Foundations of the Parton Model Description of TMDs

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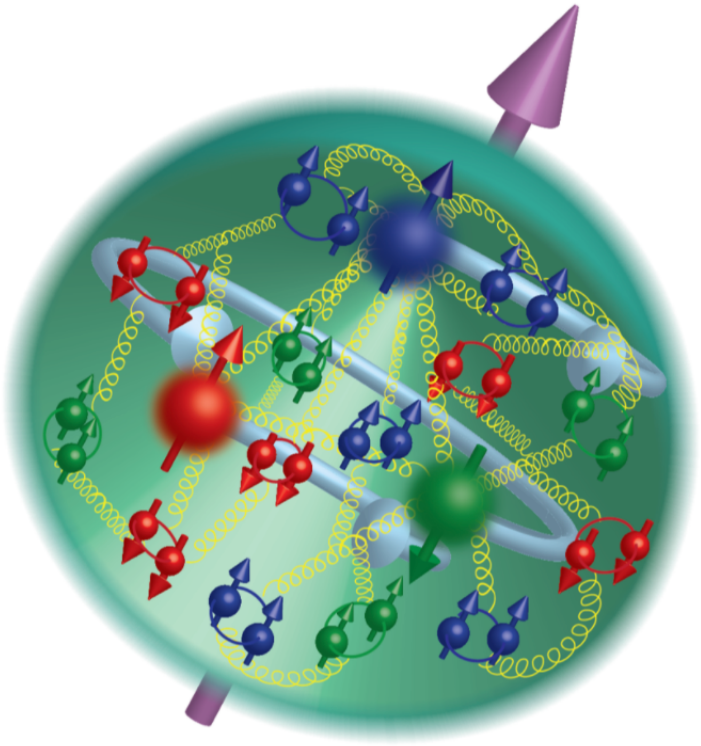
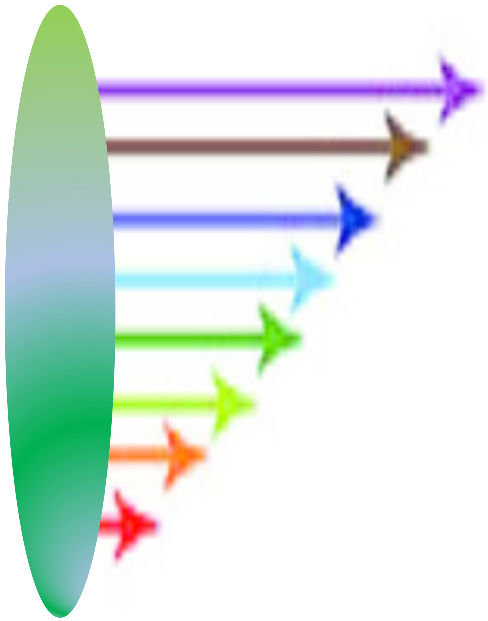
Light Cone 2021

Outline

- Motivation for a Covariant parton model
- The Assumptions of the model
- Limitations of the model
- Consistency of the model
- Summary

