

PDFs off the lightcone

Parton Densities

- Light-cone PDFs
- Pseudodistributions
on the lattice
- Link self-energy
- Renormalization
- Rest-frame density
- Higher twists
- Lattice & pPDFs

Evolution in lattice data

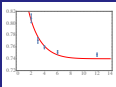
- Evolution
 z_0^2 -dependence
- Matching
- Range of applicability
- Dynamic fermions
- Recent work

Pseudodistributions: PDFs off the lightcone

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Higher twists

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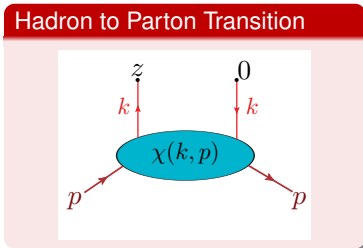
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- Experimentally, we work with hadrons
- Theoretically, we work with quarks

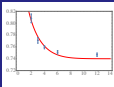


- Can be described in coordinate or momentum space

$$\langle p | \phi(0) \phi(z) | p \rangle = \frac{1}{\pi^2} \int d^4 k e^{-ikz} \chi(k, p)$$

- Concept of PDFs does not rely on spin complications

Light-cone PDFs



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Higher twists

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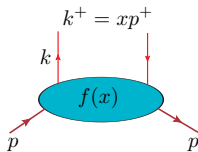
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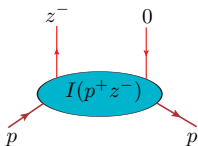
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- In momentum representation: PDF $f(x)$ gives probability that parton carries fraction xp^+ of hadron momentum component p^+



- In coordinate representation: PDF $f(x)$ is given by Fourier transform of Ioffe-time distribution

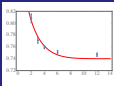
$$f(x) = \frac{p^+}{2\pi} \int_{-\infty}^{\infty} dz^- e^{-ixp^+ z^-} \mathcal{I}(p^+ z^-)$$

- Matrix element $M(z, p)$ on the light cone $z^2 = 0$, we took $z = z^-$
- Ioffe time $\nu = p^+ z^-$
- ITD: $M(z, p)|_{z=z^-} = \mathcal{I}(p^+ z^-) = \mathcal{I}(\nu)$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-ix\nu} \mathcal{I}(\nu)$$

- Observation: ν -dependence governs x -dependence

Pseudodistributions



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Parton Densities

Light-cone PDFs

Pseudodistributions on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in lattice data

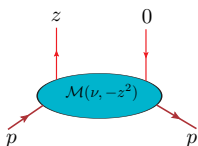
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Matching

Range of applicability

Dynamic fermions

Recent work



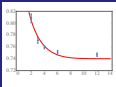
- Take z off the light cone: $z^2 < 0$
- By Lorentz invariance $M(z, p)$ depends on z through $(pz) \equiv -\nu$ and z^2 :
 $M(z, p) = \mathcal{M}(-pz, -z^2)$
- Ioffe time $\nu = -(pz)$
- $\mathcal{M}(\nu, -z^2)$: pseudo-ITD
- **Pseudo** \equiv off the light cone, $z^2 \neq 0$

- Using Schwinger's α -representation, it is possible to show that, for **any** contributing Feynman diagram, for **arbitrary** z^2 and **arbitrary** p^2

$$\mathcal{M}(\nu, -z^2) = \int_{-1}^1 dx e^{ix\nu} \mathcal{P}(x, -z^2)$$

- $\mathcal{P}(x, -z^2) =$ **pseudo-PDF**, or PDF off the light cone
- $e^{ix\nu} = e^{-ix(pz)}$: decomposition over plane waves with momentum $k = xp$
- “Canonical” limits $-1 \leq x \leq 1$
- Negative x correspond to anti-particles
- Note: x is Lorentz invariant: same “on” and “off” LC

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PDFs off the lightcone

Parton Densities

Light-cone PDFs

Pseudodistributions on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in lattice data

