Forward quark jet-nucleus scattering in a light-front Hamiltonian approach

Meijian Li

Department of Physics, University of Jyväskylä, Finland

Helsinki Institute of Physics, University of Helsinki, Finland

In collaboration with: Tuomas Lappi (U. Jyväskylä) and Xingbo Zhao (IMPCAS)

Based on work: Phys. Rev. D 104, 056014 (2021)

Light Cone 2021: Physics of Hadrons on the Light Front

November 29 - December 4, 2021, Jeju Booyoung Hotel & ZOOM

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

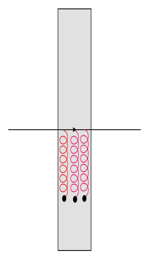
HELSINKI INSTITUTE OF PHYSICS

Motivation

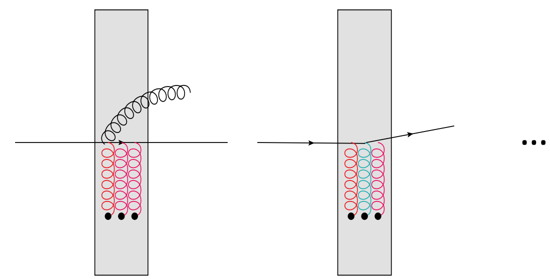
- Studying the quark-color field scattering establishes a foundation for understanding more complicated processes
  - High-energy quark nucleus scattering

    Interests: sub-eikonal effects

    The eikonal picture

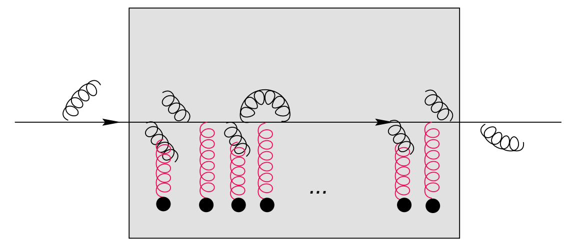


    Sub-eikonal corrections: finite-width field, gluon emission, spin flip,…



  - Jet quenching in colored medium

    Interests: interplay between coherence time of gluon emission and the timescales of the scattering centers of the medium



- Using a nonperturbative approach, we could
  - Relax the infinite-energy approximation
  - Explicitly solve the time evolution and therefore study the scattering and gluon emission/absorption at the same time

Outline

Methodology

- A time-dependent light-front Hamiltonian approach
- Application to quark-nucleus scattering in $|q\rangle + |qg\rangle$ Fock space

Results

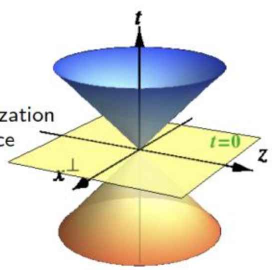
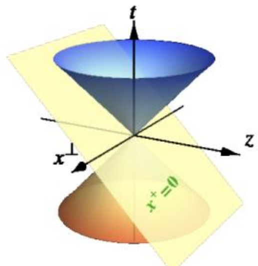
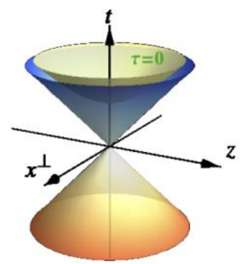
- Bare quark as an initial state [Phys. Rev. D 104, 056014 (2021)]
  1. Gluon emission and absorption
  2. Interaction with a background color field
- Dressed quark as an initial state [preliminary]
  1. QCD eigenstate
  2. Interaction with background field

Summary and outlooks

Methodology: Time-dependent Basis Light-Front Quantization (tBLFQ)[1]

- **Light-front quantization**

  The quantum field is quantized on the equal light-front time surface $x^+ = 0$

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = m v^\mu \ (v^2 = 1)$

- **Hamiltonian formalism**

  The state obeys the time-evolution equation

$$\frac{1}{2} P^-(x^+)|\psi(x^+)\rangle = i \frac{\partial}{\partial x^+} |\psi(x^+)\rangle$$

A nonperturbative treatment: the time evolution is divided into many small timesteps, each timestep is evaluated by numerical methods

$$|\psi(x^+)\rangle = \mathcal{T}_+ \exp\left[-\frac{\mathrm{i}}{2}\int_0^{x^+} dz^+ P^-(z^+)\right] |\psi(0)\rangle = \lim_{n\to\infty} \prod_{k=1}^{n} \mathcal{T}_+ \exp\left[-\frac{\mathrm{i}}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+)\right] |\psi(0)\rangle$$

