Impact of gluon saturation on dijet production at the EIC

Light Cone 2021

Farid Salazar

November 30th, 2021

R. Boussarie, H. Mäntysaari, FS, and B. Schenke. 2106.11301 (JHEP09(2021)178)

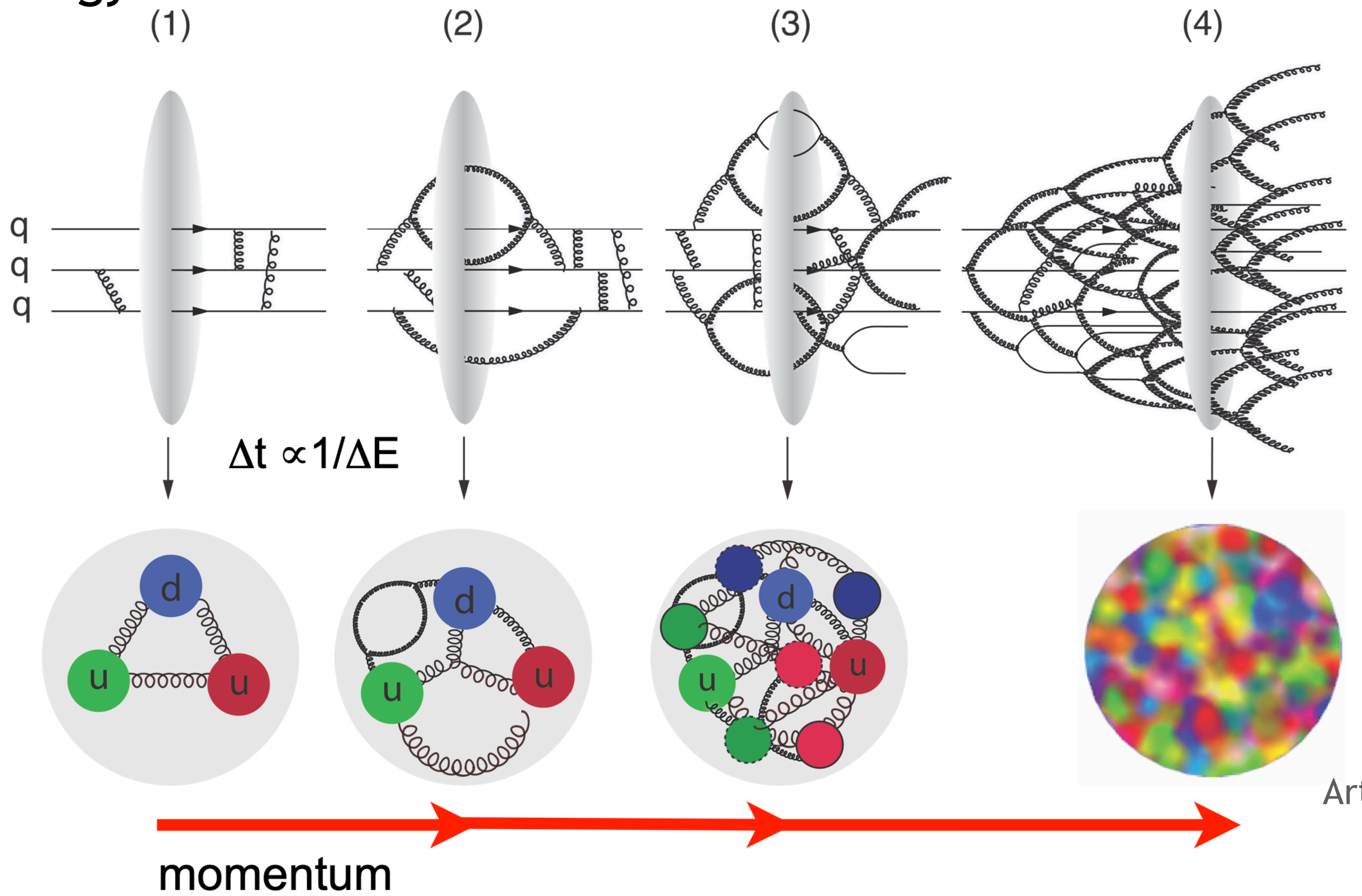
P. Caucal, FS, and R. Venugopalan. 2108.06347 (JHEP11(2021)222)

Outline

- Gluon saturation
- Dijet production beyond TMDs
- Dijet production at EIC in the CGC at NLO
- Outlook

Gluon saturation

The high energy limit of nuclear matter



Artwork: T. Ullrich

Emergence of an energy and nuclear specie dependent momentum scale (saturation scale) parametrizes importance of:

Multiple scattering (higher twist effects)

Non-linear evolution equations (BK/JIMWLK)

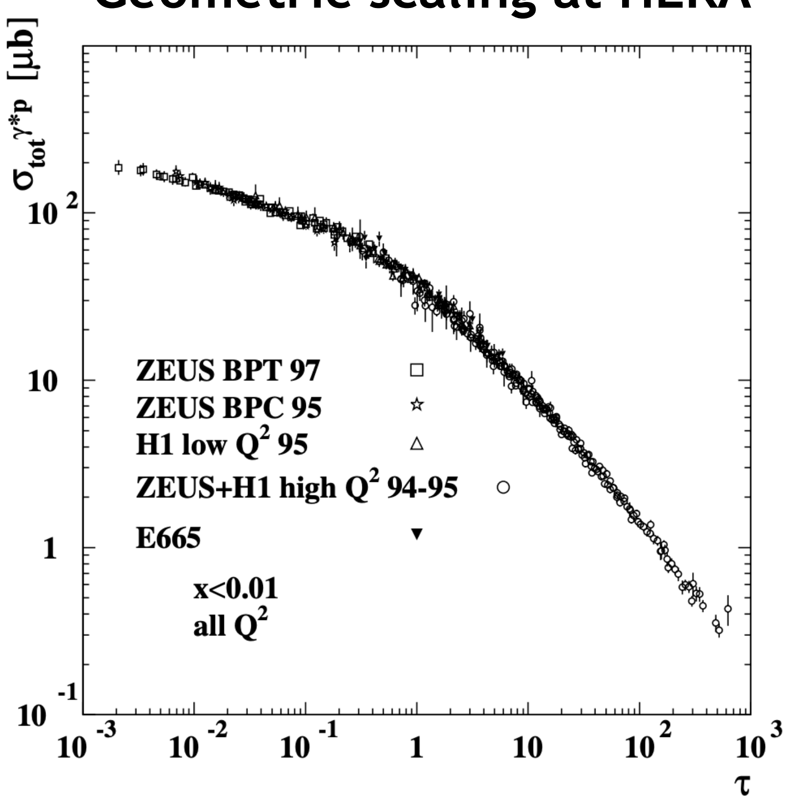
$$Q_s^2 \propto A^{1/3} s^{1/3}$$

Gluon saturation

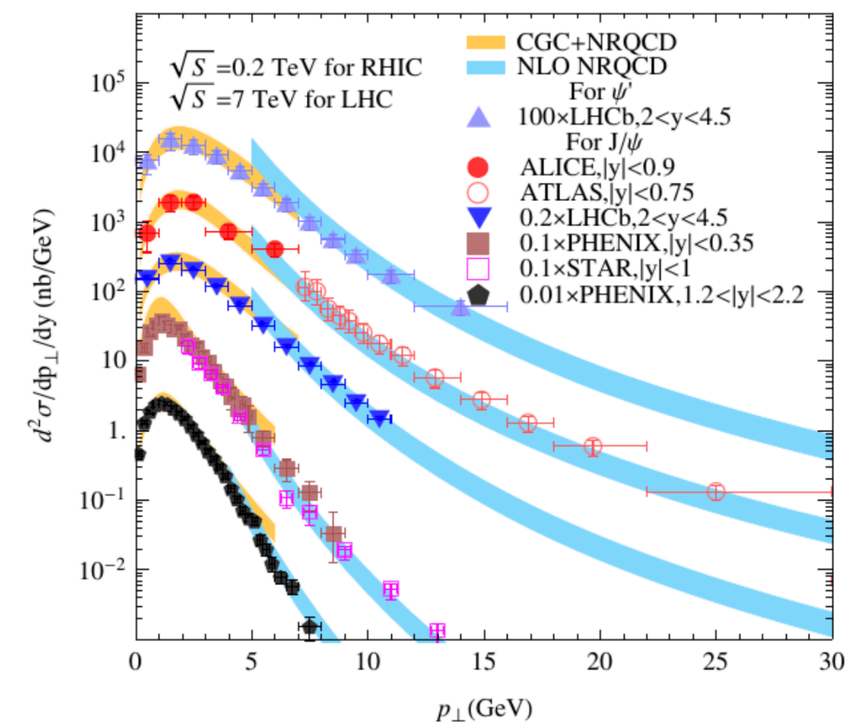
From HERA to RHIC to the LHC

For a recent review see
Astrid Morreale, and FS. 2108.08254
(*Universe* 7 (2021) 8, 312)

