Canonical Approach for Extreme QCD

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Contents based on

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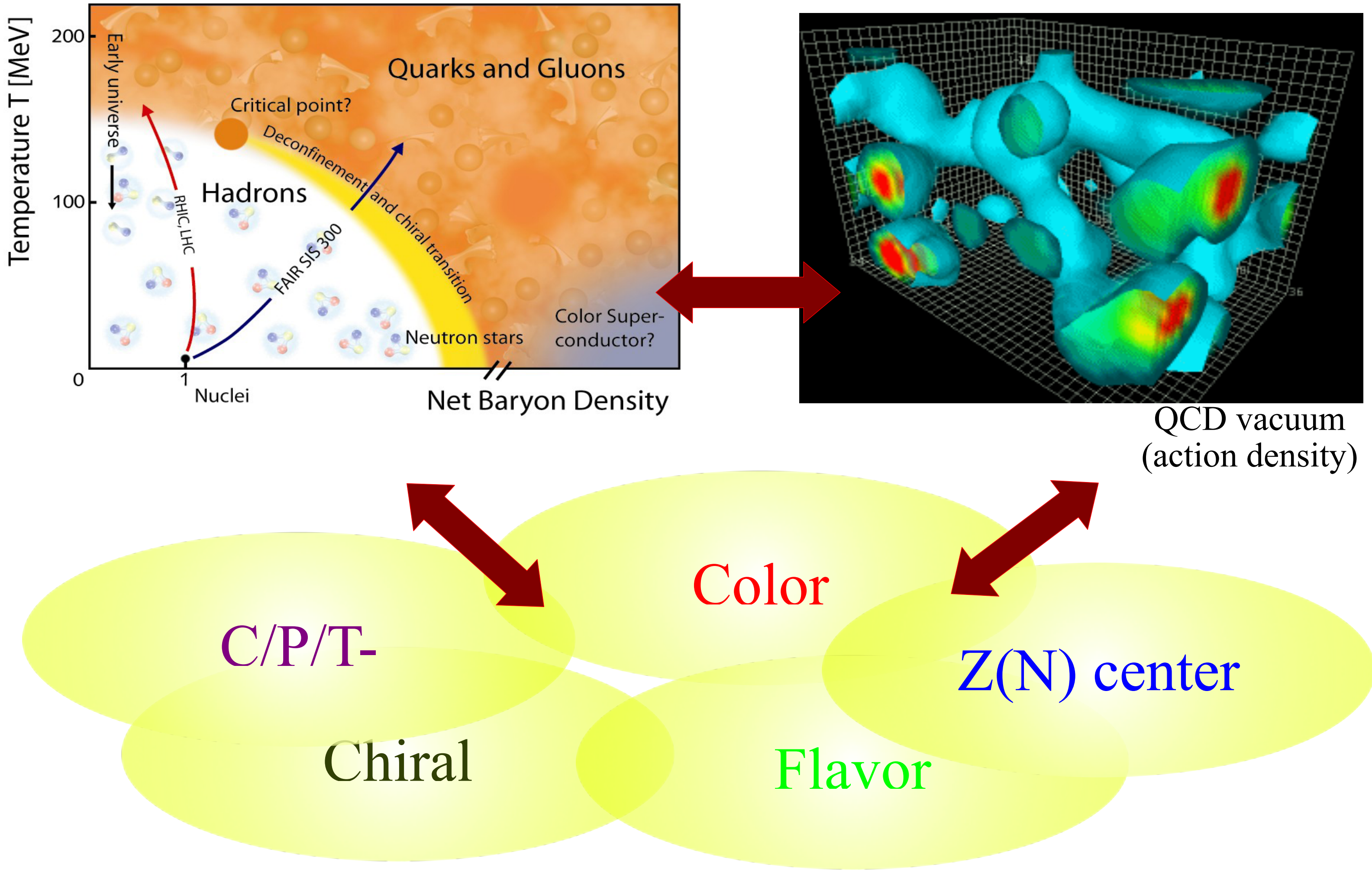
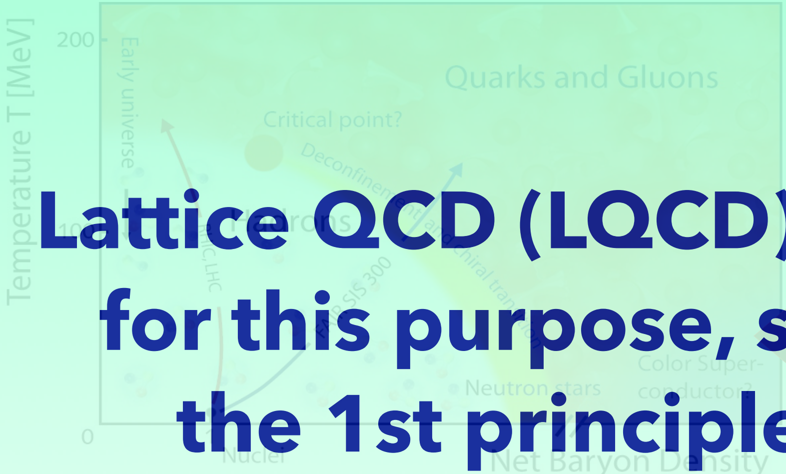
Introduction

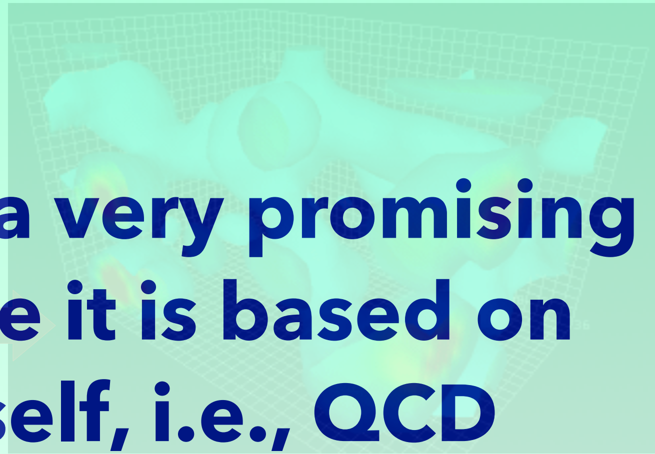
- QCD has complicated phase structure: Not fully understood



