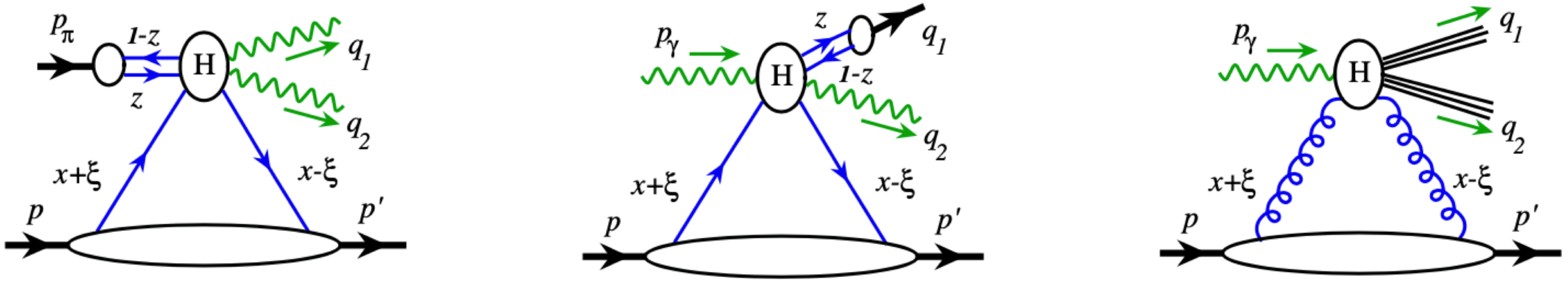
Light Cone 2021: Physics of Hadrons on the Light Front

November 28, 2021 to December 4, 2021
Jeju Booyoung Hotel, Korea

Light Cone 2021
Jeju Island, Korea

A new class of exclusive processes to better measure the x-dependence of DAs and GPDs

Exclusive production of a massive pair of high-P_T particles with $q_T \gg \sqrt{|t|} \gtrsim \Lambda_{\rm QCD}$:



+ …

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Jefferson Lab, Theory Center

In collaboration with: Zhite Yu (Michigan State University)

arXiv:2111.xxxxx

Jefferson Lab

TMD Collaboration

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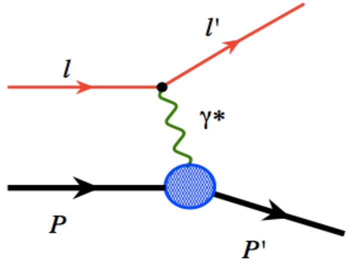
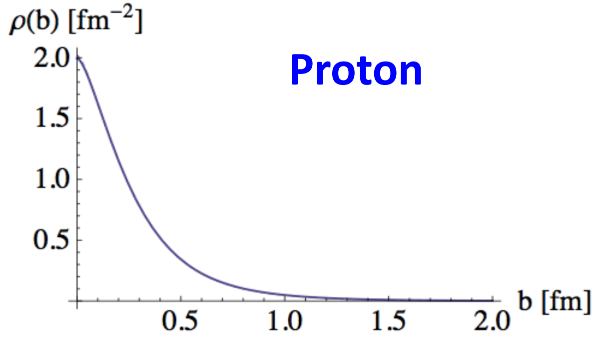
Spatial Imaging of Hadron Structure

- **Elastic e-p scattering – Electric charge distribution:**

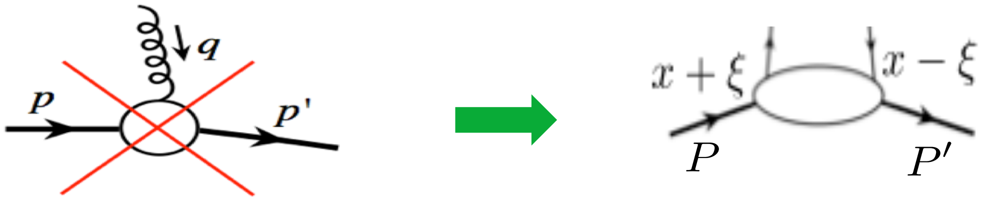




Proton EM Charge radius!

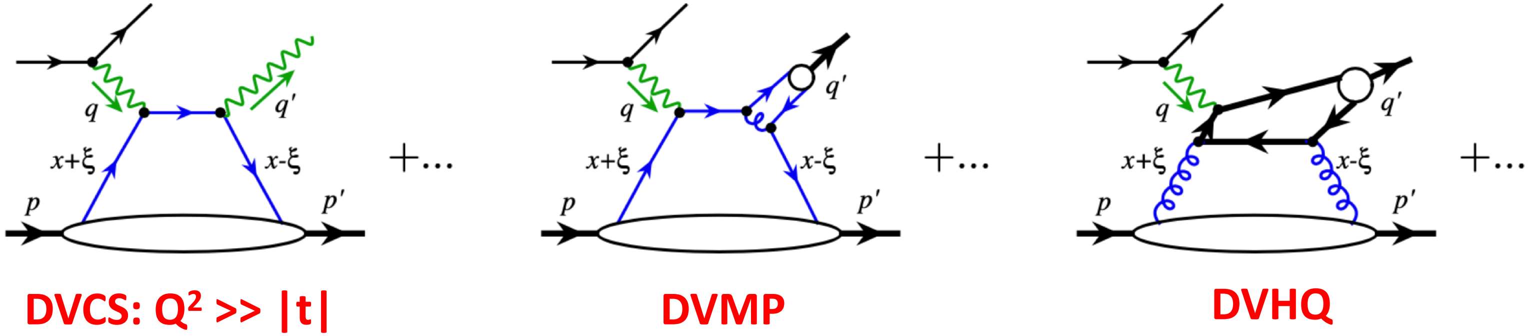
- **No color nucleon elastic form factor!**

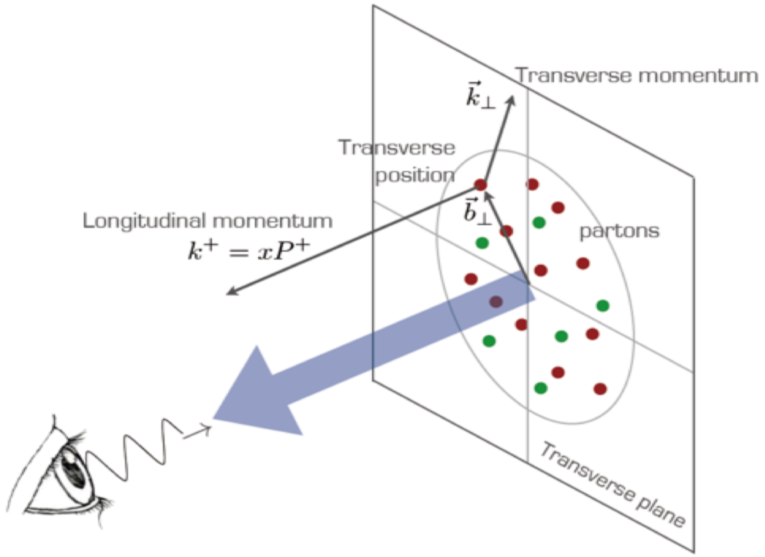
No proton color charge radius!



- **"Two-scale" exclusive observables:**

  – Localized probe, but, sensitive to details of hadron structure

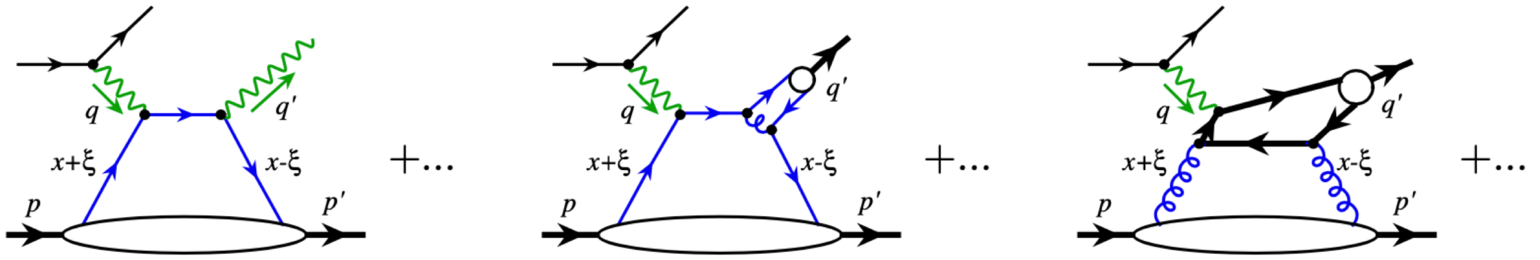




$Q^2 \equiv -q^2$ $t = (p - p')^2$ $Q^2 \gg |t|$

QCD Tomography

- Imagining spatial distribution of quarks and gluons:



DVCS: Q² >> |t| **DVMP** **DVHQ**

$$\frac{d\sigma}{dt} \qquad t=(p-p')^2$$

Factorization $Q^2 \gg |t|$

GPDs: $f_{i/h}(x,\xi,t;\mu)$

F.T. t_T **to** b_T

at $\xi \propto (p-p')^+ \to 0$

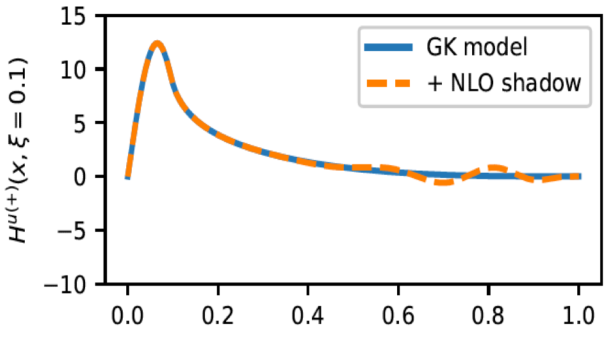
- **Proton radii of quark and gluon spatial distribution,** $r_q(x)$ **&** $r_g(x)$

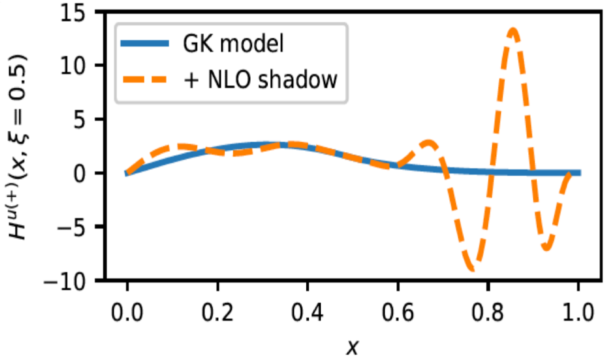
  Should $r_q(x)$ > $r_g(x)$, or vice versa? Could $r_g(x)$ saturate as $x \to 0$?