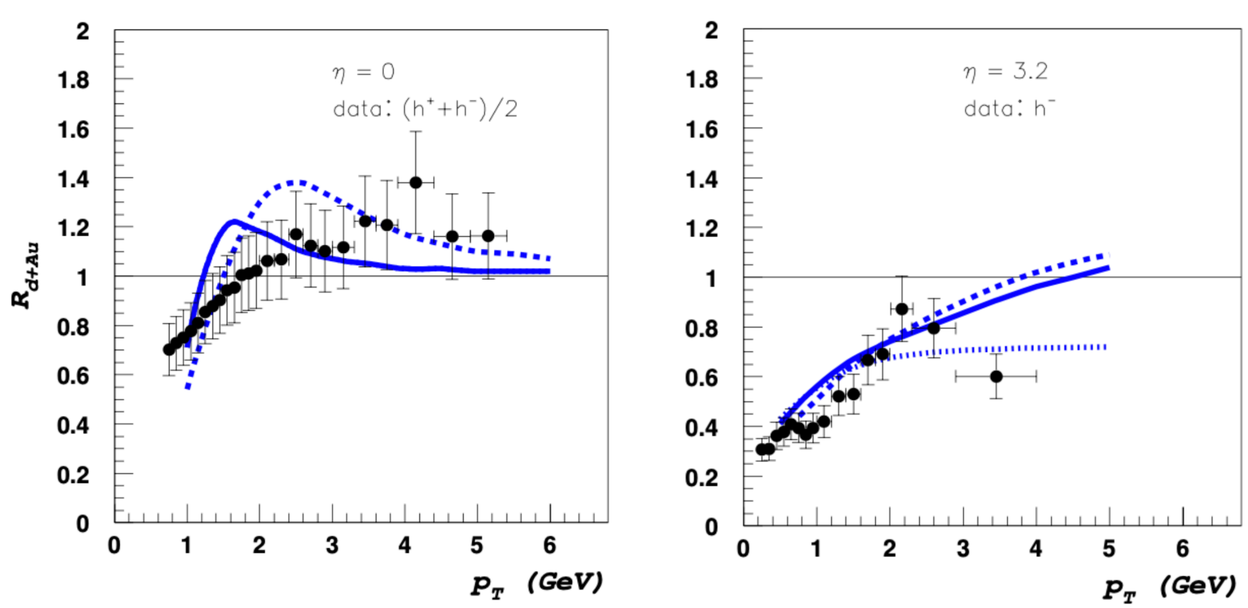
Geometric scaling at HERA



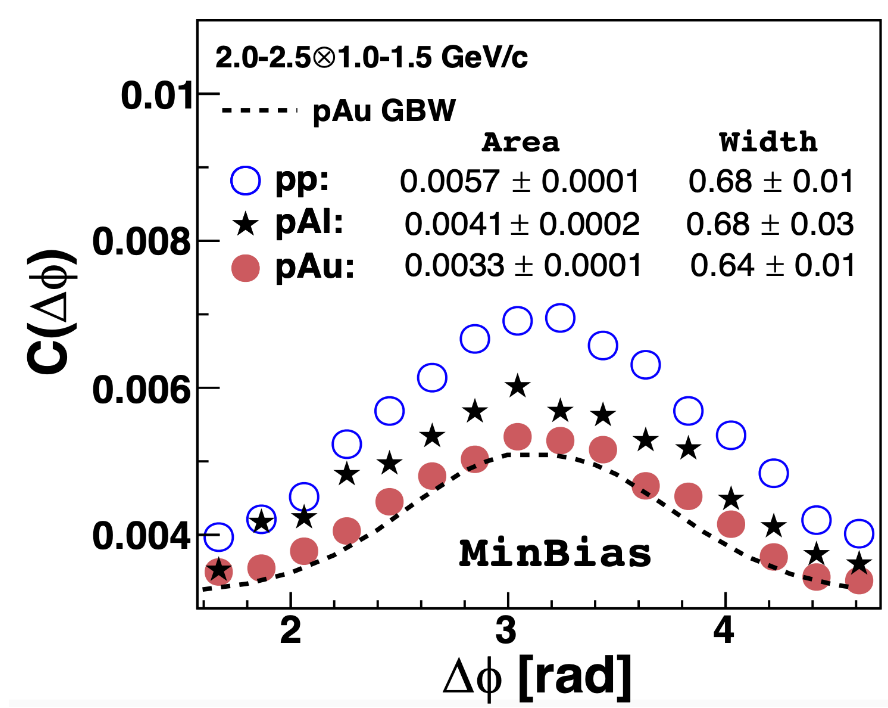
Quarkonia production at RHIC and LHC



Nuclear modification factor at RHIC



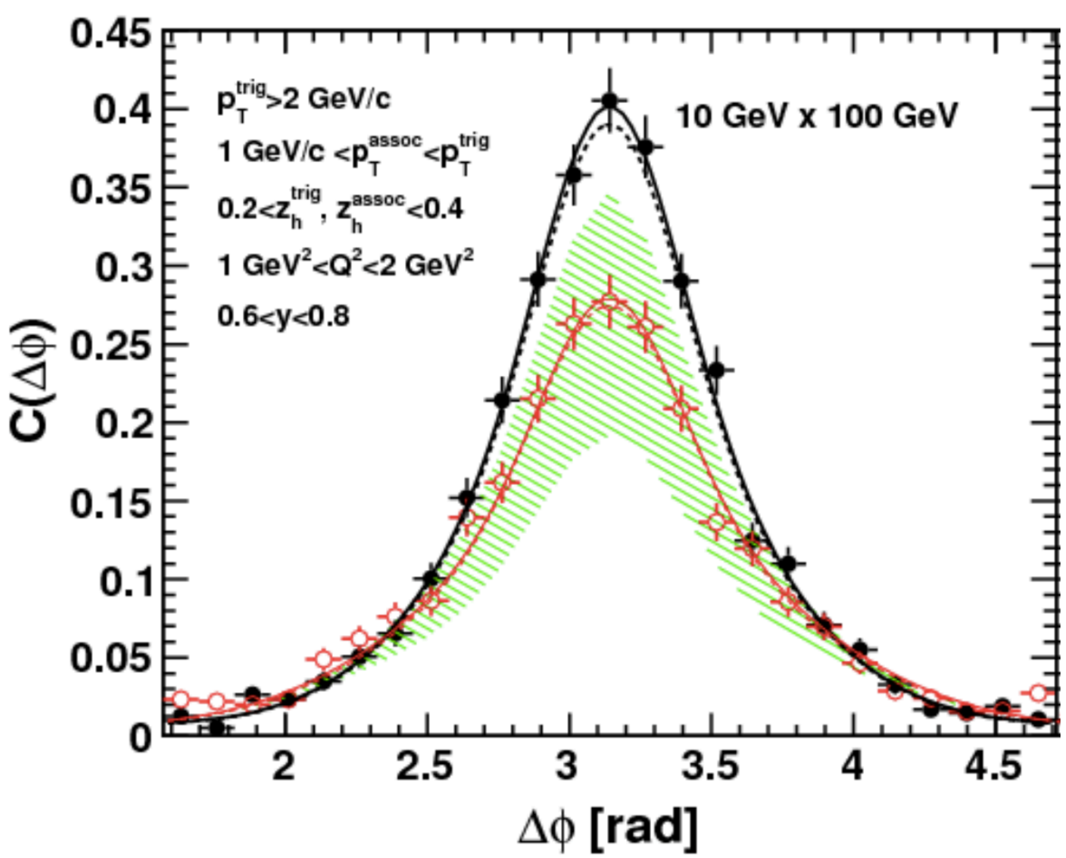
Dihadron suppression at RHIC



Gluon saturation

Observables at the EIC*

Dihadron suppression
back-to-back peak

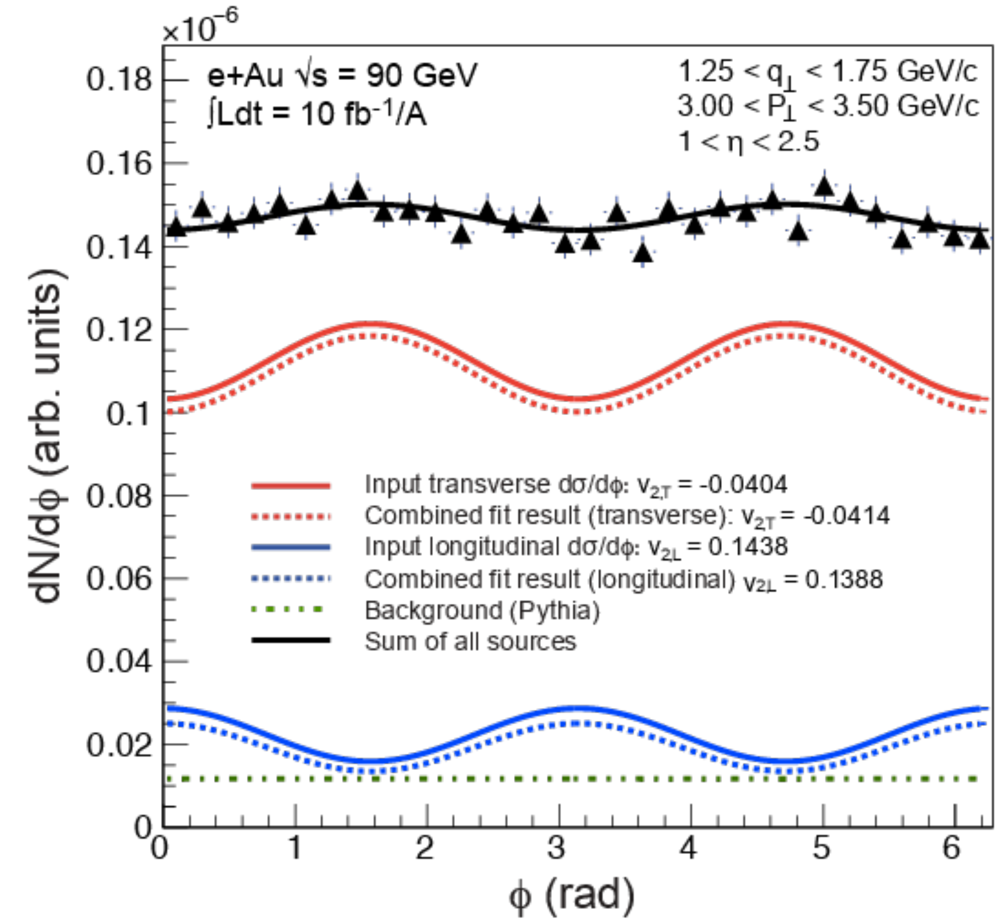


Zheng, Aschenauer, Lee, Xiao (2014)

Typical momentum transfer from proton/ nucleus to dihadron pair is $\sim Q_s$

Momentum imbalance → $k_\perp \sim Q_s$ ← Saturation scale

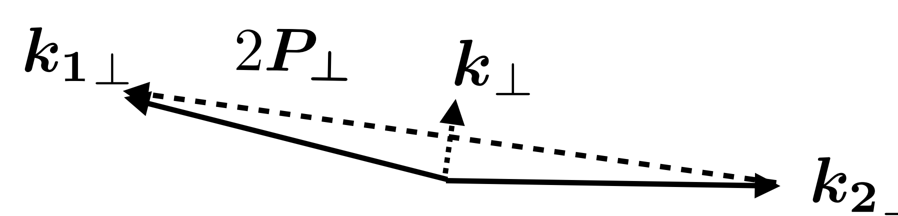
Dijet momentum imbalance
azimuthal correlations



Dumitru, Skokov, Ullrich (2018)

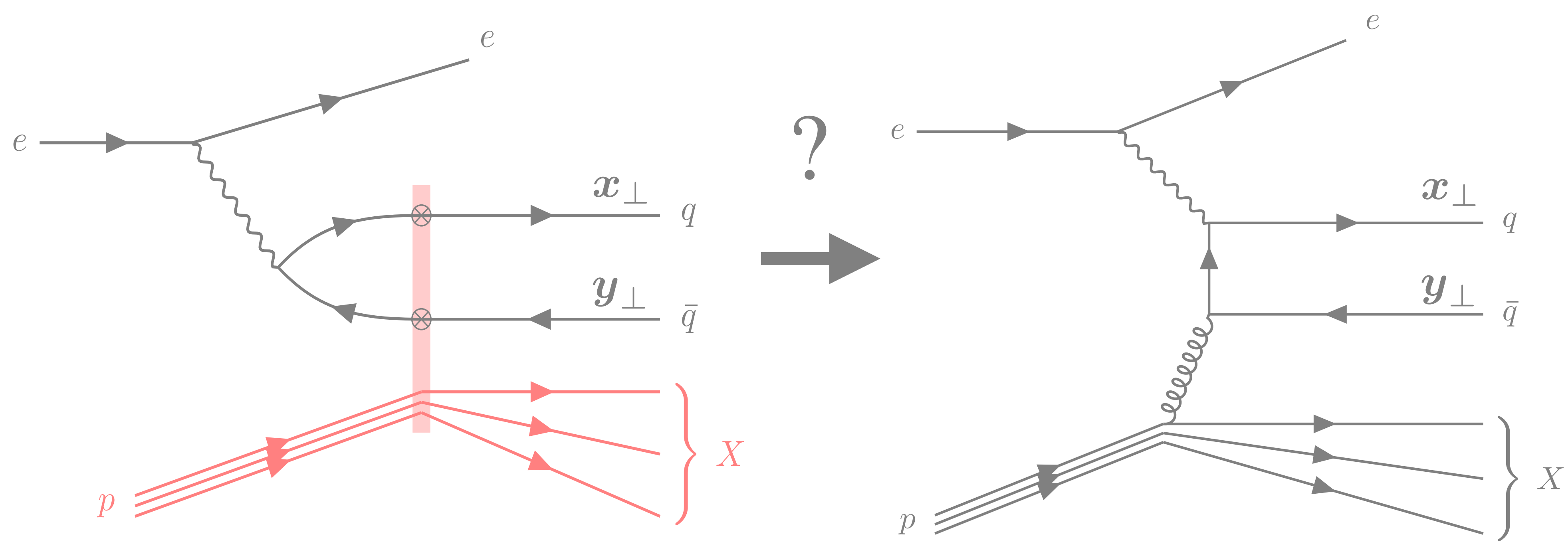
Sensitivity to linearly polarized gluons

ϕ angle between $\boldsymbol{P}_\perp$ and $\boldsymbol{k}_\perp$



*many other observables
(structure function, diffractive, …)

Dijet production beyond TMDs



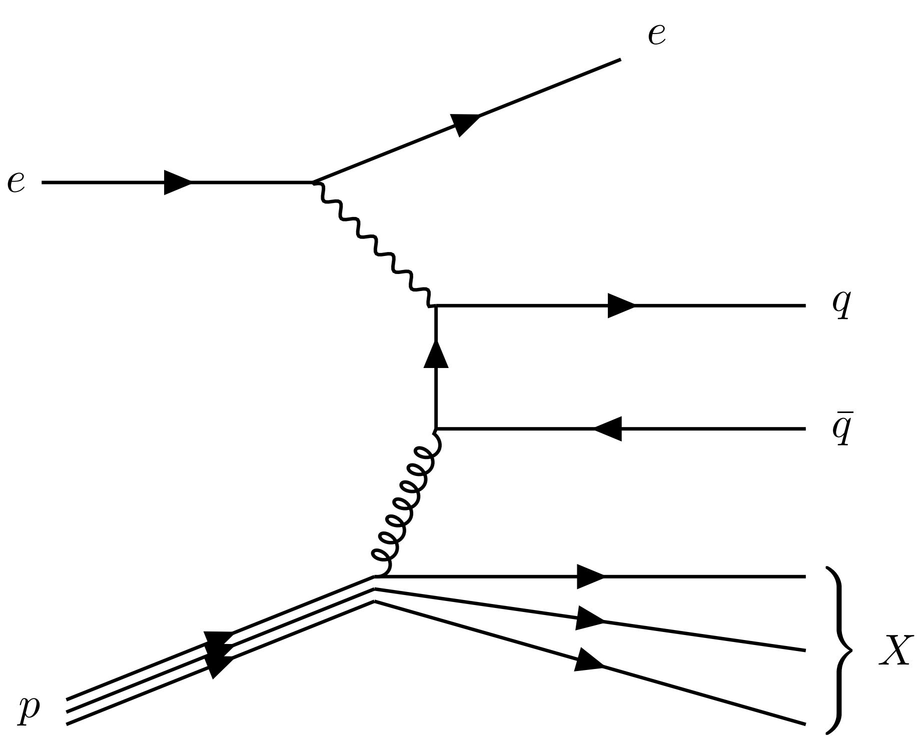
A comprehensive numerical study of the TMD/CGC correspondence

R. Boussarie, H. Mäntysaari, FS, and B. Schenke. [2106.11301](2106.11301)
(JHEP09(2021)178)

Dijet production beyond TMDs

Review of dijets in small-x TMD factorization

Bomhof, Mulders, Pijlman (2006)
Dominguez, Marquet, Xiao, Yuan (2011)
Dominguez, Qiu, Xiao, Yuan (2011)



LO diagram for $q\bar{q}$ production

$$d\sigma^{\gamma^* A \to q\bar{q}X} \sim \mathcal{H}^{ij}_{\mathrm{TMD}}(\boldsymbol{P}_\perp)\, \alpha_s x G^{ij}_{\mathrm{WW}}(x, \boldsymbol{k}_\perp)$$

Perturbatively calculable on-shell matrix element

WW gluon TMD

$$xG^{ij}_{\mathrm{WW}}(x,\boldsymbol{k}_\perp) = \frac{1}{2}\delta^{ij} xG^0_{\mathrm{WW}}(x,k_\perp) + \Pi^{ij}(\boldsymbol{k}_\perp) x h^0_{\mathrm{WW}}(x,k_\perp)$$

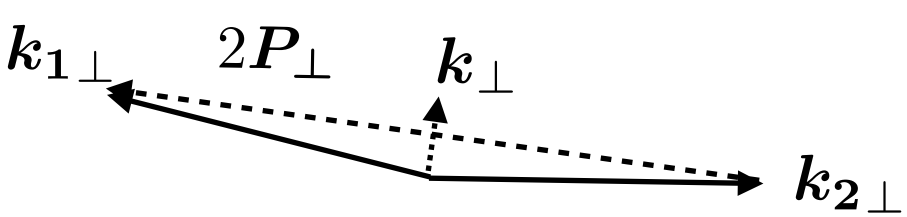
Unpolarized

Linearly polarized

$$\Pi^{ij}(\boldsymbol{k}_\perp) = \left(2\frac{\boldsymbol{k}^i_\perp \boldsymbol{k}^j_\perp}{\boldsymbol{k}^2_\perp} - \delta^{ij}\right)$$

Validity of TMD approach:

$k_\perp \ll P_\perp$ (i.e. back-to-back configuration)



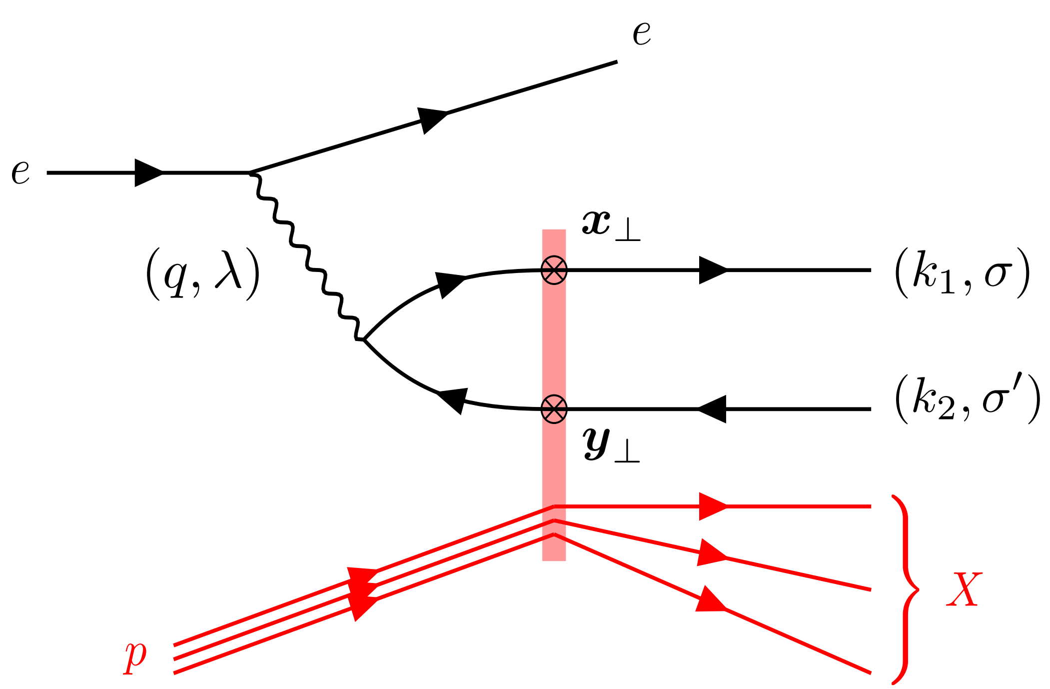
$$\boldsymbol{k}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