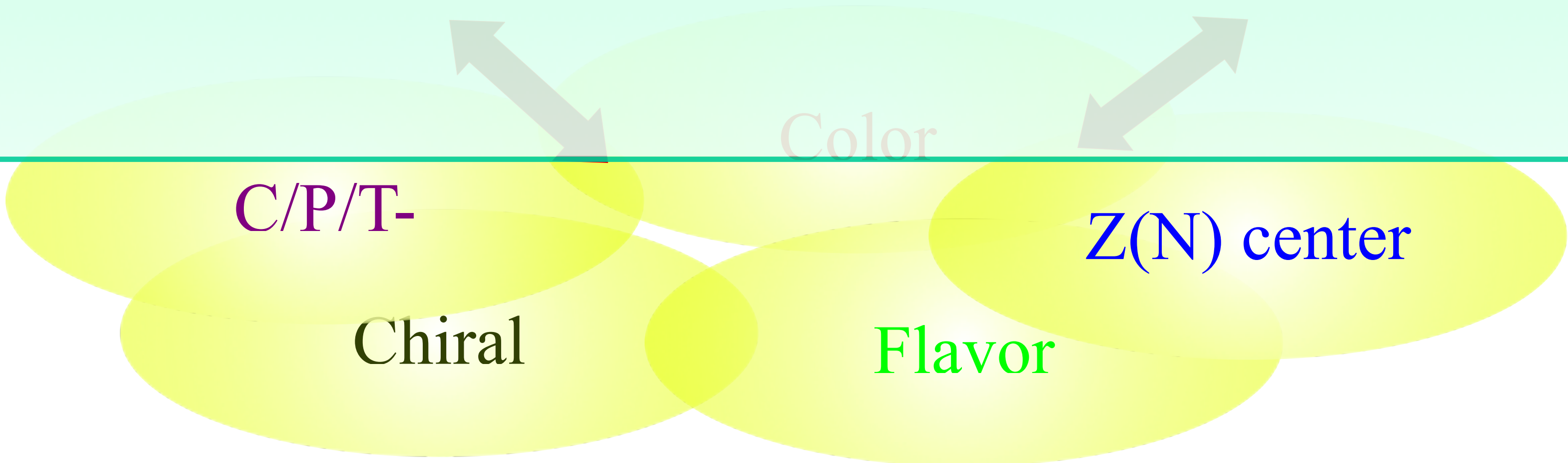
Introduction

- QCD has complicated phase structure: Not fully understood





Lattice QCD (LQCD) is a very promising for this purpose, since it is based on the 1st principle itself, i.e., QCD



Introduction

- LQCD suffers from sign problem at finite quark density
- Compute the following in LQCD with probability $(\det D(U))\ \mathrm{e}^{-S_G[U]}$

$$\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \mathcal{O}'(U_i)$$

- If quark density is finite, the quark part of the weight becomes

$$D(\mu_q) = \not{D} + m + \mu_q\gamma_0$$
$$D^\dagger(\mu_q) = -\not{D} + m + \mu_q^*\gamma_0 = \gamma_5 D^\dagger(-\mu_q^*)\gamma_5$$
$$\{\det[D(\mu_q)]\}^* = \det[D^\dagger(\mu_q)] = \det[\gamma_5 D(-\mu_q^*)\gamma_5] = \det[D(-\mu_q^*)]$$

If μ is real, det[D] is not real but complex

2. Application Lattice QCD

- In addition, if it is a complex, then we have

$$\int dU O'(U)(R+iI)e^{-S_G} \sim \int dU O'(U) e^{-S_G+i\phi}$$

- It's oscillation to cancel out the integral: Sign problem
- Notorious problem in strongly interacting fermion systems even in condensed matter, QFT, and nuclear physics as well.
- How to solve the sign problem???
- So far, there have been no cures (NP-hard problem)
- Many indirect and approximated methods developed:

Small μ expansion, complex-Langevin method, etc.

Introduction

- Canonical method to overcome this problem: $\mu_q \to i\mu_{qI}$, $\mu_{qI} \in R$