Parton Models



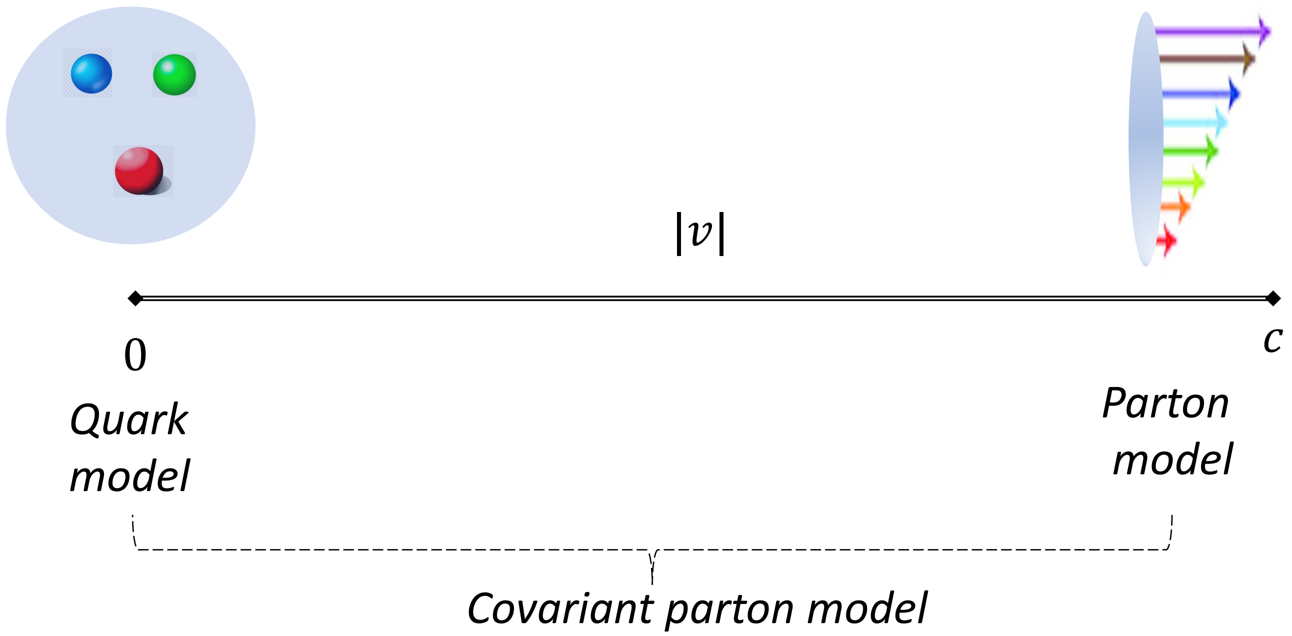


The nucleon has a complex structure

This complex structure is simplified when boosted to the IMF → Parton model

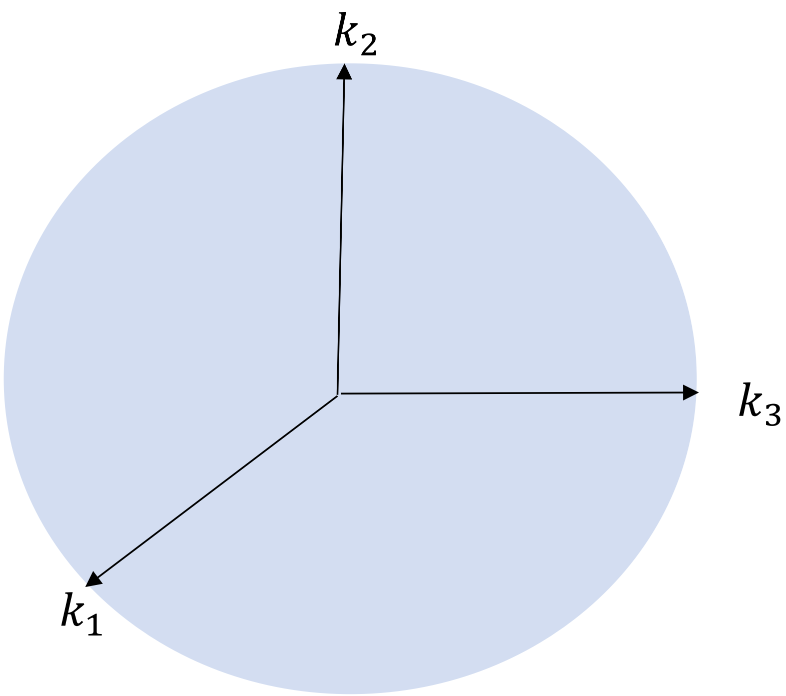
Covariant Parton Model

Formulate a covariant theory that does not prefer any special reference system (like IMF) and produces the quark model for slow hadrons, the parton model for fast hadrons…



Covariant Parton Model

Nucleon Rest Frame



$$k \equiv (k^0, k^1, k^2, k^3)$$
$$P \equiv (M, 0, 0, 0)$$

Petr Zavada-The structure functions and parton momenta distribution in the hadron rest system,1996

The assumptions of CPM

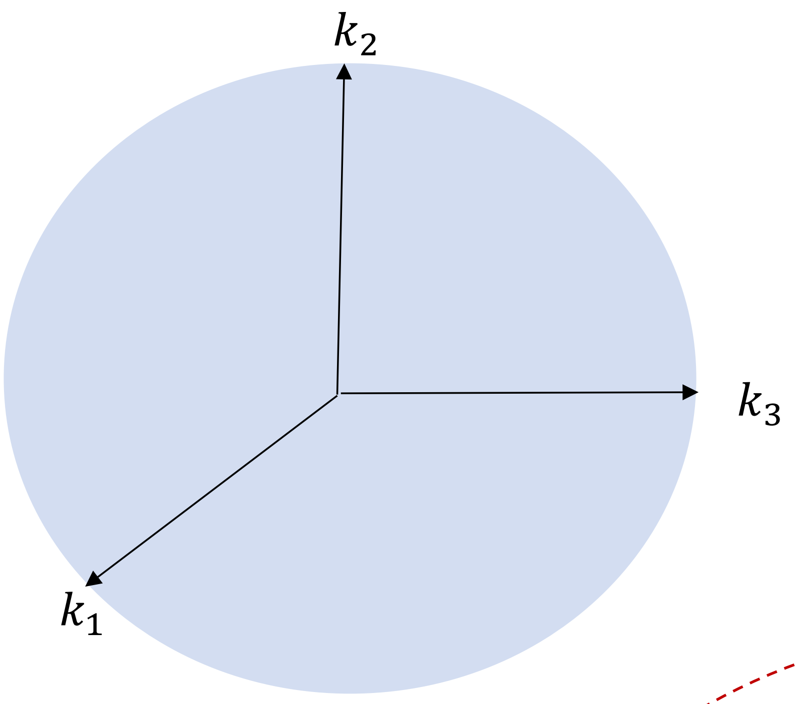
1- Spherical phase space is assumed:

$$\sqrt{k_1^2 + k_2^2 + k_3^2} \leq k_m$$

2- Quasi free partons are on mass shell: $k^2 = m^2$

Covariant Parton Model

Nucleon Rest Frame



$$k \equiv (k^0, k^1, k^2, k^3)$$

$$P \equiv (M, 0, 0, 0)$$

Now we can define a polarization vector for partons:

$$\omega_\mu = AP_\mu + BS_\mu + Ck_\mu$$

Petr Zavada-The structure functions and parton momenta distribution in the hadron rest system,1996

