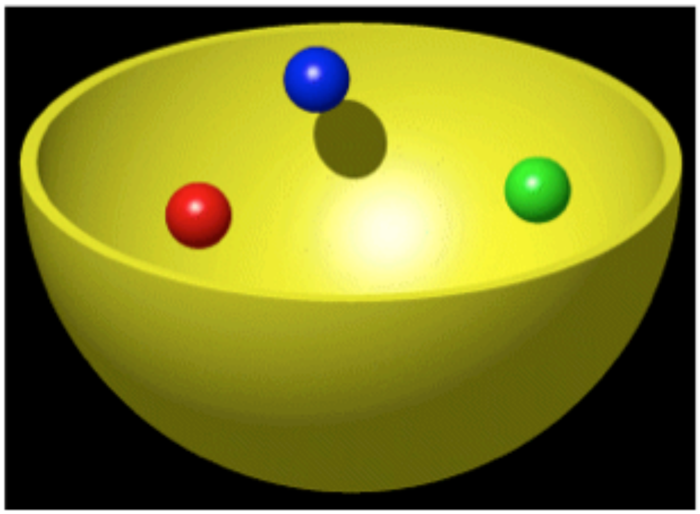
$$g \to 0, N_C \to \infty; \lambda \to finite$$

How do we understand the Quark Model in Quantum Chromodynamics?

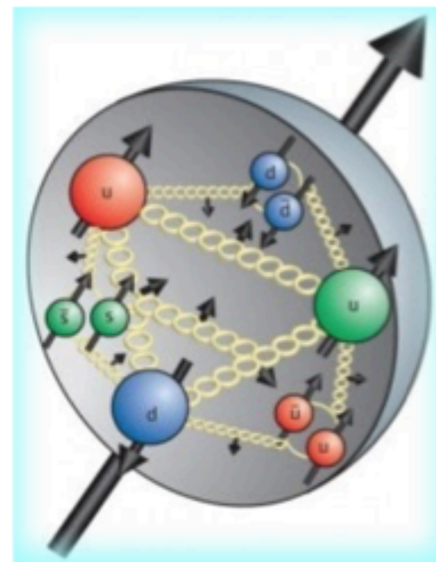


$$M_p = 938.272046 \pm 0.000021\ MeV$$

$$M_n = 939.565379 \pm 0.000021\ MeV$$



$\Longrightarrow$

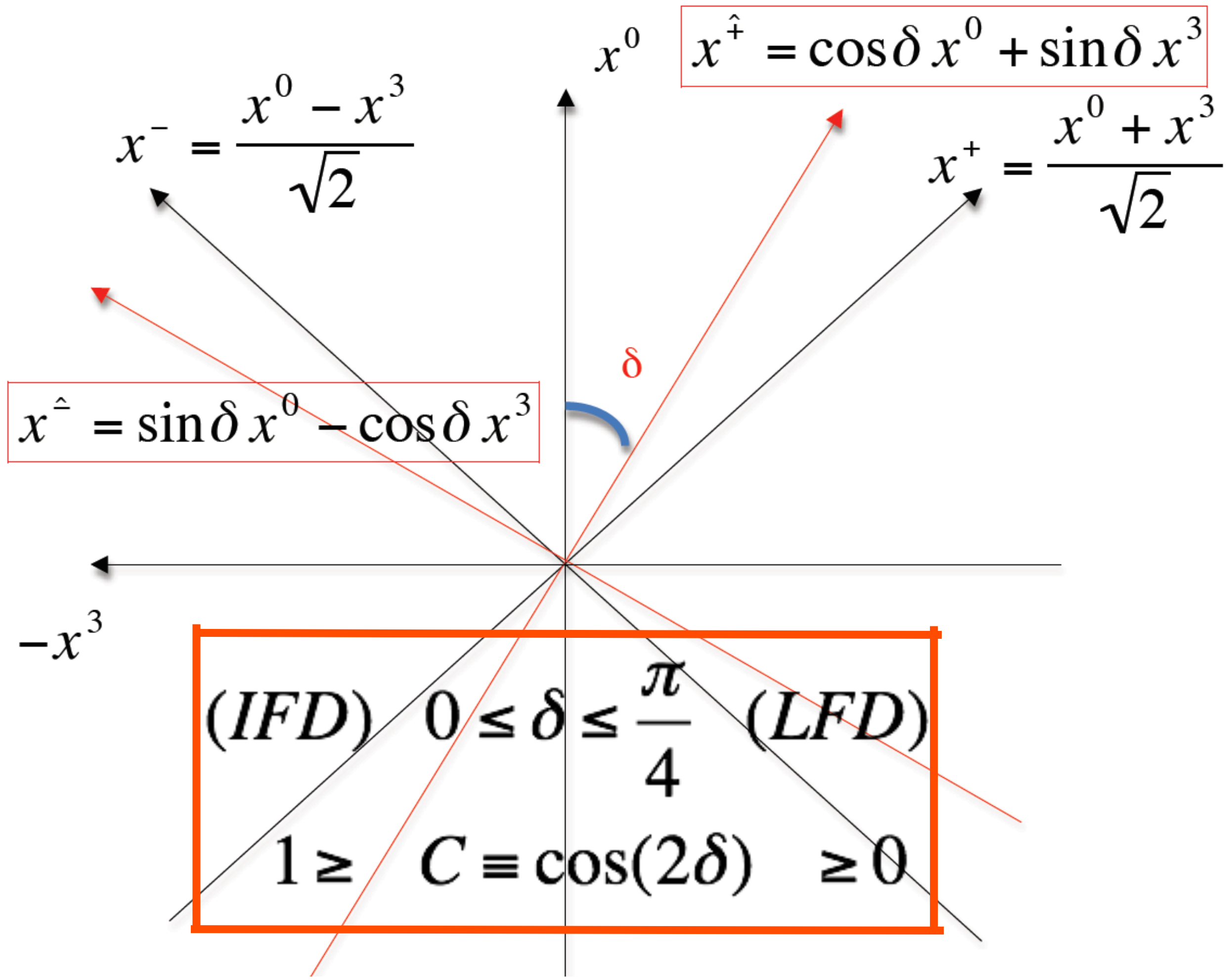


$$m_u = 2.3^{+0.7}_{-0.5}\ MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\ MeV$$