Dijet production beyond TMDs

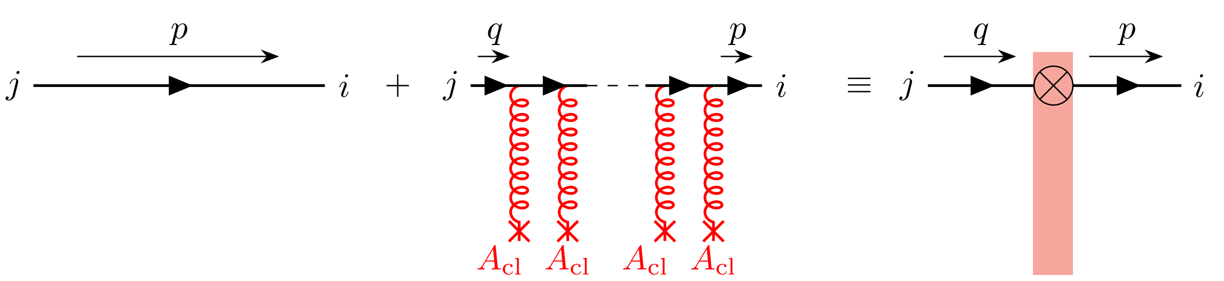
Computation in the CGC: **resummation** of multiple scatterings

Dominguez, Marquet, Xiao, Yuan (2011)



LO diagram for $q\bar{q}$ production in the CGC EFT

Dense gluon field $A_{\rm cl} \sim 1/g$ **needs <u>resummation</u> of multiple gluon interactions**



$$V_{ij}(\boldsymbol{x}) = P\exp\left\{ig\int dx^- A^{+,a}_{cl}(\boldsymbol{x},x^-)t^a\right\}$$

Amplitude (modulo leptonic part):

$$\mathcal{M}^{\lambda\sigma\sigma'}_{\rm LO} = \Psi^{\gamma^*_\lambda\to q\bar{q}}(Q,\mathbf{r}_{xy},z_q) \otimes_{\rm LO} \left[1 - V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$$

perturbatively computable — non-perturbative

$$\otimes_{\rm LO} \equiv \frac{ee_f q^-}{\pi}\int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp} e^{-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp}$$

Dijet cross-section in the CGC will contain dipoles and quadrupole:

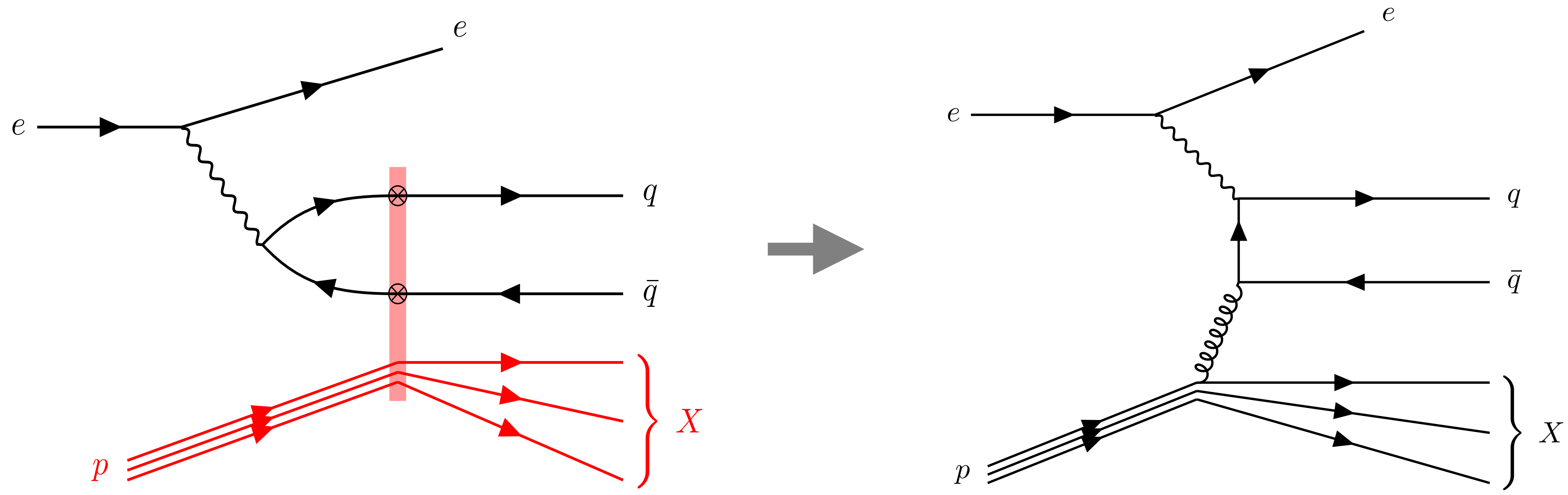
$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$$

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)V(\boldsymbol{y}'_\perp)V^\dagger(\boldsymbol{x}'_\perp)\right]\right\rangle_Y$$

Building blocks of CGC observables!

Dijet production beyond TMDs

From CGC to Improved TMD



CGC

$$V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) = \mathcal{P}\,\exp\left[-ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp}\mathrm{d}\boldsymbol{z}_\perp\cdot\tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp)\right]$$

Boussarie, Mehtar-Tani (2020)

Improved TMD

$$= 1 - ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp}\mathrm{d}\boldsymbol{z}_\perp\cdot\tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \dots$$

TMD

Altinoluk, Boussarie, Kotko (2019)

$$= 1 + ig\;\boldsymbol{r}_\perp\cdot\tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \dots$$

$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$

Dominguez, Marquet, Xiao, Yuan (2011)

Dijet production beyond TMDs

Resummation of power corrections and genuine saturation corrections

$$\mathrm{d}\sigma_{\mathrm{CGC}} = \underbrace{\mathrm{d}\sigma_{\mathrm{TMD}} + \overbrace{\mathcal{O}\left(\frac{k_\perp}{Q_\perp}\right)}^{\text{kinematic}}}_{\mathrm{d}\sigma_{\mathrm{ITMD}}} + \overbrace{\mathcal{O}\left(\frac{Q_s}{Q_\perp}\right)}^{\text{genuine}}$$

Dominguez, Marquet, Xiao, Yuan (2011)

TMD valid $k_\perp, Q_s \ll Q_\perp$

back-to-back hadrons/jets and transverse momenta larger than sat scale

Hard factor — Weizsäcker-Williams gluon TMD

$$d\sigma^{\gamma^* A \to q\bar{q}X} \sim \mathcal{H}^{ij}_{\mathrm{TMD}}(\boldsymbol{P}_\perp)\, \alpha_s x G^{ij}_{\mathrm{WW}}(x, \boldsymbol{k}_\perp)$$

$$+\mathcal{O}(k_\perp/P_\perp) + \mathcal{O}(Q_s/P_\perp)$$

Altinoluk, Boussarie, Kotko (2019)
For massive quarks see Altinoluk, Marquet, Taels. (2021)

Improved TMD valid $Q_s \ll Q_\perp$

transverse momenta larger than sat scale

Hard factor resums kinematic powers $k_\perp/P_\perp$

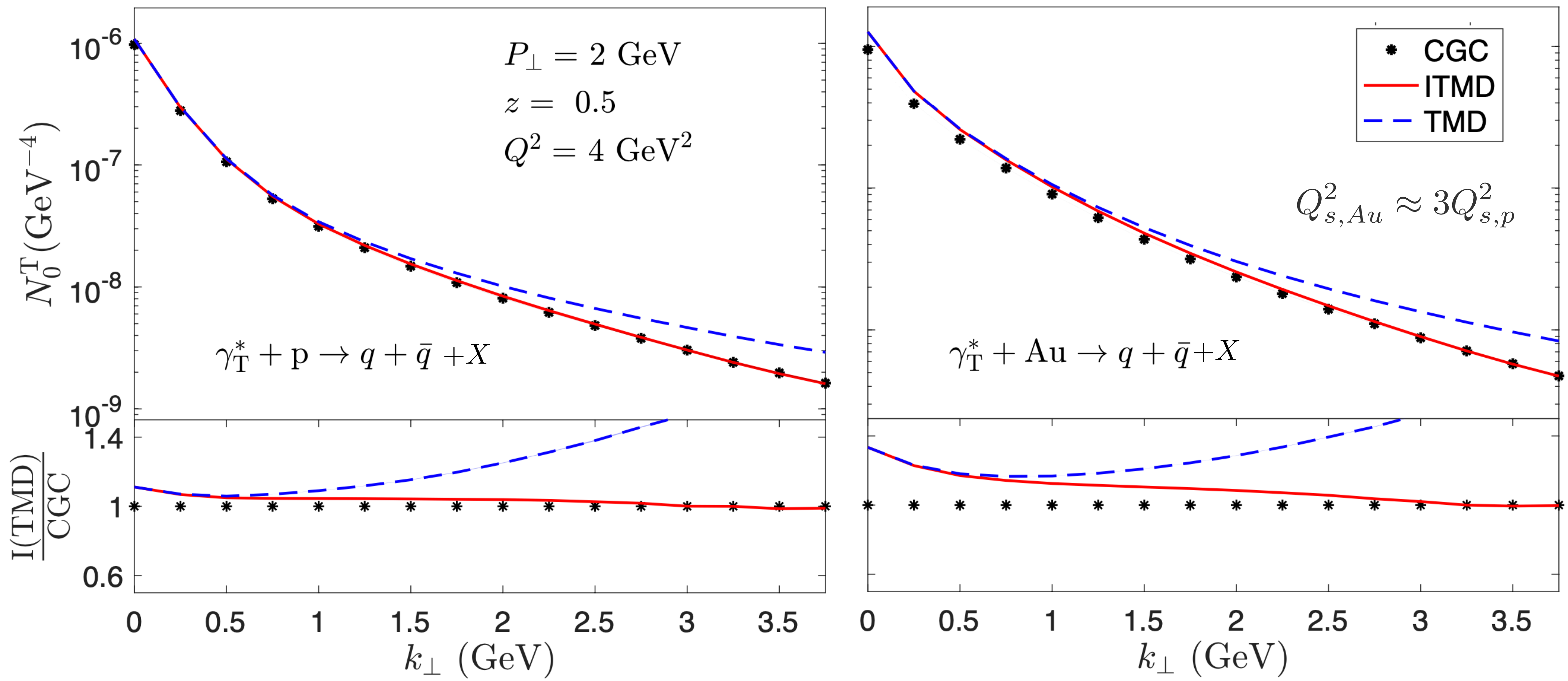
$$d\sigma^{\gamma^*_\lambda A \to q\bar{q}X} \sim \mathcal{H}^{\lambda,ij}_{\mathrm{ITMD}}(\boldsymbol{P}_\perp, \boldsymbol{k}_\perp)\, \alpha_s x G^{ij}_{\mathrm{WW}}(x, \boldsymbol{k}_\perp)$$

$$+\mathcal{O}(Q_s/P_\perp)$$

For ITMD in proton-nucleus collisions see
Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren (2015)

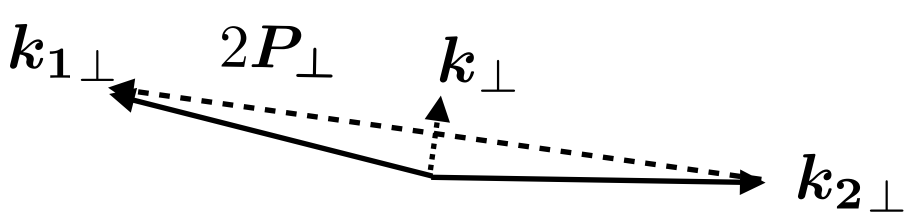
Dijet production beyond TMDs

Differential yield: TMD, ITMD and CGC



proton ~ smaller Q_s^2

Gold nucleus ~ larger Q_s^2



$$\frac{\mathrm{d}N^{\gamma_\lambda^* + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