- **Basis representation**

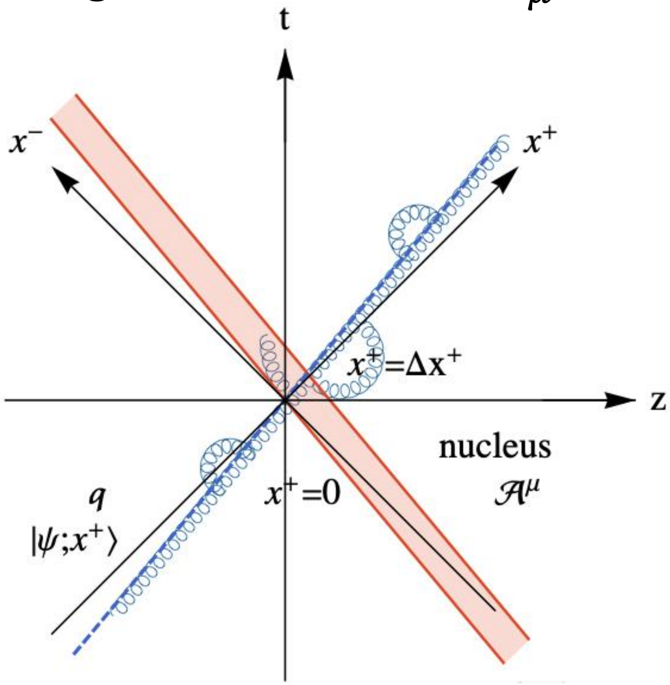
  Optimal basis has the symmetries of the system, and is key to numerical efficiency

1. J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, C. Yang., Phys. Rev. C81, 035205 (2010); X. Zhao, A. Ilderton, P. Maris, and J. P. Vary, Phys. Rev. D88, 065014 (2013).

Quark-nucleus scattering in the $|q\rangle + |qg\rangle$ Fock space: 1. the light-front Hamiltonian $P^-(x^+)$

We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.

- The light-front Hamiltonian is derived from the QCD Lagrangian with a background color field $\mathcal{A}_\mu$ in the $|q\rangle + |qg\rangle$ space



$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}{}_a F^a_{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

where $D_\mu = \partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$.

$$\mathrm{P}^-(x^+) = \mathrm{P}^-_{KE} + V_{qg} + V_{\mathcal{A}}(x^+)$$

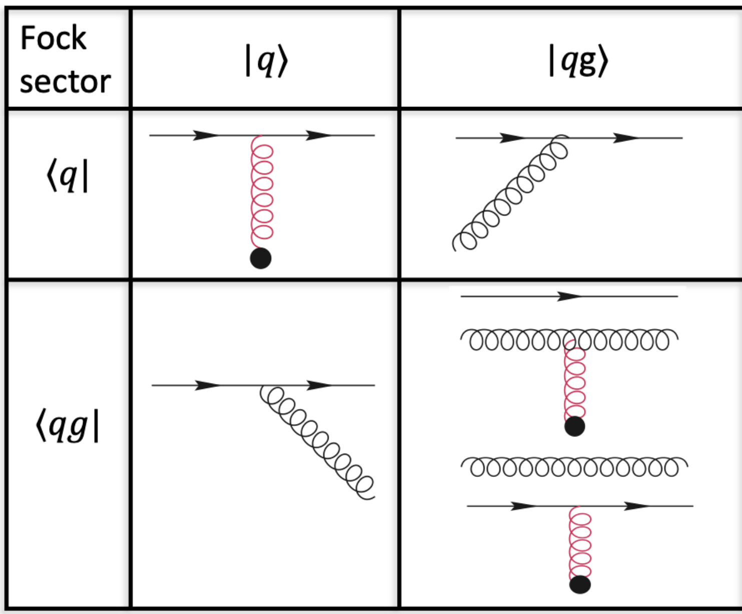
$\mathrm{P}^-_{KE} + V_{qg}$: P^-_{QCD}

$V_{\mathcal{A}}(x^+)$: background field

Quark-nucleus scattering in the $|q\rangle + |qg\rangle$ Fock space: 1. the light-front Hamiltonian $P^-(x^+)$

We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.

- The light-front Hamiltonian is derived from the QCD Lagrangian with a background color field $\mathcal{A}_\mu$ in the $|q\rangle + |qg\rangle$ space



$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}{}_a F^a_{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

where $D_\mu = \partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$.

$$\mathrm{P}^-(x^+) = \mathrm{P}^-_{KE} + V_{qg} + V_{\mathcal{A}}(x^+)$$

P^-_{QCD} (for $\mathrm{P}^-_{KE} + V_{qg}$)

background field

Interaction matrix

Quark-nucleus scattering in the $|q\rangle + |qg\rangle$ Fock space: 1.a. the background color field $\mathcal{A}_\mu$

- The background field from the nucleus, $\mathcal{A}(x^+, \vec{x}_\perp)$, is a classical gluon field described by the MV model[1]

Color charges are stochastic variables with correlations

$$< \rho_a(x^+, \vec{x}_\perp)\rho_b(y^+, \vec{y}_\perp) > = g^2\tilde{\mu}^2\delta_{ab}\delta^2(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+)$$