Short List of References

- G.'tHooft, NPB75,461(74) - LFD
- Y.Frishman, et al., PRD15(75) - Interpol Gauges IFD&LFD
- I.Bars&M.Green, PRD17,537(78) - IFD(formulation)
- A.Zhitnitsky, PLB165,405(85) - LFD(chiral sym breaking)
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)
- K.Hornbostel, Ph.D. Dissertation(88) - LFD(DLCQ)
- M.Burkardt, PRD53,933(96) - LFD(vacuum condensates)
- Y.Kalashnikova,A.Nefed'ev,Phys.-Usp.45,346('02)- IFD(rev)
- Y. Jia, et al., JHEP11, 151('17) - IFD(moving frame)
- Y. Jia, et al., PRD98, 054011('18) - IFD(quasi-PDFs)
- B.Ma&C.Ji, PRD104,036004('21) - Link IFD&LFD

Interpolation between IFD and LFD



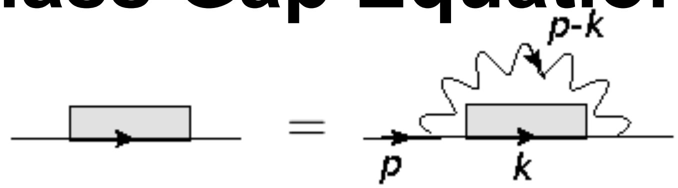
Interpolating Axial Gauge

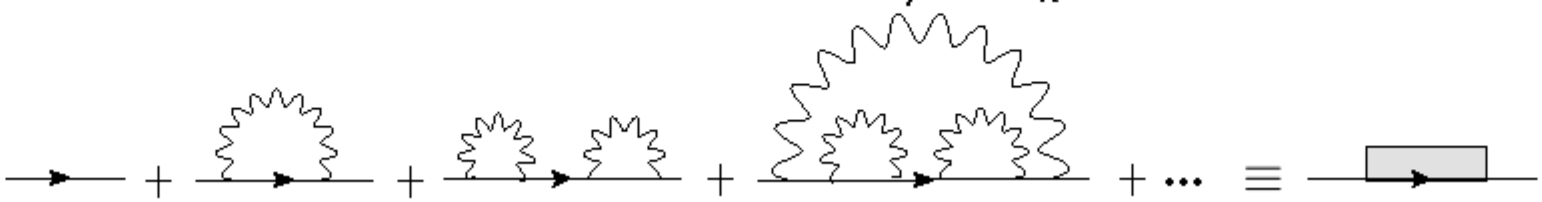
$$A^a_{\hat{-}} = 0$$

$$A^a_1 = 0 \qquad\qquad A^+_a = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\hat{-}} A^a_{\hat{+}})^2 + \bar{\psi}(i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m)\psi$$

Mass Gap Equation





$$\equiv i\,\Sigma(p)$$

$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not{p} - m + i\varepsilon}$$

$$\boxed{\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}}$$

Interacting Propagator

$$S(p) = \frac{1}{\not{p} - m - \sum(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not{p} - M(p) + i\varepsilon}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

Energy-Momentum Dispersion Relation

Free particle

$$E = \sqrt{p_z^2 + m^2}$$

$$\theta_f = \tan^{-1}(p_z / m)$$

$$\beta = p_z / E$$

$$= \sin\theta_f$$

$$= \tanh\eta$$

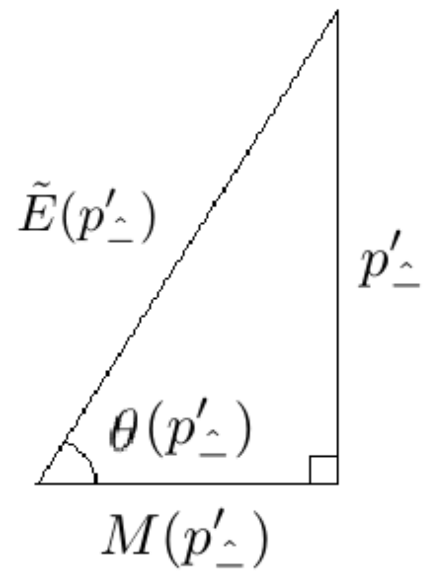
Interacting particle

$$\frac{F(p'_\perp)E(p'_\perp)}{\sqrt{C}} = \sqrt{p'^2_\perp + M(p'_\perp)^2} \equiv \tilde{E}(p'_\perp)$$

$$\theta(p'_\perp) = \theta_f(p'_\perp) + 2\xi(p'_\perp)$$

$$\begin{pmatrix} b^i(p'_\perp) \\ d^{+i}(p'_\perp) \end{pmatrix} = \begin{pmatrix} \cos\xi(p'_\perp) & -\sin\xi(p'_\perp) \\ \sin\xi(p'_\perp) & \cos\xi(p'_\perp) \end{pmatrix} \begin{pmatrix} b^i_f(p'_\perp) \\ d^{+i}_f(p'_\perp) \end{pmatrix}$$

$b^i_f |0\rangle = 0, d^i_f |0\rangle = 0$ vs. $b^i |\Omega\rangle = 0, d^i |\Omega\rangle = 0$



Interpolation

$$(E, p_z) \Rightarrow (p^+ / \sqrt{C}, p_\perp / \sqrt{C} \equiv p'_\perp)$$

Mass Gap Equation in Scaled Variables

$$\bar{p}'_{\hat{-}} = \frac{\bar{p}_{\hat{-}}}{\sqrt{\mathbb{C}}}, \quad \bar{E}' = \frac{\bar{E}}{\sqrt{\mathbb{C}}}, \bar{p}_{\hat{-}} = \frac{p_{\hat{-}}}{\sqrt{2\lambda}}, \quad \bar{E} = \frac{E}{\sqrt{2\lambda}}, \bar{m} = \frac{m}{\sqrt{2\lambda}}$$