Evolution z^2 -dependence

Matching

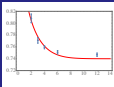
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- On the lattice: cannot take “ z ” on the light cone
Need to take it off the light cone!
- Take $z = \{0, 0, 0, z_3\}$ (X. Ji (2013), quasi-PDF approach, $p_3 \rightarrow \infty$)
- Pseudo-PDF approach is based on **key observation**:
It does not matter if ν was obtained as $-(p_+ z_-)$ or as $p_3 z_3$:
the function $\mathcal{M}(\nu, -z^2)$ is the same!
- For $z = z_3$, we have $\nu = p_3 z_3$ and $-z^2 = z_3^2$
- Pseudo-PDF strategy: map lattice data on $M(z_3, p)$
in terms of ν and z_3^2 and extrapolate $\mathcal{M}(\nu, z_3^2)$ to $z_3^2 = 0$
- Need to understand various types of z^2 -dependence of $\mathcal{M}(\nu, z_3^2)$
- Important to realize: dependence of $M(z, p)$ on z
comes (1) through dependence on (pz)
and (2) remaining dependence on z for a fixed (pz)

Link-related z_3^2 -dependence



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Light-cone PDFs

Pseudodistributions on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in lattice data

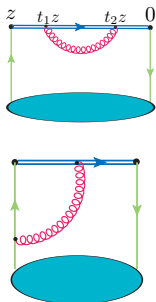
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Matching

Range of applicability

Dynamic fermions

Recent work

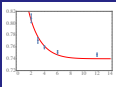


- Specific source of z^2 -dependence In QCD: gauge link $\hat{E}(0, z; A)$
- It comes together with ultraviolet divergences: linear $\sim z_3/a$ and logarithmic $\ln(z_3^2/a^2)$, where $a \sim$ UV cut-off, e.g. lattice spacing a_L
- At one loop, UV terms have been calculated in lattice perturbation theory (Ji et al., 2016)
- Result close to that obtained using Polyakov regularization $1/z^2 \rightarrow 1/(z^2 - a^2)$ for gluon propagator in coordinate space, with $a = a_L/\pi$

$$\Gamma_{\text{UV}}(z_3, a) \sim -\frac{\alpha_s}{2\pi} C_F \left[2 \frac{|z_3|}{a} \tan^{-1} \left(\frac{|z_3|}{a} \right) - \ln \left(1 + \frac{z_3^2}{a^2} \right) \right]$$

- 1-loop result exponentiates in higher orders, producing $\sim e^{-2\alpha_s z_3/3a}$ factor for large z_3
- Vertex corrections produce extra $\frac{\alpha_s}{2\pi} C_F \ln(1 + z_3^2/a^2)$ term exponentiating in higher orders

Renormalization



PDFs off the
lightcone

Parton
Densities

Light-cone PDFs

Pseudodistributions
on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in
lattice data

Evolution
 z_3^2 -dependence

Matching

Range of applicability

Dynamic fermions

Recent work

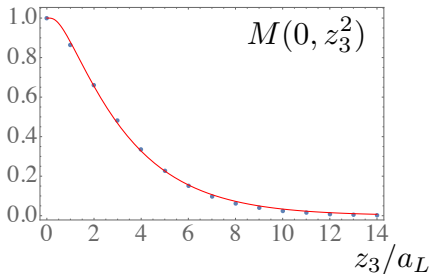
- Link-related UV divergences have the same structure as in HQET
- They are multiplicatively renormalizable (Qiu et al. , Ji et al. , Green et al. 2017)
- UV regulator a appears only in the combination z_3/a
- UV-sensitive terms form a factor $Z(z_3^2/a^2)$
- This factor is an artifact of having a non-lightlike z
- It has nothing to do with the lightcone PDFs
- We should build modified function $Z^{-1}(z_3^2/a^2)\mathcal{M}(\nu, z_3^2; a)$
- To do this, one should know the $Z(z_3^2/a^2)$ factor
- Easier way out: consider reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)} = \lim_{a \rightarrow 0} \frac{\mathcal{M}(\nu, z_3^2; a)}{\mathcal{M}(0, z_3^2; a)}$$

