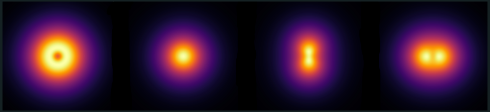
# Origins of the Proton Mass



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Argonne National Laboratory

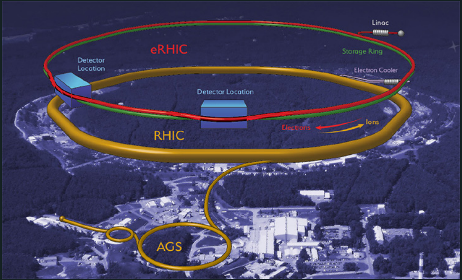
Light Cone 2021: Physics of Hadrons on the Light Front

28 November 2021 — 4 December 2021, Jeju Booyoung Hotel

Argonne NATIONAL LABORATORY

# EIC and the Proton Mass

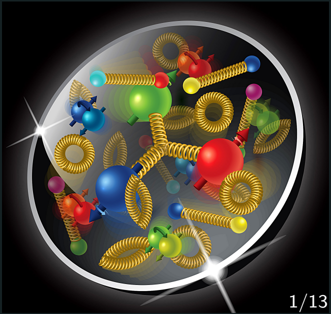
- The National Academies report *An Assessment of U.S.-Based Electron-Ion Collider Science (2018)* identified three "profound questions" about the nucleon
  - How does the mass of the nucleon arise?
  - How does the spin of the nucleon arise?
  - What are the emergent properties of dense systems of gluons?




- The proton contains an uncountable number of quarks, anti-quarks, and gluons [Gasser & Leutwyler, Phys Rept (82)]

$$M^2 = M_0^2 + m_u\, B_u + m_d\, B_d + m_s\, B_s + \ldots + \mathcal{O}(m_q^2), \qquad B_q = \langle p|\bar{q}q|p\rangle$$

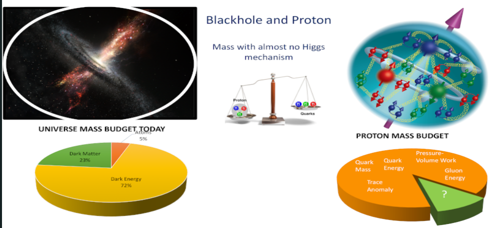
  - Even if the quarks are massless (chiral limit) the proton only gets around 10–20% lighter (sigma terms)



- The proton gets the vast bulk of its mass from the field energies of the quarks and gluons it contains
  - Deeply connected with emergent phenomena of the trace anomaly, color confinement, and dynamical chiral symmetry breaking

# Workshops Driving Progress



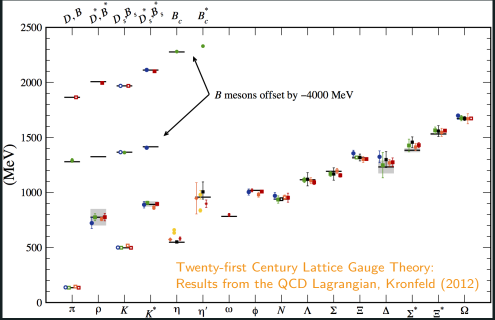

Planning 4th workshop at INT or Chicago: Origin of the Visible Universe: Unraveling the Proton Mass Organizers: Ian Cloët, Zein-Eddine Meziani, and Barbara Pasquini

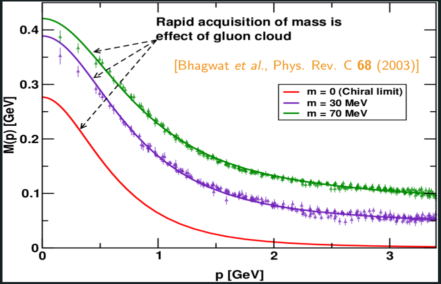
# Lattice QCD and Insights from Models

- Lattice QCD has had remarkable success are reproducing the hadron spectrum
  - Just a few inputs associated with the quark masses in the QCD Lagrangian (only mass dimension parameters in QCD)
- But key questions remain: How does the quark-gluon dynamics of QCD produce hadron mass? Can we learn more? For example, is something analogous to the spin decomposition possible:

$$\frac{1}{2} = \frac{1}{2}\Sigma + L_q + \Delta G + L_g$$

- Models can also offer important insight and intuition, e.g., bag models, NJL, CQS, DSEs, etc.
  - Mass is usually generated by confinement and/or DCSB
  - However, mechanisms like the bag radius and dressed quark masses are not observables
  - Also, model, scheme, gauge, truncation, etc. dependent