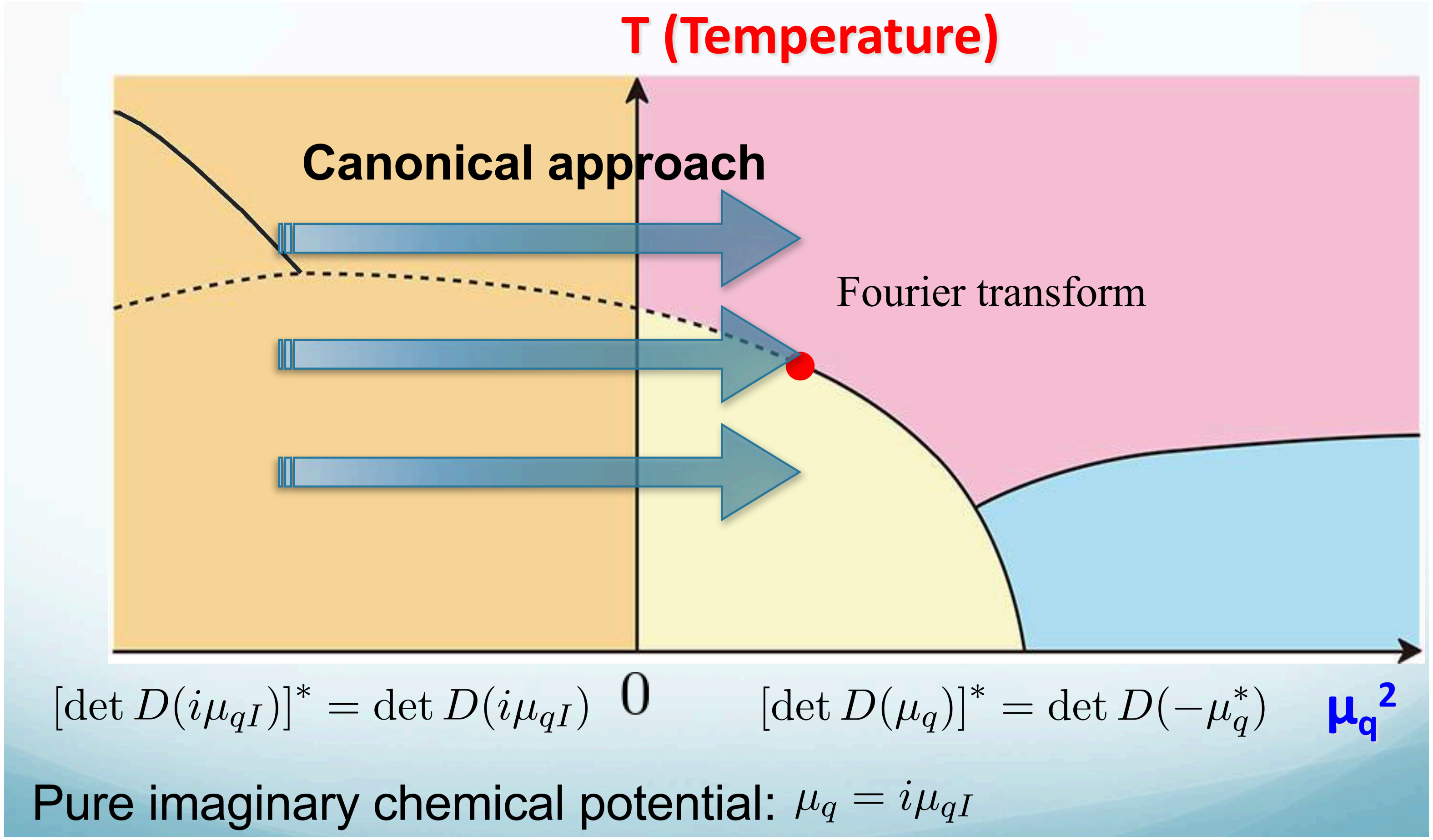


Figure by Dr. Wakayama

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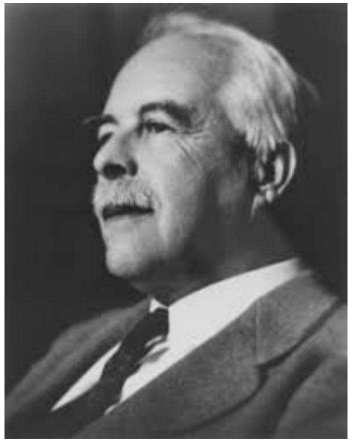
- Fugacity expansion of grand canonical partition function

$$Z_{GC}(\mu, T, V) = \sum_{n=-\infty}^{\infty} Z_C(n, T, V)\xi^n$$

$\xi(\equiv e^{\mu/T})$

Fugacity



Gilbert Newton Lewis

- Obtain canonical partition function by Fourier transform

$$Z_C(n, T, V) = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-in\theta} Z_{GC}(\mu = i\mu_I, T, V) \qquad \theta = \mu_I/T.$$

Without sign problem

- For imaginary chemical potential, there is no SIGN problem

One can do MCMC or Metropolis-Hastings MC

Then, we obtain Z_{GC} on LQCD

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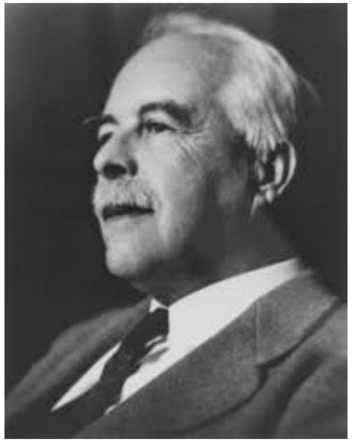

Gilbert Newton Lewis

- Fugacity expansion of grand canonical partition function

$$Z_{\mathrm{GC}}(\mu, T, V) = \sum_{n=-\infty}^{\infty} Z_C(n, T, V)\xi^n$$

$\xi(\equiv e^{\mu/T})$ Fugacity

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Canonical approach developed

Lattice QCD

$$n_q(\mu_q = i\mu_{qI}, T, V)$$

Number density formulation

V. Bornyakov et al., PRD95(2017)

$$\frac{n_q}{T^3} = \frac{1}{VT^2}\frac{\partial}{\partial \mu_q}\ln Z_{\mathrm{GC}}$$

$$Z_{\mathrm{GC}}(\mu_q = i\mu_{qI}, T, V)$$

Fourier transform

$$Z(n, T, V)$$

$$Z_{\mathrm{GC}}(\mu_q, T, V) = \sum_{n=-\infty}^{\infty} Z(n, T, V)\xi^n \qquad \xi = e^{\mu_q/T}$$

If we get Z_n for all n, we can search at ANY density!

Slide by Dr. Wakayama

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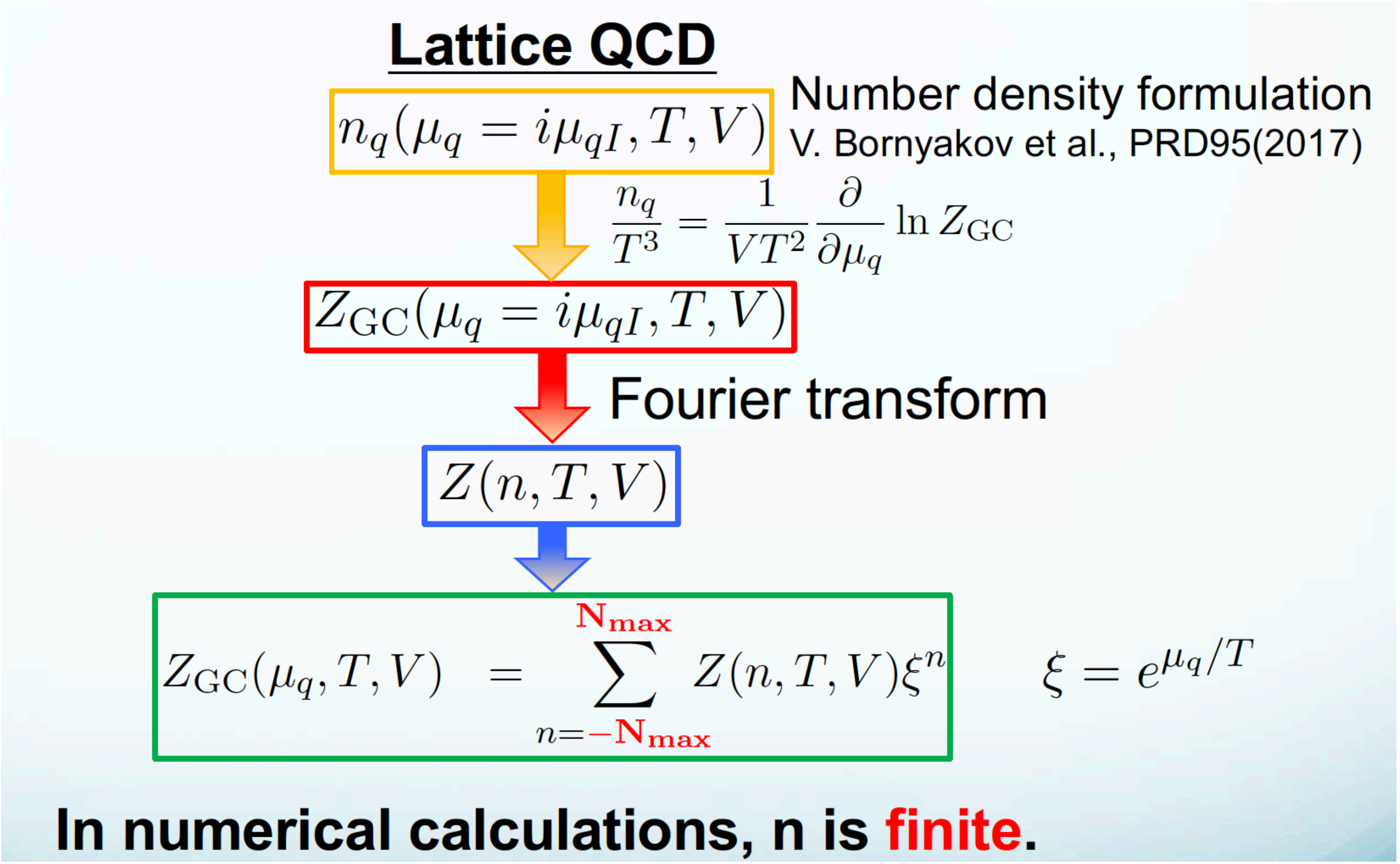
Canonical approach developed