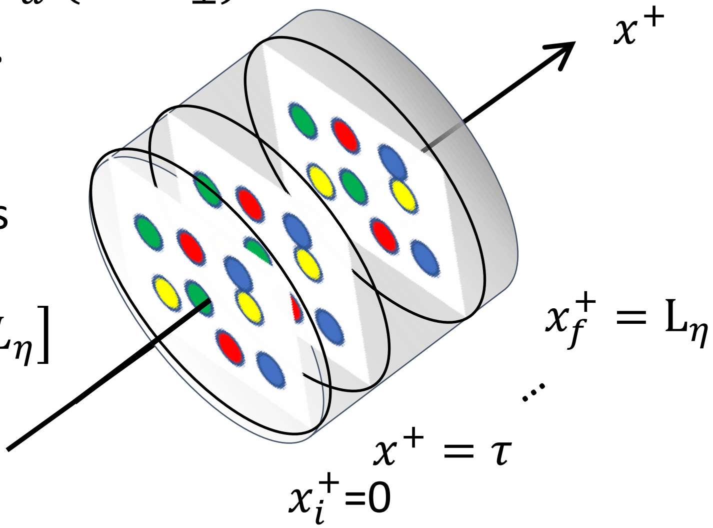
The color field is solved from

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(x^+, \vec{x}_\perp) = \rho_a\,(x^+, \vec{x}_\perp)$$

where m_g is a chosen infrared (IR) regulator.

The color sources of different layers are independent of each other, simulating the quarks from different nucleons of the heavy ion

- The duration of the interaction: $x^+ = [0, L_\eta]$
- Number of layers: N_η
- The duration of each layer: $\tau = L_\eta/N_\eta$



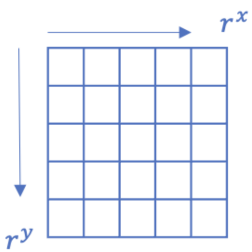
[1]L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 3352 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D50, 2225 (1994).

Quark-nucleus scattering in the $|q\rangle + |qg\rangle$ Fock space: 2. basis representation

- We choose the momentum states as the basis states: $P^-_{\mathrm{KE}}|\beta\rangle = P^-_\beta|\beta\rangle$

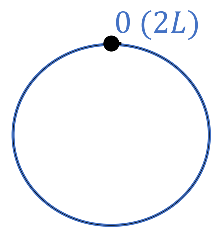
  (1) Each single particle state carries five quantum numbers: $\beta_l = \{k_l^x, k_l^y, k_l^+, \lambda_l, c_l\}, (l = q, g)$, the transverse momenta, the longitudinal momentum, helicity, and color

  - *The transverse space:*



$$r_l^\perp = \frac{L_\perp}{N_\perp}[-N_\perp, \ldots, N_\perp - 1]$$
$$p_l^\perp = \frac{2\pi}{2L_\perp} k_l^\perp$$
$$k_l^\perp = -N_\perp, \ldots, N_\perp - 1$$

  - *The longitudinal space:*



$$x^- = [0, 2L]$$
$$p_l^+ = \frac{2\pi}{L} k_l^+$$
$$k_q^+ = \frac{1}{2}, \frac{3}{2}, \ldots, K + \frac{1}{2}$$
$$k_g^+ = 1, 2, \ldots, K$$

  (2) In each Fock sector, the many-particle basis states are direct products of single particle states: $|\beta_q\rangle, |\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$

The quark state is expanded in the basis space, and the information of the state is encoded in the wavefunction as basis coefficients:

$$|\psi; x^+\rangle = \sum_\beta c_\beta(x^+)|\beta\rangle$$

Basis size: $N_{tot} = (2N_\perp)^2 \times 2 \times 3 + K \times (2N_\perp)^4 \times 4 \times 24$

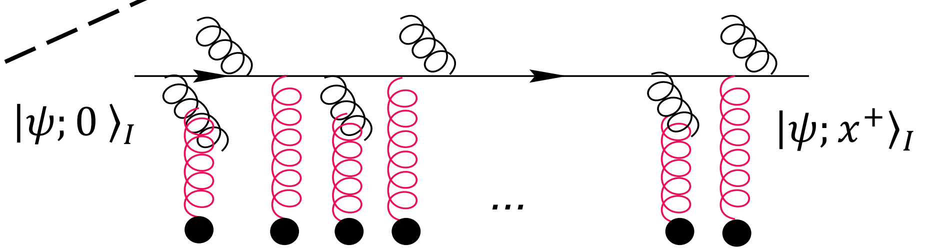
Quark-nucleus scattering in the $|q\rangle + |qg\rangle$ Fock space: 3. time evolution

- Solve the time-evolution equation in the interaction picture

$$\frac{1}{2}V_I(x^+)|\psi; x^+\rangle_\mathrm{I} = i\frac{\partial}{\partial x^+}|\psi; x^+\rangle_\mathrm{I}$$

- States: $|\psi; x^+\rangle_\mathrm{I} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}\, x^+}|\psi; x^+\rangle$
- Interaction: $\mathrm{V_I(x^+)} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}\, x^+} V(x^+) e^{-\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}\, x^+}$
- Evolution: $|\psi; x^+\rangle_\mathrm{I} = \lim_{n\to\infty}\prod_{k=1}^{n} \mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k} dz^+ V_I(z^+)\}|\psi; 0\,\rangle_\mathrm{I}$