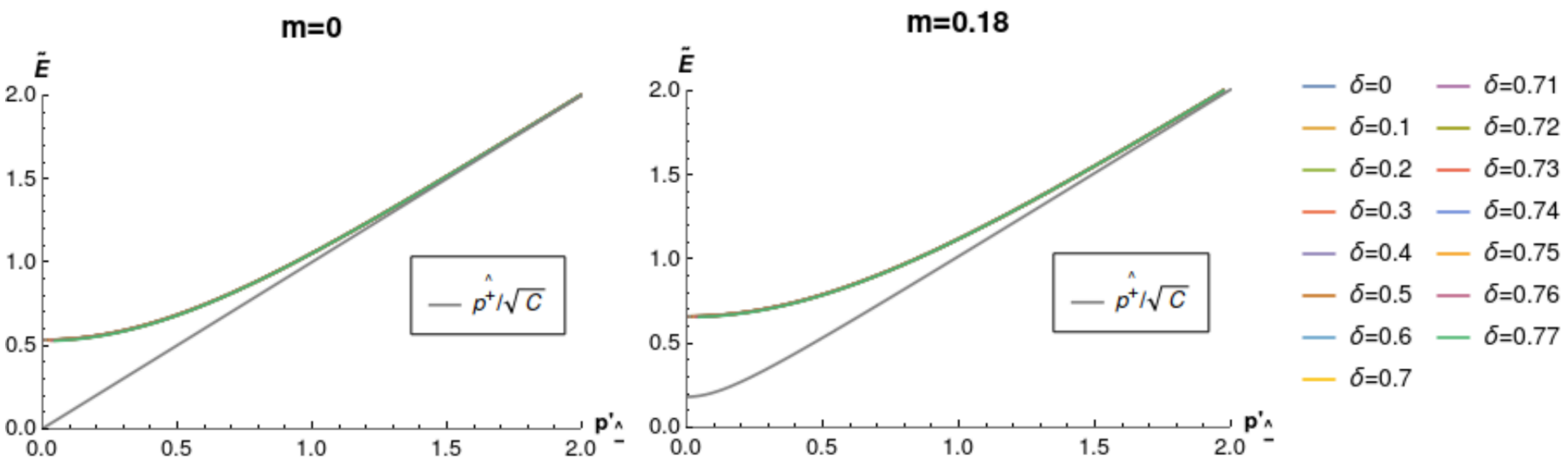
$$\bar{p}'_{\hat{-}} \cos\theta(\bar{p}'_{\hat{-}}) - \bar{m}\sin\theta(\bar{p}'_{\hat{-}}) = \frac{1}{4}\int \frac{d\bar{k}'_{\hat{-}}}{(\bar{p}'_{\hat{-}} - \bar{k}'_{\hat{-}})^2} \sin\left(\theta(\bar{p}'_{\hat{-}}) - \theta(\bar{k}'_{\hat{-}})\right)$$

$$\bar{E}'(\bar{p}'_{\hat{-}}) = \bar{p}'_{\hat{-}} \sin\theta(\bar{p}'_{\hat{-}}) + \bar{m}\cos\theta(\bar{p}'_{\hat{-}}) + \frac{1}{4}\int \frac{d\bar{k}'_{\hat{-}}}{(\bar{p}'_{\hat{-}} - \bar{k}'_{\hat{-}})^2} \cos\left(\theta(\bar{p}'_{\hat{-}}) - \theta(\bar{k}'_{\hat{-}})\right)$$

$$\frac{p_{\hat{-}}}{\mathbb{C}} \cos\theta(p_{\hat{-}}) - \frac{m}{\sqrt{\mathbb{C}}}\sin\theta(p_{\hat{-}}) = \frac{\lambda}{2}\int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2} \sin\left(\theta(p_{\hat{-}}) - \theta(k_{\hat{-}})\right)$$

$$E(p_{\hat{-}}) = p_{\hat{-}} \sin\theta(p_{\hat{-}}) + \sqrt{\mathbb{C}}m\cos\theta(p_{\hat{-}}) + \frac{\mathbb{C}\lambda}{2}\int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2} \cos\left(\theta(p_{\hat{-}}) - \theta(k_{\hat{-}})\right)$$