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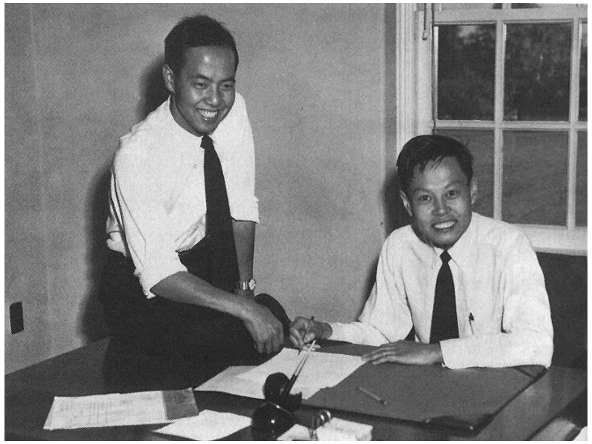
2. Application Lattice QCD

- Application of canonical method: **Lee-Yang zeros**

- Zeros of Z_{GC} so-called Lee-Yang Zeros (LYZ) contain a valuable information on the phase transitions of a system.

T.D. Lee & C.N. Yang, Phys. Rev. 87, 404&410 (1952)



$$Z_{\mathrm{GC}}(\mu_q, T, V) = \sum_{n=-N_{\mathrm{max}}}^{N_{\mathrm{max}}} Z_{\mathrm{c}}(n, T, V)\xi^n = 0$$

Physically, at LYZ, critical-end point (CEP) appears!!

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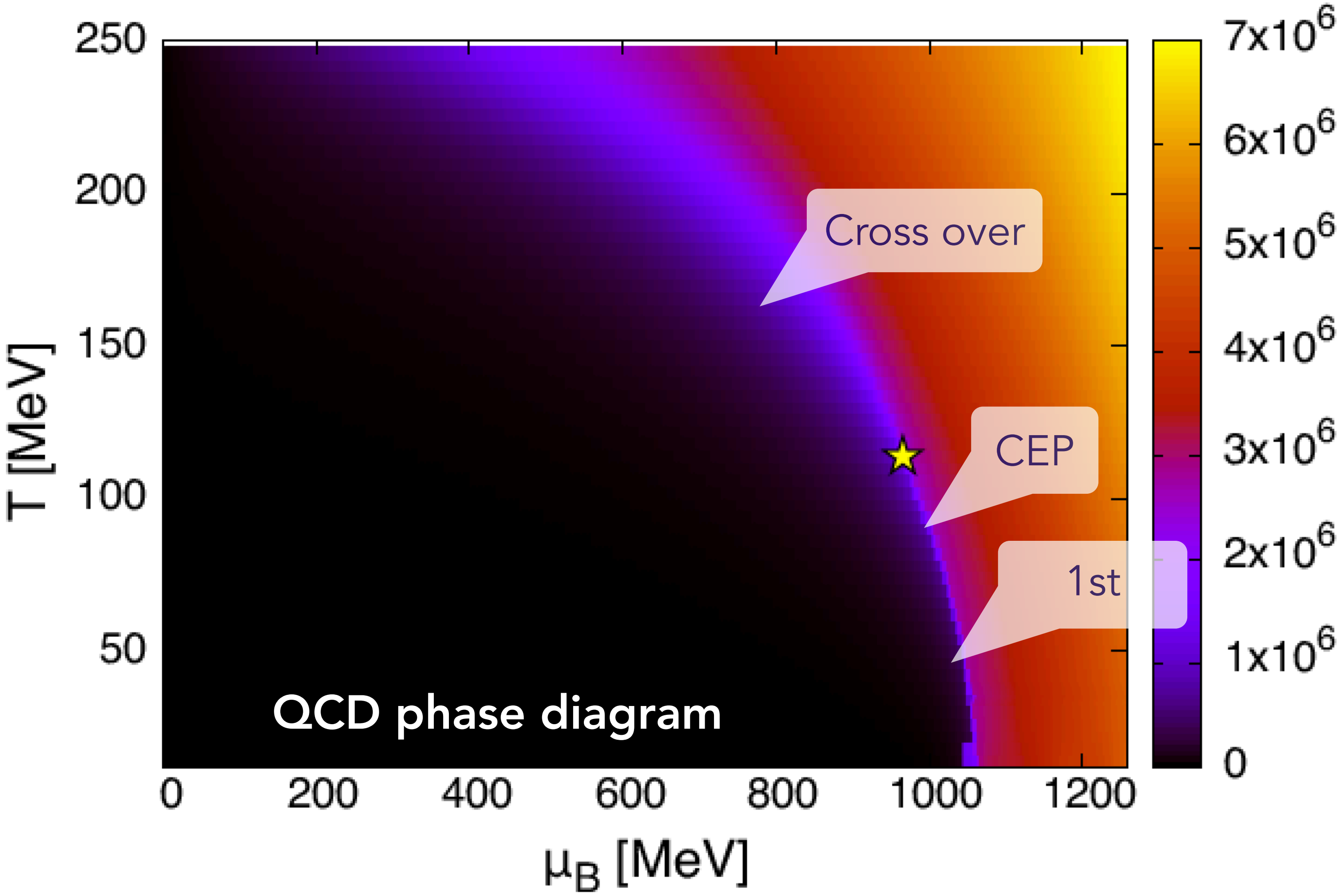
- What is critical-end point (CEP)??