The assumptions of CPM

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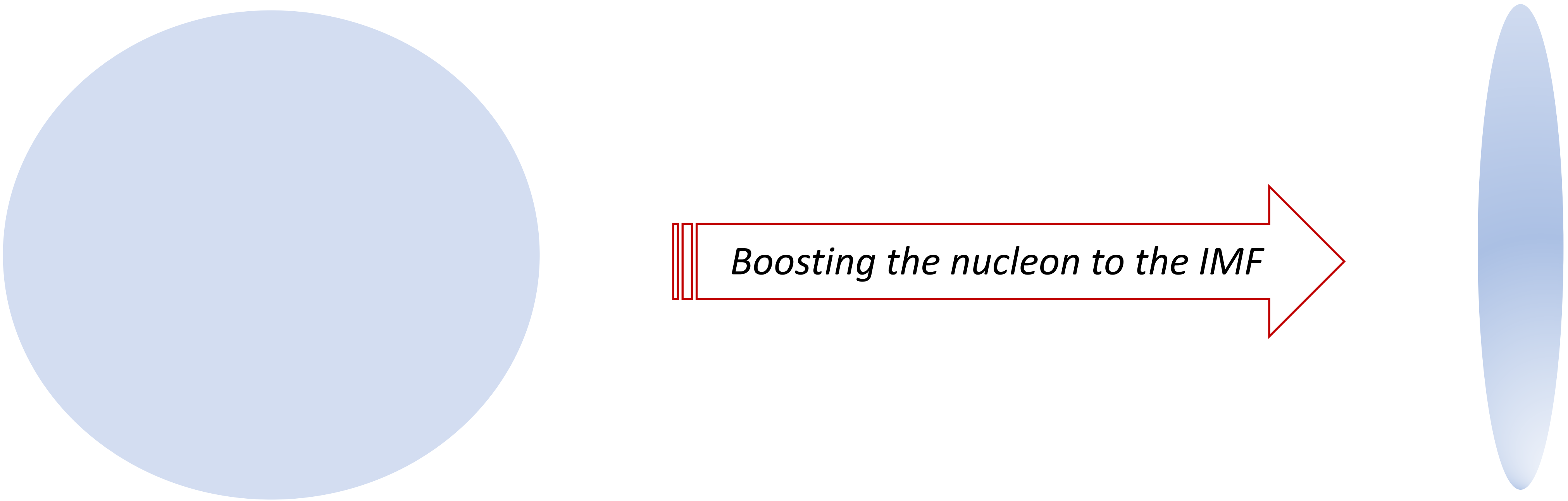
Covariant Parton Model

Nucleon Rest Frame

Infinite Momentum Frame



Nucleon Rest Frame:

$$k \equiv (k^0, k^1, k^2, k^3)$$
$$P \equiv (M, 0, 0, 0)$$
$$x = \frac{k^0 + k^3}{M}$$

Infinite Momentum Frame:

$$k' \equiv (k'^0, k^1, k^2, k'^3)$$
$$P' \equiv (P'^0, P^1, P^2, P'^3)$$
$$x = \frac{k^0 + k^3}{P^0 + P^3}$$

Covariant Parton Model

Assumptions

- Spherical phase space
- On-shell partons
- Consistency with IMF

Limits

$$\frac{m^2}{M^2} \leq x \leq 1$$

$$\frac{m^2 - M^2}{2M} \leq k_3 \leq \frac{M^2 - m^2}{2M}$$

$$0 \leq k_T^2 \leq M^2\left(x - \frac{m^2}{M^2}\right)(1 - x)$$

Covariant Parton Model

P. Zavada, Phys. Rev. D 55, 4290 (1997) [hep-ph/9609372]

P. Zavada, Phys. Rev. D 65, 054040 (2002) [hep-ph/0106215]

P. Zavada, Phys. Rev. D 67, 014019 (2003) [hep-ph/0210141]

$\Rightarrow$ $f_1(x),\ g_1(x),\ g_T(x)$

A. V. Efremov, O. V. Teryaev and P. Zavada,
B. Phys. Rev. D 70, 054018 (2004) [hep-ph/0405225].

$\Rightarrow$ $\ldots + \ h_1(x)$

A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada,
Phys. Rev. D 80, 014021 (2009) [arXiv:0903.3490 [hep-ph]].