- $Z(z_3^2/a^2)$ factors cancel, and $\mathfrak{M}(\nu, z_3^2)$ has finite $a \rightarrow 0$ limit

Rest-frame density and Z factor

- Exploratory study in quenched approximation (Orginos et al. 2017), is still the most precise pPDF calculation
- Allows to study basic aspects of hadron dynamics on the lattice
- Rest-frame density $\mathcal{M}(0, z_3^2)$ is produced by data at $p_3 = 0$



- $\mathcal{M}(0, z_3^2)$ serves as the UV renormalization factor
- Red line is exponential of 1-loop result for link self-energy and vertex corrections with $\alpha_s = 0.19$
- Very strong effect from $Z(z_3^2) \sim e^{-c|z_3|/a}$

PDFs off the
lightcone

Parton
Densities

Light-cone PDFs

Pseudodistributions
on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in
lattice data

Evolution
 z_3^2 -dependence

Matching

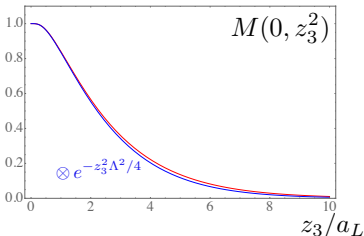
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Dynamic fermions

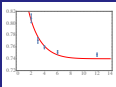
Recent work

Higher-twist effects

- From phenomenology: $f(x, k_\perp) \sim e^{-k_\perp^2/\Lambda^2} f(x)$, with $\Lambda \sim 300$ MeV
- Reflect finite hadron size
- Translates into $\mathcal{P}(x, z_3^2) \sim e^{-z_3^2\Lambda^2/4} f(x)$ for pPDF
- Translates into $\mathcal{M}(\nu, z_3^2) \sim e^{-z_3^2\Lambda^2/4} I(\nu)$ for pITD



- Small correction compared to $Z(z_3^2)$
- Also: cancels in the $\mathcal{M}(\nu, z_3^2)/\mathcal{M}(0, z_3^2)$ ratio
- If $\mathcal{M}(\nu, z_3^2) \sim e^{-z_3^2\Lambda^2/4} I(\nu)$ is not perfect, some residual HT terms $\sim z_3^2\lambda^2$ may remain, with $\lambda \lesssim 100$ MeV
- Strategy: fit residual HT from data



PDFs off the lightcone

Parton Densities

Light-cone PDFs

Pseudodistributions on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in lattice data

Evolution z_3^2 -dependence

Matching

Range of applicability

Dynamic fermions

Recent work

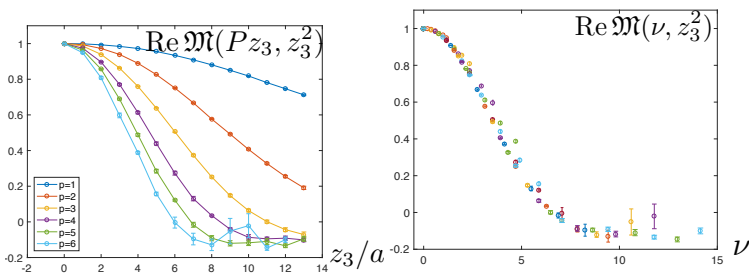
- Exploratory lattice study of reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ for the valence $u_v - d_v$ parton distribution in the nucleon [Orginos et al. 2017]
- Lattice QCD calculations in quenched approximation
- $32^3 \times 64$ lattices, lattice spacing $a = 0.093$ fm
- Pion mass 601(1) MeV and nucleon mass 1411(4) MeV
- Six lattice momenta p_i ($2\pi/L$), with 2.5 GeV maximal momentum
- Real part of lightcone ITD $\mathcal{I}(\nu)$ corresponds to cosine Fourier transform of $q_v(x) = u_v(x) - d_v(x)$

$$\mathcal{R}(\nu) \equiv \text{Re} \mathcal{I}(\nu) = \int_0^1 dx \cos(\nu x) q_v(x)$$

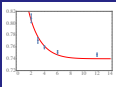
- On the lattice, we extract the reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$$