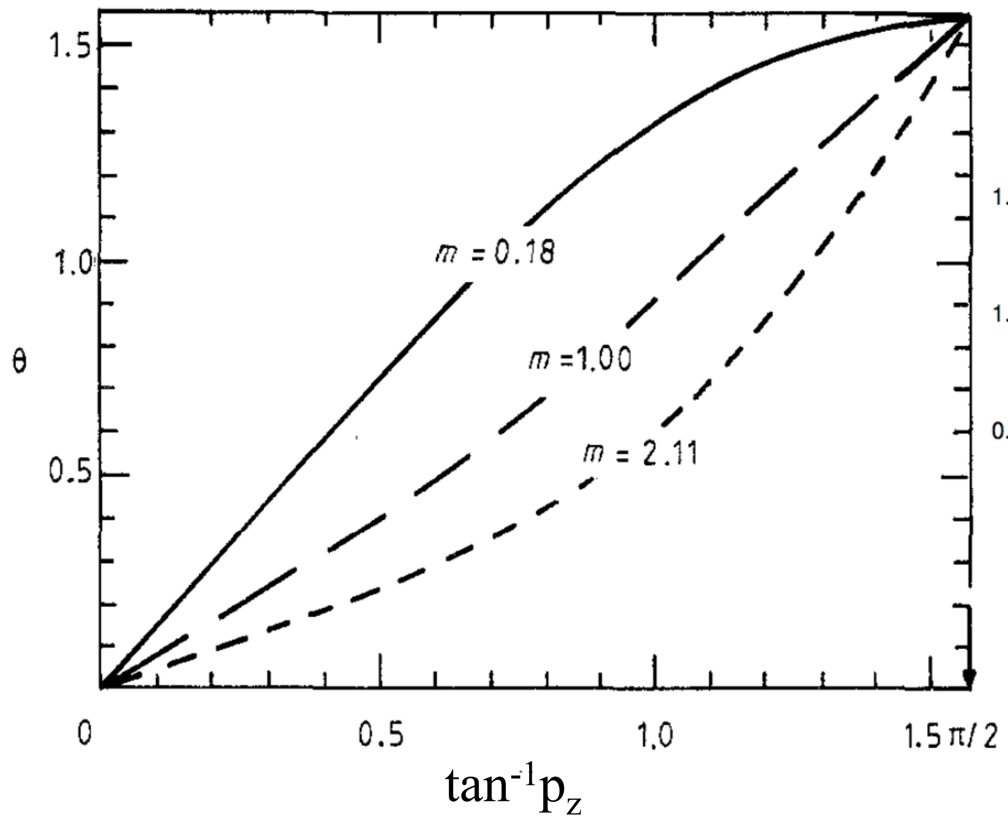
Mass Gap Solutions

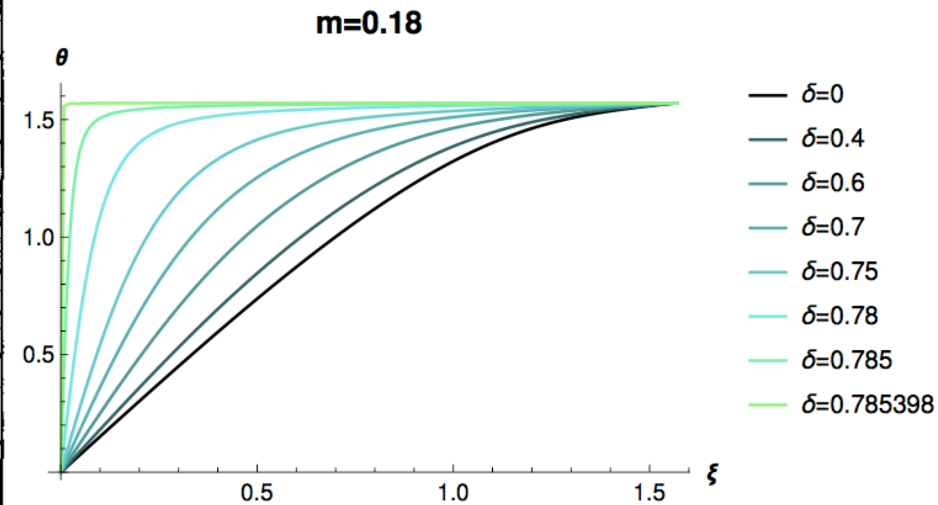


$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

$$m \lesssim 0.56$$

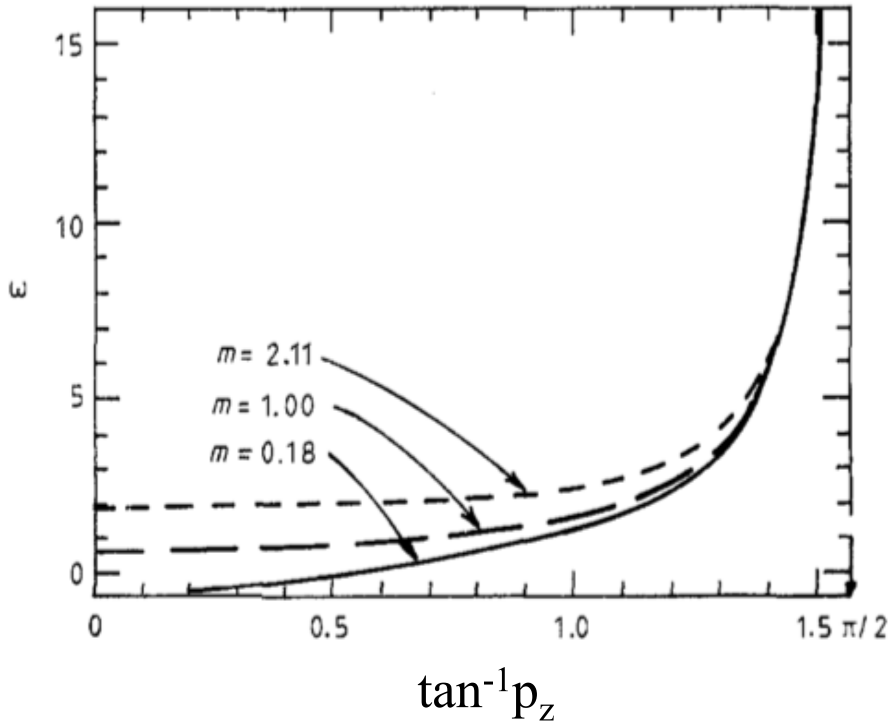


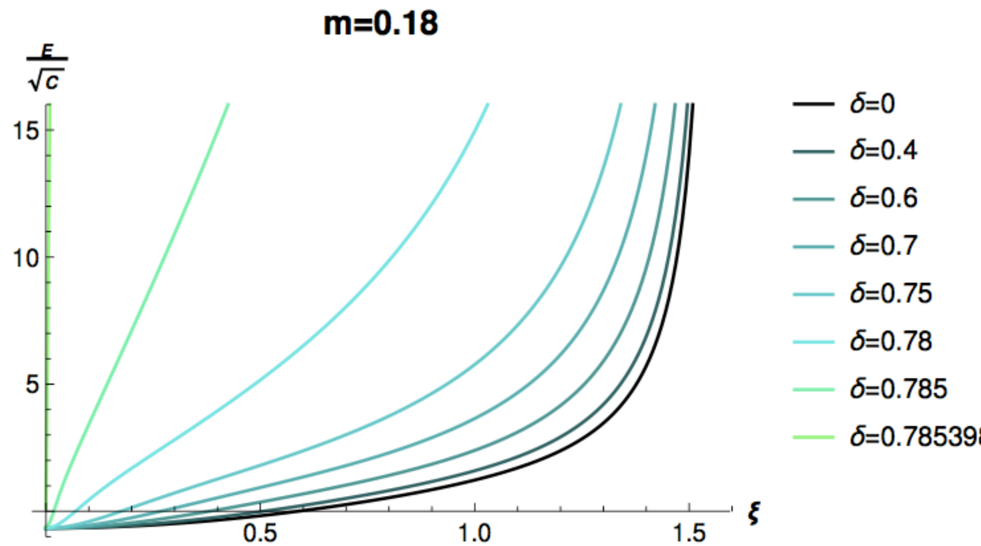


$$\xi = \tan^{-1} p_{\hat{-}}$$

- M.Li, et al., JPG13, 915(87) - IFD(rest frame)

$$\frac{0.785398}{\pi/4} \approx 0.999999792$$



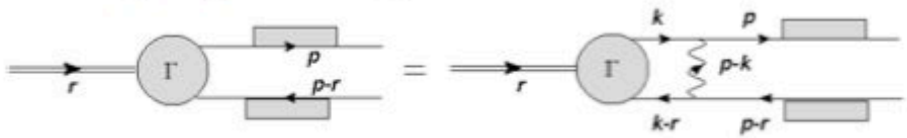


$$\xi = \tan^{-1} p_{\hat{-}}$$

- M.Li, et al., JPG13, 915(87) - IFD(rest frame)

BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$

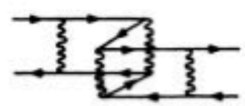


$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$

$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$

$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$

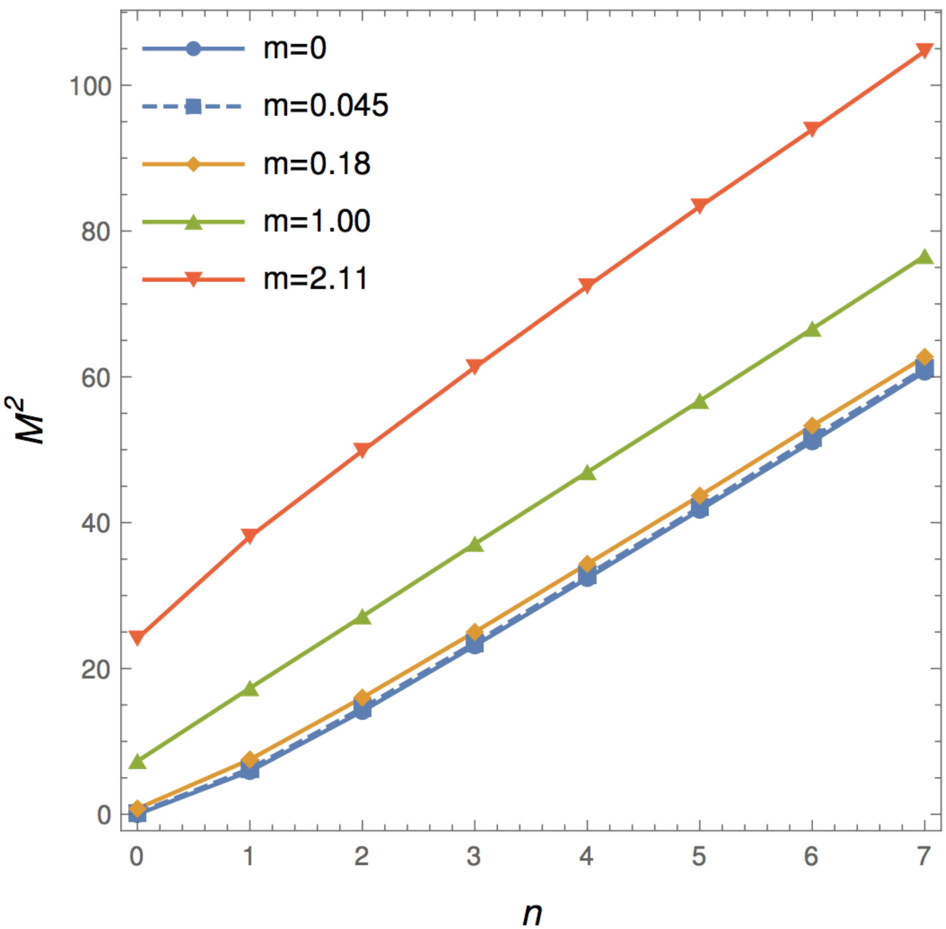
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$



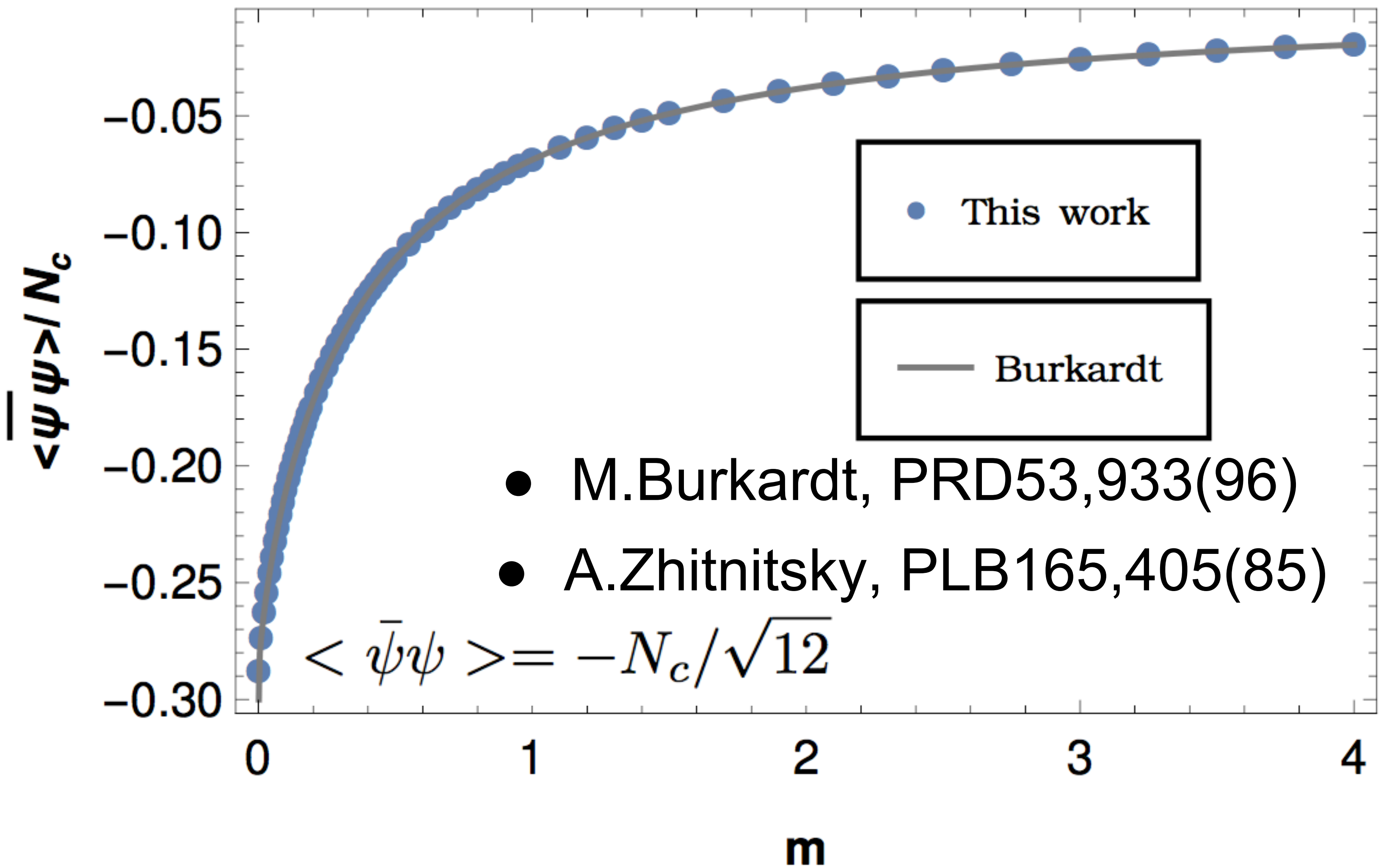
LFD $$\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right]\phi(x) = -2\lambda \fint_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

