Origins of the Proton Mass

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Light Cone 2021: Physics of Hadrons on the Light Front
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EIC and the Proton Mass

- The National Academies report *An Assessment of U.S.-Based Electron-Ion Collider Science (2018)* identified three “profound questions” about the nucleon:
  - How does the mass of the nucleon arise?
  - How does the spin of the nucleon arise?
  - What are the emergent properties of dense systems of gluons?

- The proton contains an uncountable number of quarks, antiquarks, and gluons [Gasser & Leutwyler, Phys Rept (82)]

\[
M^2 = M_0^2 + m_u B_u + m_d B_d + m_s B_s + \ldots + O(m_q^2), \quad B_q = \langle p | \bar{q} q | p \rangle
\]

- Even if the quarks are massless (chiral limit) the proton only gets around 10–20% lighter (sigma terms)

- The proton gets the vast bulk of its mass from the field energies of the quarks and gluons it contains

- Deeply connected with emergent phenomena of the trace anomaly, color confinement, and dynamical chiral symmetry breaking
Workshops Driving Progress

The Proton Mass
At the heart of most visible matter.
Temple University, March 28-29, 2016

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EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY
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Planning 4th workshop at INT or Chicago: Origin of the Visible Universe: Unraveling the Proton Mass
Organizers: Ian Cloët, Zein-Eddine Meziani, and Barbara Pasquini
Lattice QCD and Insights from Models

- Lattice QCD has had remarkable success in reproducing the hadron spectrum.
- Just a few inputs associated with the quark masses in the QCD Lagrangian (only mass dimension parameters in QCD).
- But key questions remain: How does the quark-gluon dynamics of QCD produce hadron mass? Can we learn more? For example, is something analogous to the spin decomposition possible:
  \[
  \frac{1}{2} = \frac{1}{2} \Sigma + L_q + \Delta G + L_g
  \]
- Models can also offer important insight and intuition, e.g., bag models, NJL, CQS, DSEs, etc.
  - Mass is usually generated by confinement and/or DCSB.
  - However, mechanisms like the bag radius and dressed quark masses are not observables.
  - Also, model, scheme, gauge, truncation, etc. dependent.

New Insights are possible

- Pioneering papers by Xiangdong Ji (did not generate significant interest at the time)

- Important paper by Cédric Lorcé stimulated by 2nd Proton Mass Workshop:

- A lot of recent progress, for example:
  - Y. Hatta, A. Rajan and K. Tanaka, JHEP 12, 008 (2018)
  - Y. B. Yang, J. Liang, Y. J. Bi, et al., PRL 121 (2018)
  - S. Rodini, A. Metz and B. Pasquini, JHEP 09, 067 (2020)
  - X. Ji, Front. Phys. (Beijing) 16, no.6, 64601 (2021)
  - C. Lorcé, A. Metz, B. Pasquini and S. Rodini, JHEP 11 (2021)
QCD's EMT and the Trace Anomaly

• All mass decompositions are related (in some way) to QCD's Energy-Momentum Tensor (EMT), which classically reads

\[ T^{\mu\nu} = \frac{1}{2} \bar{\psi} iD^{(\mu\gamma)} \psi + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F_{\alpha}^{\nu}, \quad T^{\mu\nu} = T^{\nu\mu}, \quad \partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\mu} = m \bar{\psi} \psi \]

• In the chiral limit \((m \to 0)\) classical EMT is traceless. What does this mean?

• The QCD Lagrangian in the chiral limit is invariant under scale transformations (dilatations) \(x \to e^\lambda x\) associated with a dilatation current

\[ s^\mu = x_\nu T^{\mu\nu} \quad \text{where} \quad \partial_\mu s^\mu = T^{\mu}_{\mu} \]

• QCD is scale invariant if EMT is traceless — scale-invariant theories can only have massless states

• Cannot be true in QCD, as the proton has mass in the chiral limit (with massless Goldstone bosons from DCSB). QCD has a trace anomaly

\[ T^{\mu}_{\mu} = m_q \bar{\psi}_q \psi_q + \gamma_m m_q \bar{\psi}_q \psi_q + \frac{\tilde{\beta}(g)}{2g} F^2 \]

• QCD’s trace anomaly breaks scale invariance and is therefore responsible for hadron masses
EMT (gravitational) Form Factors