- Left: Real part of the ratio $\mathcal{M}(Pz_3, z_3^2)/\mathcal{M}(0, z_3^2)$ as a function of z_3
- Taken at six values of $P \Rightarrow$ curves have Gaussian-like shape
- $\Rightarrow Z(z_3^2)$ link factor cancels in the ratio



- Right: Same data, as functions of $\nu = Pz_3$ (z_3^2 varies from point to point)
- Data practically fall on the same universal curve
- Data show no polynomial z_3 -dependence for large z_3 though z_3^2/a^2 changes from 1 to ~ 200
- Apparently no higher-twist terms in the reduced pseudo-ITD



PDFs off the lightcone

Parton Densities

Light-cone PDFs

Pseudodistributions on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in lattice data

Evolution z_3^2 -dependence

Matching

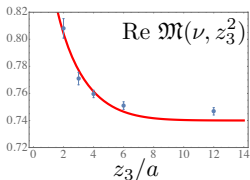
Range of applicability

Dynamic fermions

Recent work

- After cancellation of z_3^2 -dependence from $Z(z_3^2)$ and (hopefully) HT:
- Remaining z_3^2 -dependence corresponds to perturbative (DGLAP) evolution
- At one loop,

$$\mathfrak{M}^{(1)}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi} C_F \ln(z_3^2) \int_0^1 du B(u) \mathfrak{M}^{(0)}(u\nu)$$



- Altarelli-Parisi (AP) evolution kernel

$$B(u) = \left[\frac{1+u^2}{1-u} \right]_+$$

- Example of z_3 -dependence for “magic” loffe-time value $\nu = 3\pi/4 \approx 2.36$

- Shows “perturbative” $\ln(1/z_3^2)$ for small z_3
- Close to a constant for $z_3 > 6a$
- Finite-size (“HT”) effect in 1-loop terms

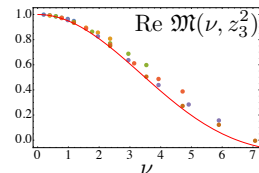
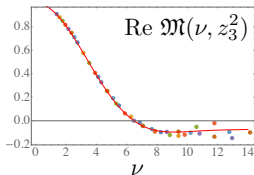
PDFs off the lightcone

Parton Densities

Light-cone PDFs
 Pseudodistributions on the lattice
 Link self-energy
 Renormalization
 Rest-frame density
 Higher twists
 Lattice & pPDFs

Evolution in lattice data

Evolution z_3^2 -dependence
 Matching
 Range of applicability
 Dynamic fermions
 Recent work



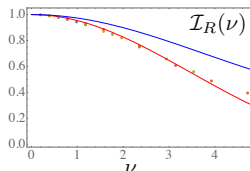
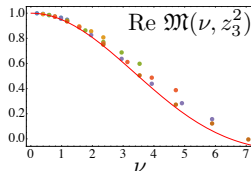
- Points corresponding to $7a \leq z_3 \leq 13a$ values
- Some scatter for points with $\nu \gtrsim 10$
- Otherwise, practically all the points lie on a universal curve
- No z_3^2 -evolution visible in large- z_3 data
- Points in $a \leq z_3 \leq 6a$ region
- All points lie higher than the curve based on the $z_3 \geq 7a$ data
- Perturbative evolution increases real part of the pseudo-ITD when z_3 decreases
- Observed higher values of $\text{Re}\mathfrak{M}$ for smaller- z_3 points are a consequence of evolution

Matching relations

- Matching condition between reduced pseudo-ITD and $\overline{\text{MS}}$ ITD (Y. Zhao 2017, A.R. 2017)

$$\mathfrak{M}(\nu, z_3^2) = \mathcal{I}(\nu, \mu^2) - \frac{\alpha_s(\mu)}{2\pi} C_F \int_0^1 dw \mathcal{I}(w\nu, \mu^2) \times \left\{ B(w) \left[\ln \left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4 \frac{\ln(1-w)}{1-w} - 2(1-w) \right]_+ \right\}$$