  …

- **But, all these observables are not very sensitive to the *x*-dependence!**

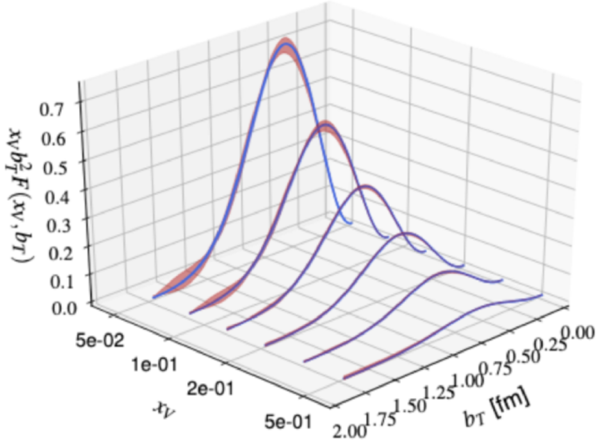
  Sensitive to the total momentum of the pair, not the relative momentum





Blue and dashed
Fit the same CFFs !

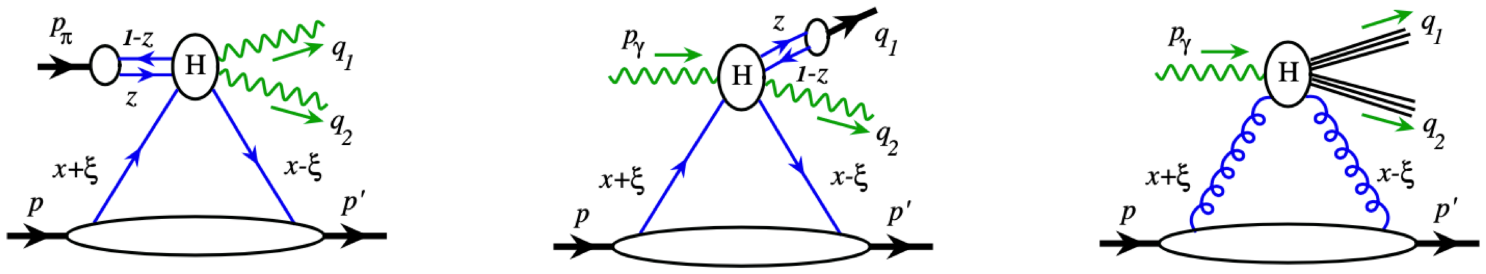
Phys.Rev. D103 (2021) 114019



Jefferson Lab

Exclusive Massive Pair Production

- Exclusive massive pair production with high-P_T (two-scale observables):



\+ …

Introduced by G. Duplancic et al.
JHEP 11 (2018) 179

Introduced by Y. Hatta et al.
Phys.Rev.Lett. 116 (2016) 202301

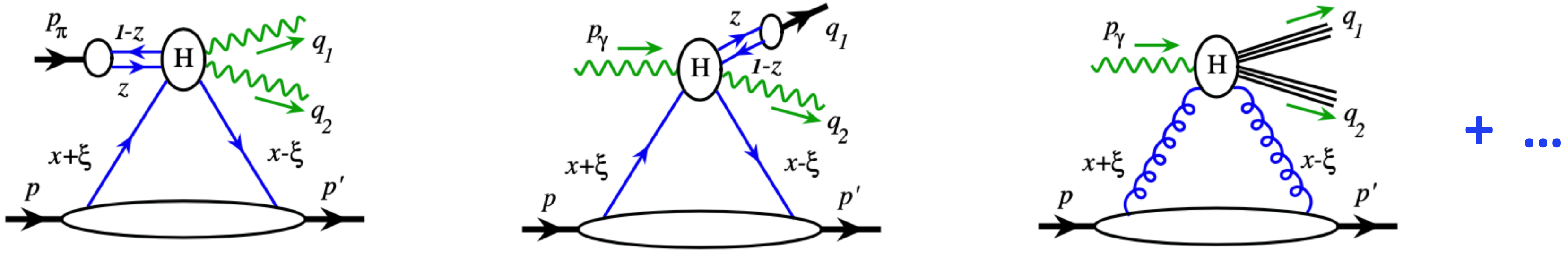
Hard scale: $q_T \gg \Lambda_{\rm QCD}$
in $p_\pi - (p - p')$ frame

Soft scale: $t = (p - p')^2$

Factorization: $q_T \gg \sqrt{|t|}$

Exclusive Massive Pair Production

- **Exclusive massive pair production with high-P_T (two-scale observables):**



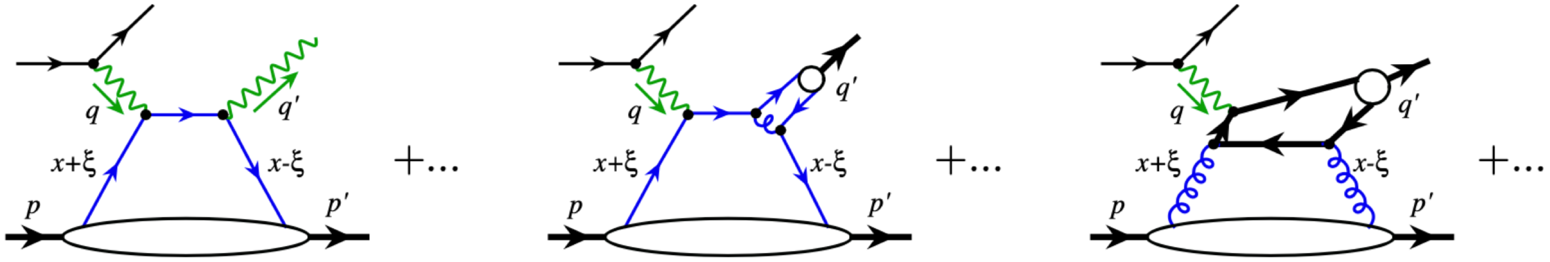
Hard scale: $q_T \gg \Lambda_{\rm QCD}$

in $p_\pi - (p - p')$ frame

Soft scale: $t = (p - p')^2$

Factorization: $q_T \gg \sqrt{|t|}$

- **Similarity and difference from lepton-hadron exclusive processes:**



Hard scale: $Q^2 = -q^2 \gg \Lambda^2_{\rm QCD}$

Soft scale: $t = (p - p')^2$

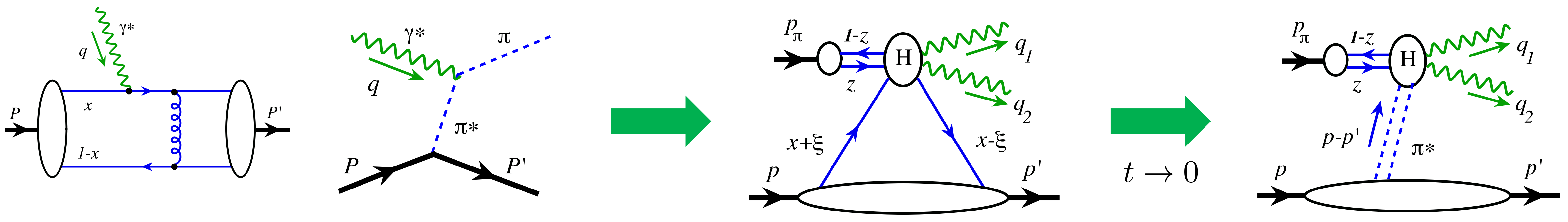
Factorization: $Q \gg \sqrt{|t|}$

- Both are $2 \to 3$ exclusive processes
- Key difference is the source of the hard scale (single virtual photon vs. massive two-particle pair)
- Allow x-dependence to flow through the production of the “pair”
- Additional sensitivity from angular distribution of q_1 or q_T in the pair’s rest frame