- The form factors of the quark and gluon pieces of the EMT \((T^\mu = T^\mu_q + T^\mu_g)\) read

\[
\langle p' | T^\mu_{ij} | p \rangle = \bar{u}(p') \left[ A_i(t) \frac{P^\mu P^\nu}{M} + D_i(t) \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^\mu\nu}{4M} + J_i(t) \frac{P^{(\mu} i \sigma^\nu) \alpha \Delta^\alpha}{2M} + \bar{c}_i(t) M g^\mu\nu \right] u(p)
\]

- \(A_q(0) + A_g(0) = 1, \quad J(t) = \frac{1}{2} [A(t) + B(t)], \quad B_q(0) + B_g(0) = 0, \quad \text{and} \quad \bar{c}_q(t) + \bar{c}_g(t) = 0\)

- Related to mass and angular momentum distributions, and pressure and shear forces

- EMT form factors are related to GPDs

\[
\int_{-1}^{1} dx \left[ x H_i(x, \xi, t), x E_i(x, \xi, t) \right] = \left[ A_i(t) + \xi^2 D_i(t), \quad B_i(t) - \xi^2 D_i(t) \right]
\]

- In the forward limit

\[
\langle p | T^\mu_{ij} | p \rangle = 2 \left[ A_i(0) P^\mu P^\nu + \bar{c}_i(0) M^2 g^\mu\nu \right], \quad \langle p | T^\mu_{ij} | p \rangle = 2M^2 \left[ A_i(0) + 4 \bar{c}_i(0) \right], \quad \langle p | T^\mu_{ij} | p \rangle = 2M^2
\]

- Any hadron mass decomposition should depend on at most two quantities

- Recall trace anomaly \(T^\mu_{\mu} = (1 + \gamma_m) m_q \bar{\psi}_q \psi_q + \frac{\bar{\beta}(g)}{2g} F^2\), gives trace decomposition

- This decomposition implies that in chiral limit entire hadron mass from gluons

- In pion \(\langle \pi | T^\mu_{ij} | \pi \rangle = 2m^2_\pi \to 0\) because of DCSB, however, \(A_i(t)\) and \(\bar{c}_i(t)\) can be finite
NJL Results for Proton GPDs and EMT form factors

- Charge and EMT form factors
  \[ \int_{-1}^{1} dx \left[ 1, x \right] H_q(x, 0, t) = \left[ F_1^q(t), A_q(t) \right] \]
  - expect mass radius to be smaller than charge radius
- We find (light cone) radii of:
  \[ \langle r^2 \rangle_C = (0.61 \text{ fm})^2 \quad \langle r^2 \rangle_A = (0.45 \text{ fm})^2 \]
  \[ D(0) = -1.08 \]
Following Kharzeev PRD 104 (2021), the production of $J/\psi$ and $\Upsilon$ are threshold may be sensitive to the trace anomaly

$$\langle p' \big| T_\mu^\mu \big| p \rangle \propto G(t)$$

- $G(t)$ thought of as a scalar EMT/gravitational form factor
- Factorization of two gluons into $F_2$ not yet proven
- Assuming vector meson dominance can relate quarkonium production to the scalar EMT form factor
- Taking a dipole form for the scalar EMT form factor gives

$$G(t) = \frac{M}{[1 - t/m_s^2]} \quad \langle r_m^2 \rangle = \frac{6}{M} \frac{dG(t)}{dt} \bigg|_{t=0} \rightarrow \frac{12}{m_s^2}$$

- Fitting to GlueX data Kharzeev finds

$$r_m = 0.55 \pm 0.03 \text{ fm} \quad c.f \quad r_c = 0.8409 \pm 0.0004 \text{ fm}$$

- for a light cone radius: $r_m^{LC} = 2/\sqrt{6} \ r_m \simeq 0.45 \text{ fm}$
- Kharzeev argues smaller mass radius is because "pion cloud" decouples from $T_\mu^\mu$ — $\langle 0 \big| T \big| \pi^+\pi^- \rangle = q^2$
Ji’s (four-term) Mass Decomposition

• Proposed a rest frame hadron mass decomposition based on $T^{00}$ [PRL 74 (1995); PRD 52 (1995)]

\[
M = \left. \frac{\langle p | \int d^3x \ T^{00}(0, x) | p \rangle}{\langle p | p \rangle} \right|_{\text{at rest}} = M_q + M_g + M_m + M_a
\]

$M_q = \frac{3}{4} (a - b) M$, $M_g = \frac{3}{4} (1 - a) M$, $M_m = b M$, $M_a = \frac{1}{4} (1 - b) M$,

• $a =$ quark momentum fraction, $b$ related to sigma-terms

• Decomposition is obtained by first breaking EMT into traceless and trace pieces