- Building $\overline{\text{MS}}$ ITD



- Points in $a \leq z_3 \leq 4a$ region $\mu = 1/a_L \approx 2.15 \text{ GeV}$, $\alpha_s/\pi = 0.1$
- Evolved points have a rather small scatter
- The curve corresponds to the cosine transform of a normalized $\sim x^a(1-x)^b$ distribution with $a = 0.35$ and $b = 3$
- Upper curve: ITD of the CJ15 global fit PDF for $\mu = 2.15 \text{ GeV}$

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lightcone

Parton
Densities

Light-cone PDFs

Pseudodistributions
on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in
lattice data

Evolution
 z_3^2 -dependence

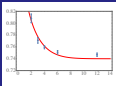
Matching

Range of applicability

Dynamic fermions

Recent work

Range of applicability



PDFs off the
lightcone

Parton
Densities

Light-cone PDFs

Pseudodistributions
on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in
lattice data

Evolution
 z_3^2 dependence

Matching

Range of applicability

Dynamic fermions

Recent work

- Rule of thumb: use perturbation theory when correction is small

$$\mathfrak{M}(\nu, z_3^2) = \mathcal{I}(\nu, \mu^2) - \frac{\alpha_s(\mu)}{2\pi} C_F \int_0^1 dw \mathcal{I}(w\nu, \mu^2) \times \left\{ B(w) \left[\ln \left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4 \frac{\ln(1-w)}{1-w} - 2(1-w) \right]_+ \right\}$$

- Factor $e^{2\gamma_E}/4 \approx 1/1.2$ relates scales in $\overline{\text{MS}}$ and “ z^2 ” scheme
- Suggesting $\Lambda_{z^2} \approx \Lambda_{\overline{\text{MS}}}/1.1$
- Next step: $\mathfrak{M}(\nu, z_3^2) = \mathcal{I}(\nu, \mu^2)$ when α_s correction is zero
- This happens when $\mu \approx 4/z_3$, because of large correction from $\ln(1-w)$
- Numerically: $\mathcal{I}(\nu, (2 \text{ GeV})^2) \approx \mathfrak{M}(\nu, (0.4 \text{ fm})^2)$
- Take $\mu = 1 \text{ GeV}$: $\mathcal{I}(\nu, (1 \text{ GeV})^2) \approx \mathfrak{M}(\nu, (0.8 \text{ fm})^2)$
- \Rightarrow for $a_L \sim 0.1 \text{ fm}$, PT is formally applicable till $z_3 \sim 8a_L$
- Caution: data show deviation from $\ln(z_3^2)$ for $z_3 \gtrsim 5a_L$
- Finite hadron size effects in $\mathcal{O}(\alpha_s)$ terms

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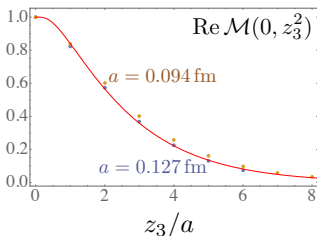
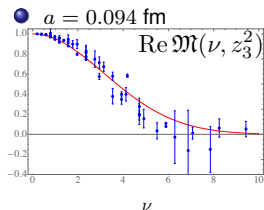
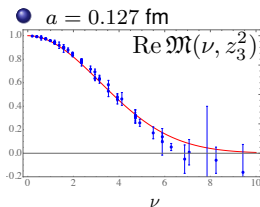
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Light-cone PDFs
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Link self-energy
Renormalization
Rest-frame density
Higher twists
Lattice & pPDFs

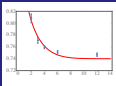
Evolution in lattice data

Evolution z_3^2 -dependence
Matching
Range of applicability
Dynamic fermions
Recent work

Reduced ITD for two lattice spacings



- Z-factor $\text{Re } \mathcal{M}(0, z_3^2)$ for two lattice spacings
- Essentially universal function of z/a
- Curve is given by perturbative formula for the link $Z(z/a)$ factor with $\alpha_s = 0.26$
- $a_L = 0.094$ data are described by PT formula with $\alpha_s = 0.24$