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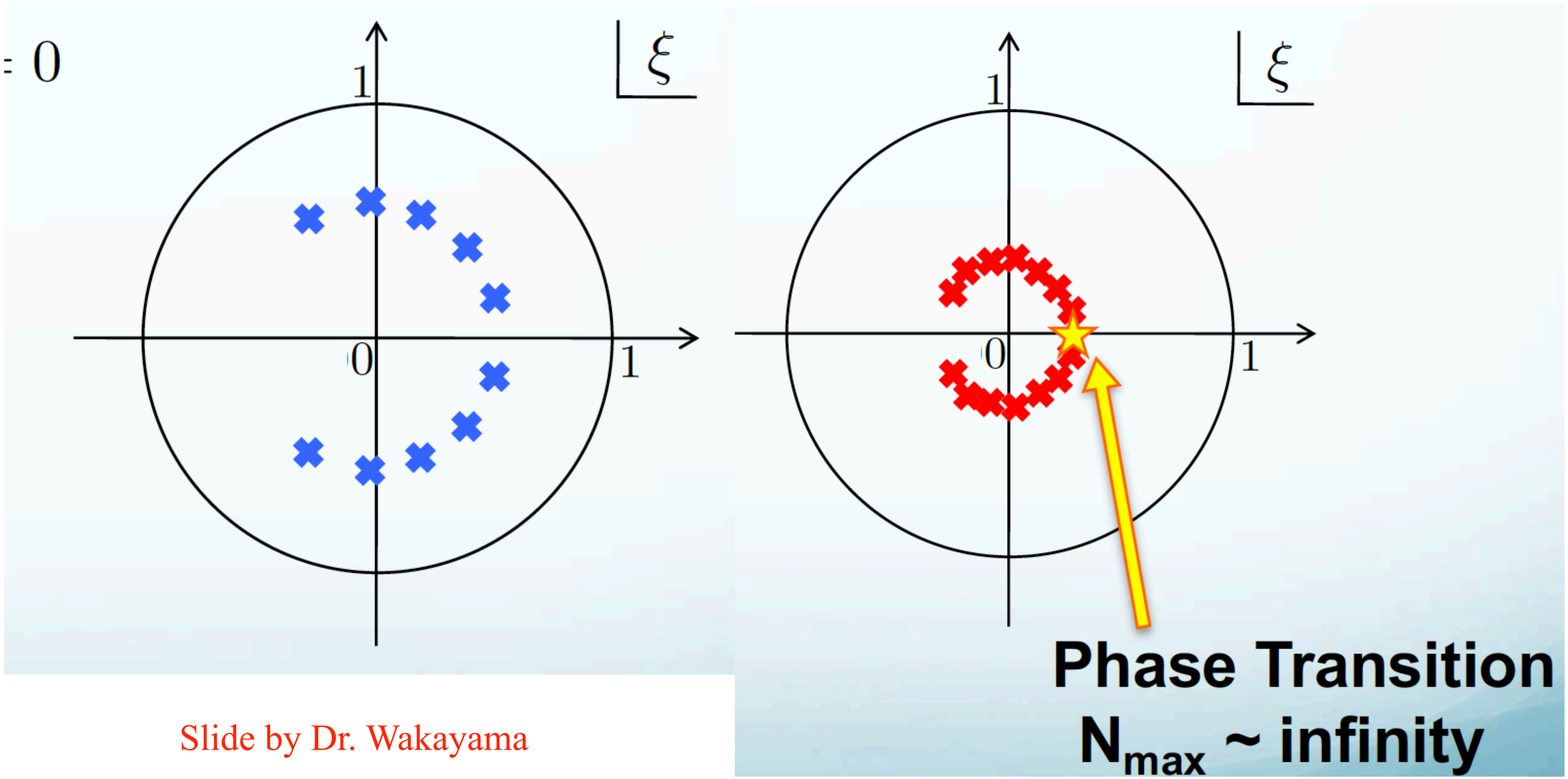
- There are 2Nmax LYZs in complex fugacity plane



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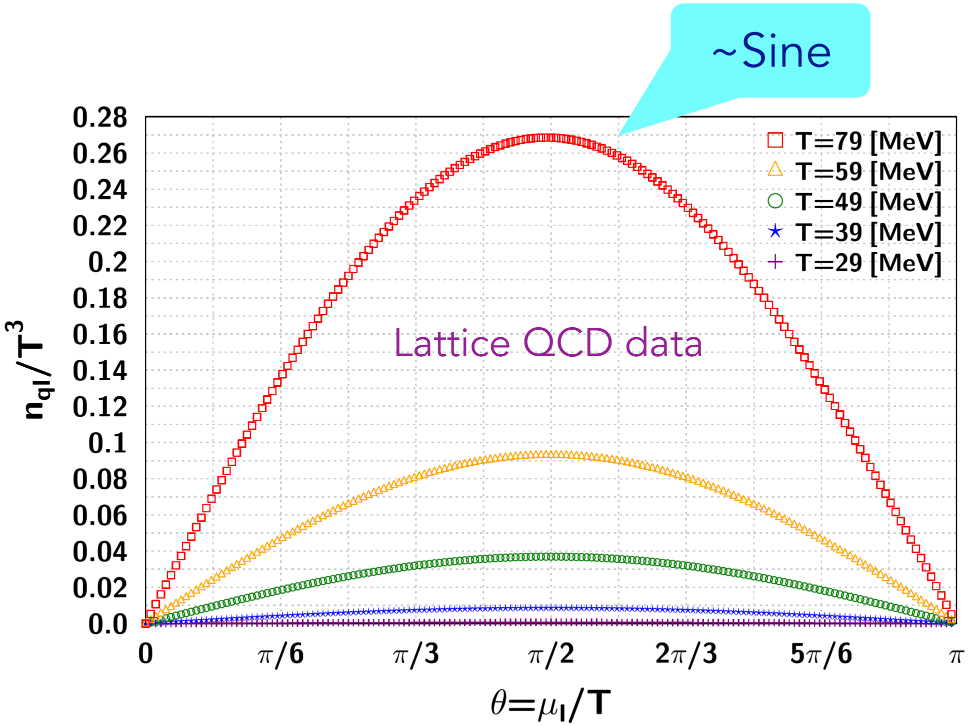
2. Application Lattice QCD

- First, we parameterize number density with sine function for more reliable numerical treatment in lattice QCD

$$\frac{n_{qI}}{T^3}(\theta) = \sum_{k=1}^{N_{\sin}} f_k \sin(k\theta)$$

$$Z_{\rm GC}(i\mu_I, T, V) = C\exp\left\{-V\int_0^\theta d\theta'\, n_{qI}(\theta')\right\} = C\exp\left\{VT^3\sum_{k=1}^{N_{\sin}}\frac{f_k}{k}\cos(k\theta)\right\},$$