Each timestep contains two successive operations, computed with optimal numerical methods:



$$\mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k} dz^+ V_{\mathcal{A},I}(z^+)\}\, \mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k} dz^+ V_{qg,I}(z^+)\}$$

matrix exponential in coordinate space + Fast Fourier Transform, $\sim O(N_{tot}\log N_{tot})$

4th-order Runge-Kutta method, $\sim O(N_{tot})$

Total computational complexity of each timestep: $O(N_{tot} log\ N_{tot})$

Outline

Methodology

- A time-dependent light-front Hamiltonian approach
- Application to quark-nucleus scattering in $|q\rangle + |qg\rangle$ Fock space

Results

- Bare quark as an initial state [Phys. Rev. D 104, 056014 (2021)]
  1. Gluon emission and absorption
  2. Interaction with a background color field
- Dressed quark as an initial state [preliminary]
  1. QCD eigenstate
  2. Interaction with background field

Summary and outlooks

Results: 1.a. Gluon emission and absorption

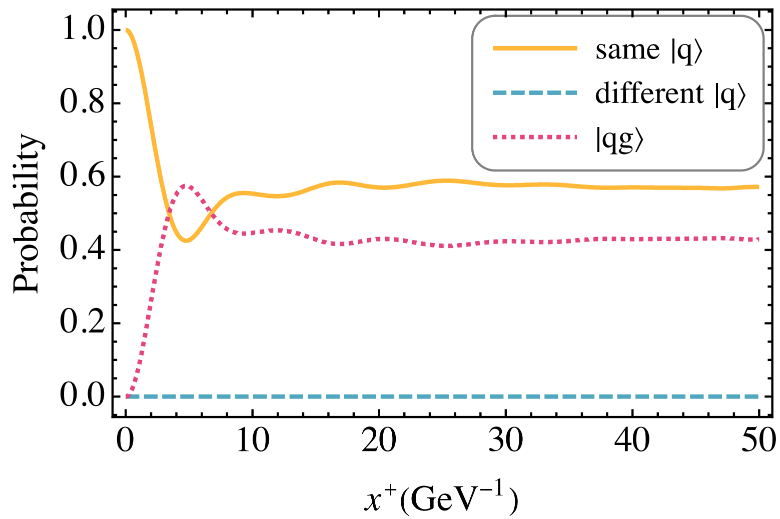
➢ The initial state is a single quark state, testing the physical effects of different parts of the Hamiltonian in a clean and tractable setup.

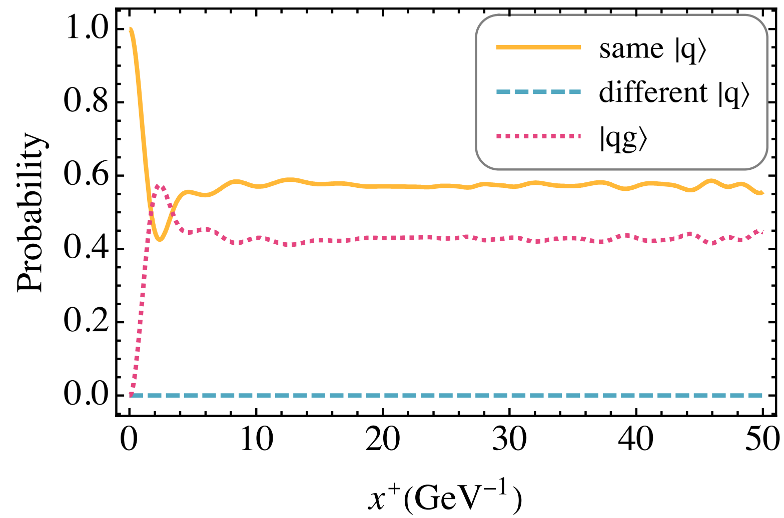
The interaction contains the gluon emission/absorption term,

$$V_I(x^+) = e^{\frac{i}{2}P^-_{KE}\, x^+} V_{qg} e^{-\frac{i}{2}P^-_{KE}\, x^+}$$

- Transition *Initial state:* $|q\rangle$ *with* $\vec{p}_\perp = \vec{0}_\perp,\ \lambda_q = \uparrow,\ c_q = 2$