# New Insights are possible

- Pioneering papers by Xiangdong Ji (did not generate significant interest at the time)
  - X. D. Ji, Phys. Rev. Lett. **74**, 1071-1074 (1995)
  - X. D. Ji, Phys. Rev. D **52**, 271-281 (1995)
- Important paper by Cédric Lorcé stimulated by 2nd Proton Mass Workshop:
  - C. Lorcé, Eur. Phys. J. C **78**, no.2, 120 (2018)
- A lot of recent progress, for example:
  - Y. Hatta, A. Rajan and K. Tanaka, JHEP **12**, 008 (2018)
  - Y. B. Yang, J. Liang, Y. J. Bi, *et al.*, PRL **121** (2018)
  - S. Rodini, A. Metz and B. Pasquini, JHEP **09**, 067 (2020)
  - A. Metz, B. Pasquini and S. Rodini, PRD **102** (2020)
  - D. E. Kharzeev, Phys. Rev. D **104**, no.5, 054015 (2021)
  - X. Ji, Front. Phys. (Beijing) **16**, no.6, 64601 (2021)
  - I. Zahed, Phys. Rev. D **104**, no.5, 054031 (2021)
  - X. Ji, Y. Liu and A. Schäfer, Nucl. Phys. B **971** (2021)
  - C. Lorcé, A. Metz, B. Pasquini and S. Rodini, JHEP **11** (2021)

$$H_q' = \int d^3x \left[ \psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi + \tfrac{3}{4}\bar{\psi}m\psi \right], \quad (22)$$

$$H_g = \int d^3x \tfrac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \quad (23)$$

$$H_m' = \int d^3x \tfrac{1}{4}\bar{\psi}m\psi, \quad (24)$$

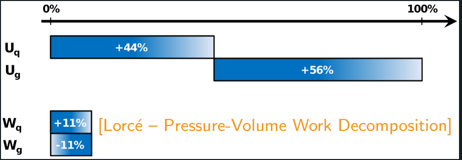
$$H_a = \int d^3x \frac{9\alpha_s}{16\pi}(\mathbf{E}^2 + \mathbf{B}^2), \quad (25)$$

$$H_q = \int d^3x\, \psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi, \quad (26)$$

$$H_m = \int d^3x\, \bar{\psi}m\psi, \quad (27)$$

QCD Hamiltonian becomes

$$H_{\rm QCD} = H_q + H_m + H_g + H_a. \quad (28)$$



# QCD's EMT and the Trace Anomaly

- All mass decompositions are related (in some way) to QCD's Energy-Momentum Tensor (EMT), which classically reads

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi}\, iD^{(\mu}\gamma^{\nu)}\,\psi + \frac{1}{4}\,g^{\mu\nu}\,F^2 - F^{\mu\alpha}F^{\nu}_{\alpha}, \qquad T^{\mu\nu} = T^{\nu\mu}, \qquad \partial_\mu T^{\mu\nu} = 0, \qquad T^{\mu}_{\mu} = m\,\overline{\psi}\psi$$

  - In the chiral limit ($m \to 0$) classical EMT is traceless. What does this mean?

- The QCD Lagrangian in the chiral limit is invariant under scale transformations (dilatations) $x \to e^{\lambda}x$ associated with a dilatation current

$$s^{\mu} = x_\nu\, T^{\mu\nu} \qquad \text{where} \qquad \partial_\mu s^{\mu} = T^{\mu}_{\mu}$$

  - QCD is scale invariant if EMT is traceless — scale-invariant theories can only have massless states

- Cannot be true in QCD, as the proton has mass in the chiral limit (with massless Goldstone bosons from DCSB). QCD has a trace anomaly

$$T^{\mu}_{\mu} = m_q\,\overline{\psi}_q\psi_q + \gamma_m\, m_q\,\overline{\psi}_q\psi_q + \frac{\tilde{\beta}(g)}{2\,g}\,F^2$$