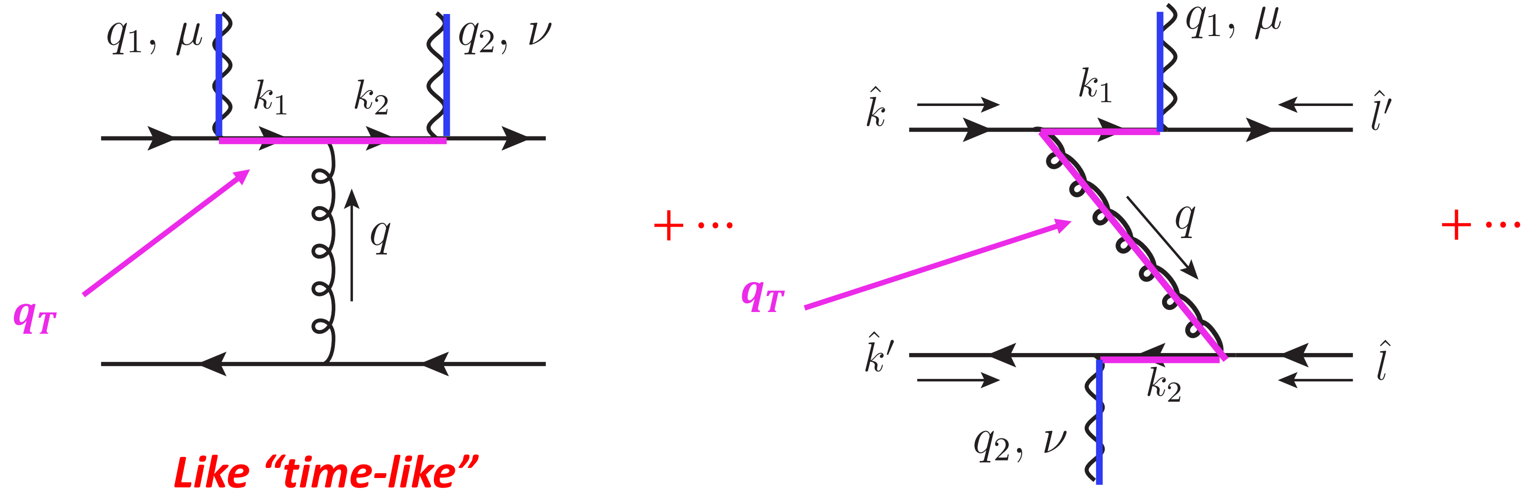
Jefferson Lab

Exclusive Massive Photon-Pair Production with High-P_T

- **Form factor to GPDs:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$



- **Much more sensitive to the x-dependence of DAs:**



Like "time-like" form factor

What about the factorization?

Hard scale: $q_T \le \sqrt{\hat{s}_{\gamma\gamma}}$

Soft scale: $t = (p - p')^2$

$$\xi = \frac{(p-p')^+}{(p+p')^+} \qquad P^+ = \frac{(p+p')^+}{2}$$

Momentum transfer: $\Delta \equiv p - p'$

Leading power:

$$\Delta^+ = 2\xi P^+ = (p-p')^+$$

Exclusive Massive Photon-Pair Production in Meson-Baryon Collision

- **Massive photon pair:**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

- **Observed momentum scales:**
  (in $\gamma\gamma$ CM, with π^- in $-\hat{z}$ direction)

$$s = (p_\pi + p)^2 \qquad \hat{s}_{\gamma\gamma} = (q_1 + q_2)^2$$

$$t = (p - p')^2 \qquad \mathbf{q}_{1T} = -\mathbf{q}_{2T} \equiv \mathbf{q}_T$$

- **Factorization – *necessary conditions*:**

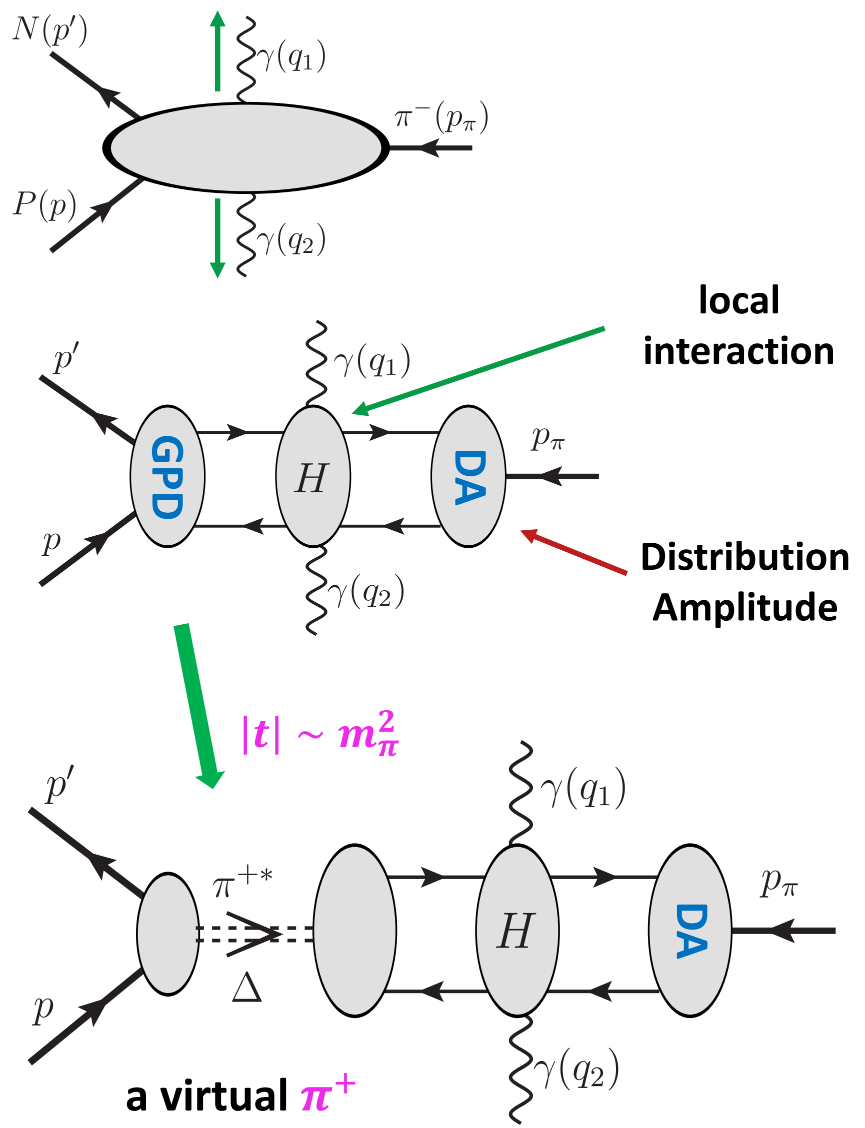
$$q_T \gg \Lambda_{\rm QCD}$$

Requires the time of the hard collision $\sim 1/q_T$

$$\Delta^+ = (p - p')^+ \gg \sqrt{|t|}$$

to be much shorter than the lifetime of the exchanged $q\bar{q}$ (or gg) state

Also needed to ensure perturbative pinch singularities to separate the physics at different scales



Exclusive Massive Photon-Pair Production in Meson-Meson Collision

- **A simpler process:** $\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$

$$s = (p_\pi + p)^2 = (q_1 + q_2)^2 = \hat{s}_{\gamma\gamma}$$

$$p_1 = \left(p_1^+,\, \frac{m_\pi^2}{2p_1^+},\, \mathbf{0}_T\right) \simeq (p_1^+,\, 0^-,\, \mathbf{0}_T)$$

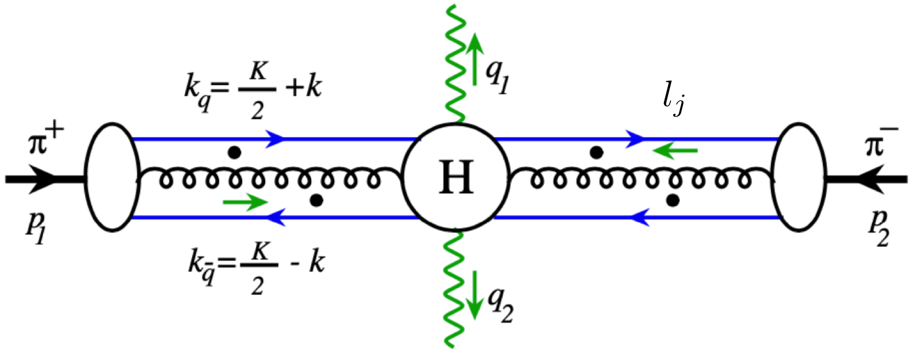
$$p_2 = \left(\frac{m_\pi^2}{2p_2^-},\, p_2^-,\, \mathbf{0}_T\right) \simeq (0^+,\, p_2^-,\, \mathbf{0}_T)$$

$p_1^+ = p_2^- = \sqrt{s/2}$ in the CM frame

$$q_1 = \left(\frac{p_1^+}{2}\left(1 \pm \sqrt{1-\kappa}\right),\, \frac{p_2^-}{2}\left(1 \mp \sqrt{1-\kappa}\right),\, -\boldsymbol{q}_T\right)$$

$$q_2 = \left(\frac{p_1^+}{2}\left(1 \mp \sqrt{1-\kappa}\right),\, \frac{p_2^-}{2}\left(1 \pm \sqrt{1-\kappa}\right),\, \boldsymbol{q}_T\right)$$

$$\kappa = 4q_T^2/s \le 1$$



$$\frac{d\sigma}{dq_T}(\kappa, s) \qquad \frac{d\sigma}{d\cos\theta}(\kappa, s)$$