$\Rightarrow$ $\ldots + f_1(x,k_T), g_1(x,k_T), h_1(x,k_T), g_{1T}^{\perp}(x,k_T)$

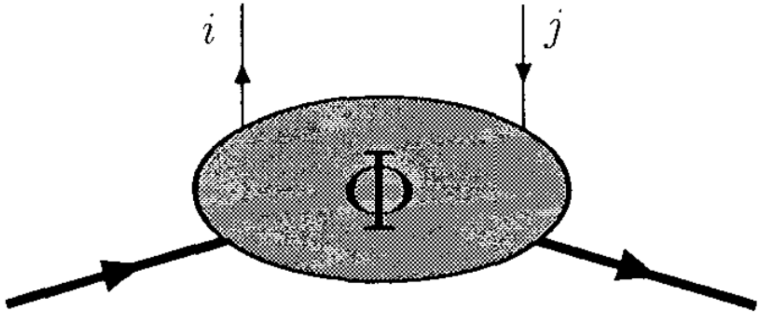
$, h_{1L}^{\perp}(x,k_T), h_{1T}^{\perp}(x,k_T)$

Covariant Parton Model

Structure of the nucleon at leading and subleading twist in the covariant parton model - Bastami, Efremov, Schweitzer, Teryaev, Zavada-2020

$$\Phi(P,k,S,\eta) = MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{1}{2M}[\not{P},\not{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$

$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$



k: Parton momentum

P, S: Hadron momentum, spin

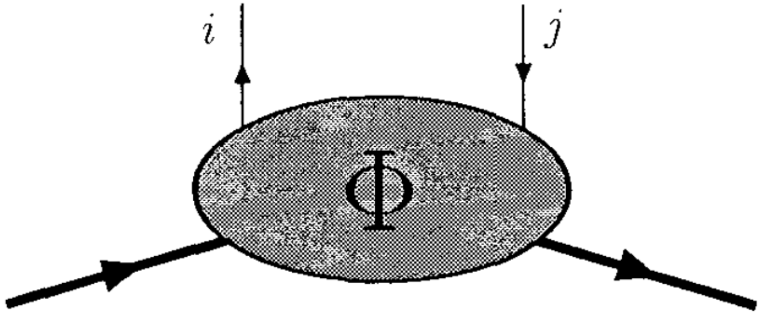
M: Hadron mass

$A_j \equiv A_j(k^2, k.P)$

Covariant Parton Model

$$\Phi(P,k,S,\eta) = MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{1}{2M}[\not{P},\not{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$

$$+ \frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$

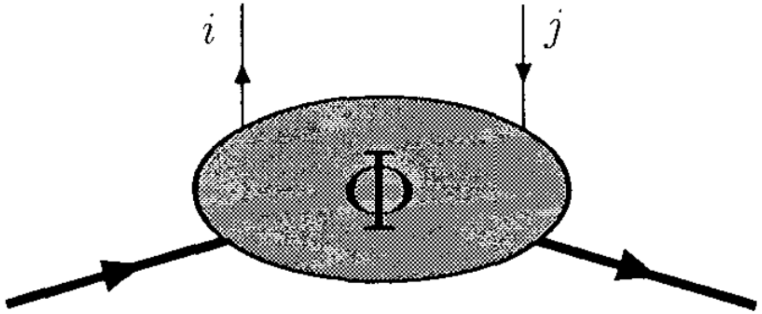


In models without gauge field degrees of freedom
T-odd amplitudes, A_4 , A_5 , A_{12} *and*
all the B_i *amplitudes are absent*

Covariant Parton Model

$$\Phi(P,k,S) = MA_1 + \not{P}A_2 + \not{k}A_3 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$

$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11}$$

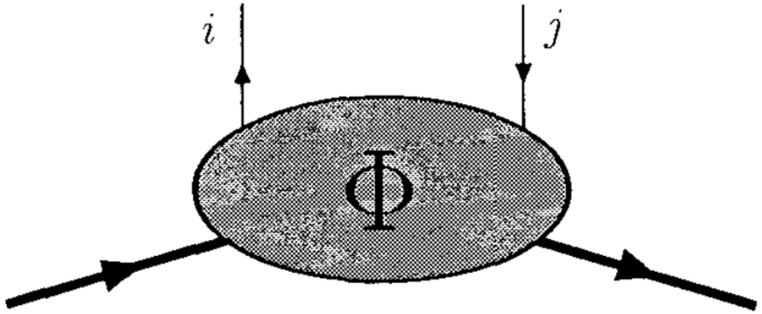


No gauge field degrees of freedom → Down to 9 amplitudes

Covariant Parton Model

$$\Phi(P,k,S) = MA_1 + \not{P}A_2 + \not{k}A_3 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$

$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11}$$



Assuming the partons are on-shell, $Tr[\Phi\Gamma(\not{k}+m)] = 0$

leads to the following relations between the amplitudes

$$A_2 = A_9 = 0$$

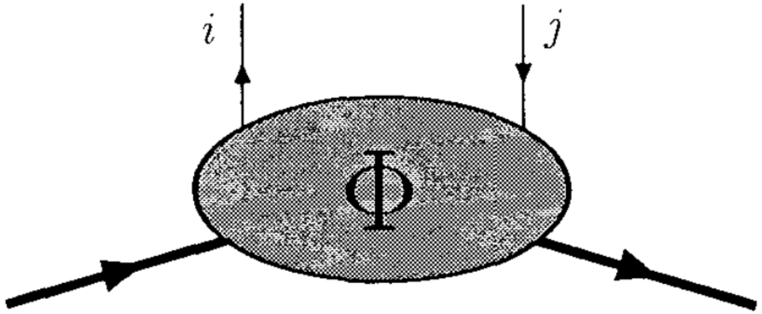
$$A_1 = \frac{m}{M}A_3 \qquad A_6 = \frac{m}{M}A_{10} \qquad A_7 = -\frac{m}{M}A_{11}$$

$$A_{10} = \frac{(P\cdot k)}{M^2}A_{11} - \frac{m}{M}A_8$$

Covariant Parton Model

$$\Phi(P,k,S) = MA_1 + \not{P}A_2 + \not{k}A_3 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$