- QCD's trace anomaly breaks scale invariance and is therefore responsible for hadron masses

# EMT (gravitational) Form Factors

- The form factors of the quark and gluon pieces of the EMT ($T^\mu = T^\mu_q + T^\mu_g$) read

$$\langle p' \,|\, T_i^{\mu\nu} \,|\, p\rangle = \bar{u}(p') \left[A_i(t)\, \frac{P^\mu P^\nu}{M} + D_i(t)\, \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}}{4\,M} + J_i(t)\, \frac{P^{\{\mu} i\sigma^{\nu\}\alpha}\Delta_\alpha}{2\,M} + \bar{c}_i(t)\, M\, g^{\mu\nu}\right] u(p)$$

  - $A_q(0) + A_g(0) = 1, \quad J(t) = \frac{1}{2}\,[A(t) + B(t)], \quad B_q(0) + B_g(0) = 0, \quad \text{and} \quad \bar{c}_q(t) + \bar{c}_g(t) = 0$
  - Related to mass and angular momentum distributions, and pressure and shear forces

- EMT form factors are related to GPDs

$$\int_{-1}^{1} \mathrm{d}x\, [x\, H_i(x,\xi,t), x\, E_i(x,\xi,t)] = \left[A_i(t) + \xi^2 D_i(t),\ B_i(t) - \xi^2 D_i(t)\right]$$

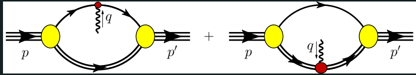
- In the forward limit

$$\langle p \,|\, T_i^{\mu\nu} \,|\, p\rangle = 2\left[A_i(0)\ p^\mu p^\nu + \bar{c}_i(0)\, M^2\, g^{\mu\nu}\right], \quad \langle p \,|\, T_{i\,\mu}^{\mu} \,|\, p\rangle = 2\,M^2\,[A_i(0) + 4\,\bar{c}_i(0)], \quad \langle p \,|\, T^\mu_\mu \,|\, p\rangle = 2\,M^2$$

  - Any hadron mass decomposition should depend on at most two quantities

- Recall trace anomaly $T^\mu_\mu = (1+\gamma_m)\, m_q\, \overline{\psi}_q \psi_q + \frac{\tilde{\beta}(g)}{2\,g}\, F^2$, gives trace decomposition
  - This decomposition implies that in chiral limit entire hadron mass from gluons
  - In pion $\langle \pi \,|\, T^\mu_\mu \,|\, \pi\rangle = 2\, m_\pi^2 \to 0$ because of DCSB, however, $A_i(t)$ and $\bar{c}_i(t)$ can be finite

# NJL Results for Proton GPDs and EMT form factors




[A. Freese and ICC, Phys. Rev. C **101**, no.3, 035203 (2020); A. Freese and ICC, Phys. Rev. C **103**, no.4, 045204 (2021)]

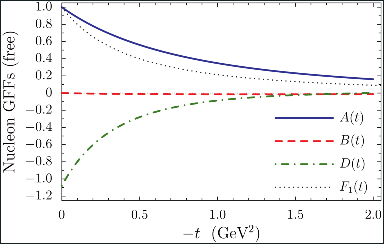
- Charge and EMT form factors

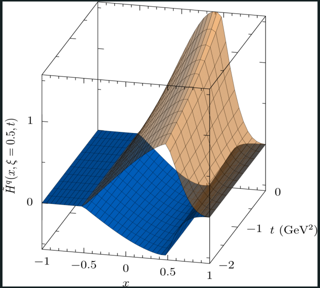
$$\int_{-1}^{1} dx\ [1, x]\, H_q(x,0,t) = [F_1^q(t),\ A_q(t)]$$

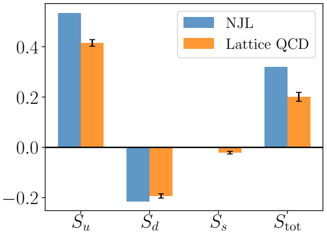
  - expect mass radius to be smaller than charge radius

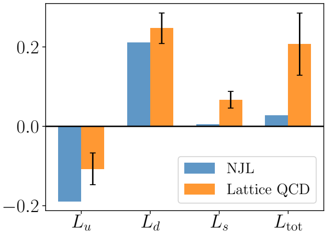
- We find (light cone) radii of:

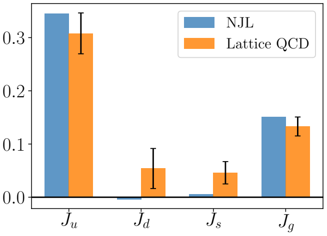
$$\langle r^2 \rangle_C = (0.61\,\text{fm})^2 \quad \langle r^2 \rangle_A = (0.45\,\text{fm})^2 \quad D(0) = -1.08$$



# Trace Anomaly and a Mass Radius from Experiment

- Following Kharzeev PRD **104** (2021), the production of $J/\psi$ and $\Upsilon$ are threshold may be sensitive to the trace anomaly

$$\langle p' \,|\, T^{\mu}_{\mu} \,|\, p \rangle \propto G(t)$$

  - $G(t)$ thought of as a scalar EMT/gravitational form factor
  - Factorization of two gluons into $F^2$ not yet proven

- Assuming vector meson dominance can relate quarkonium production to the scalar EMT form factor

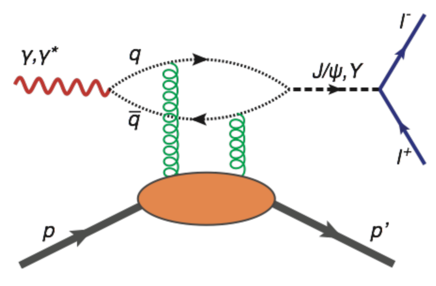
- Taking a dipole form for the scalar EMT form factor gives

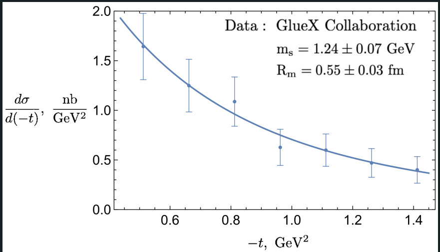
$$G(t) = \frac{M}{[1 - t/m_s^2]^2}, \qquad \langle r_m^2 \rangle = \frac{6}{M} \frac{dG(t)}{dt}\bigg|_{t=0} \to \frac{12}{m_s^2}$$

- Fitting to GlueX data Kharzeev finds

$$r_m = 0.55 \pm 0.03\,\text{fm} \quad \text{c.f} \quad r_c = 0.8409 \pm 0.0004\,\text{fm}$$

  - for a light cone radius: $r_m^{\text{LC}} = 2/\sqrt{6}\, r_m \simeq 0.45\,\text{fm}$
  - Kharzeev argues smaller mass radius is because "pion cloud" decouples from $T^{\mu}_{\mu}$ — $\langle 0 \,|\, T \,|\, \pi^+\pi^- \rangle = q^2$



# Ji's (four-term) Mass Decomposition

- Proposed a rest frame hadron mass decomposition based on $T^{00}$ [PRL **74** (1995); PRD **52** (1995)]

$$M = \frac{\langle p \left| \int d^3x\, T^{00}(0,\boldsymbol{x}) \right| p\rangle}{\langle p|p\rangle}\Bigg|_{\text{at rest}} = \underbrace{M_q + M_g}_{\text{quark \& gluon energies}} + \underbrace{M_m}_{\text{quark mass}} + \underbrace{M_a}_{\text{trace anomaly}}$$

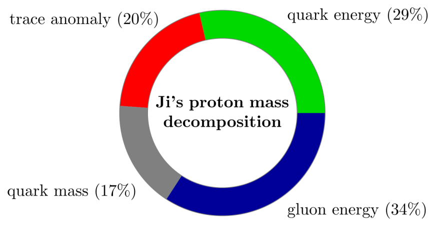
$$M_q = \tfrac{3}{4}(a-b)\,M, \quad M_g = \tfrac{3}{4}(1-a)\,M, \quad M_m = b\,M, \quad M_a = \tfrac{1}{4}(1-b)\,M,$$

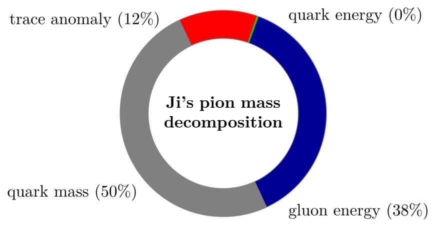
  - $a =$ quark momentum fraction, $b$ related to sigma-terms