$T^{\mu\nu} = \tilde{T}^{\mu\nu} + \hat{T}^{\mu\nu}$,

$\langle \tilde{T}^{00} \rangle = \frac{3}{4} M$, $\langle \hat{T}^{00} \rangle = \frac{1}{4} M$,

• $\tilde{T}^{\mu\nu}$, $\hat{T}^{\mu\nu}$ transform under different representations of Lorentz group – do not mix under renormalization

![Ji’s proton mass decomposition](image1)

![Ji’s pion mass decomposition](image2)
Lattice Results for Ji’s Mass Decomposition

- All terms in the Ji decomposition can (in principle) be simulated on the lattice

\[ H_q = \int d^3x \, \psi^\dagger (-i \mathbf{D} \cdot \alpha) \psi \quad H_g = \int d^3x \, \frac{1}{2} (\mathbf{E}^2 + B^2) \]

\[ H_m = \int d^3x \, \left(1 + \frac{1}{4} \gamma_m\right) \bar{\psi} m \psi \]

\[ H_a = \int d^3x \, \frac{1}{4} \beta(g) (\mathbf{E}^2 - B^2) \]

- however, the trace anomaly term is currently constrained by sum rules
- the anomaly term, or quantum anomalous energy \( \langle p | F^2 | p \rangle \), is twist-4 and difficult to calculate

- Including the anomalous dimension the mass components read

\[ M_q = \frac{3}{4} \left(a - \frac{b}{1 + \gamma_m}\right) M, \quad M_g = \frac{3}{4} (1 - a) M, \]

\[ M_m = \frac{4 + \gamma_m}{4(1 + \gamma_m)} b M, \quad M_a = \frac{1}{4} (1 - b) M \]
Recent work from Rodini, Metz, and Pasquini, and later Lorcé is advocating a new mass decomposition based on $T^{00}$

- S. Rodini, A. Metz and B. Pasquini, JHEP 09, 067 (2020)

In this mass decomposition there is no trace anomaly contribution to the mass
- This is contrary to expectations based on the breaking of scale invariance by the trace anomaly
- If there was no trace anomaly in QCD, all states must be massless in chiral limit

The apparent contradiction to expectation and the Ji’s decomposition has caused a lot of discussion
- Formally both decompositions appear correct
- Differences likely arise from renormalization procedures
- Discussions still underway, e.g., Ji, Liu, Schäfer, NPB 971 (2021)
EMT Form Factors in Nuclear Matter

- Recall, the 3 nucleon EMT/gravitational form factors:
  \[ \langle p' | T^{\mu\nu} | p \rangle = \bar{u}(p') \left[ A(t) \frac{p^\mu p^\nu}{M} + D(t) \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^\mu\nu}{4M} + J(t) \frac{\epsilon^{\mu\nu\alpha\beta} \Delta_{\alpha\beta}}{2M} \right] u(p) \]
  \[ \sum_{i=q,g} \int_{-1}^{1} dx \left[ H_i(x, \xi, t), E_i(x, \xi, t) \right] = \left[ A(t) + \xi^2 D(t), B(t) - \xi^2 D(t) \right] \]

- Our NJL model framework can be extended to self-consistent nuclear matter calculations
  - mean scalar and vector fields couple to the quarks inside the bound nucleon
  - self-consistently modifies their internal structure

- We find (light front) charge and mass radii of:
  - \text{free} \quad \langle r^2 \rangle_C = (0.61 \text{ fm})^2, \quad \langle r^2 \rangle_A = (0.45 \text{ fm})^2, \quad D(0) = -1.08
  - \text{NM} \quad \langle r^2 \rangle_C = (0.66 \text{ fm})^2, \quad \langle r^2 \rangle_A = (0.46 \text{ fm})^2, \quad D(0) = -1.21

  - mass radius changes much less than the charge radius
  - pressure and shear forces on the nucleon increase by around 10%
  - small mass radius may help explain success of traditional NP
Conclusion and Outlook

- Understanding the origin of the nucleon mass is one of the most important and profound questions in nuclear physics
- Impacts almost every aspect of the visible universe

- Still a lot to learn and explore
  - Are there additional mass decompositions that provide new insights?
  - Can experiment unambiguously access the trace anomaly contribution to the proton mass?
  - Will lattice QCD deliver an independent calculation of the anomaly contribution to the proton mass and provide a check of the sum rules?

- Some related open questions:
  - What determines the QCD scale and how does it affect the visible universe?
  - Does the trace anomaly in QCD reflect both color confinement and dimensional transmutation?
  - What is the interplay between DCSB and color confinement in determining hadron masses?