Single scale observable
– QCD collinear factorization

- **Perturbative pinch singularities:**

$$\mathcal{M} \propto \int \frac{d^4K}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\Big[\hat{R}_{\pi^-}(p_2, l_j) \otimes_{l_j} \hat{H}(q_T, s; l_j; K, k, k_i) \otimes_{k_i} \frac{\gamma\cdot(K/2+k)}{(K/2+k)^2 + i\epsilon} \hat{D}_{\pi^+}(K, k, k_i) \frac{-\gamma\cdot(K/2-k)}{(K/2-k)^2 + i\epsilon}\Big]$$

$$k^- = \frac{k_T^2 - k^2}{K^+ + 2k^+} - i\epsilon\theta(K^+ + 2k^+) \quad \to \quad 0 - i\epsilon$$

$$k^- = -\frac{k_T^2 - k^2}{K^+ - 2k^+} + i\epsilon\theta(K^+ - 2k^+) \quad \to -0 + i\epsilon$$

Requirements:

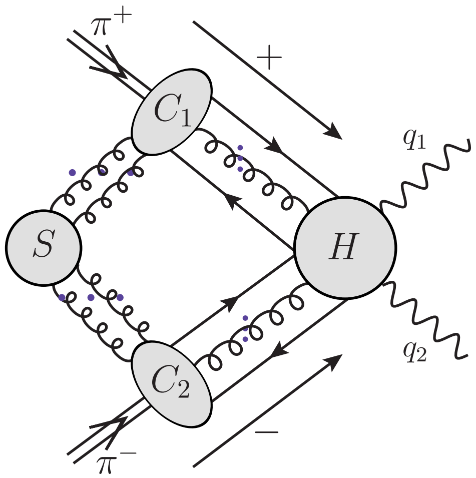
$q_T \gg \Lambda_{\mathrm{QCD}}$ **and**

$\Delta^+ = (p - p')^+ \gg \sqrt{|t|}$

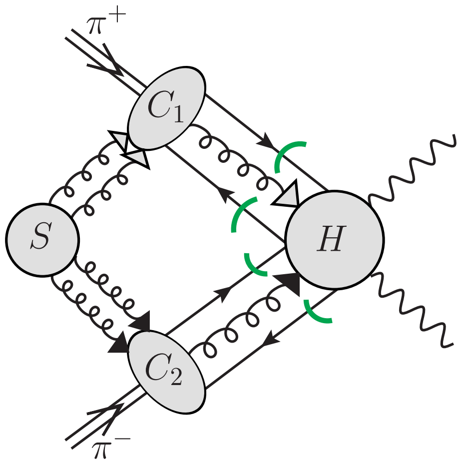
Exclusive Massive Photon-Pair Production in Meson-Meson Collision

❑ **Factorization:** $\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$

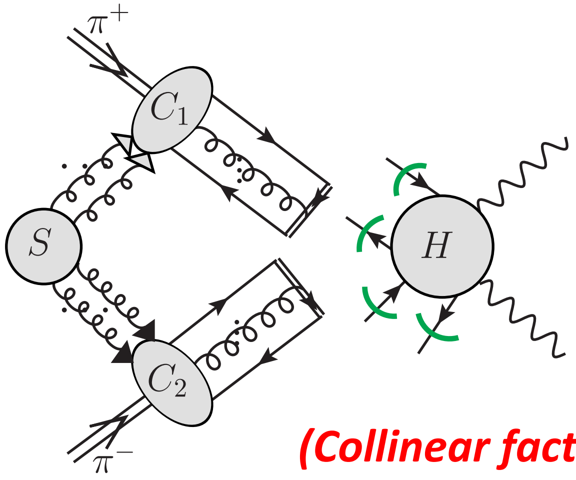
- **Leading region**



- **Approximations**



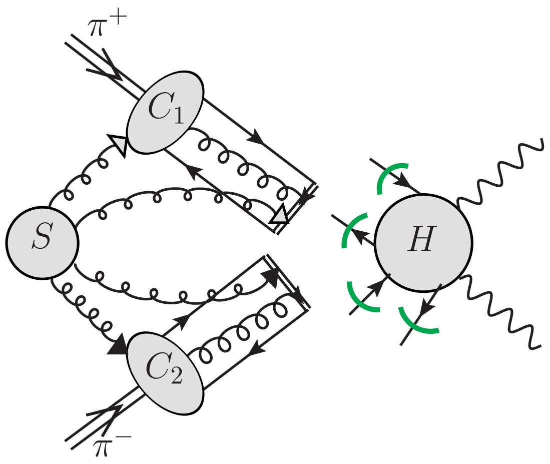
- **Ward identity for collinear gluons**



(Collinear factorization)

- **Ward identity for soft gluons**
  - **Soft gluons are as if attached to a "*closed fermion loop*"**
  - **Sum over diagrams**

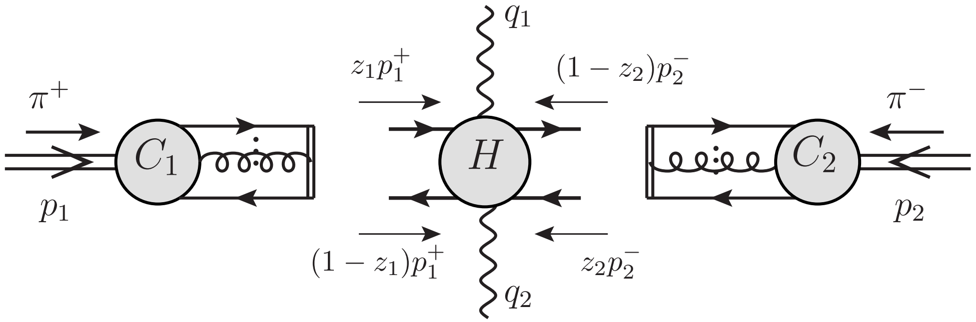
    $\Longrightarrow \boldsymbol{S} = \mathbf{0}$



Soft gluons cancel because collinear parton lines are in color singlet states.

Exclusive Massive Photon-Pair Production in Meson-Meson Collision

- **Factorization:** $\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$



$$\mathcal{M} = \frac{s}{2}\int_0^1 \mathrm{d}z_1 \mathrm{d}z_2\, \phi_{\pi^+}(z_1)\phi_{\pi^-}(z_2)\cdot \mathrm{Tr}\left[\frac{\gamma_5\gamma^-}{2} H(\hat{k}_1,\hat{k}_2;q_1,q_2;\mu)\frac{\gamma_5\gamma^+}{2}\right] + \mathcal{O}\left(\frac{m_\pi}{q_T}\right) \longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}q_T} \propto |\mathcal{M}|^2$$

- **Hadron functions: distribution amplitudes (DA):**

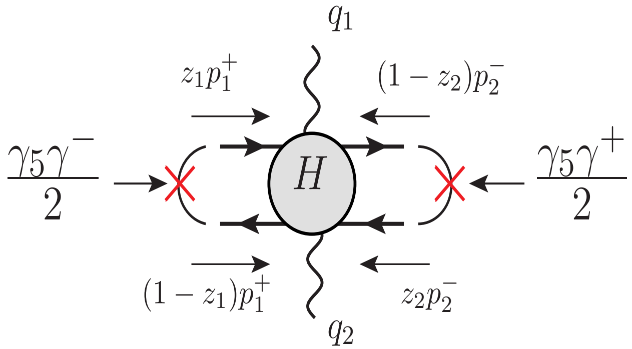
$$\phi_{\pi^+}(z_1) = \int \frac{dx^-}{4\pi} e^{i\, z_1\, p_1^+ x^-} \langle 0|\, \bar{d}(0)\, \gamma^+\gamma_5\, W(0,x^-)\, u(x^-)\, |\pi^+(p_1)\rangle$$

$$\phi_{\pi^-}(z_2) = \int \frac{dx^+}{4\pi} e^{i\, z_2\, p_2^- x^+} \langle 0|\, \bar{u}(0)\, \gamma^-\gamma_5\, W(0,x^+)\, d(x^+)\, |\pi^-(p_2)\rangle$$

$\boldsymbol{\phi_{\pi^+}(z) = \phi_{\pi^-}(z) = \phi(z)}$
are universal DAs

- **Hard coefficient**

$$C\left(z_1, z_2; \frac{q_T^2}{s}; \frac{q_T^2}{\mu^2}\right) =$$

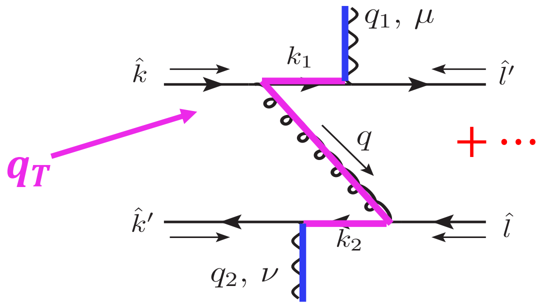


Projections (for $\pi^{\mp}$):

1. **Spin-0** $\gamma_5\gamma^{\pm}$
2. **P-odd**

Exclusive Massive Photon-Pair Production in Meson-Meson Collision

❑ Hard part for A-type:



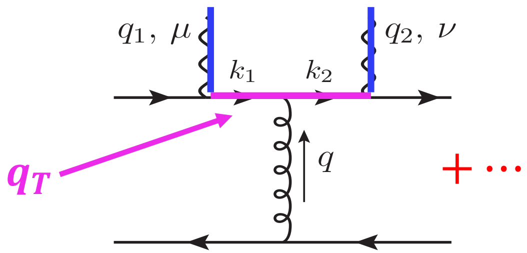
New

- **Gluon propagator** $q^2 = -\frac{\hat{s}}{4}\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]$ $\quad \kappa = 4\,q_T^2/\hat{s}$

$$\Rightarrow \mathcal{M} \propto \int_0^1 \mathrm{d}z_1\,\mathrm{d}z_2 \frac{\phi(z_1)\phi(z_2)}{(1-z_1)\,(1-z_2)\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]}$$