$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11}$$

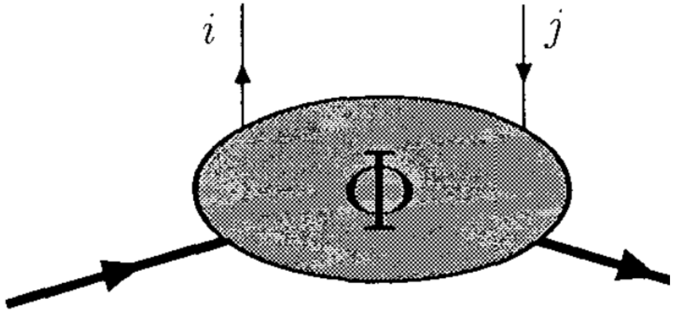


On mass shell quarks → Down to 3 amplitudes

$$\Phi(P,k,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)A_{11} - mMA_8\Big]\left\{\not{S} - \frac{(k.S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{P}A_{11} + \frac{M}{m}\frac{M(k\cdot S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{k}A_8\right\}\gamma_5$$

Covariant Parton Model

$$\Phi(P,k,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)A_{11} - mMA_8\Big]\left\{\not{S} - \frac{(k.S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{P}A_{11} + \frac{M}{m}\frac{M(k\cdot S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{k}A_8\right\}\gamma_5$$



$$\Phi(k,P,S)_{ij} = 2P^0\Theta(k^0)\delta(k^2-m^2)\bar{u}_j(k)u_i(k)\times\begin{cases}\mathcal{G}(kP) & \text{unpolarized partons}\\ \mathcal{H}(kP) & \text{polarized partons.}\end{cases}$$

$$Tr[\Phi(P,k,S)\Gamma] = P^0\Theta(k^0)\delta(k^2-m^2)Tr\Big[(\not{k}+m)(\mathcal{G}(kP)+\mathcal{H}(kP)\gamma^5\not{u})\Gamma\Big]$$

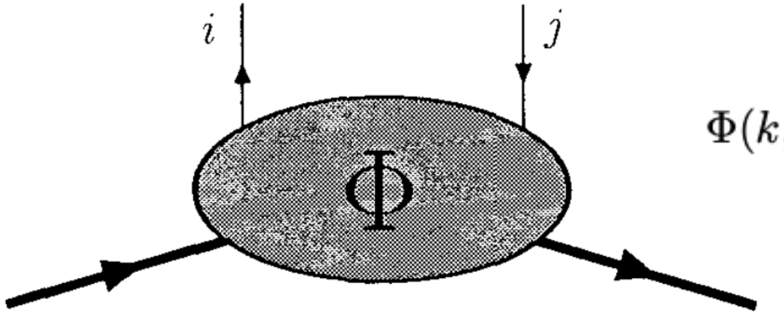
Covariant Parton Model

$$\Phi(P,k,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)A_{11} - mMA_8\Big]\left\{\not{S} - \frac{(k.S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{P}A_{11} + \frac{M}{m}\frac{M(k\cdot S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{k}A_8\right\}\gamma_5$$

$$\underbrace{\left\{\cdots\right\}}_{\not{w}}$$

$$\mathcal{G}(kP) \geq 0$$

$$|\mathcal{H}(kP)| \leq \mathcal{G}(kP)$$



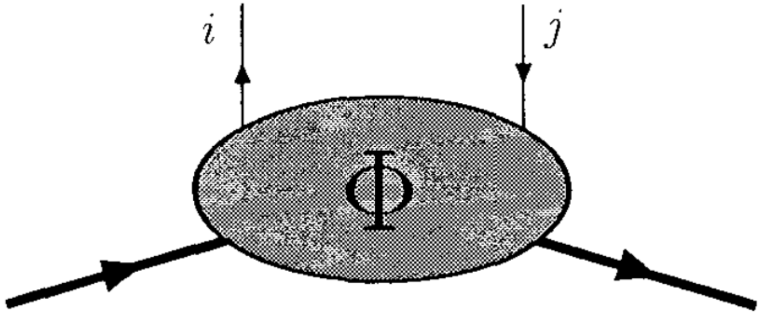
$$\Phi(k,P,S)_{ij} = 2P^0\Theta(k^0)\delta(k^2-m^2)\bar{u}_j(k)u_i(k) \times \begin{cases} \mathcal{G}(kP) & \text{unpolarized partons} \\ \mathcal{H}(kP) & \text{polarized partons.} \end{cases}$$

$$Tr[\Phi(P,k,S)\Gamma] = P^0\Theta(k^0)\delta(k^2-m^2)Tr\Big[(\not{k}+m)(\mathcal{G}(kP) + \mathcal{H}(kP)\gamma^5\not{w})\Gamma\Big]$$