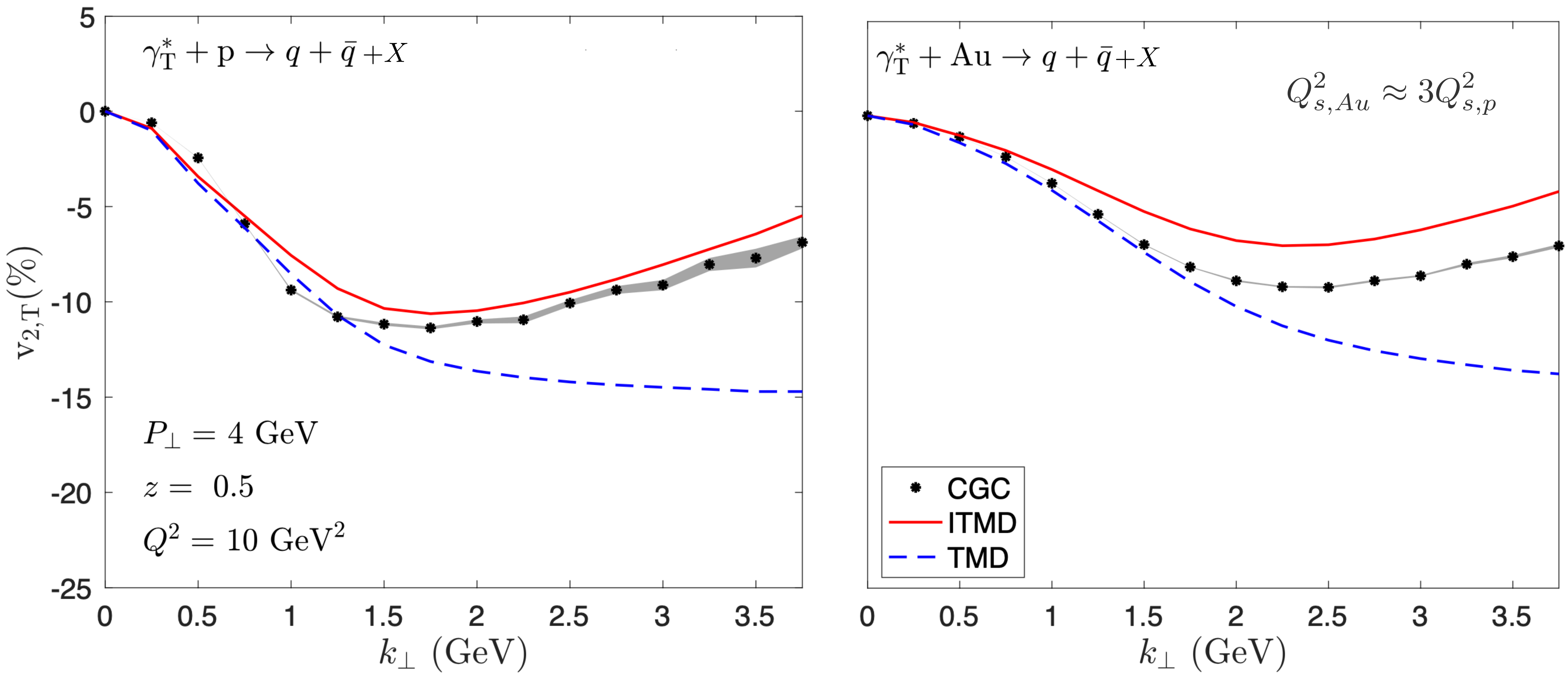
$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

For a comparison of TMD, ITMD and CGC in pA see Fuji, Watanabe, Marquet (2020)

Dijet production beyond TMDs

Momentum imbalance **elliptic** anisotropies:
TMD vs ITMD vs CGC



proton ~ smaller Q_s^2

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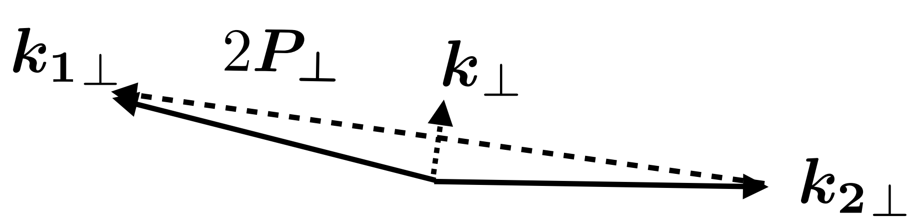


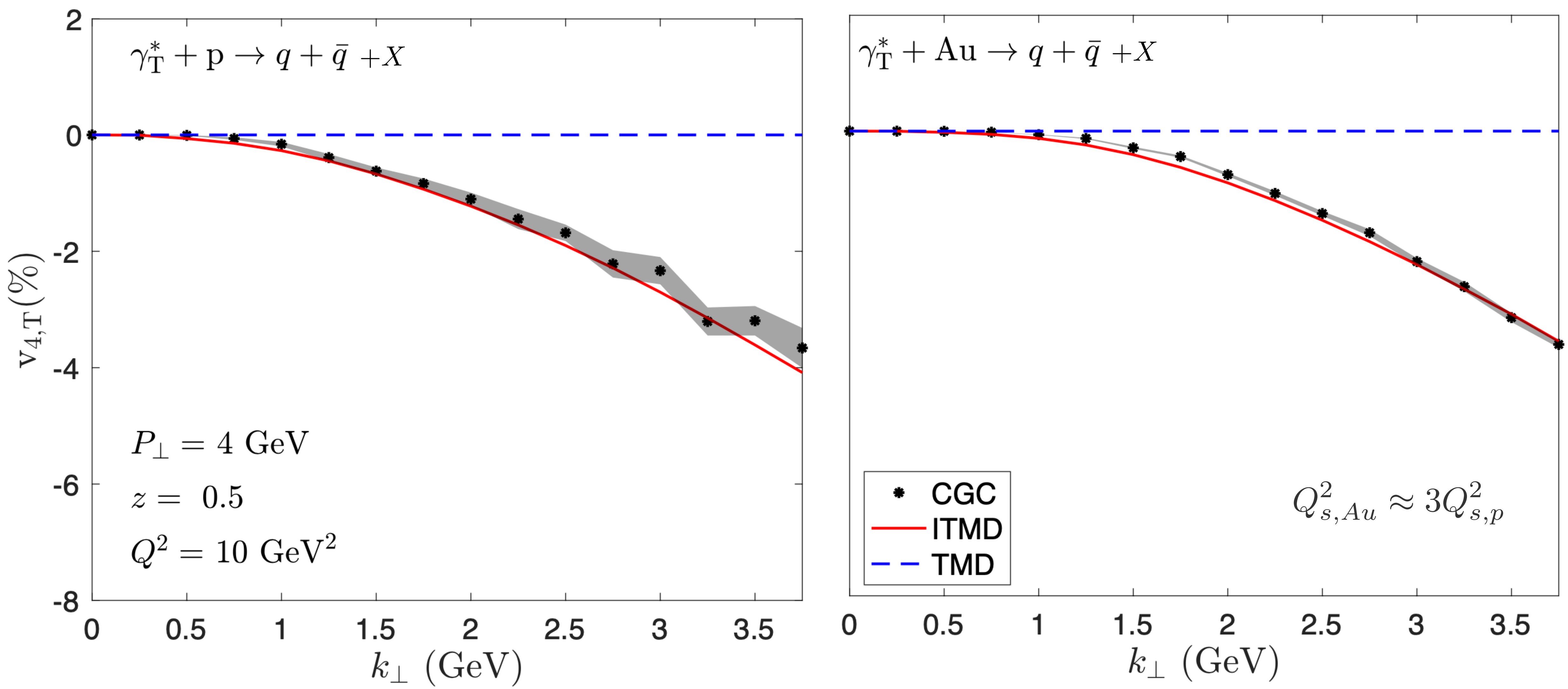
$$\frac{\mathrm{d}N^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

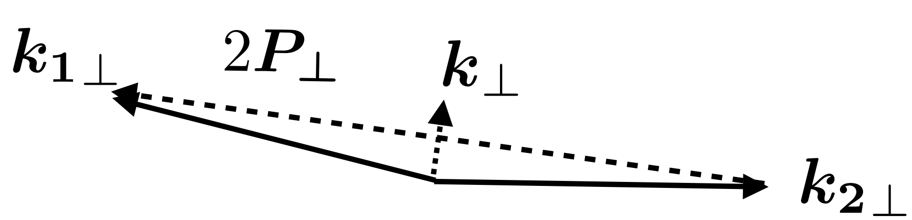
R. Boussarie, H. Mäntysaari,
FS, B. Schenke (2021)

Dijet production beyond TMDs

Momentum imbalance **quadrangular** anisotropies:
TMD vs ITMD vs CGC



proton ~ smaller Q_s^2 **Gold nucleus ~ larger** Q_s^2

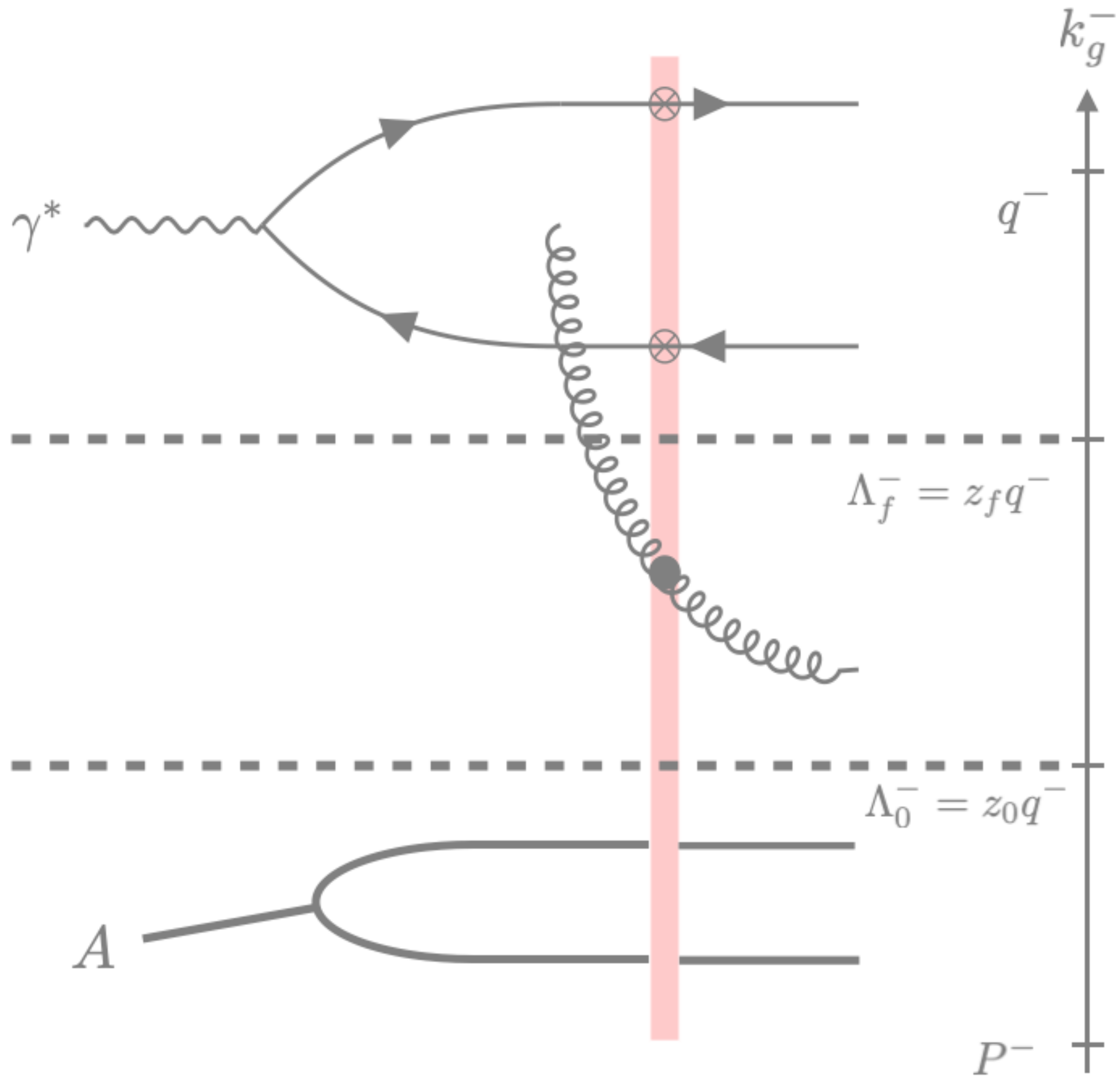


$$\frac{\mathrm{d}N^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

R. Boussarie, H. Mäntysaari,
FS, B. Schenke (2021)

Dijet production in the CGC at NLO



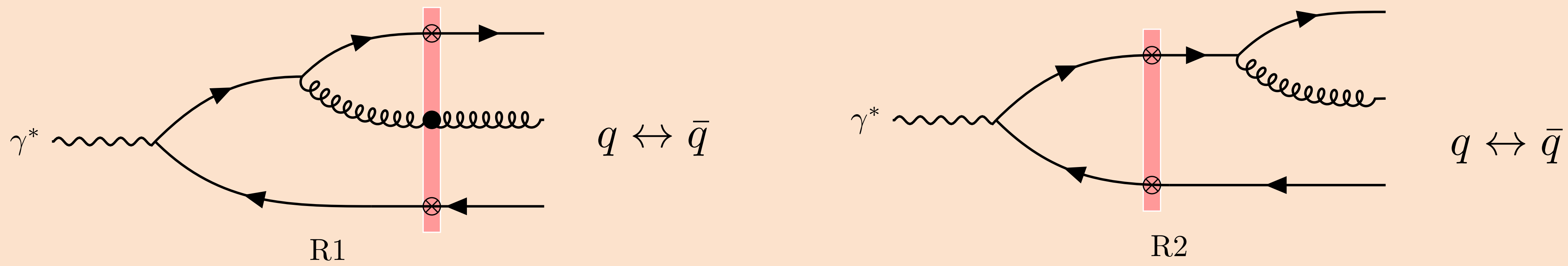
Rapidity factorization and NLO impact factor

P. Caucal, FS, and R. Venugopalan. 2108.06347
(JHEP11(2021)222)

Dijet production in the CGC at NLO

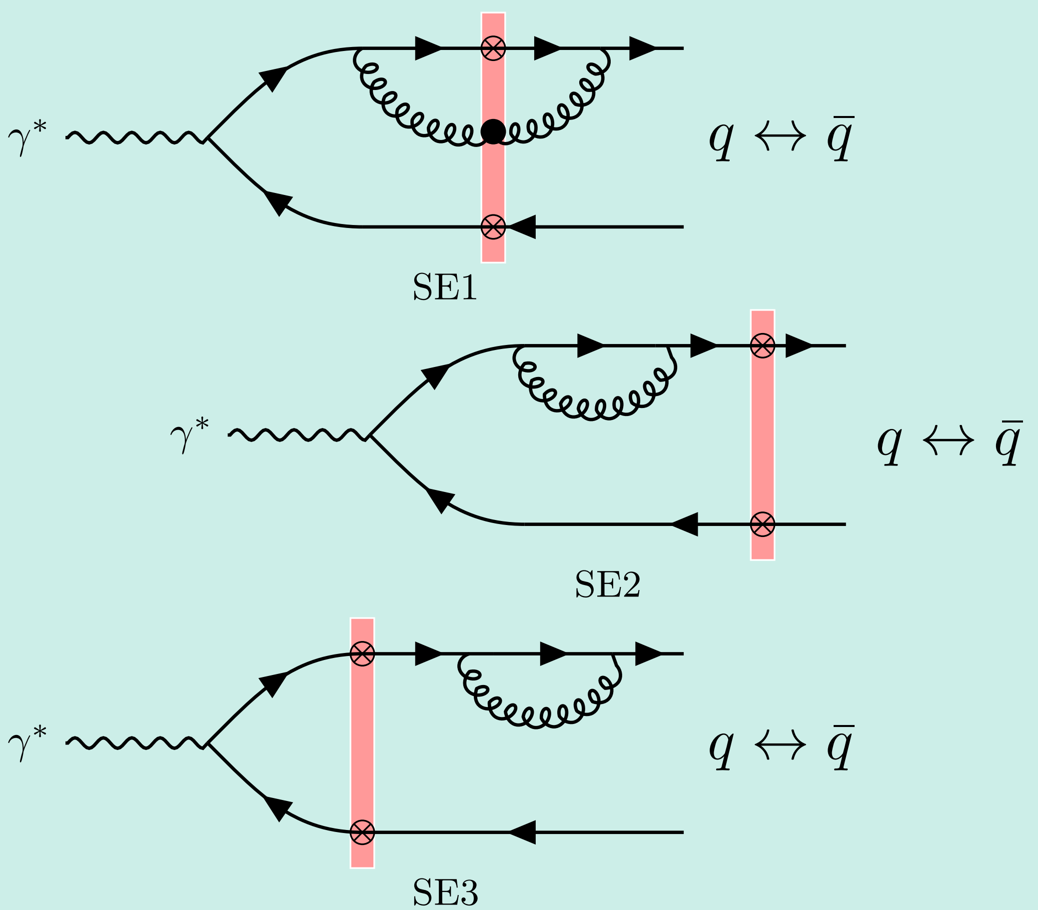
Real and virtual emissions

Real emission diagrams (loop opens in DA and closes in the CCA)