- **Change q_T changes the z_1-z_2 integral.**
- **$\mathrm{d}\sigma/\mathrm{d}q_T^2$ provides sensitivity to the DA's functional form of z.**

❑ Hard part for B-type:



Like "time-like" form factor

- **Gluon propagator** $q^2 = z_2(1 - z_1)\hat{s}$

$$\Rightarrow \mathcal{M} \propto \int_0^1 \mathrm{d}z_1\,\mathrm{d}z_1 \frac{\phi(z_1)\phi(z_2)}{z_1\,(1-z_1)\,z_2\,(1-z_2)} \sim \left[\int_0^1 \mathrm{d}z\,\frac{\phi(z)}{z\,(1-z)}\right]^2$$

- ***Not* sensitive to DA functional form.**
- **Relies on $\phi(z) = 0$ at end points.**
- **Sudakov resummation could suppress the end-point sensitivity.**

Li, Sterman, 1992

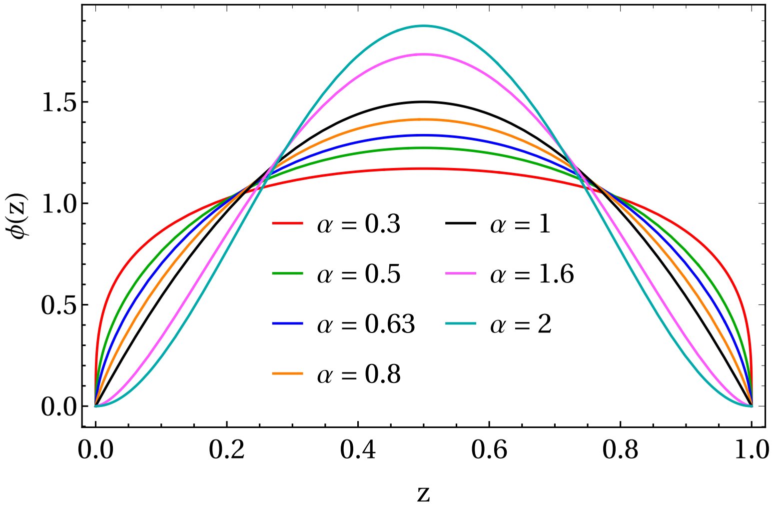
Exclusive Massive Photon-Pair Production in Meson-Meson Collision

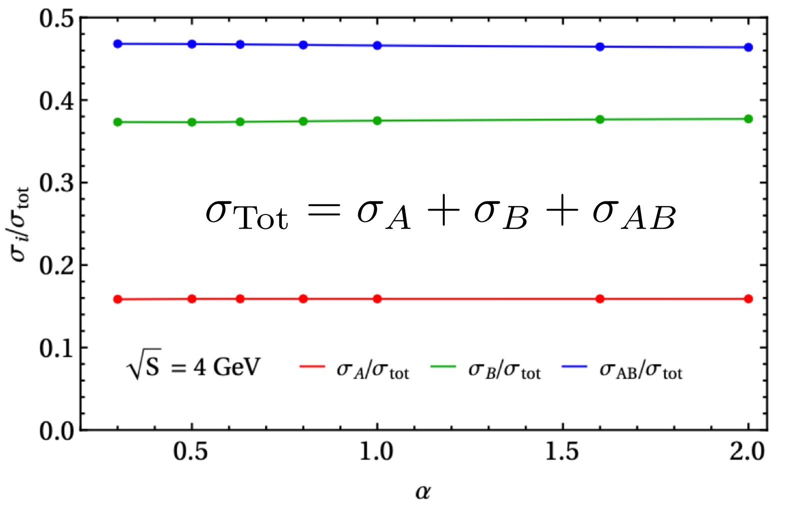
- **DA parametrization:**

$$\phi_\alpha(z) = \frac{i\, f_\pi}{2} \cdot \left[\frac{z^\alpha (1-z)^\alpha}{B(1+\alpha, 1+\alpha)} \right]$$

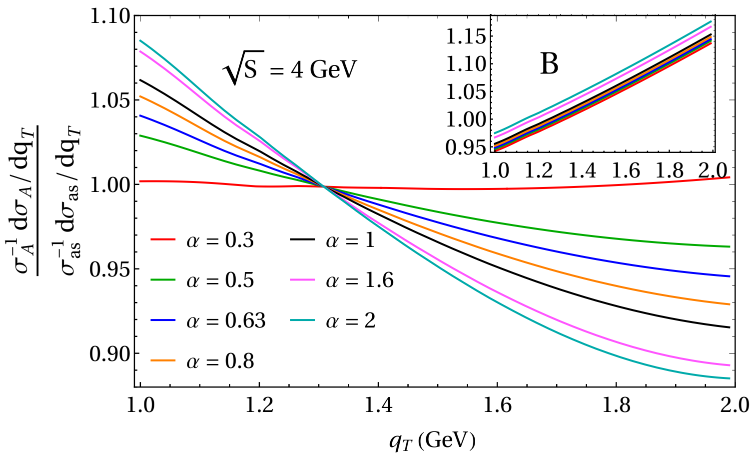
Change α

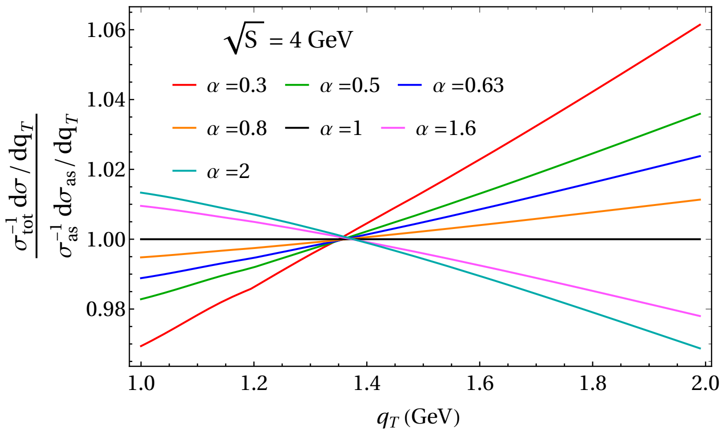
$\Longrightarrow$ Change z dependence

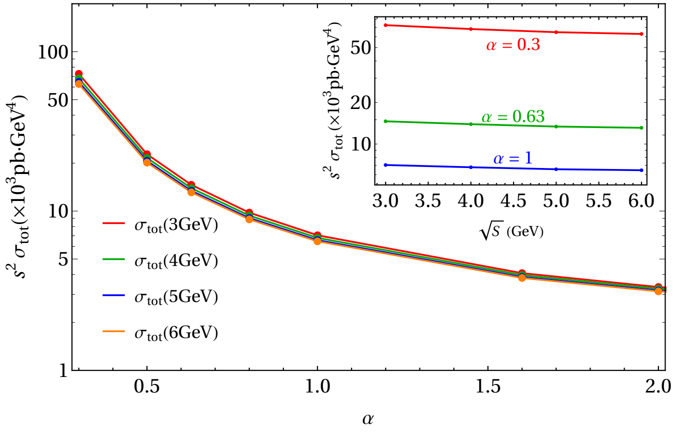




- **q_T distribution:** $\dfrac{d\sigma}{dq_T} \sim |\phi(z)|^2$







A: photons from *two* quark lines

B: photons from *one* quark line

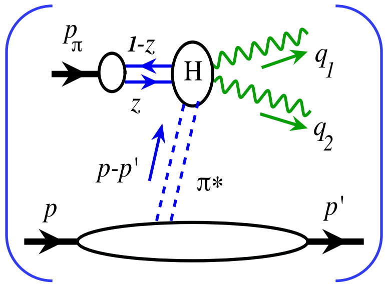
Total: $\sigma_{Tot} = \sigma_A + \sigma_B + \sigma_{AB}$

$$\sigma_{tot} = \int_{1\ \text{GeV}}^{\sqrt{s}/2} dq_T\, \frac{d\sigma}{dq_T}$$

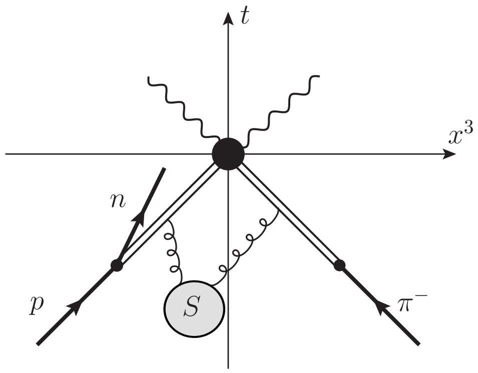
Exclusive Massive Photon-Pair Production in Meson-Baryon Collision

- **Factorization:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

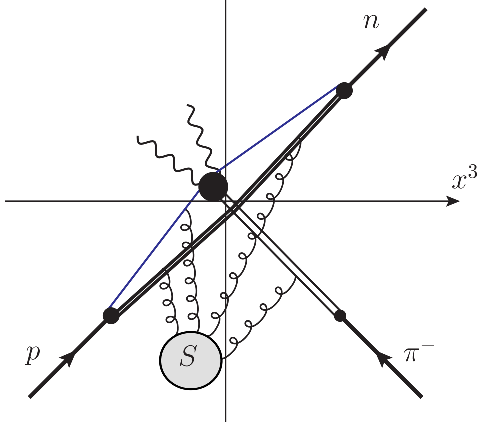
Additional region: DGLAP region!