Covariant Parton Model

$$\Phi(P,k,S) = (\not k + m)A_3 + \frac{(\not k + m)}{M^2}\Big[(P\cdot k)A_{11} - mMA_8\Big]\left\{\not S - \frac{(k.S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not P A_{11} + \frac{M}{m}\frac{M(k\cdot S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not k A_8\right\}\gamma_5$$

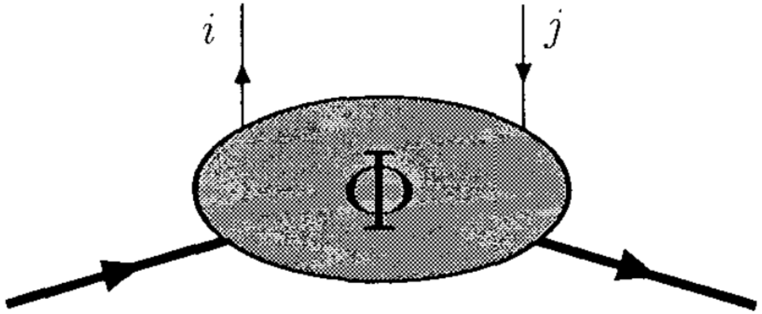
$\not\omega$



ω^2	
Mixed spin state	$-1 \leq \omega^2 \leq 0$
Pure spin state	$\omega^2 = -1$

Covariant Parton Model

$$\Phi(P,k,S) = (\not k + m)A_3 + \frac{(\not k + m)}{M^2}\Big[(P\cdot k)A_{11} - mMA_8\Big]\underbrace{\left\{\not S - \frac{(k.S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not P A_{11} + \frac{M}{m}\frac{M(k\cdot S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not k A_8\right\}}_{\not\omega}\gamma_5$$



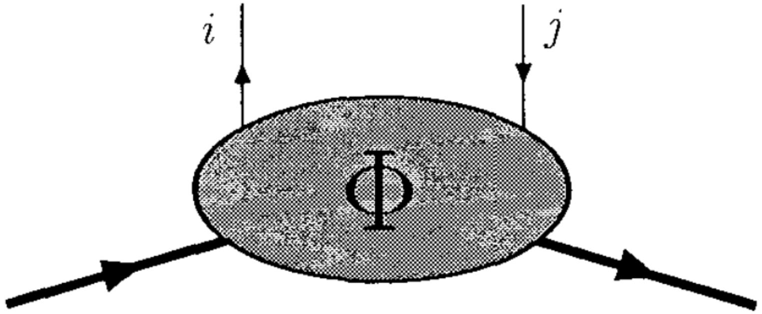
Assuming pure spin states, ω^2=-1

leads to the following relations between the amplitudes

$$A_8 = \mp A_{11}$$

Covariant Parton Model

$$\Phi(P,k,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)A_{11} - mMA_8\Big]\underbrace{\left\{\not{S} - \frac{(k.S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{P}A_{11} + \frac{M}{m}\frac{M(k\cdot S)}{\Big[(P\cdot k)A_{11} - mMA_8\Big]}\not{k}A_8\right\}}_{\not{\psi}}\gamma_5$$



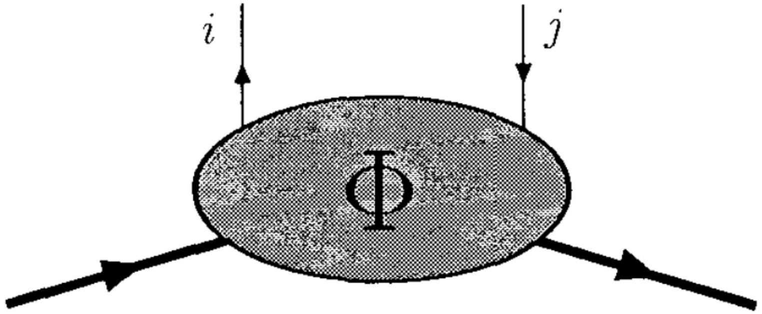
Pure spin states → Down to 2 amplitudes

$$\Phi(P,k,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k) + mM\Big]\left\{\not{S} - \frac{(k.S)}{\Big[(P\cdot k) + mM\Big]}\not{P} - \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k) + mM\Big]}\not{k}\right\}\gamma_5 A_{11}$$

Covariant Parton Model

$$\Phi(P,k,S,\eta) = MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{1}{2M}[\not{P},\not{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$

$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$



- *No gauge field degrees of freedom*
- *On mass shell quarks*
- *Pure quark spin states*

$$\Phi(P,k,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]\left\{\not{S} - \frac{(k.S)}{\Big[(P\cdot k)+mM\Big]}\not{P} - \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)+mM\Big]}\not{k}\right\}\gamma_5 A_{11}$$

Twist-2 TMDs in Covariant Parton Model

Nucleon polarization / Quark polarization	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	h_1 $h_{1T}^{\perp}$

T-even TMDs (in blue color) can be computed in models based on quark degrees of freedom only. T-odd TMDs (in red color) require explicit gauge field degrees of freedom, and cannot be modeled in the approach used in this model.