Meson Spectroscopy

- G.'tHooft, NPB75, 461(74) - LFD

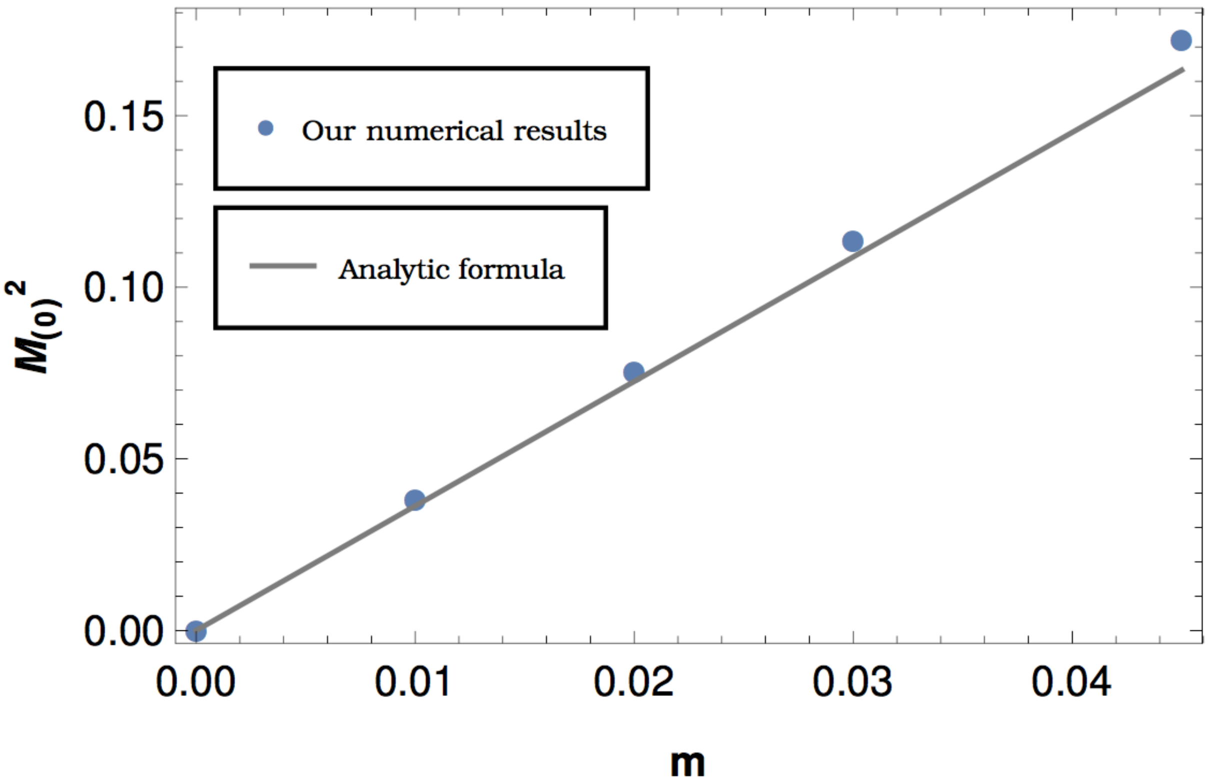


- M.Li, et al., JPG13, 915(87) - IFD (rest frame)
- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)



−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
0
1
2
3
4
m
$<\bar{\psi}\psi>/N_c$
This work
Burkardt
● M.Burkardt, PRD53,933(96)
● A.Zhitnitsky, PLB165,405(85)
$<\bar{\psi}\psi>=-N_c/\sqrt{12}$