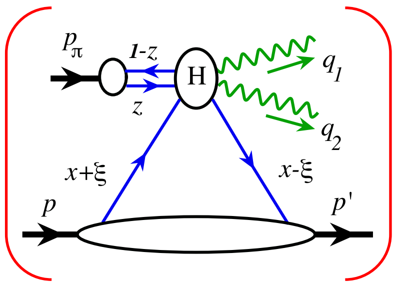
(Efremov, Radyushkin, Brodsky, Lepage)



ERBL region



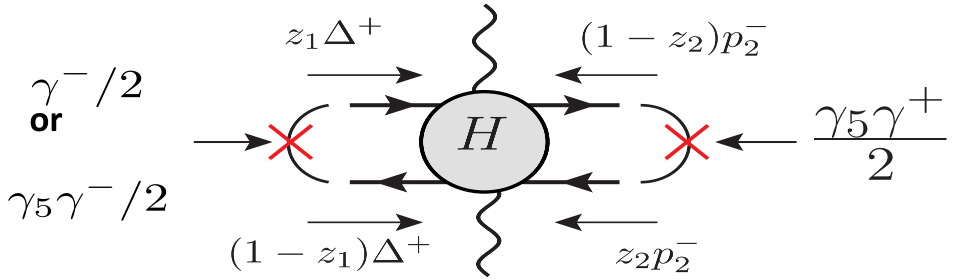
DGLAP region



- Different soft structures
- Factorization proof needs to be modified

$$\mathcal{M} = \mathrm{GPD} \otimes \mathrm{DA} \otimes \mathrm{Hard}$$

- **Additional channels – more GPDs:**



$\gamma^-/2$ corresponds to $F^u_{pn} \supset (H, E)$

$\gamma_5\gamma^-/2$ corresponds to $\widetilde{F}^u_{pn} \supset (\widetilde{H}, \widetilde{E})$

- **Factorization formula:**

$$N = -2ig^2(C_F/N_c)(1/\hat{s})$$

$$x_L = \frac{\xi - 1}{2\xi}, \quad x_R = \frac{\xi + 1}{2\xi}$$

$$\mathcal{M}_{\lambda\lambda'} = N \int_{x_L}^{x_R} \mathrm{d}x \int_0^1 \mathrm{d}z\, \widetilde{H}(x,\xi,t) D(z) O_{\lambda\lambda'}(x,z)$$

$$\widetilde{\mathcal{M}}_{\lambda\lambda'} = N \int_{x_L}^{x_R} \mathrm{d}x \int_0^1 \mathrm{d}z\, H(x,\xi,t) D(z) \widetilde{O}_{\lambda\lambda'}(x,z)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}\xi\,\mathrm{d}q_T^2} \propto |\mathcal{M}|^2$$

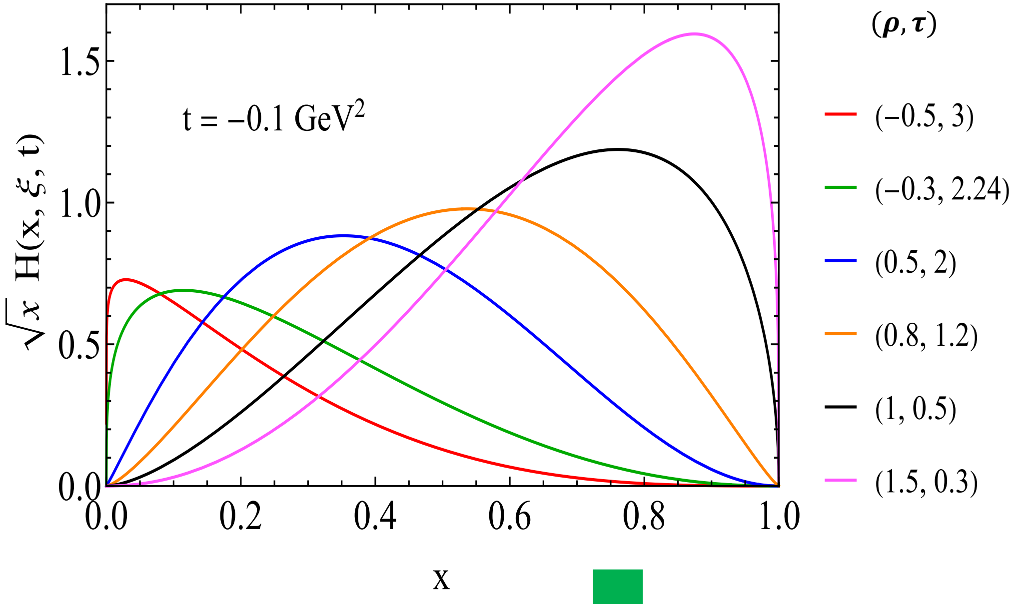
Exclusive Massive Photon-Pair Production in Meson-Baryon Collision

❑ GPD models – simplified GK model:

$$H_{pn}(x,\xi,t) = \theta(x)\, x^{-0.9\,(t/\mathrm{GeV}^2)} \frac{x^\rho (1-x)^\tau}{B(1+\rho, 1+\tau)}$$

$$\widetilde{H}_{pn}(x,\xi,t) = \theta(x)\, x^{-0.45\,(t/\mathrm{GeV}^2)} \frac{1.267\, x^\rho (1-x)^\tau}{B(1+\rho, 1+\tau)}$$

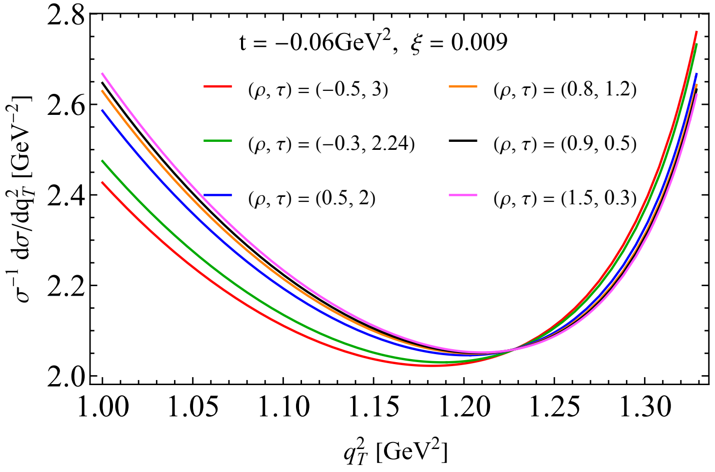
Goloskokov, Kroll
hep-ph/0501242
arXiv: 0708.3569
arXiv: 0906.0460

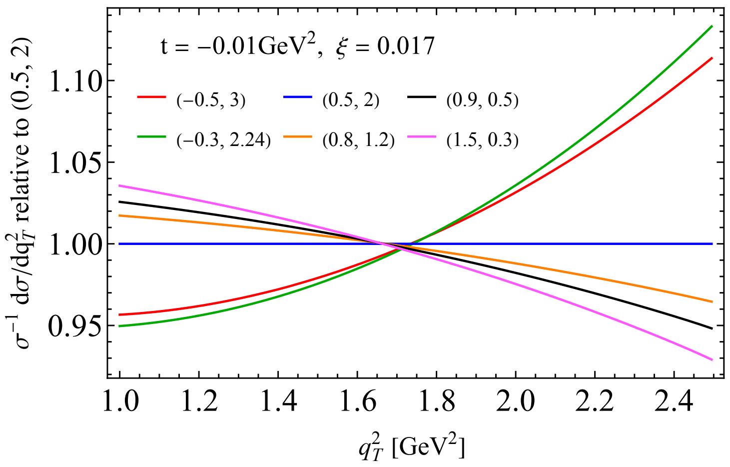


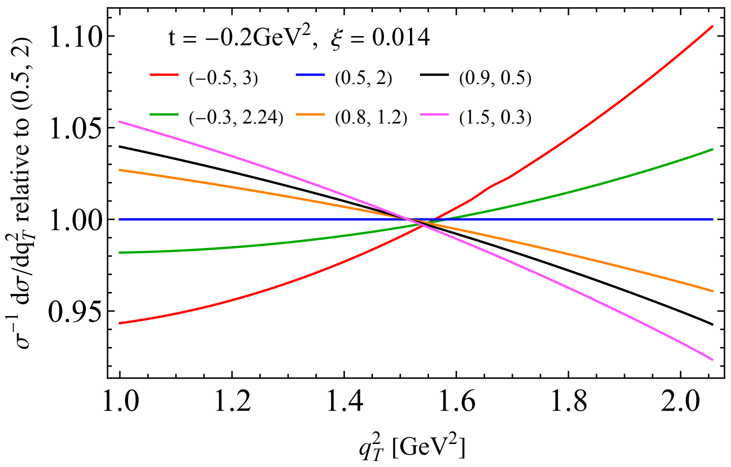
- Neglect $E, \widetilde{E}$. Neglect evolution effect.
- Tune (ρ, τ) to control x shape.
- Fix DA: $D(z) = N\, z^{0.63}(1-z)^{0.63}$

❑ Normalized q_T distribution:

$$\frac{d\sigma}{dt\, d\xi\, dq_T^2} \sim |H(x,\xi,t)|^2$$







$E_\pi = 150$ GeV

Exclusive Massive Photon-Pair Production in Meson-Baryon Collision

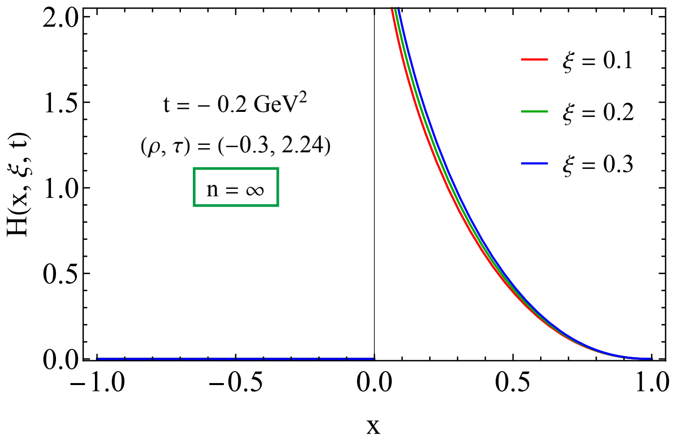
❑ GPD models – modified GK model:

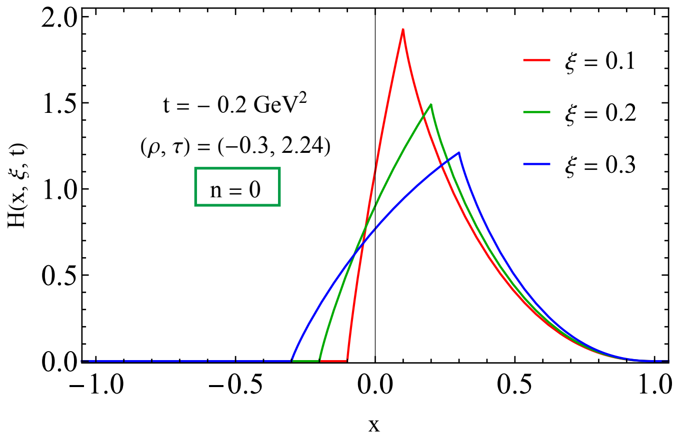
$$H(x,\xi,t) = \int \mathrm{d}\beta\,\mathrm{d}\alpha\;\delta(x-\beta-\xi\alpha)\,f(\beta,\alpha,t)$$

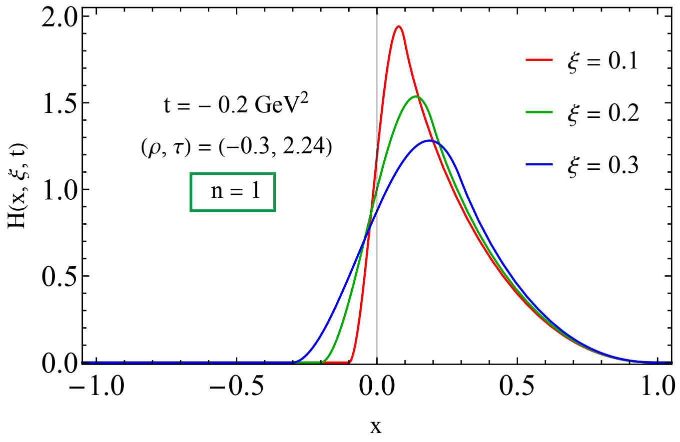
$$f(\beta,\alpha,t) = e^{(b+\alpha'\ln|\beta|^{-1})t}\cdot h(\beta)\cdot w(\beta,\alpha)$$

$$w(\beta,\alpha) = \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)}\,\frac{\left[(1-|\beta|)^2-\alpha^2\right]^n}{(1-|\beta|)^{2n+1}}$$

- **Change n to change ξ dependence**
- **Choose $n = 0, 1, \infty$**



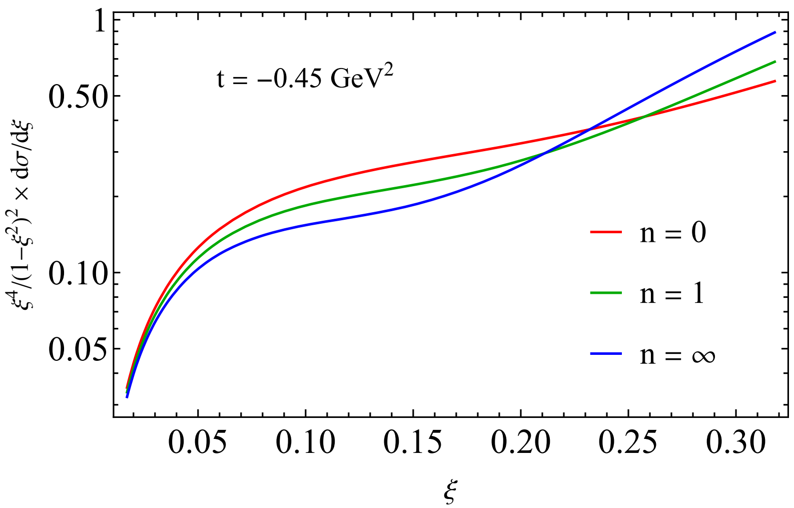


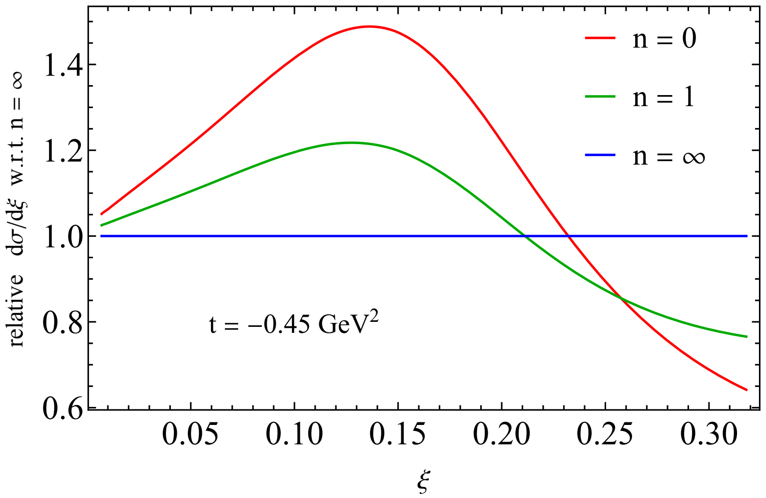


❑ ξ distribution (integrate out q_T):

$$q_T \geq 1\ \mathrm{GeV}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}\xi\,\mathrm{d}q_T^2} \sim |H(x,\xi,t)|^2$$

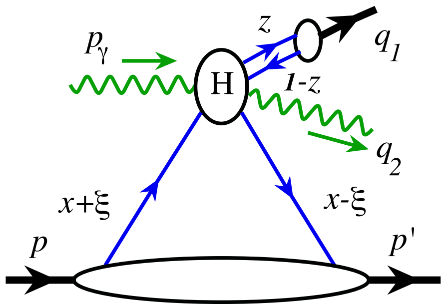




Exclusive Photo-Production of a $\pi\gamma$ Pair

- **Process:** $\gamma(p_\gamma) + h(p) \to \pi^{\pm}(q_1) + \gamma(q_2) + h'(p')$

  Introduced by G. Duplancic et al. [JHEP 11 (2018) 179],
  No contribution from gluon GPDs



Hall D at JLab

$$\frac{d\sigma}{dt\,d\xi\,dq_T^2} \quad \text{or} \quad \frac{d\sigma}{dt\,d\xi\,d\cos\theta}$$

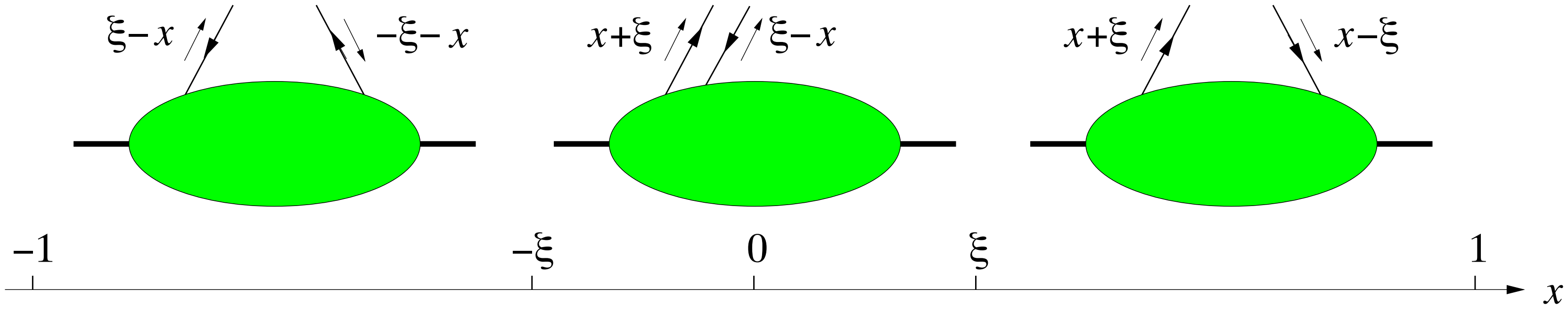
- **Factorization:**

  Proved to be valid when $q_T \gg \sqrt{|t|} \gtrsim \Lambda_{\rm QCD}$

- **Cancellation of unwanted propagators & cosθ dependence:**

$$\mathrm{Re}\,O_{++} = (e_1 - e_2)^2 \left[\frac{1-\cos\theta}{1+\cos\theta} \cdot P\frac{x+z-2xz}{2\,x\,z\,(1-x)\,(1-z)}\right] + (e_1^2 - e_2^2)\left[\frac{2}{1-\cos\theta} \cdot P\frac{x-z}{x\,z\,(1-x)\,(1-z)}\right]$$
$$- e_1 e_2 P\frac{1-\cos\theta}{x\,z\,(1-x)\,(1-z)} \cdot \frac{(xz+(1-x)(1-z))\,(x(1-x)+z(1-z))}{(2(1-x)(1-z)-(1+\cos\theta)xz)\,(2xz-(1+\cos\theta)(1-x)(1-z))}$$

- **Sensitive to ERBL region (complementary)**



Also sensitive to DA in the bulk region.