$p^+ = 8.5$ GeV V_{qg} —— boost invariant ——→ $p^+ = 4.25$ GeV V_{qg}

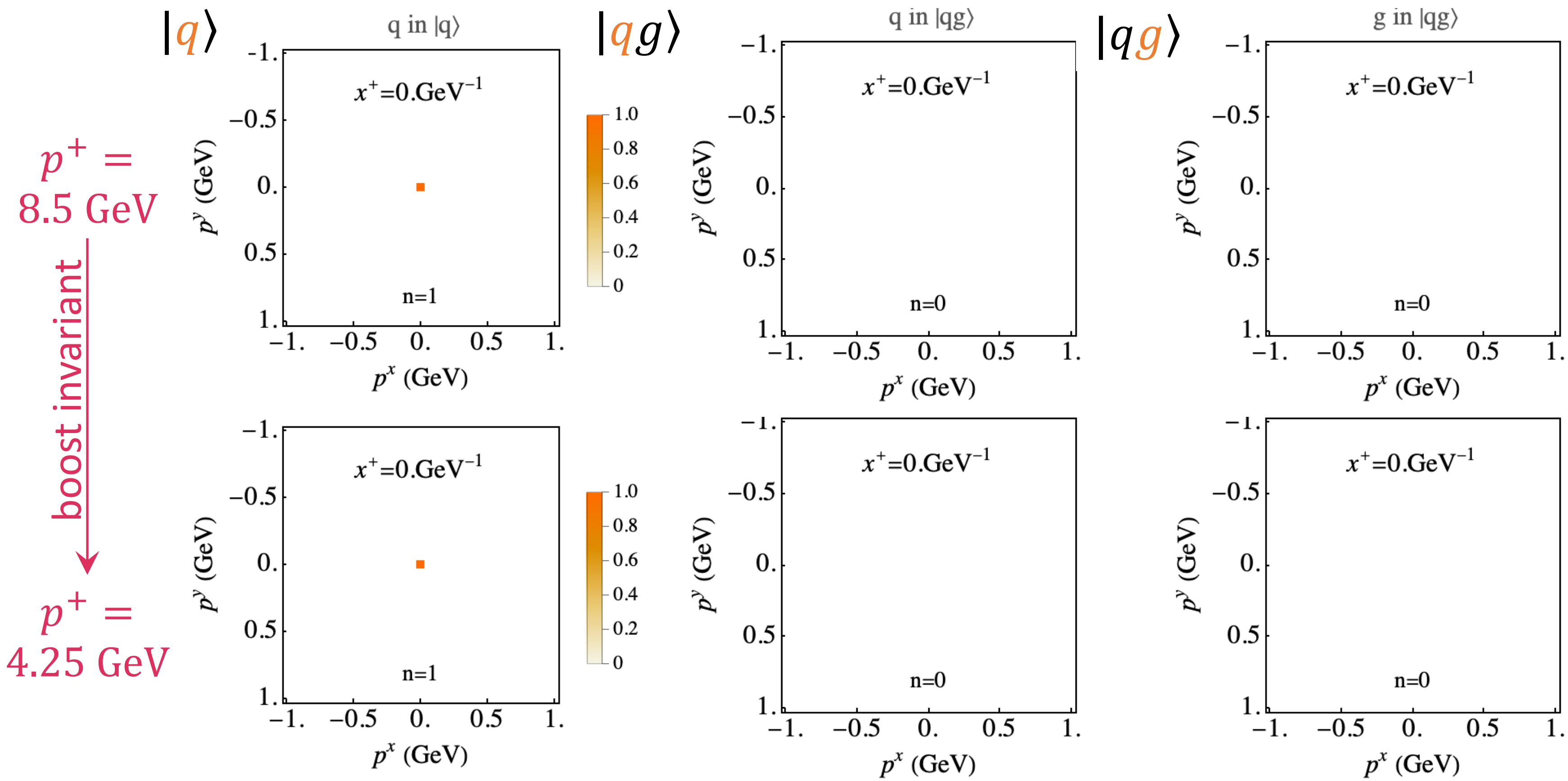




❖ The probability of the initial $|q\rangle$ state decreases, the $|qg\rangle$ sector increases.

Results: 1.b. Gluon emission and absorption

- Evolution in $\vec{p}_\perp$ space *Initial state:* $|q\rangle$ *with* $\vec{p}_\perp = \vec{0}_\perp,\ \lambda_q = \uparrow,\ c_q = 2$



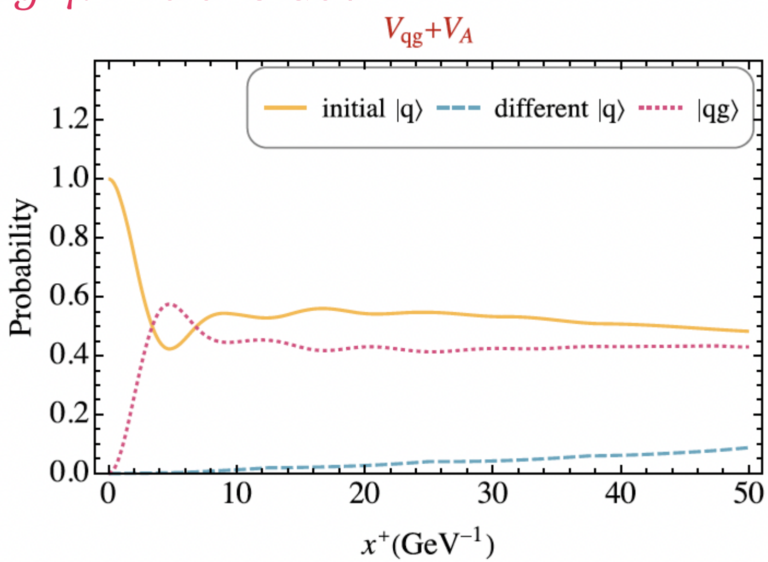
❖ The total momentum is conserved. Different $\vec{p}_\perp$ modes oscillate out of phase due to phase factor $e^{-\frac{i}{2}P^-_{KE}\,x^+}$.