Gell-Mann - Oaks - Renner Relation

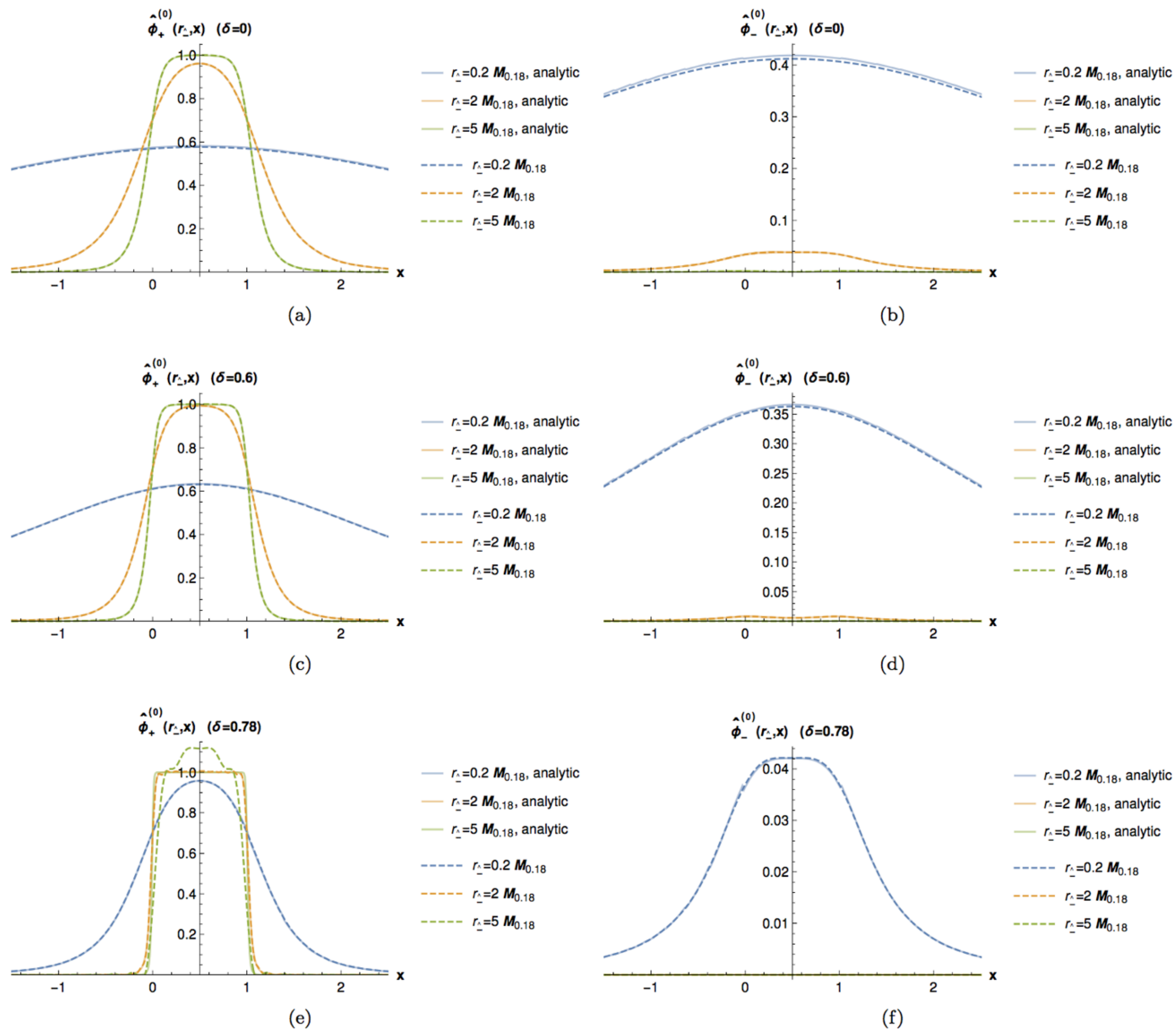


$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

Meson Ground-state Wave-function for m=0 case

$$\hat{\phi}^{(0)}_{\pm}(r_{\hat{-}}, p_{\hat{-}}) = \frac{1}{2}\left(\cos\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) - \theta(p_{\hat{-}})}{2} \pm \sin\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) + \theta(p_{\hat{-}})}{2}\right)$$



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-,0]\psi(0)|P_n^-, P^+\rangle_C,$$

$$\mathcal{W}[\xi^-,0] = \mathcal{P}\left[\exp\left(-ig_s\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

A⁺=0 Gauge in LFD

Quasi-PDFs

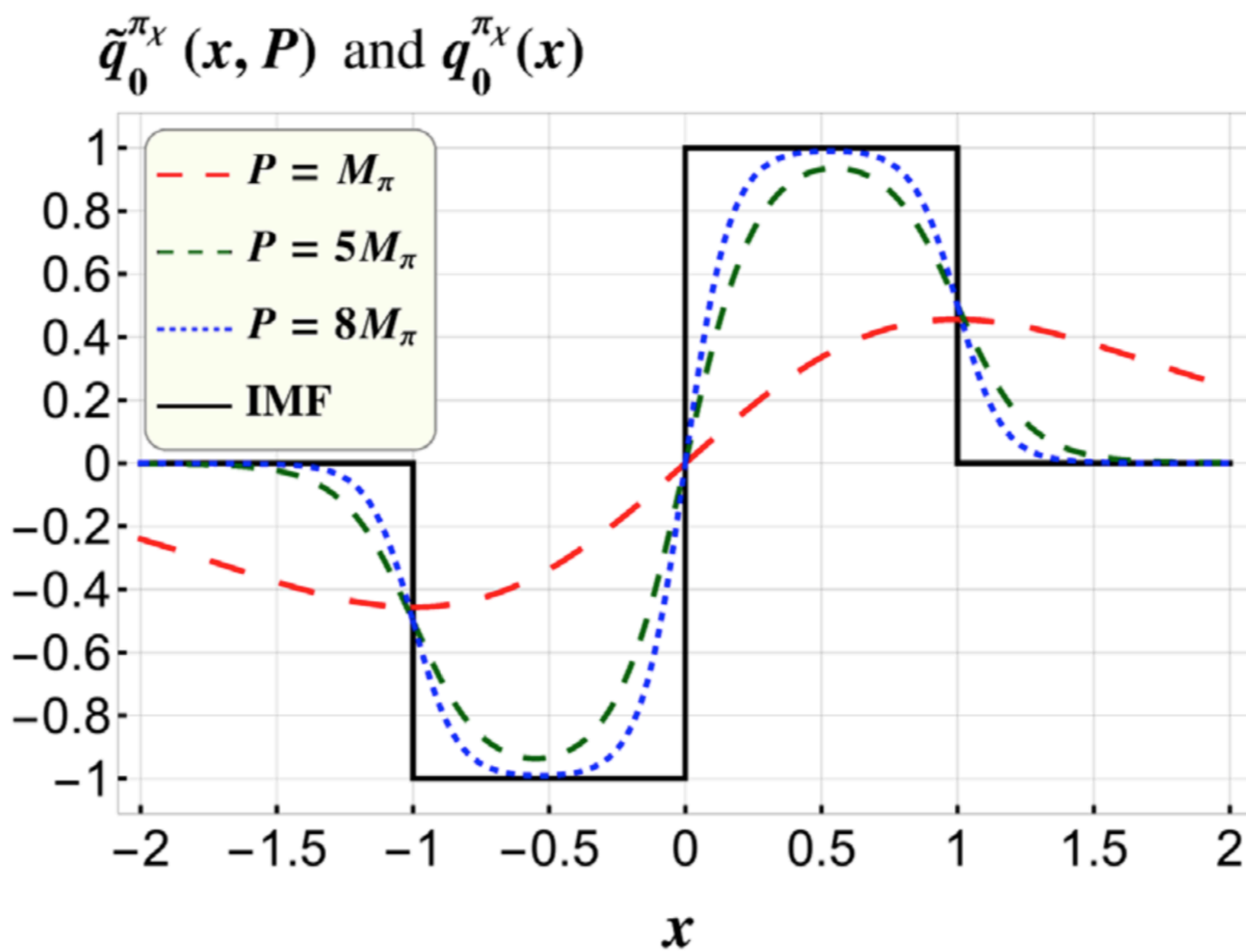
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \, \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