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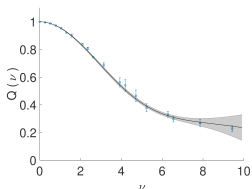
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Densities

Light-cone PDFs
Pseudodistributions
on the lattice
Link self-energy
Renormalization
Rest-frame density
Higher twists
Lattice & pPDFs

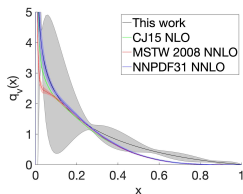
Evolution in
lattice data

Evolution
 z_0^2 -dependence
Matching
Range of applicability
Dynamic fermions
Recent work

- Light-cone ITD for $\mu = 2$ GeV extracted from $a = 0.127$ fm data



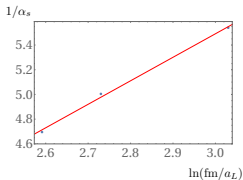
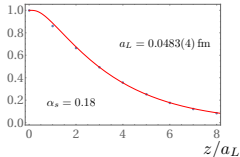
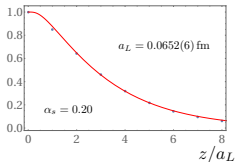
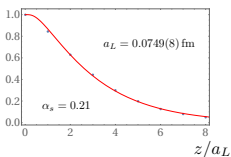
- PDF compared to global fits



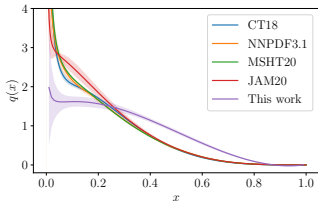
3 lattice spacings (Karpie et al. 2021)

18/20

● Z -factor $\text{Re } \mathcal{M}(0, z_3^2)$ is clearly a function of z_3/a_L



- α_s decreases with a_L . Check if it is $\alpha_s(1/a_L)$
- Since $\alpha_s(1/a_L) = 2\pi/[b_0 \ln(1/a\Lambda)]$, we plot $1/\alpha_s$ versus $\ln(1/a_L)$
- Fit corresponds to $\Lambda = 200$ MeV, and $\beta_0 = 11.4$
- Since $\beta_0 = 11 - 2N_f/3$, contribution of quark loops into α_s in this simulation is not visible



- Comparison with global fits
- Lattice result is smaller for small x
- Pion mass was taken 440 MeV
- Too large to give realistic PDF for small x
- Higher twists $\lesssim 0.15 \Lambda_{\text{QCD}}^2 z_3^2$

PDFs off the lightcone

Parton Densities

Light-cone PDFs
Pseudodistributions on the lattice

Link self-energy

Renormalization

Rest-frame density

Higher twists

Lattice & pPDFs

Evolution in lattice data

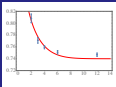
Evolution z_3^2 -dependence

Matching

Range of applicability

Dynamic fermions

Recent work



PDFs off the lightcone

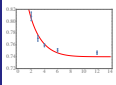
Parton Densities

Light-cone PDFs
Pseudodistributions on the lattice
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Evolution in lattice data

Evolution z_0^2 -dependence
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Dynamic fermions
Recent work

- Collaboration with NNPDF: fitting lattice points like experimental data (2020)
- Unpolarized gluon PDFs
Theory: Balitsky, Morris, A.R. (2019)
Lattice: HadStruc (2021)
- Transversity distributions
Theory and Lattice: HadStruc (2021)
- Polarized gluon PDFs: In progress
Theory: Balitsky, Morris, A.R. (2021)
Lattice: HadStruc, ongoing



PDFs off the lightcone

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Recent work

- Pseudodistribution approach allows to study hadron structure in a way similar to experimental study of DIS
- Instead of structure functions $W(x, Q^2)$, we study loffe-time distributions $\mathcal{M}(\nu, z_3^2)$
- loffe time ν is Fourier-conjugate to x
- z_3 is probing scale, like $1/Q$ in DIS
- Detailed studies of ν - and z_3^2 -dependence decipher subtleties of hadron dynamics