Twist-2 TMDs in Covariant Parton Model

Nucleon polarization \ Quark polarization	U	L	T
U	f_1		
L		g_1	$h_{1L}^{\perp}$
T		$g_{1T}^{\perp}$	h_1 $h_{1T}^{\perp}$

T-even TMDs (in blue color) can be computed in models based on quark degrees of freedom only. T-odd TMDs (in red color) require explicit gauge field degrees of freedom, and cannot be modeled in the approach used in this model.

$h_1^{\perp}$ *and* $f_{1T}^{\perp}$ *cannot be calculated in CPM because they are T-odd*

Twist-3 TMDs in Covariant Parton Model

$$
\begin{aligned}
\phi^{[1]} &= \frac{M}{P^+}\left[e - \frac{\varepsilon^{jk}k_T^j S_T^k}{M} e_T^\perp\right], \\
\phi^{[i\gamma^5]} &= \frac{M}{P^+}\left[S_L e_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} e_T\right], \\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[\frac{k_T^j}{M} f^\perp + \varepsilon^{jk} S_T^k f_T + S_L \frac{\varepsilon^{jk}k_T^k}{M} f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl}S_T^l}{M^2} f_T^\perp\right], \\
\phi^{[\gamma^j\gamma^5]} &= \frac{M}{P^+}\left[S_T^j g_T + S_L \frac{k_T^j}{M} g_L^\perp + \frac{\kappa^{jk}S_T^k}{M^2} g_T^\perp + \frac{\varepsilon^{jk}k_T^k}{M} g^\perp\right], \\
\phi^{[i\sigma^{jk}\gamma^5]} &= \frac{M}{P^+}\left[\frac{S_T^j k_T^k - S_T^k k_t^j}{M} h_T^\perp - \varepsilon^{jk} h\right], \\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[S_L h_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} h_T\right].
\end{aligned}
$$

Twist-3 TMDs in Covariant Parton Model

$$
\begin{aligned}
\phi^{[1]} &= \frac{M}{P^+}\left[e - \frac{\varepsilon^{jk} k_T^j S_T^k}{M} e_T^\perp\right], \\
\phi^{[i\gamma^5]} &= \frac{M}{P^+}\left[S_L e_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} e_T\right], \\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[\frac{k_T^j}{M} f^\perp + \varepsilon^{jk} S_T^k f_T + S_L \frac{\varepsilon^{jk} k_T^k}{M} f_L^\perp - \frac{\kappa^{jk} \varepsilon^{kl} S_T^l}{M^2} f_T^\perp\right], \\
\phi^{[\gamma^j \gamma^5]} &= \frac{M}{P^+}\left[S_T^j g_T + S_L \frac{k_T^j}{M} g_L^\perp + \frac{\kappa^{jk} S_T^k}{M^2} g_T^\perp + \frac{\varepsilon^{jk} k_T^k}{M} g^\perp\right], \\
\phi^{[i\sigma^{jk}\gamma^5]} &= \frac{M}{P^+}\left[\frac{S_T^j k_T^k - S_T^k k_t^j}{M} h_T^\perp - \varepsilon^{jk} h\right], \\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[S_L h_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} h_T\right].
\end{aligned}
$$

$e_T^\perp$, e_L, e_T, f_T, $f_L^\perp$, $f_T^\perp$, g_T, and h cannot be calculated in CPM because they are T-odd

Limitations in Covariant Parton Model

Twist-2 TMDs

		Quark polarization		
Nucleon polarization		U	L	T
	U	f_1		
	L		g_1	$h_{1L}^{\perp}$
	T		$g_{1T}^{\perp}$	h_1 $h_{1T}^{\perp}$

Twist-3 TMDs

$$
\begin{aligned}
\phi^{[1]} &= \frac{M}{P^+}\left[e - \frac{\varepsilon^{jk} k_T^j S_T^k}{M} e_T^{\perp}\right], \\
\phi^{[i\gamma^5]} &= \frac{M}{P^+}\left[S_L e_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} e_T\right], \\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[\frac{k_T^j}{M} f^{\perp} + \varepsilon^{jk} S_T^k f_T + S_L \frac{\varepsilon^{jk} k_T^k}{M} f_L^{\perp} - \frac{\kappa^{jk} \varepsilon^{kl} S_T^l}{M^2} f_T^{\perp}\right], \\
\phi^{[\gamma^j \gamma^5]} &= \frac{M}{P^+}\left[S_T^j g_T + S_L \frac{k_T^j}{M} g_L^{\perp} + \frac{\kappa^{jk} S_T^k}{M^2} g_T^{\perp} + \frac{\varepsilon^{jk} k_T^k}{M} g^{\perp}\right], \\
\phi^{[i\sigma^{jk}\gamma^5]} &= \frac{M}{P^+}\left[\frac{S_T^j k_T^k - S_T^k k_t^j}{M} h_T^{\perp} - \varepsilon^{jk} h\right], \\
\phi^{[\gamma^j]} &= \frac{M}{P^+}\left[S_L h_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} h_T\right].
\end{aligned}
$$

No access to $T-odd$ TMDs

Lorentz Invariance Relations in Covariant Parton Model

Lorentz invariance relations (LIRs) **connect** the twist-2 and twist-3 parton distribution functions (PDFs) and weighted moments of transverse momentum dependent (TMD) correlation functions