Wakayama and Hosaka, PRD (2019)

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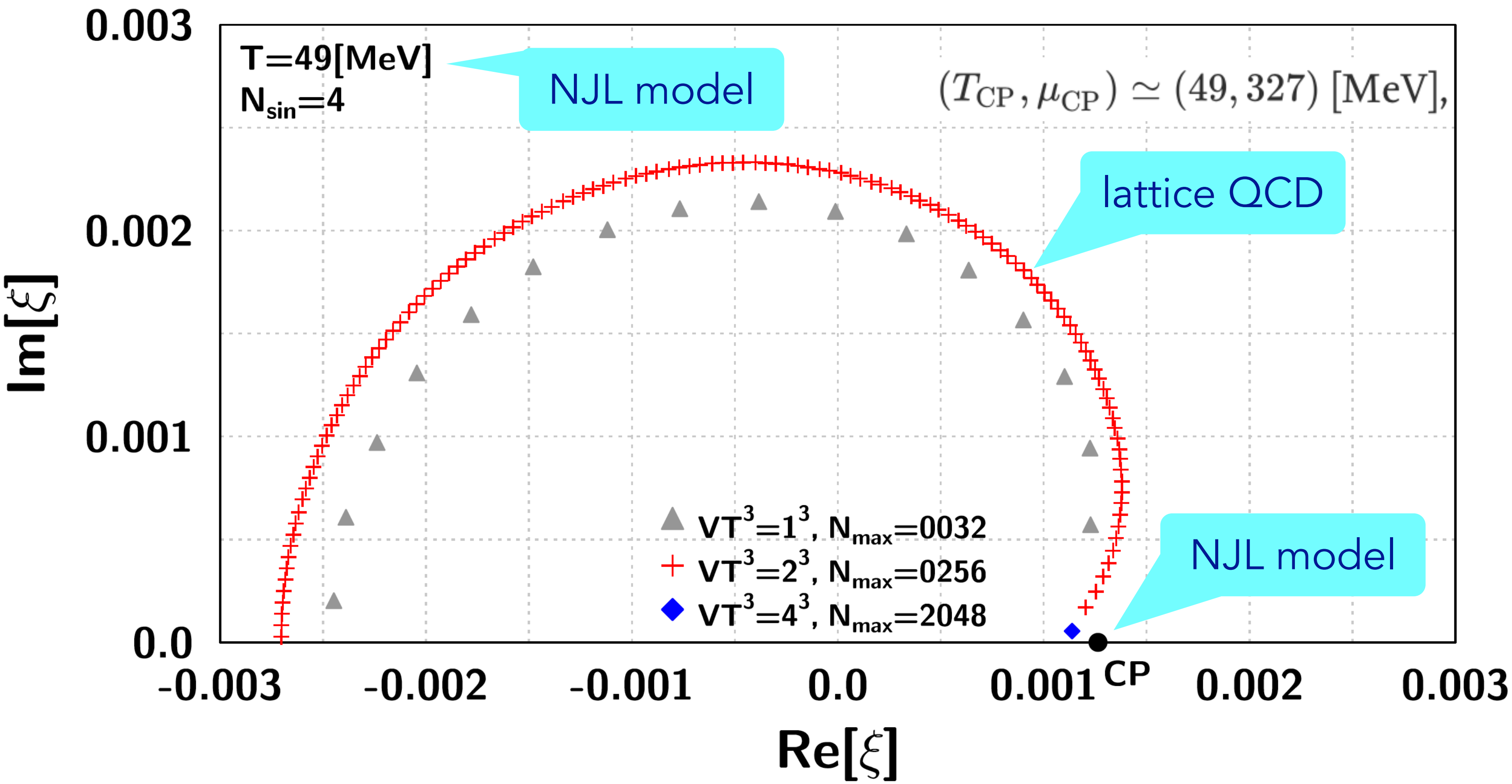
- We observe LYZs cross the Im[ξ]=0 line: CEP



Wakayama and Hosaka, PRD (2019)

2. Application Lattice QCD

- Application of canonical method: **QCD phase structure**
- This method is not full lattice QCD but mimics it closely
- Then, can we describe QCD phase diagram???: Yes!!!
- Before doing lattice QCD with canonical method, we test it in effective models: NJL and PNJL

Thermodynamic potential of PNJL

Quark

Quark-Gluon

$$\omega = \frac{1}{2G}(M - m_q)^2 - 2N_cN_f\int\frac{d^3p}{(2\pi)^3}E_p \; -2N_fT\int\frac{d^3p}{(2\pi)^3}\left\{\mathrm{Tr}_c\ln\left[1 + Le^{-\frac{E_p-\mu}{T}}\right]\right.$$

$$\left.+\mathrm{Tr}_c\ln\left[1 + L^\dagger e^{-\frac{E_p+\mu}{T}}\right]\right\} + T^4\left[-\frac{b_2(T)}{2}\ell\bar{\ell} - \frac{b_3}{6}\left(\ell^3 + \bar{\ell}^3\right) + \frac{b_4}{4}\left(\ell\bar{\ell}\right)^2\right]$$