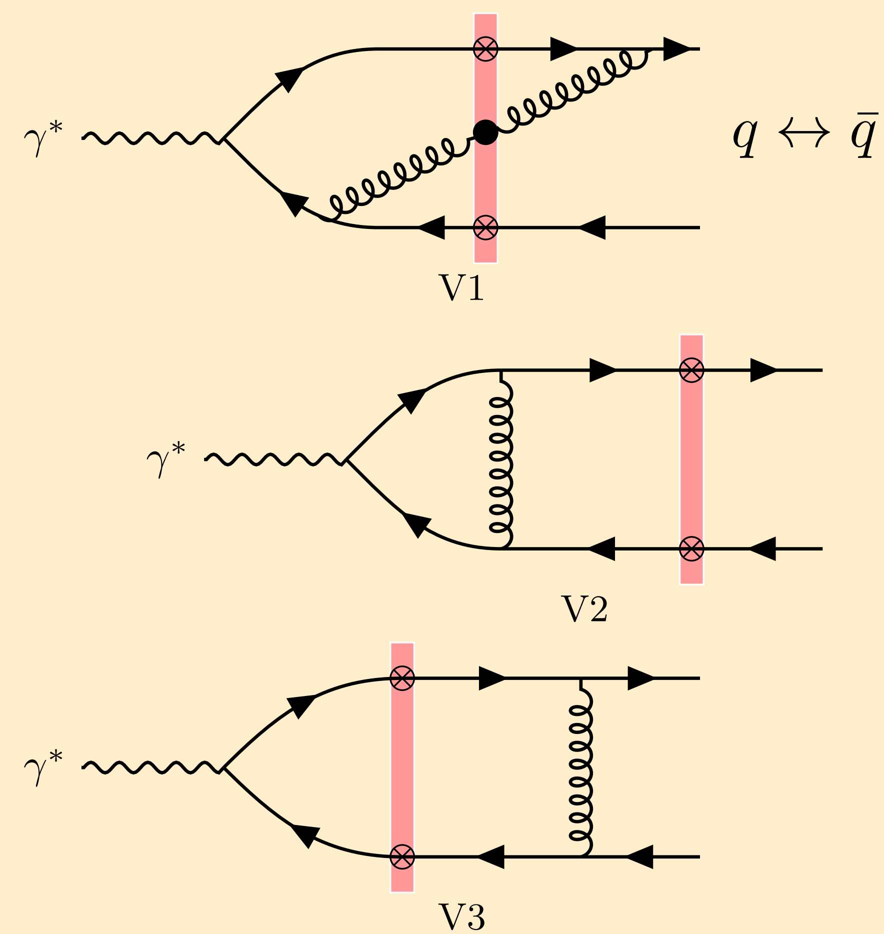


Virtual emission (loop open and closes in DA or CCA)

Self-energy contributions



Vertex contributions



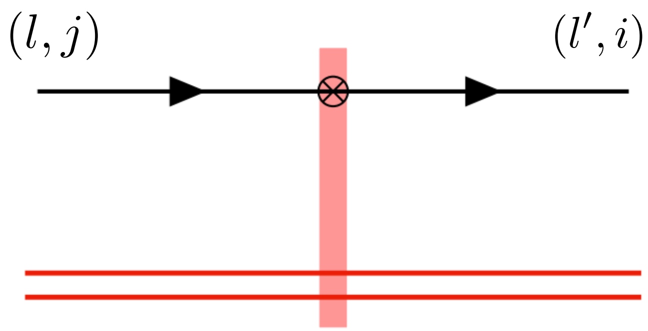
Dijet production in the CGC at NLO

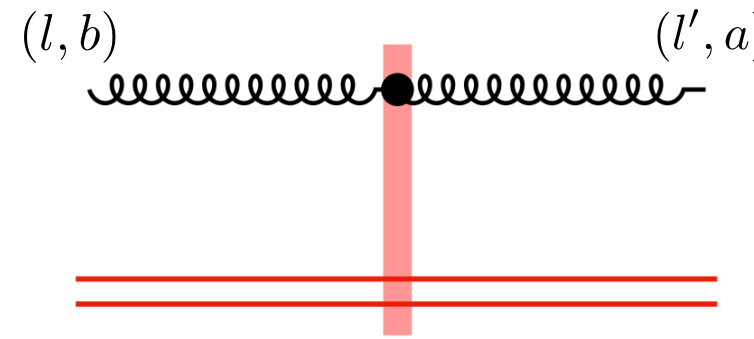
Setup for the calculation

- Covariant perturbation theory in momentum space
  (another popular approach is LCPT)

  Standard QED, QCD rules: propagators, vertices, polarization vectors, etc

- Vertices for (eikonally) coupling to the CGC background field (in $A^-_{\rm cl} = 0$ gauge)





$$\mathcal{T}^q_{ij}(l, l') = (2\pi)\delta(l^- - l'^-)\gamma^- \mathrm{sgn}(l^-) \times \int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp)\cdot \boldsymbol{z}_\perp} V^{\mathrm{sgn}(l^-)}_{ij}(\boldsymbol{z}_\perp)$$

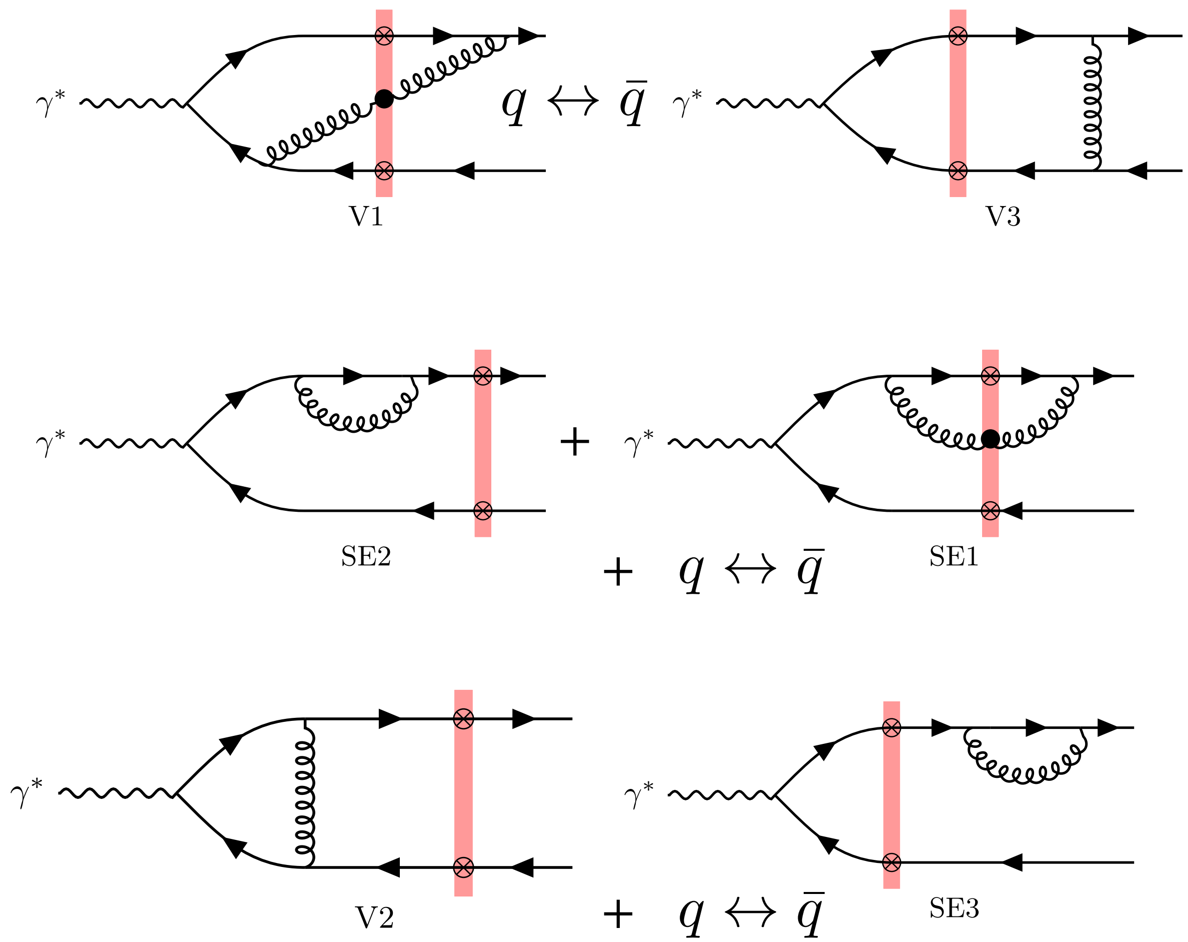
$$\mathcal{T}^g_{ab}(l, l') = -(2\pi)\delta(l^- - l'^-)(2l^-) g_{\mu\nu} \mathrm{sgn}(l^-) \times \int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp)\cdot \boldsymbol{z}_\perp} U^{\mathrm{sgn}(l^-)}_{ab}(\boldsymbol{z}_\perp)$$

- Regularization schemes

  dimensional regularization + rapidity cut-off

Dijet production in the CGC at NLO

Cancellation of divergences of UV divergences



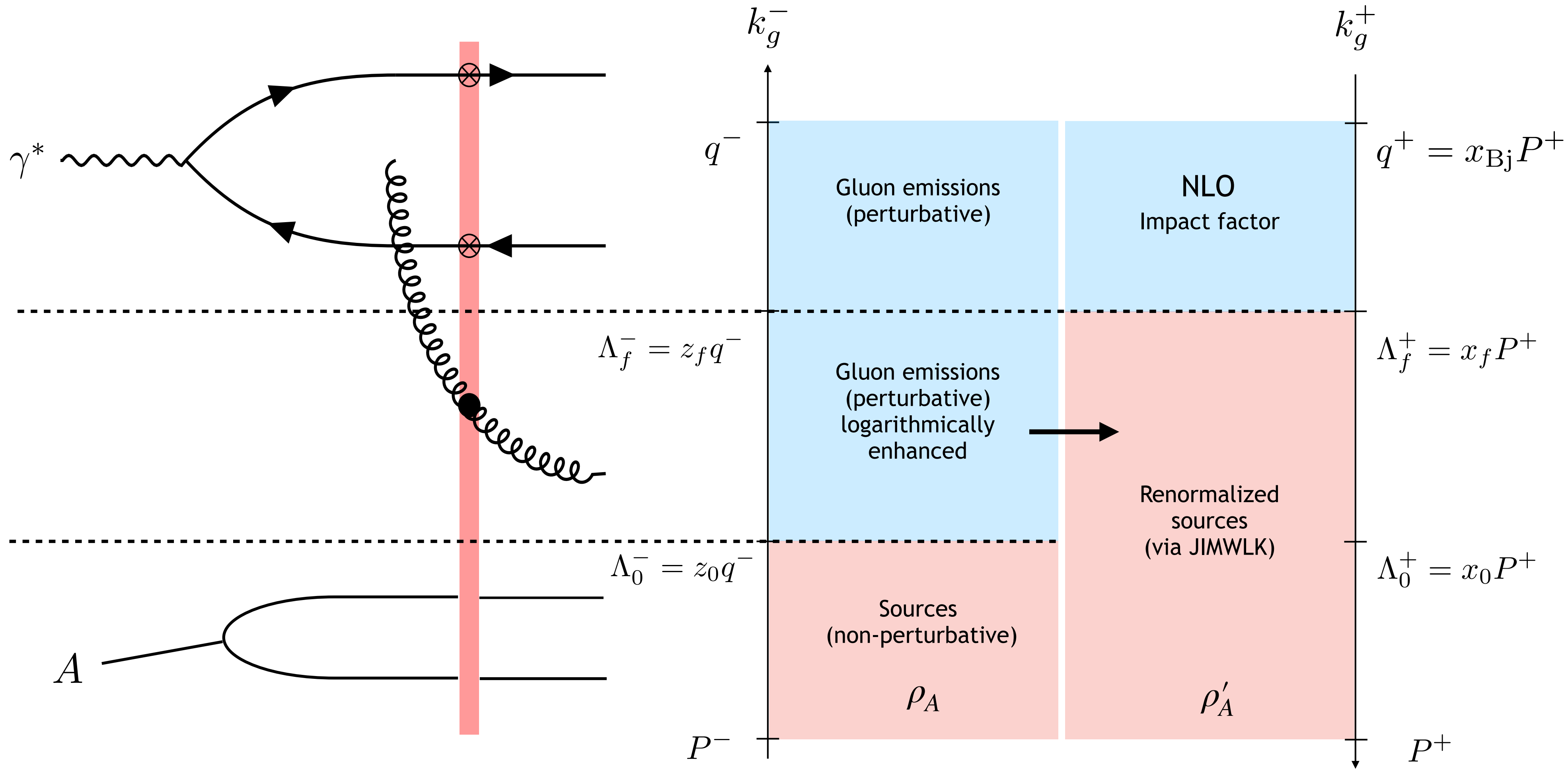
- UV finite diagrams
- UV divergences cancel among self energies contributions (before SW and crossing SW)
- UV divergence cancel in vertex contribution before SW and self energy contribution after SW

- UV finite, no need for counter-terms at this order in PT.
- Overall IR divergence is left in sum of virtual diagrams, and soft divergence left in V3. Both cancel with real emissions.