- Decomposition is obtained by first breaking EMT into traceless and trace pieces

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}, \qquad \langle \bar{T}^{00} \rangle = \tfrac{3}{4}\,M, \qquad \langle \hat{T}^{00} \rangle = \tfrac{1}{4}\,M,$$

  - $\bar{T}^{\mu\nu}$, $\hat{T}^{\mu\nu}$ transform under different representations of Lorentz group – do not mix under renormalization



# Lattice Results for Ji's Mass Decomposition

- All terms in the Ji decomposition can (in principle) be simulated on the lattice

$$H_q = \int d^3x\, \psi^\dagger \left(-i\, \boldsymbol{D} \cdot \alpha\right) \psi \qquad H_g = \int d^3x\, \tfrac{1}{2} \left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$H_m = \int d^3x \left(1 + \frac{1}{4}\gamma_m\right) \bar{\psi}\, m\, \psi$$

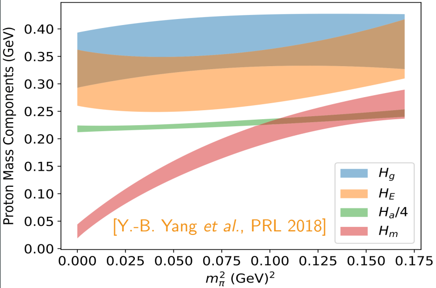
$$H_a = \int d^3x\, \tfrac{1}{4}\, \beta(g) \left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right)$$

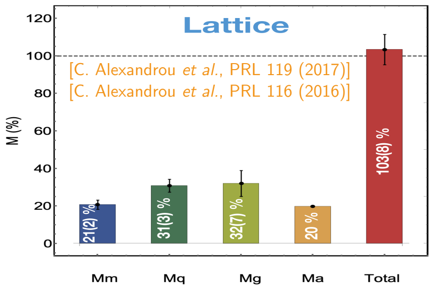
  - however, the trace anomaly term is currently constrained by sum rules
  - the anomaly term, or quantum anomalous energy $\langle p | F^2 | p \rangle$, is twist-4 and difficult to calculate

- Including the anomalous dimension the mass components read

$$M_q = \frac{3}{4}\left(a - \frac{b}{1+\gamma_m}\right) M, \qquad M_g = \frac{3}{4}\left(1 - a\right) M,$$

$$M_m = \frac{4+\gamma_m}{4\left(1+\gamma_m\right)}\, b\, M, \qquad M_a = \frac{1}{4}\left(1 - b\right) M$$



# Three Term Mass Decomposition

- Recent work from Rodini, Metz, and Pasquini, and later Lorcé is advocating a new mass decomposition based on $T^{00}$
  - S. Rodini, A. Metz and B. Pasquini, JHEP **09**, 067 (2020)
  - A. Metz, B. Pasquini and S. Rodini, PRD **102** (2020)
  - C. Lorcé, A. Metz, B. Pasquini & S. Rodini, JHEP **11** (2021)
- In this mass decomposition there is no trace anomaly contribution to the mass
  - This is contrary to expectations based on the breaking of scale invariance by the trace anomaly
  - If there was no trace anomaly in QCD, all states must be massless in chiral limit
- The apparent contradiction to expectation and the Ji's decomposition has caused a lot of discussion
  - Formally both decompositions appear correct
  - Differences likely arise from of renormalization procedures
  - Discussions still underway, e.g., Ji, Liu, Schäfer, NPB **971** (2021)

$$M = M_q + M_m + M_g + M_a, \tag{51}$$

where

$$M_i = \left.\frac{\langle H_i \rangle}{\langle P|P \rangle}\right|_{\boldsymbol{P}=0}, \qquad i = q, m, g, a, \tag{52}$$

with the individual operators given by [37]

$$\mathcal{H}_q = (\psi^\dagger i\boldsymbol{D}\cdot\boldsymbol{\alpha}\psi)_R, \tag{53}$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R, \tag{54}$$