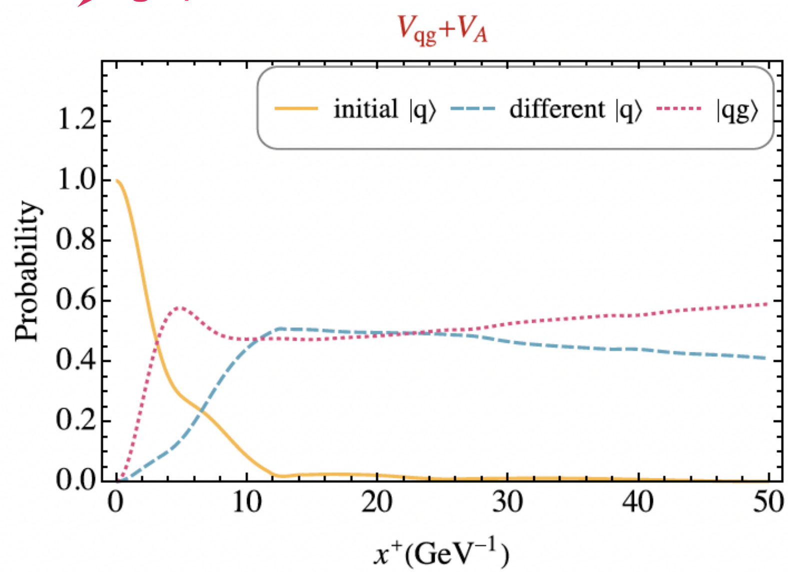
Results: 2.a. Interaction with a background field

The interaction contains the gluon emission/absorption, and the background field term $V_I(x^+) = e^{\frac{i}{2}P^-_{KE}x^+}[V_{qg} + V_{\mathcal{A}}(x^+)]e^{-\frac{i}{2}P^-_{KE}x^+}$

- Transition *Initial state:* $|q\rangle$ *with* $p^+ = 8.5\ \text{GeV},\ \vec{p}_\perp = \vec{0}_\perp,\ \lambda_q = \uparrow,\ c_q = 2$

$g^2\tilde{\mu} = 0.018\ \text{GeV}^{3/2}$ —Stronger background→ $g^2\tilde{\mu} = 0.144\ \text{GeV}^{3/2}$