P. Caucal, FS, and R. Venugopalan (2021)

Inclusive dijet production at NLO

Rapidity (slow gluon) divergences and JIMWLK **factorization**



JIMWLK LL Hamiltonian

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \alpha_s \ln\left(\frac{z_f}{z_0}\right) \mathcal{H}_{\mathrm{LL}} \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_s \mathrm{d}\sigma_{\mathrm{NLO,i.f.}}$$

Large logs need to be **resummed**!

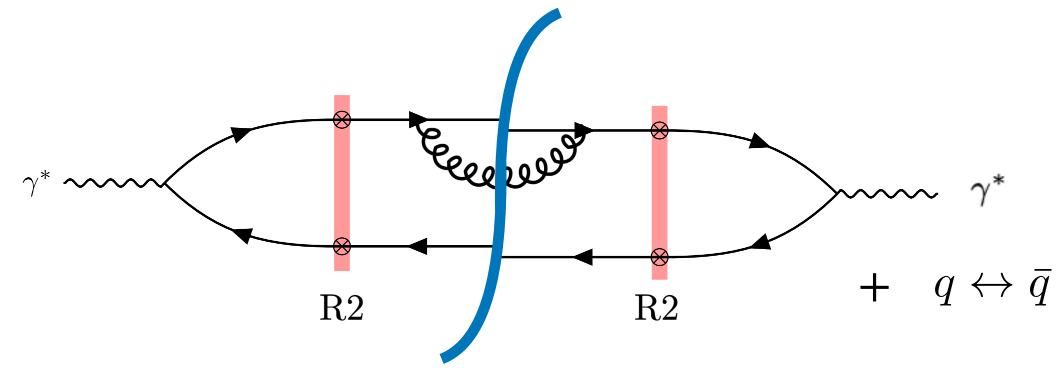
$$\alpha_s \ln\left(\frac{z_f}{z_0}\right) \sim \alpha_s \ln(s)$$

Evolution of sources (weight functional)

$$W_{\Lambda_0^-}[\rho_A] \to W_{\Lambda_f^-}[\rho'_A]$$

Dijet production in the CGC at NLO

Infrared and collinear safety



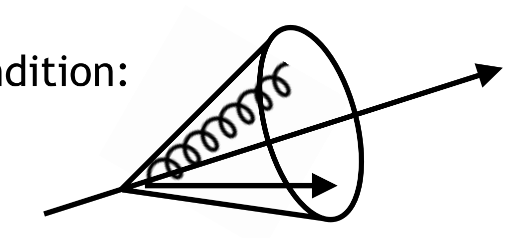
Collinear non-slow divergences

- Implement a jet algorithm* (small cone) excluding slow gluon divergence

Phase space for collinear non-slow gluon

$$\int_{z_f}^{z_j} \frac{\mathrm{d}z_g}{z_g}\mu^{\varepsilon} \int \frac{\mathrm{d}^{2-\varepsilon}\mathcal{C}_{qg,\perp}}{(2\pi)^{2-\varepsilon}} \frac{1}{\mathcal{C}^2_{qg,\perp}}$$

Small-cone condition:

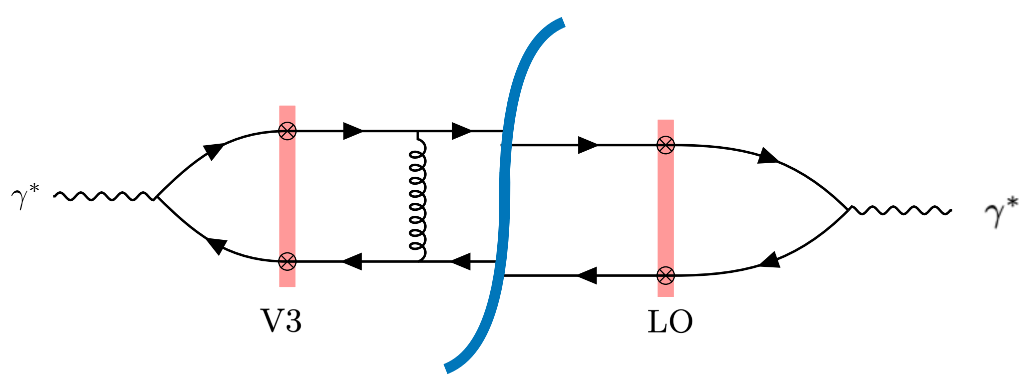


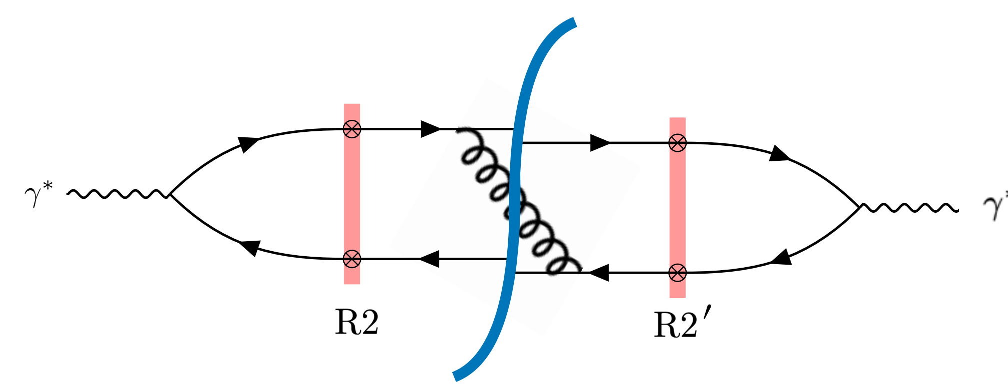
$$\mathcal{C}^2_{qg,\perp} \leq \mathcal{C}^2_{qg,\perp}\big|_{\max} = R^2 \boldsymbol{p}_j^2 \min\left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2}\right)$$

- Collinear divergence cancels against IR divergence left in virtual contributions

Soft divergence

- Remaining soft divergence cancel between vertex correction after SW, and cross term real gluon emission after SW





P. Caucal, FS, and R. Venugopalan (2021)

Summary

- Gluon saturation
  CGC effective theory for high energy, applied to a variety processes in different colliders, and exciting opportunities at the EIC

- Dijet production at EIC beyond TMDs
  Resummation of kinematic and genuine saturation corrections expected to be significant at the EIC

- Dijet production at EIC in the CGC at NLO
  Exact cancellation of UV divergence, IRC safe, JIMWLK rapidity factorization, and impact factor isolated

Outlook

- Couple our partonic cross-sections to event generators

  How much of the kinematic power and genuine saturation
  corrections survives in the actual observable?

- Investigate dijet production at NLO in the back-to-back limit

  Match to TMD factorization at NLO
  Is the Improved TMD framework valid at NLO?

- Numerical implementation of dijet production at NLO

  Promoting saturation physics to a precision science

  For single hadron production in pA collisions at NLO in the CGC see Hao-yu Liu (Parallel Session 2-A)!

- Employ modern techniques such SCET to the CGC at NLO

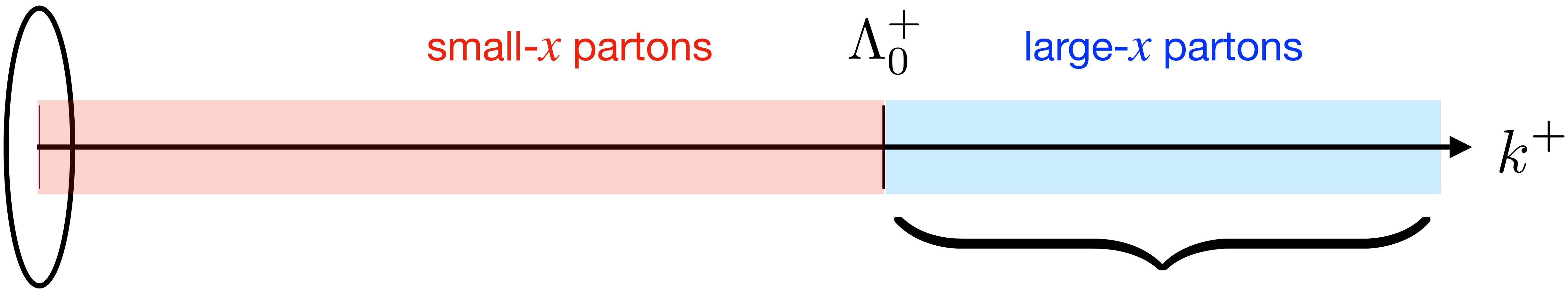
  Extend existing SCET studies (focused on moderate-x)
  to the small-x regime

Back-up slides

What is the Color Glass Condensate?

Separating sources and fields

Gelis, Iancu, Jalilian-Marian, Venugopalan (2003)



Back-ground field ← Integrate out current (along light-cone)

A double average:

$$\langle\langle \mathcal{O} \rangle\rangle = \underbrace{\int [\mathcal{D}\rho]\, W_{\Lambda_0}[\rho]}_{\text{CGC average for } \rho} \underbrace{\int^{\Lambda_0} [\mathcal{D}A]\, \mathcal{O} e^{i\mathcal{S}[A,\rho]}}_{\text{Path integral in the presence of } \rho}$$

At leading order:

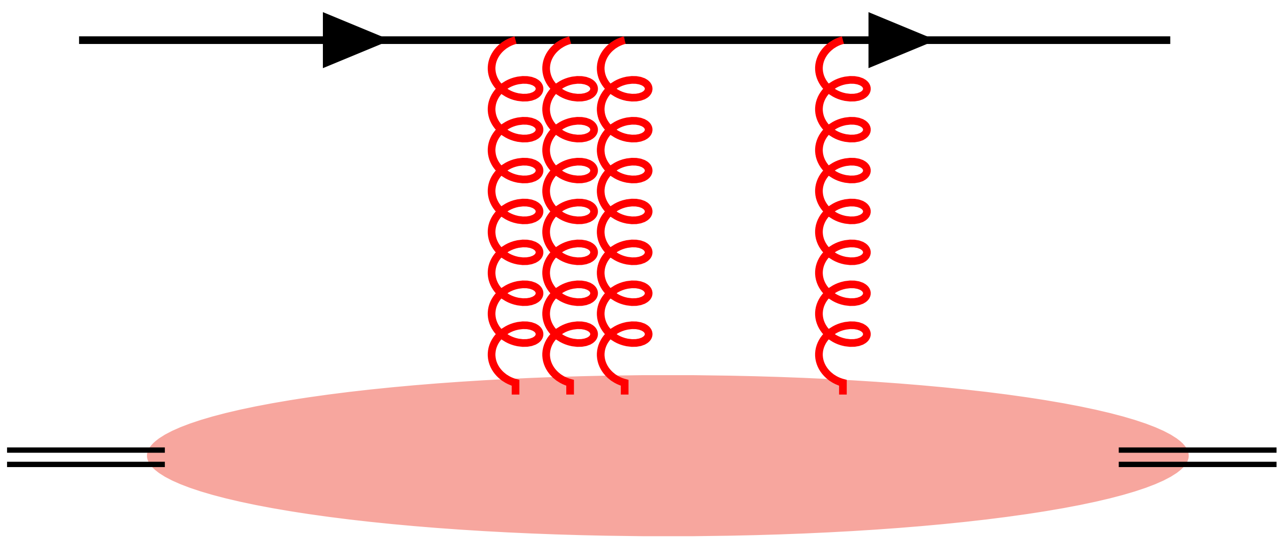
$$\langle\langle \mathcal{O} \rangle\rangle = \int [\mathcal{D}\rho]\, W_{\Lambda_0}[\rho] \mathcal{O}[A_{\rm cl}]$$

← Classical solution in presence of ρ

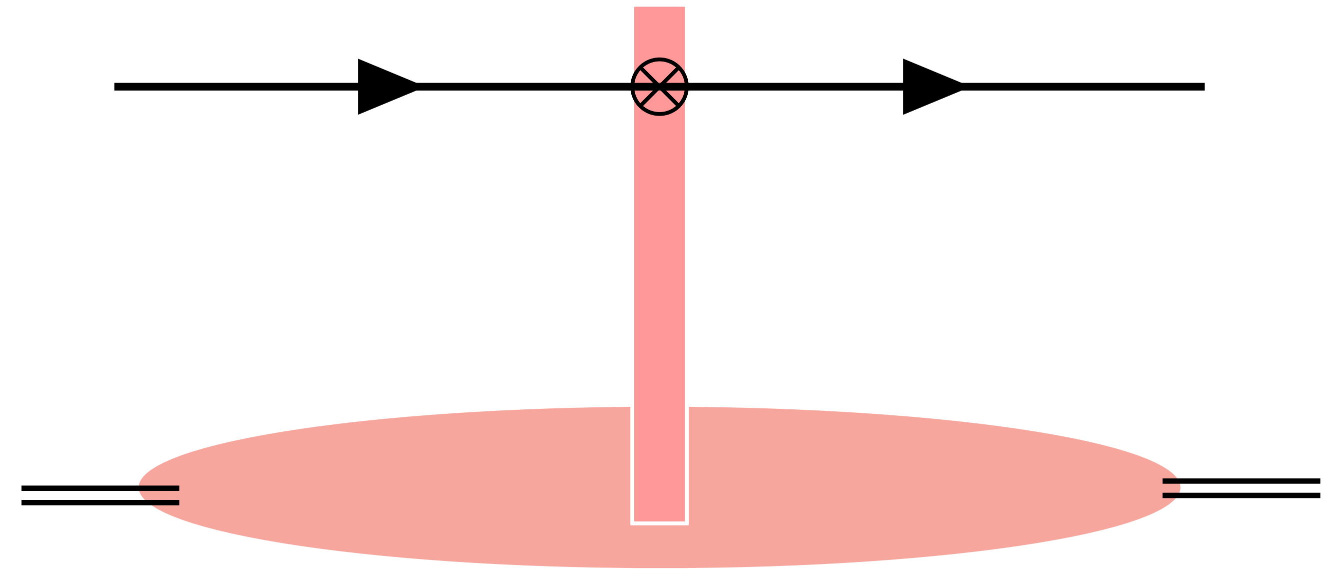
What is the Color Glass Condensate?