$$\mathcal{H}_g = \frac{1}{2}(E^2 + B^2)_R, \tag{55}$$

$$\mathcal{H}_a = 0. \tag{56}$$

$$m_e = \frac{3}{4}ma + \frac{m}{4}\left(x(1-b)\frac{2e}{\beta(e)} + b\frac{y-3}{1+\gamma_m}\right), \tag{4.45}$$

$$m_m = \frac{mb}{1+\gamma_m}, \tag{4.46}$$

$$m_\gamma = \frac{3}{4}m(1-a) + \frac{m(1-b)}{4}\left(1 - x\frac{2e}{\beta(e)}\right) + mb\frac{\gamma_m - y}{4(1+\gamma_m)}, \tag{4.47}$$

$$m_a = 0, \tag{4.48}$$

# EMT Form Factors in Nuclear Matter

- Recall, the 3 nucleon EMT/gravitational form factors:

$$\langle p' \,|\, T^{\mu\nu} \,|\, p\rangle = \bar{u}(p')\left[A(t)\,\frac{P^\mu P^\nu}{M} + D(t)\,\frac{\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu}}{4\,M} + J(t)\,\frac{P^{\{\mu} i\sigma^{\nu\}\alpha}\Delta_\alpha}{2\,M}\right]u(p)$$

$$\sum_{i=q,g}\int_{-1}^{1} \mathrm{d}x\, x\,\left[H_i(x,\xi,t), E_i(x,\xi,t)\right] = \left[A(t) + \xi^2 D(t),\;\; B(t) - \xi^2 D(t)\right]$$

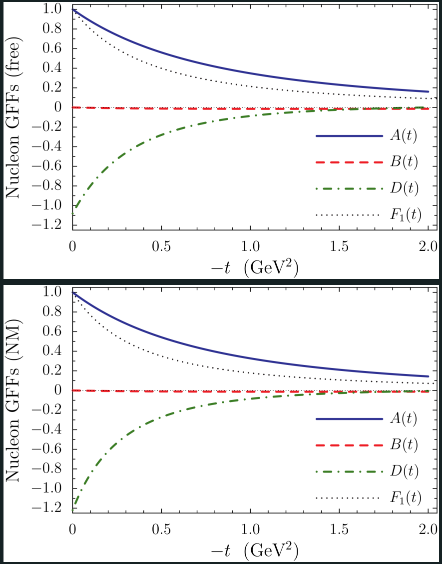
- Our NJL model framework can be extended to self-consistent nuclear matter calculations
  - mean scalar and vector fields couple to the quarks inside the bound nucleon
  - self-consistently modifies their internal structure
- We find (light front) charge and mass radii of:

free $\langle r^2\rangle_C = (0.61\,\text{fm})^2, \quad \langle r^2\rangle_A = (0.45\,\text{fm})^2, \quad D(0) = -1.08$

NM $\langle r^2\rangle_C = (0.66\,\text{fm})^2, \quad \langle r^2\rangle_A = (0.46\,\text{fm})^2, \quad D(0) = -1.21$

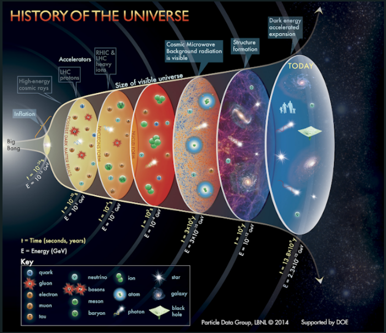
  - mass radius changes much less than the charge radius
  - pressure and shear forces on the nucleon increase by around 10%
  - small mass radius may help explain success of traditional NP

[Adam Freese and ICC, *to appear* (2021)]



# Conclusion and Outlook

- Understanding the origin of the nucleon mass is one of the most important and profound questions in nuclear physics
  - impacts almost every aspect of the visible universe
- Still a lot to learn and explore
  - Are there additional mass decompositions that provide new insights
  - Can experiment unambiguously access the trace anomaly contribution to the proton mass
  - Will lattice QCD deliver an independent calculation of the anomaly contribution to the proton mass and provide a check of the sum rules




- Some related open questions:
  - What determines the QCD scale and how does it affect the visible universe?
  - Does the trace anomaly in QCD reflect both color confinement and dimensional transmutation?
  - What is the interplay between DCSB and color confinement in determining hadron masses?