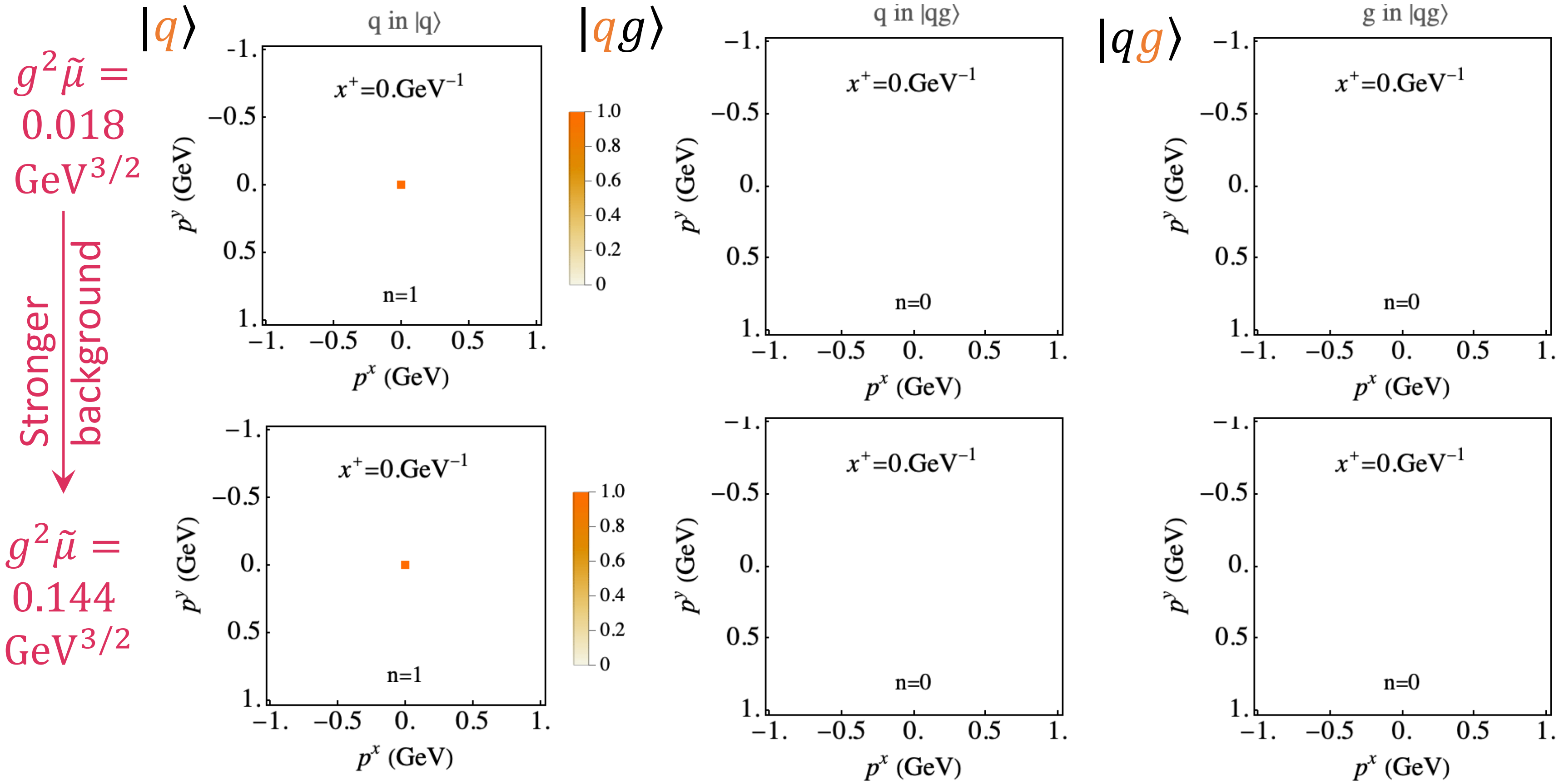




- The probability of the initial $|q\rangle$ state decreases, those of other $|q\rangle$ states and the $|qg\rangle$ sector increases.

Results: 2.b. Interaction with a background field

- Evolution in $\vec{p}_\perp$ space *Initial state:* $|q\rangle$ *with* $p^+ = 8.5$ GeV, $\vec{p}_\perp = \vec{0}_\perp$, $\lambda_q = \uparrow$, $c_q = 2$



❖ The background field interaction changes the momentum distribution.
Different $\vec{p}_\perp$ modes oscillate out of phase due to phase factor $e^{-\frac{i}{2}P^-_{KE}\,x^+}$.

Outline

Methodology

- A time-dependent light-front Hamiltonian approach
- Application to quark-nucleus scattering in $|q\rangle + |qg\rangle$ Fock space

Results

- Bare quark as an initial state [Phys. Rev. D 104, 056014 (2021)]
  1. Gluon emission and absorption
  2. Interaction with a background color field
- Dressed quark as an initial state [preliminary]
  1. QCD eigenstate
  2. Interaction with background field

Summary and outlooks

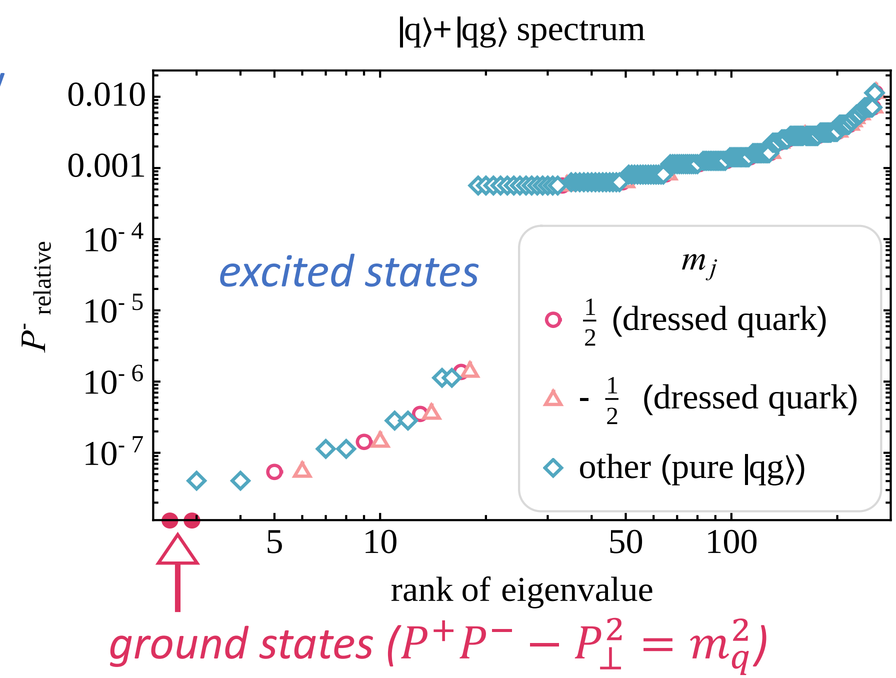
Results (preliminary): 3. Dressed quark

- In a physical high-energy scattering process, the initial quark state has already developed a gluon cloud before the interaction.