High energy scattering: Shockwave and Wilson lines

Multiple-eikonal scattering



Quark-shockwave vertex



Light-like Wilson line

$$V_{ij}(\boldsymbol{x}) = P\exp\left\{ig\int dx^- A_{cl}^{+,a}(\boldsymbol{x},x^-)t^a\right\}$$

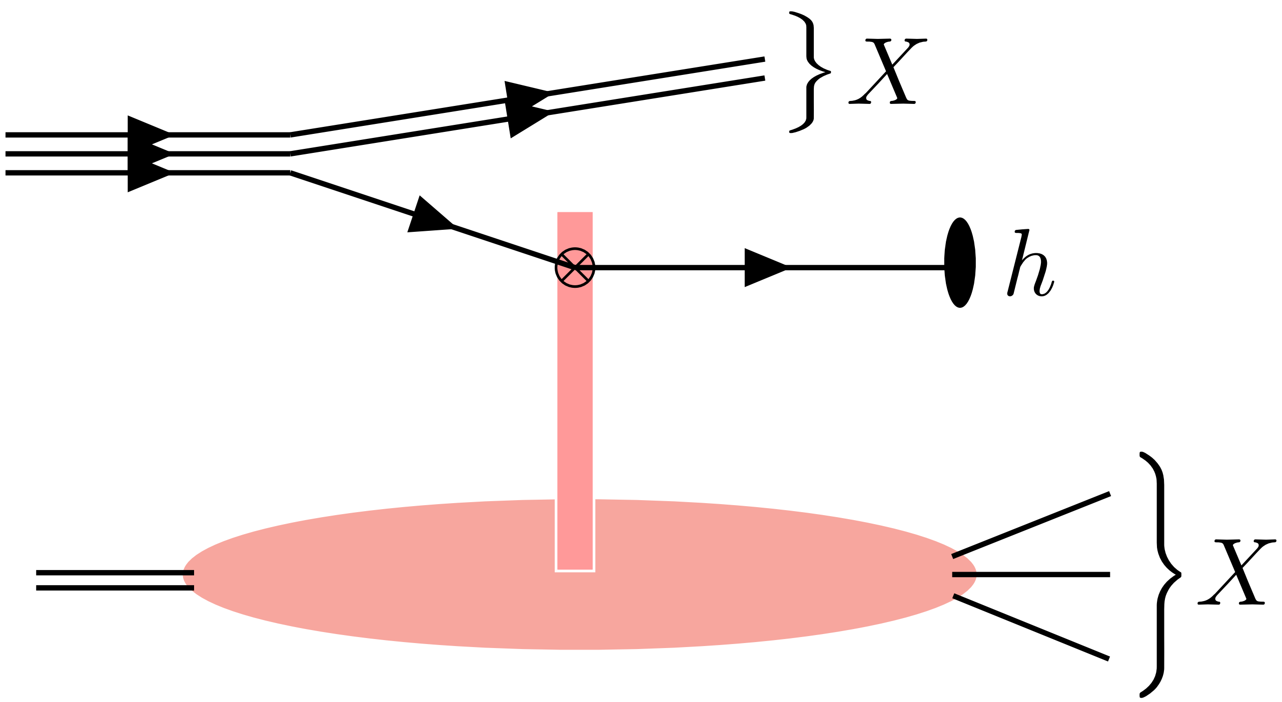
Observables built from Wilson lines, derivatives, etc… convoluted with perturbative factor (splitting functions)

$$\langle\mathcal{O}\rangle = \langle VV^\dagger ...\rangle$$

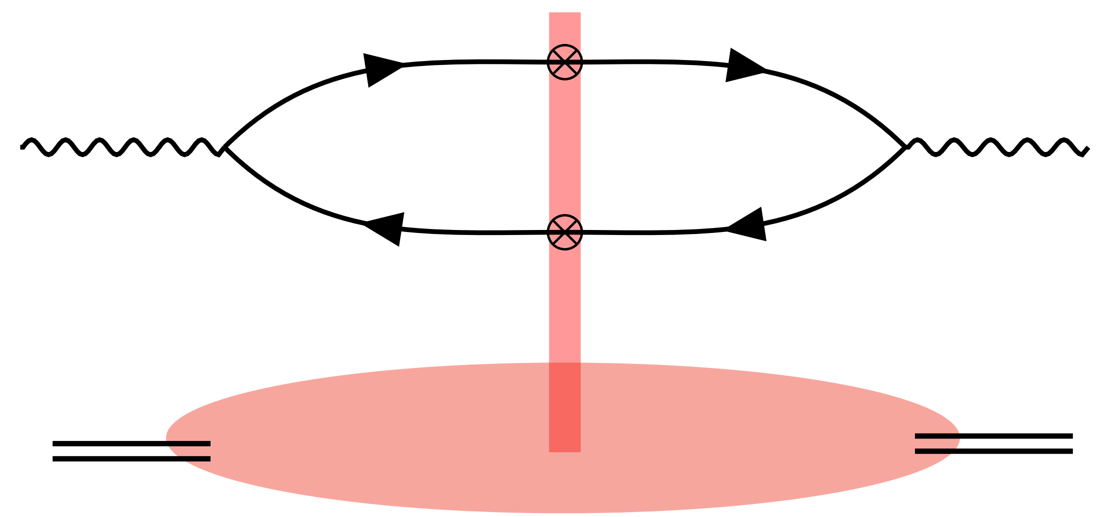
What is the Color Glass Condensate?

Universality from proton-nucleus to DIS and more

$$pA \to h + X$$

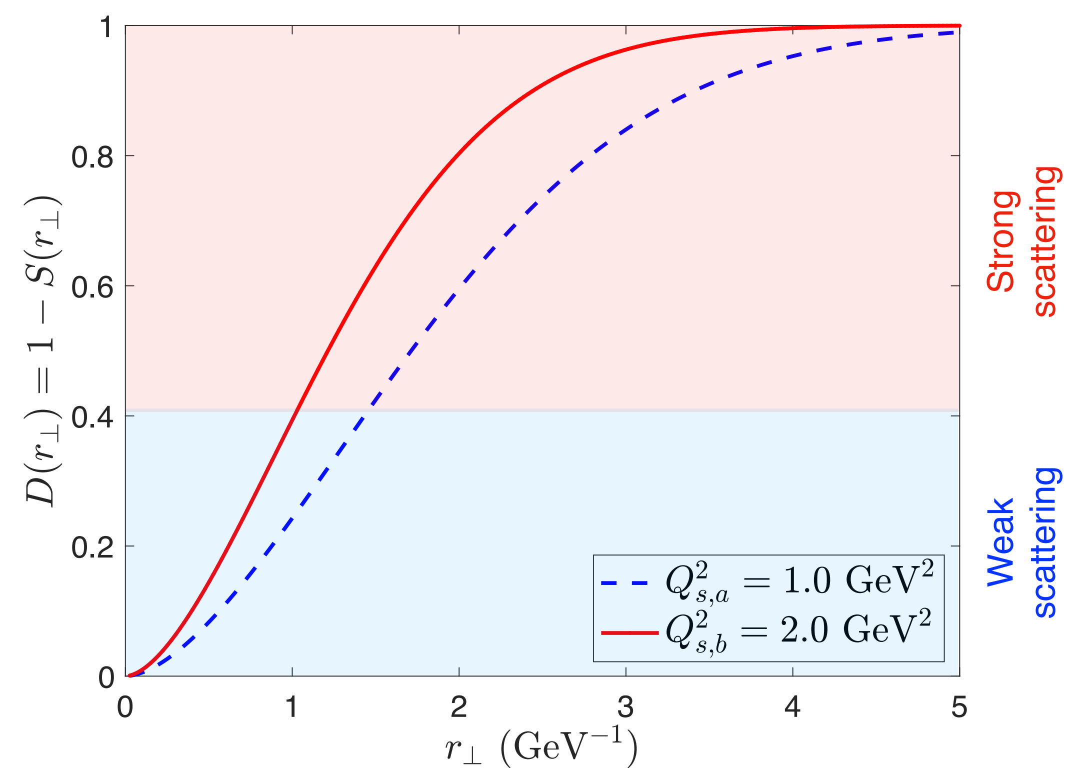


$$eA \to e + X$$



Both processes will depend on the “dipole”

$$S(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = \left\langle \mathrm{Tr}[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)] \right\rangle$$



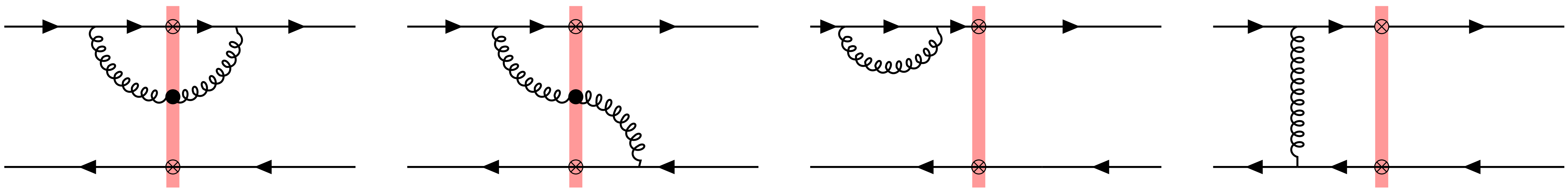
See for example: Mäntysaari, Lappi (2013)

What is the Color Glass Condensate?

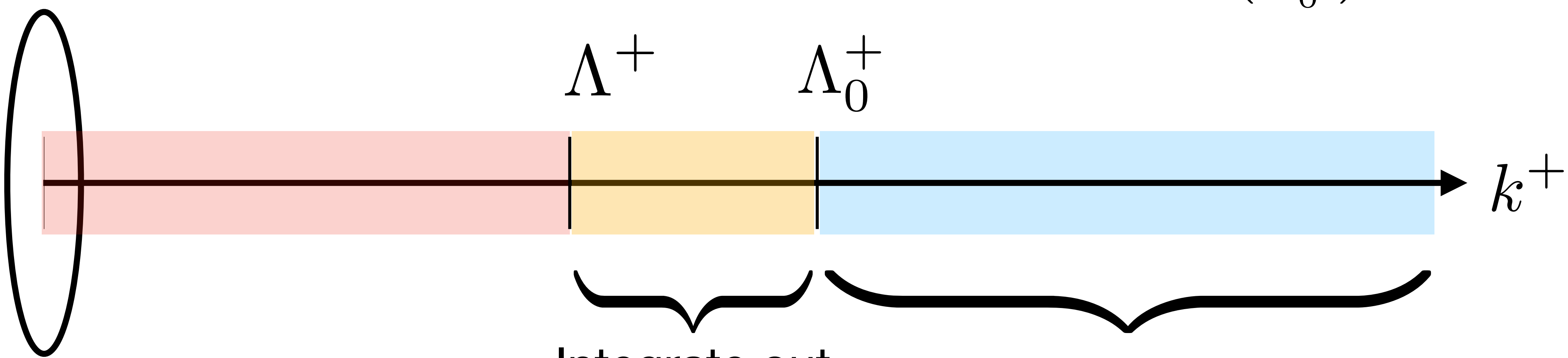
Quantum evolution

Gelis, Iancu, Jalilian-Marian, Venugopalan (2003)

Beyond classical picture, dress Wilson lines with loops (gluons)



Corrections enhanced by rapidity logs $\alpha_s Y$ $\quad Y = \ln\left(\frac{\Lambda^+}{\Lambda_0^+}\right)$



Small-x BK-JIMWLK resummation

$$W_{\Lambda_0}[\rho] \to W_{\Lambda}[\rho] \qquad \langle\langle \mathcal{O} \rangle\rangle = \int [\mathcal{D}\rho]\, W_{\Lambda}[\rho] \mathcal{O}[A_{\rm cl}]$$

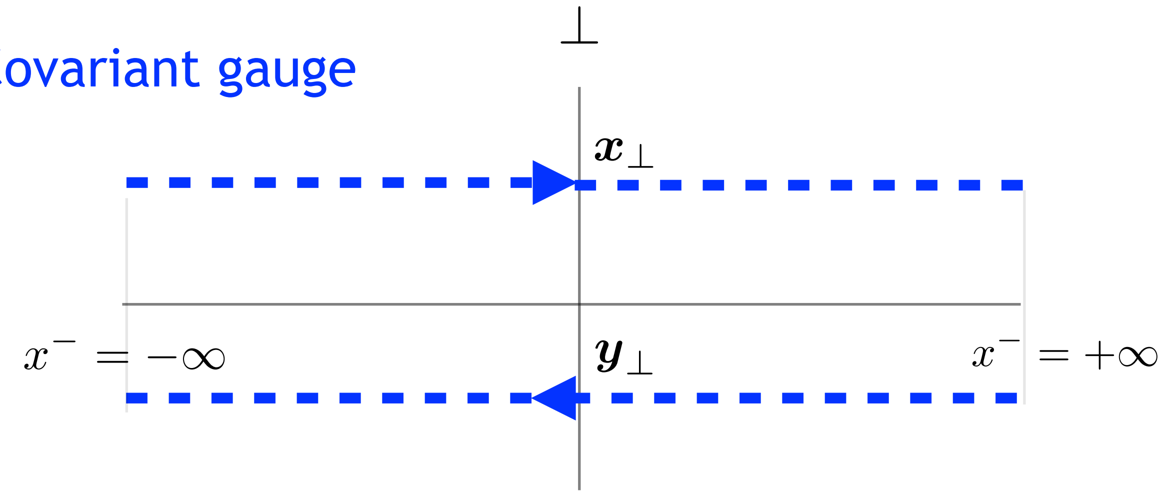
Dijet production beyond TMDs

Choosing the gauge: light-like Wilson lines vs transverse gauge link

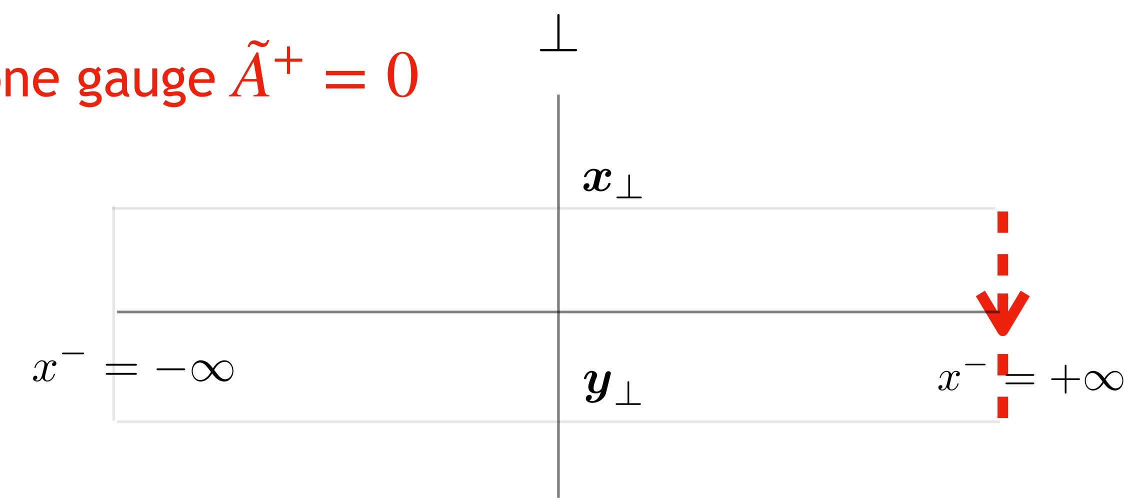
Boussarie, Mehtar-Tani (2020)