The superscript (1) indicates the k_T^2 **moment** of the TMD

$$g_{1T}^{(1)}(x) = \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} g_{1T}(x, \mathbf{k}_T^2), \quad \text{etc.}$$

$$g_T^q(x) \overset{\text{LIR}}{=} g_1^q(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)q}(x)\,,$$

$$h_L^q(x) \overset{\text{LIR}}{=} h_1^q(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)q}(x)\,,$$

$$h_T^q(x) \overset{\text{LIR}}{=} -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)q}(x)\,,$$

$$g_L^{\perp q}(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)q}(x) \overset{\text{LIR}}{=} 0\,,$$

$$h_T^q(x, p_T) - h_T^{\perp q}(x, p_T) \overset{\text{LIR}}{=} h_{1L}^{\perp q}(x, p_T)\,,$$

P.J. Mulders and R.D. Tangerman, Nucl. Phys. B 461, 197-237 (1996).
D. Boer and P.J. Mulders, Phys. Rev. D 57, 5780 (1998).

EoM Relations in Covariant Parton Model

$$xe = x\tilde{e} + \frac{m}{M} f_1$$

$$xf^{\perp} = x\tilde{f}^{\perp} + f_1$$

$$xg_L^{\perp} = x\tilde{g}_L^{\perp} + g_1 + \frac{m}{M} h_{1L}^{\perp}$$

$$xg_T = \tilde{g}_T + g_{1T}^{\perp(1)} + \frac{m}{M} h_1$$

$$xg_T^{\perp} = x\tilde{g}_T^{\perp} + g_{1T}^{\perp} + \frac{m}{M} h_{1T}^{\perp}$$

$$xh_L = x\tilde{h}_L - 2h_{1L}^{\perp(1)} + \frac{m}{M} g_1$$

$$xh_T = x\tilde{h}_T - h_1 - h_{1T}^{\perp(1)} + \frac{m}{M} g_{1T}^{\perp}$$

$$xh_T^{\perp} = x\tilde{h}_T^{\perp} + h_1 - h_{1T}^{\perp(1)}$$

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$$xh_T^{\perp} = x\tilde{h}_T^{\perp} + h_1 - h_{1T}^{\perp(1)}$$

WW Relations in Covariant Parton Model

$$g_T^q(x) \overset{\text{WW}}{=} \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^q(y) + \frac{m}{M}\left[-\frac{h_1^q(x)}{x} + \int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^q(y)\right],$$

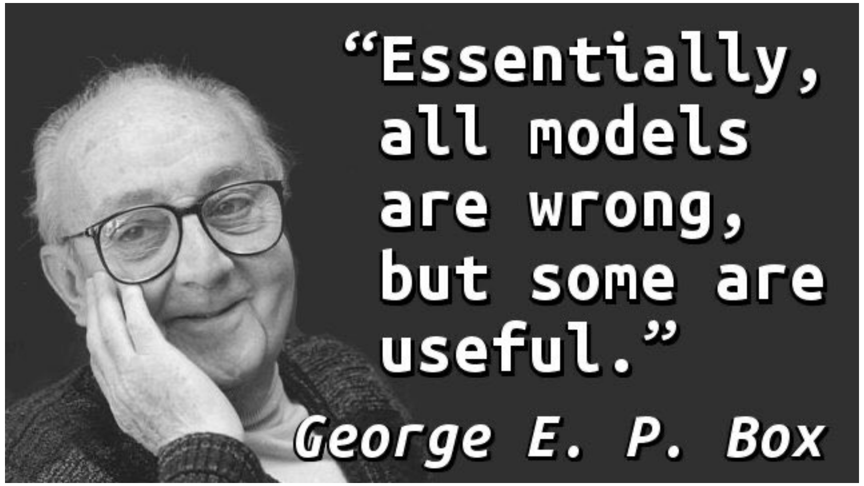
$$h_L^q(x) \overset{\text{WW}}{=} 2x\int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^q(y) + \frac{m}{M}\left[\frac{g_1^q(x)}{x} - 2x\int_x^1 \frac{\mathrm{d}y}{y^3}\, g_1^q(y)\right].$$

Quark model relations in Covariant Parton Model

$$\begin{aligned}
g_{1T}^{\perp q}(x,p_T) &= -h_{1L}^{\perp q}(x,p_T),\\
g_T^{\perp q}(x,p_T) &= -h_{1T}^{\perp q}(x,p_T),\\
g_L^{\perp q}(x,p_T) &= -h_T^{q}(x,p_T),\\
g_1^q(x,p_T) - h_1^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
g_T^q(x,p_T) - h_L^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
h_T^q(x,p_T) - h_T^{\perp q}(x,p_T) &= h_{1L}^{\perp q}(x,p_T).
\end{aligned}$$

These relations are valid in a large class of quark models, including spectator models, bag model, light-front constituent quark model

One value of the models is that they allow us to quantify how much of an observed phenomenon can be attributed to a specific model concept



Summary

- All polarized and unpolarized T-even TMDs are systematically obtained,
- TMD relations expected in QCD or supported by other quark models are satisfied

Outlook

- Exploring the anti-quark distributions
- Generalization to include off-shell-ness effects
- Wish to access T-odd TMDs