Mass gap

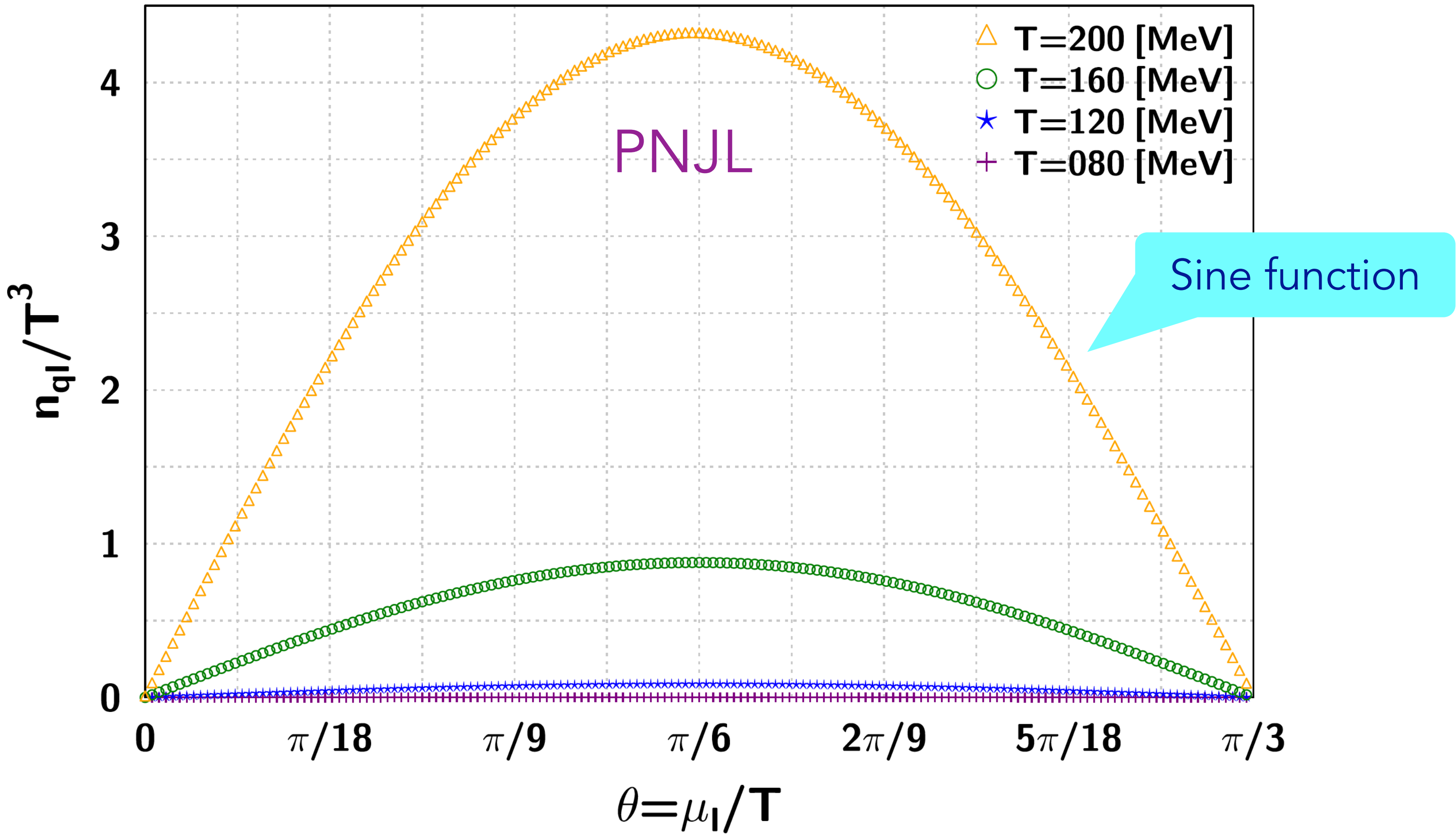
Quark-Gluon

Gluon ~ Z(Nc)

Wakayama, Nam, and Hosaka, PRD (2020)

Numerical results

- QCD phase structure from Polyakov-loop NJL model
- Quark number density from PNJL at $i\mu$



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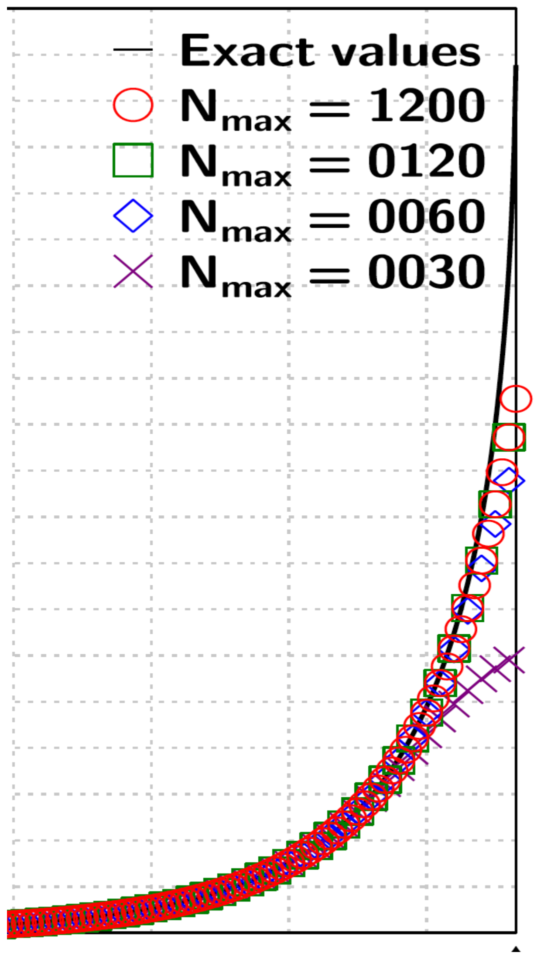
Wakayama, Nam, and Hosaka, PRD (2020)

- Application of canonical method: **QCD phase structure**
- As N_{max} increases, results from canonical method reaches to exact value
- Nonetheless, canonical method does not coincide with exact one: limitation of the present method.
- Then, how do we quantify phase transition in this method?: Taking tolerance