Exclusive $\pi^0\gamma$ Pair Production

❑ Phenomenology:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}|t|\,\mathrm{d}\xi\,\mathrm{d}\cos\theta_\pi\,\mathrm{d}\phi_\pi} = \frac{|\mathcal{A}|^2}{32\,s\,(2\pi)^4\,(1+\xi)^2}$$

$$\frac{1}{2}\overline{|\mathcal{A}|^2} = \left(\frac{2\pi\alpha_s}{s}f_\pi\right)^2\left(\frac{C_F}{N_c}\right)^2\left(\frac{1+\xi}{\xi}\right)^2(1-\xi^2) \times\left[|O_{++}^{[\widetilde{H}]}|^2 + |O_{+-}^{[\widetilde{H}]}|^2 + |\widetilde{O}_{++}^{[H]}|^2 + |\widetilde{O}_{+-}^{[H]}|^2\right]$$

- **Factorized helicity amplitude:**

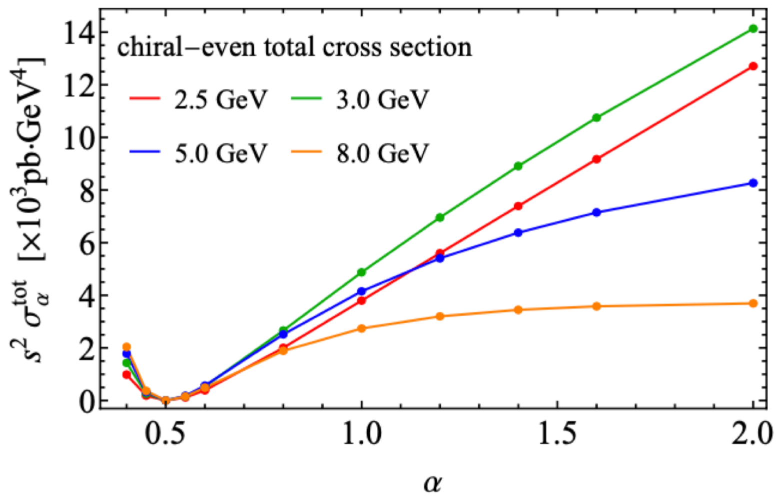
$$O_{\lambda\lambda'}^{[\widetilde{H}]} = \sum_q \int_{x_L}^{x_R}\mathrm{d}x\int_0^1 \mathrm{d}z\,\widetilde{H}^q(x,\xi,t)\,\phi_\pi^q(z)\,O_{\lambda\lambda'}^q(x,z)$$

- **Pion distribution amplitude:**

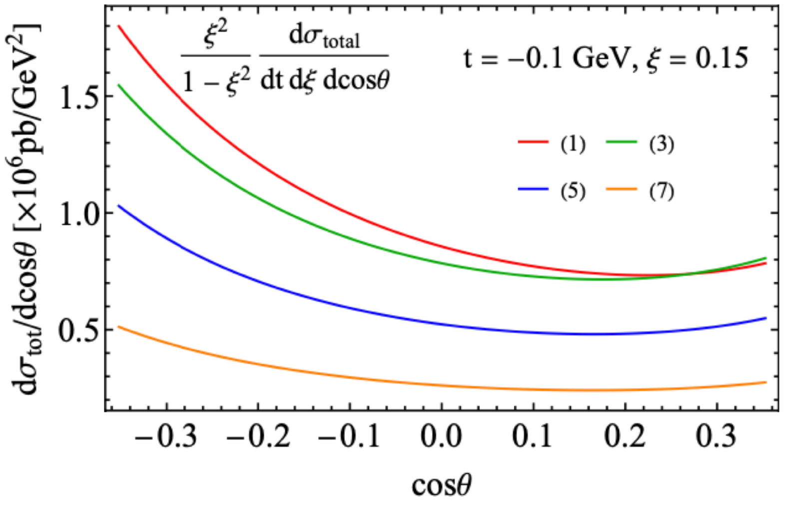
$$\phi_{\pi^0}^d(z) = \phi_{\pi^0}^u(z) = \frac{1}{\sqrt{2}}\frac{z^\alpha(1-z)^\alpha}{\mathrm{B}(1+\alpha,1+\alpha)}, \quad (\alpha > 0)$$

- **Model GPDs = simplified GK model:**
  - Taking $n_i = 0$
  - Parametrizing the forward limit as $x^a(1-x)^b$
  - Neglecting the D-term

❑ Sensitivity on DAs (total – q_T > 1 GeV):



❑ Sensitivity on GPDs (α = 0.63):



Exclusive $\pi^0\gamma$ Pair Production

Phenomenology:

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$$\frac{1}{2}\overline{|\mathcal{A}|^2} = \left(\frac{2\pi\alpha_s}{s}f_\pi\right)^2\left(\frac{C_F}{N_c}\right)^2\left(\frac{1+\xi}{\xi}\right)^2(1-\xi^2) \times \left[|O_{++}^{[\widetilde{H}]}|^2 + |O_{+-}^{[\widetilde{H}]}|^2 + |\widetilde{O}_{++}^{[H]}|^2 + |\widetilde{O}_{+-}^{[H]}|^2\right]$$

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$$O_{\lambda\lambda'}^{[\widetilde{H}]} = \sum_q \int_{x_L}^{x_R} dx \int_0^1 dz\,\widetilde{H}^q(x,\xi,t)\,\phi_\pi^q(z)\,O_{\lambda\lambda'}^q(x,z)$$

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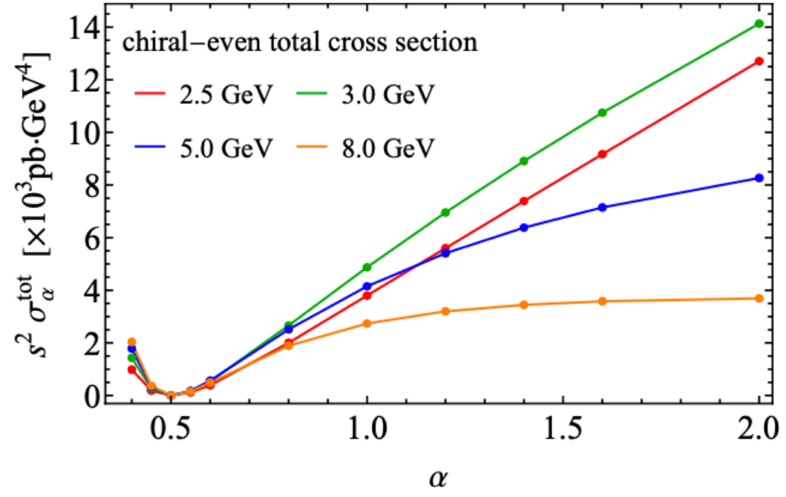
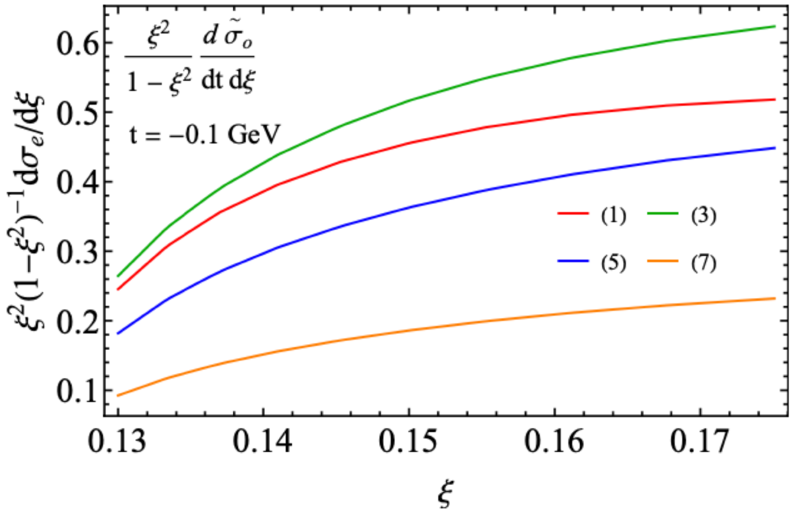
  - Taking $n_i = 0$
  - Parametrizing the forward limit as $x^a(1-x)^b$
  - Neglecting the D-term

Sensitivity on DAs (total – q_T > 1 GeV):



Sensitivity on GPDs (α = 0.63):



Jefferson Lab

Summary and Outlook

- Prove QCD factorization for a new type of exclusive two-scale observables
  – exclusive production of a pair of high-P_T photons in meson-meson and meson-baryon collisions

  - This process is factorizable and sensitive to pion DAs and hadron GPDs
  - Complementary to the exclusive deep virtual lepton-hadron scattering processes, such as DVCS, DVMP, …
  - The hard scale of the process is given by the transverse momentum of produced photon in the lab frame,
    not by a virtual photon in the exclusive lepton-hadron scattering
  - More sensitive to the x-dependence of pion DA and hadron GPDs, …

- This process can be generated to similar factorizable exclusive two-scale observables that could be measured at JLab, J-PARC, Amber, EIC, EICC, …

  - Photoproduction of $\pi\gamma$, introduced by G. Duplancic et al.
  - Polarization asymmetries of photoproduction can provide even more sensitive information GPDs
  - More observables could be explored – hard part (the probe) should be sensitive to the momentum difference of the two active partons from the diffracted hadron

Thank you!