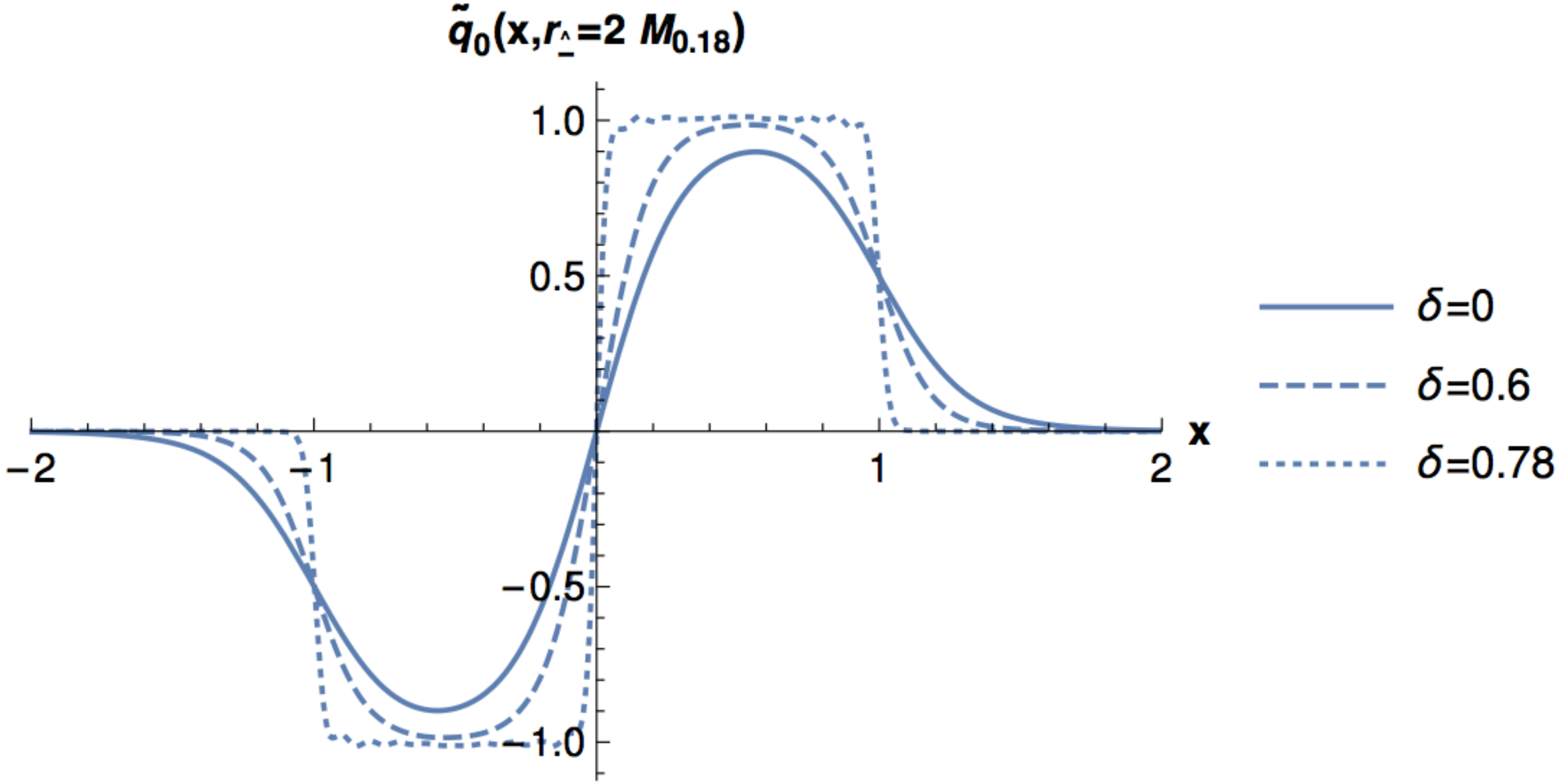
$$\mathcal{W}[x^{\hat{-}},0] = \mathcal{P}\left[\exp\left(-ig\int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

Interpolating dynamics

- Y. Jia, et al., PRD98, 054011('18)
  - IFD (quasi-PDFs)



- B.Ma&C.Ji, PRD104,036004(2021)



Extended Wick Rotation

$$p^0 \to \tilde{P}^0 = ip^0 \quad (\delta = 0)$$

$$For \quad 0 < \delta < \pi / 4 \, ,$$

$$p^{\hat{+}} / \sqrt{C} \to \tilde{P}^{\hat{+}} / \sqrt{C} = ip^{\hat{+}} / \sqrt{C} \, .$$

Correspondence to Euclidean Space

$$p'^2_{\hat{\perp}} = p^2_{\hat{\perp}} / C \leftrightarrow -\tilde{P}^2$$

Conclusions and Outlook

- QCD(1+1) in large Nc "tHooft model' [as well as QED(3+1)] was interpolated between IFD and LFD.
- Interpolation angle independent energy function, chiral condensate, mass spectra, etc. were found indicating the persistence of nontrivial vacuum even in LFD.
- Applying to quasi-PDFs in the interpolating formulation, we note a possibility of utilizing not only the reference frame dependence but also the interpolation angle dependence to get an alternative effective approach to the LFD's PDFs.
- QCD(3+1) extension is highly non-trivial but should be explored.
- Investigation of the link between IFD and LFD appears useful.