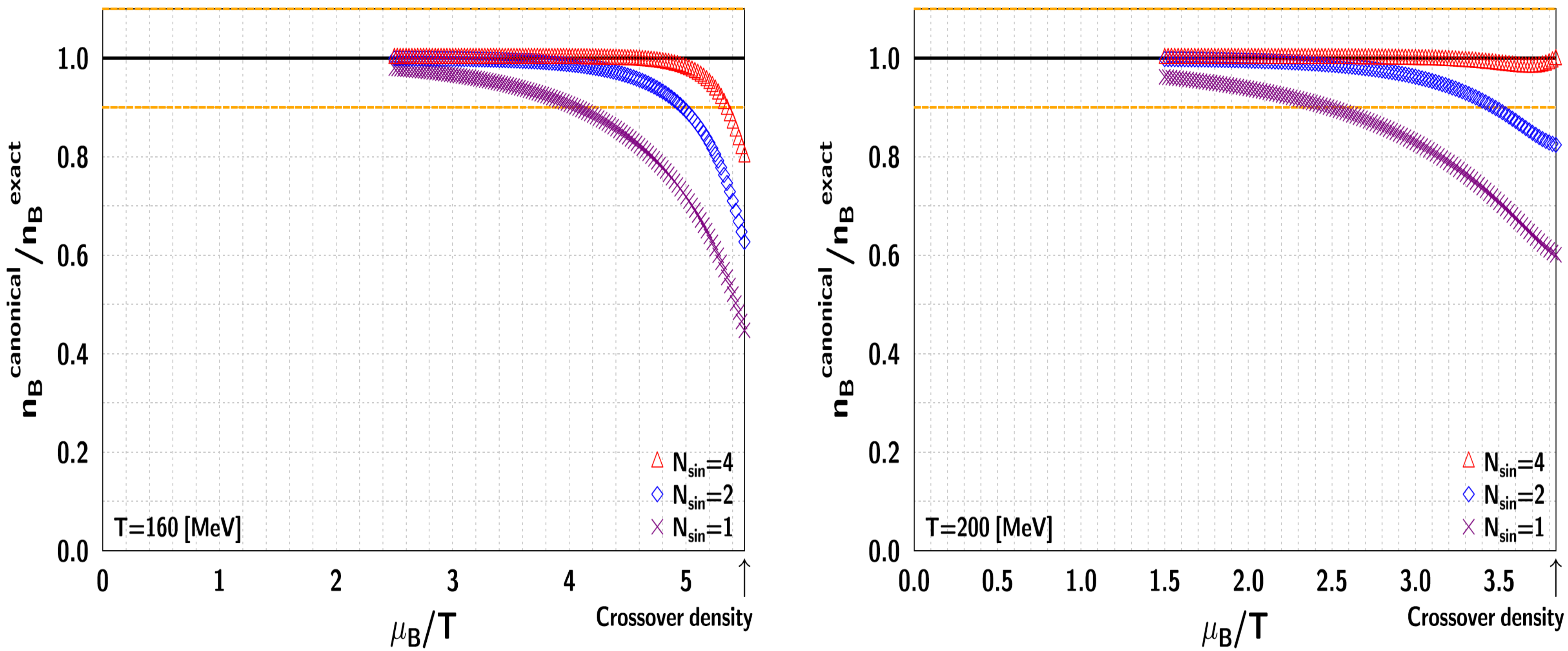
$$\frac{n_B^{\text{PNJL}}}{n_B^{\text{Canonical}}} < 10\%$$



2. Application Lattice QCD

Wakayama, Nam, and Hosaka, PRD (2020)

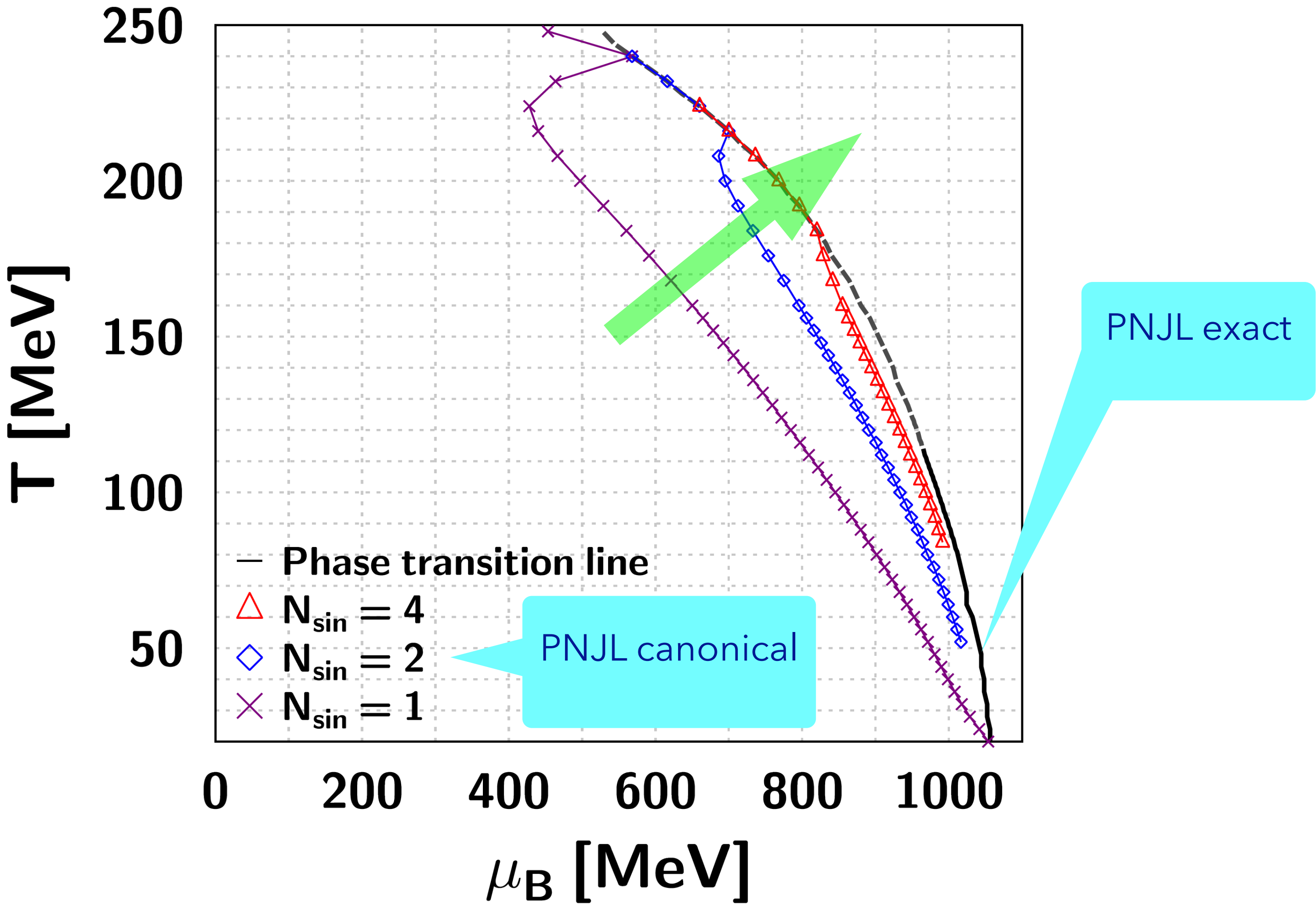
■Application of canonical method: **QCD phase structure**



$$\frac{n_B^{\text{PNJL}}}{n_B^{\text{Canonical}}} < 10\%$$

Numerical results

- QCD phase structure from Polyakov-loop NJL model
- Chiral boundary is described very well!!!



Summary

- QCD phase diagram investigated via canonical method
- To verify the method, we compare $\mathrm{PNJL}_{\mathrm{exact}}$ and $\mathrm{PNJL}_{\mathrm{canonical}}$
- Taking 10% tolerance between them
- Sufficiently small parameters can reproduce QCD phases: 4!
- At finite μ, not on phase transition line, LQCD can be used!
- QCD phase diagram via LQCD (in progress)
- LQCD data for nuclear matter (in progress)
- Interesting new phase found at $i\mu$ (in progress)

Thank you for your attention!!

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