The dressed quark state is solved from the eigenvalue equation in the $|q\rangle + |qg\rangle$ space by matrix diagonalization with mass renormalization scheme[1] :

$$\hat{P}^-_{QCD}|\psi\rangle = P^-|\psi\rangle, \qquad \hat{P}^-_{QCD} = P^-_{KE} + V_{qg}$$

- Spectrum in q-g relative energy ($P^-_{relative} = P^- - P^-_{CM}$)
  - Boost-invariant
    Same at different center-of-mass transverse momentum
  - SU(3) gauge invariant
    Degenerate in color space
  - Rotational symmetry
    Eigenstates of J_z, m_j



1. V.A. Karmanov, J.-F. Mathiot, A.V. Smirnov, Phys.Rev.D 77 (2008), 085028; Phys.Rev.D 86 (2012) 085006.

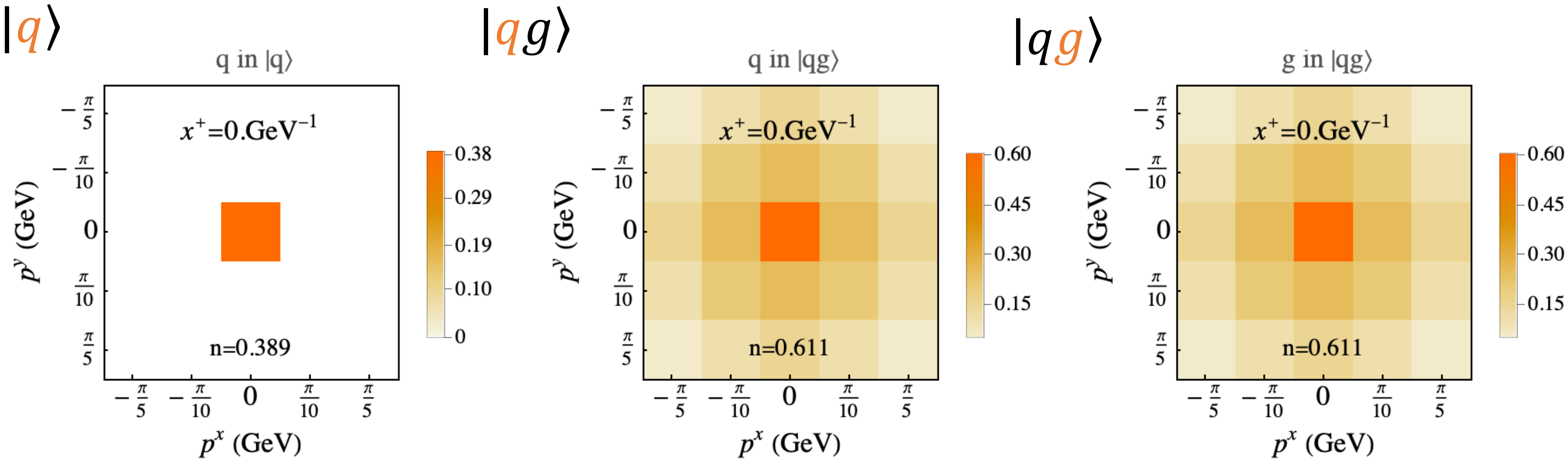
Results (preliminary): 3. Dressed quark

- In a physical high-energy scattering process, the initial quark state has already developed a gluon cloud before the interaction.

The dressed quark state is solved from the eigenvalue equation in the $|q\rangle + |qg\rangle$ space by matrix diagonalization with mass renormalization scheme[1] :

$$\hat{P}^-_{QCD}|\psi\rangle = P^-|\psi\rangle, \qquad \hat{P}^-_{QCD} = P^-_{KE} + V_{qg}$$

- Evolution in $\vec{p}_\perp$ space   *Initial state is* the $|q\rangle + |qg\rangle$ *ground state*



❖ The background field interaction stimulates the ground state to excited states.

1. V.A. Karmanov, J.-F. Mathiot, A.V. Smirnov, Phys.Rev.D 77 (2008), 085028; Phys.Rev.D 86 (2012) 085006.

Summary and outlooks

- We demonstrated a nonperturbative method to investigate time-evolution problems with light-front Hamiltonian formalism
  - We studied the quark-nucleus scattering in the $|q\rangle + |qg\rangle$ space:
    1. Gluon emission and absorption
    2. Interaction with a background color field
  - A main advantage of this method: one can smoothly change the magnitudes of the above effects separately to match different physics regimes
- Ongoing and future works
  - Investigate high-energy scattering with the initial quark state as an eigenstate of the QCD Hamiltonian in the $|q\rangle + |qg\rangle$ space
  - Study jet quenching in the quark-gluon plasma

Thank you!

[Backup] Results (preliminary): Dressed quark

- Distribution amplitudes of the two $|q\rangle + |qg\rangle$ ground states in the relative transverse-momentum space, summed over p_q^+ states

Relative momentum:

$$\vec{\Delta}_m = -\vec{p}_{\perp,q} + \frac{p_q^+}{P^+}\vec{P}_\perp$$

Center-of-mass:

$$\vec{P}_\perp = \vec{p}_{\perp,q} + \vec{p}_{\perp,g}$$

$$P^+ = p_q^+ + p_g^+$$