Pair of Wilson lines as transverse gauge link

Covariant gauge



Light-cone gauge $\tilde{A}^+ = 0$



$$V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) = \mathcal{P}\,\exp\left[-ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp)\right]$$

gA expansion:

$$= 1 - ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

Small dipole expansion:

Altinoluk, Boussarie, Kotko (2019)

$$= 1 + ig\,\boldsymbol{r}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$

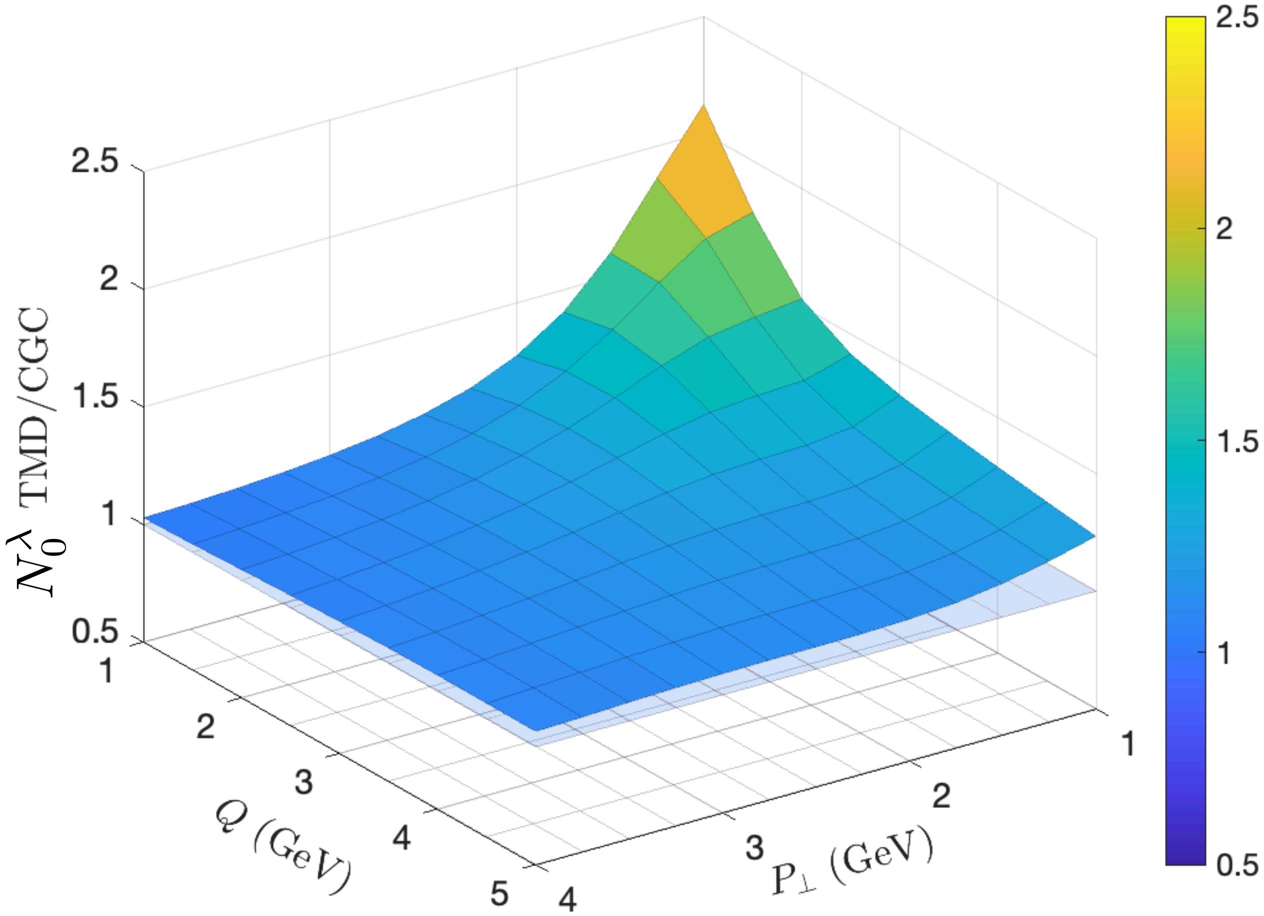
Dominguez, Marquet, Xiao, Yuan (2011)

Dijet production beyond TMDs

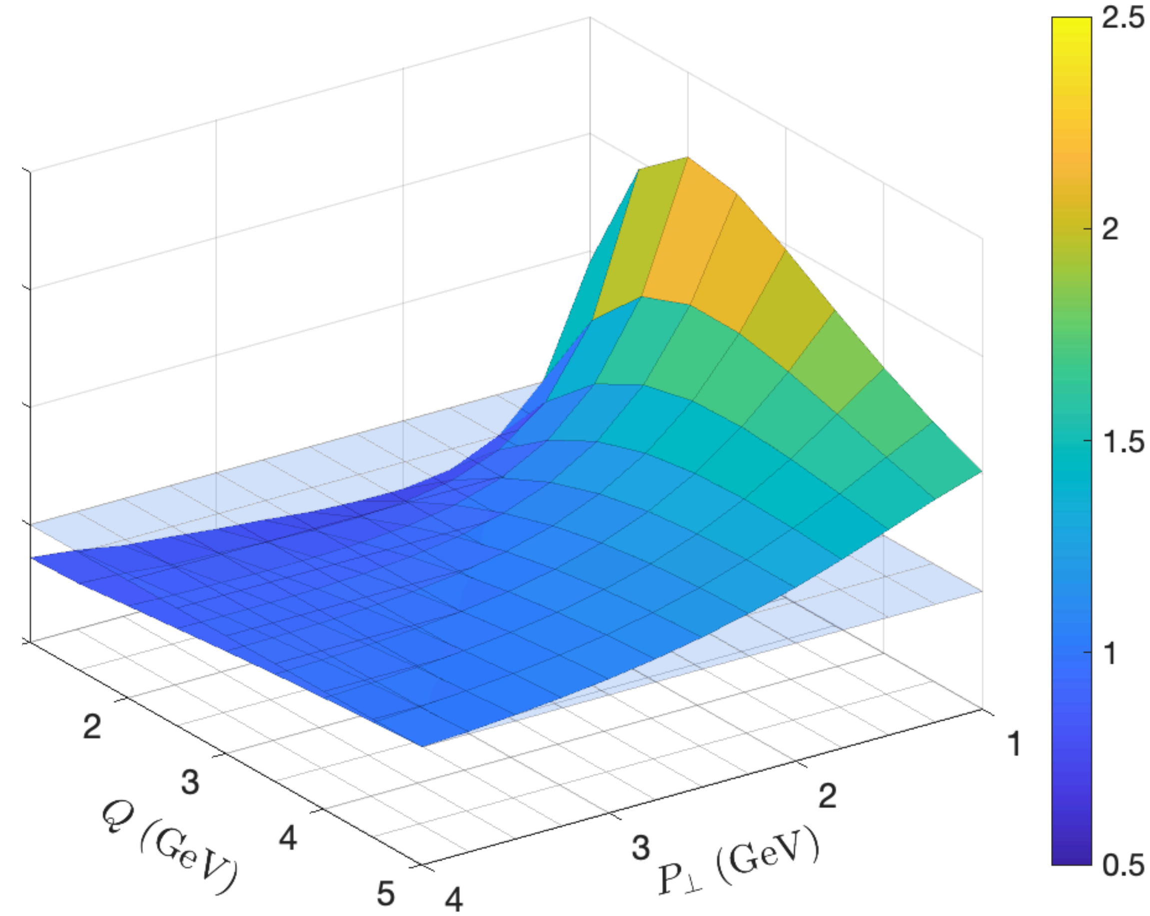
Q^2 and $P_\perp$ dependence of genuine saturation

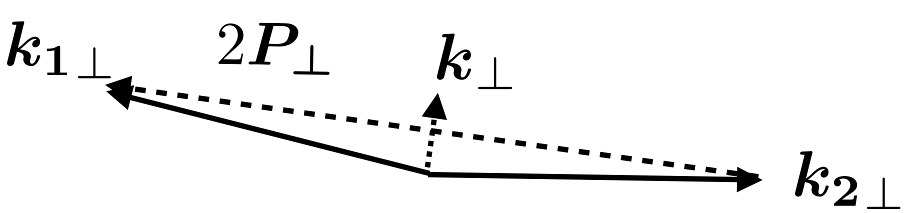
At exactly back-to-back $k_\perp \approx 0$ the ratio of CGC/TMD is sensitive to genuine twists

$\gamma_T^* + \mathrm{Au} \to q + \bar{q} + X$



$\gamma_L^* + \mathrm{Au} \to q + \bar{q} + X$





$$\frac{\mathrm{d}N^{\gamma_\lambda^* + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

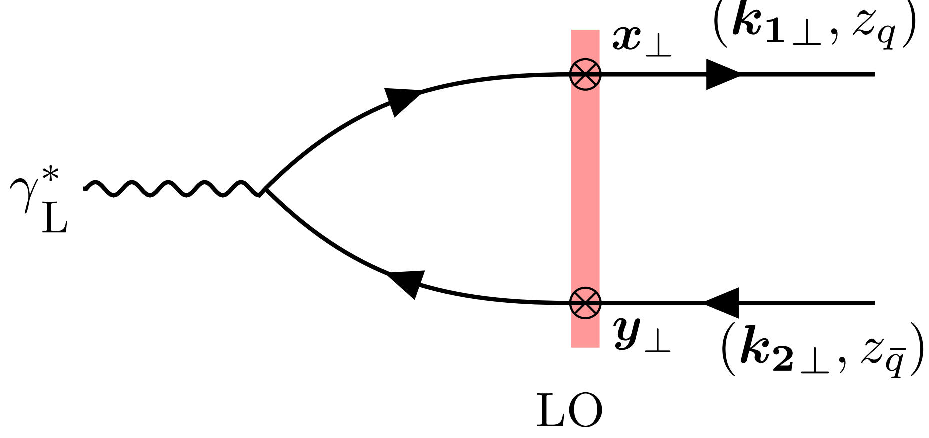
$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

Dijet production in the CGC at NLO

An example of structure of LO vs NLO amplitudes

LO amplitude



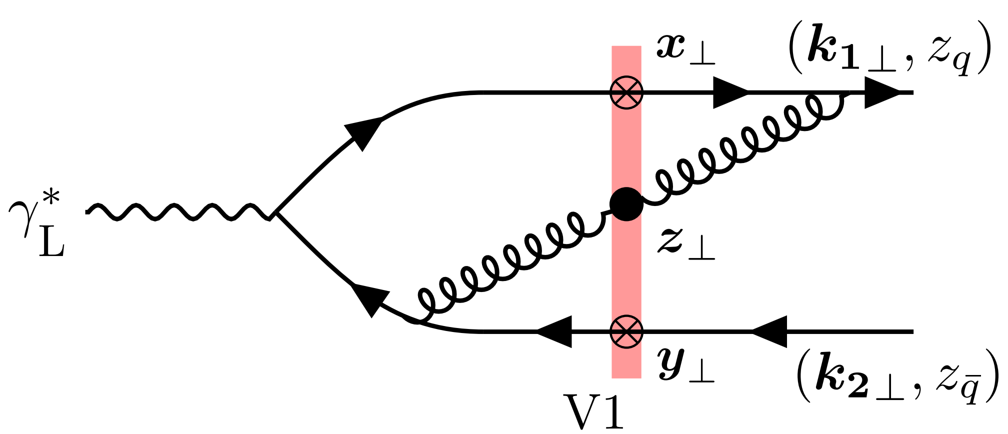
$$\otimes_{\rm LO} \equiv \frac{e e_f q^-}{\pi} \int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp e^{-i\boldsymbol{k_1}_\perp\cdot\boldsymbol{x}_\perp} e^{-i\boldsymbol{k_2}_\perp\cdot\boldsymbol{y}_\perp}$$

$$X_{q\bar{q}}^2 = z_q z_{\bar{q}} r_{xy}^2$$ effective dipole size

$$\mathcal{M}_{\rm LO}^{{\rm L},\sigma\sigma'} = \left[1 - V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right] \otimes_{\rm LO} 2(z_q z_{\bar{q}})^{3/2} Q K_0(Q X_{q\bar{q}}) \delta^{\sigma,-\sigma'}$$

non-perturbative — perturbatively computable

Dressed vertex amplitude



$$\otimes_{\rm V} \equiv \frac{e e_f q^-}{\pi} \int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \mathrm{d}^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k_1}_\perp\cdot\boldsymbol{x}_\perp} e^{-i\boldsymbol{k_2}_\perp\cdot\boldsymbol{y}_\perp}$$

$$X_{q\bar{q}g}^2 = (z_q - z_g) z_{\bar{q}} r_{xy}^2 + (z_q - z_g) z_g r_{xz}^2 + z_g z_{\bar{q}} r_{zy}^2$$ effective dipole size

$$\mathcal{M}_{\rm V1}^{{\rm L},\sigma\sigma'} = \left[C_{\rm F}\mathbb{1} - t^a V(\boldsymbol{x}_\perp) t^b V^\dagger(\boldsymbol{y}_\perp) U_{ab}(\boldsymbol{z}_\perp)\right] \otimes_{\rm V} \frac{\alpha_s}{\pi^2} \int_{z_0}^{z_q} \frac{\mathrm{d}z_g}{z_g} \frac{\mathbf{r}_{zx}\cdot\mathbf{r}_{zy}}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy}^2} \left[\left(1 - \frac{z_g}{z_q}\right)\left(1 + \frac{z_g}{z_{\bar{q}}}\right)\left(1 - \frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right) + \dots\right] e^{-i\frac{z_g}{z_q}\boldsymbol{k}_\perp\cdot\mathbf{r}_{zx}}$$

$$\times\ 2(z_q z_{\bar{q}})^{3/2} Q K_0(Q X_{q\bar{q}g}) \delta^{\sigma,-\sigma'}$$

non-perturbative — perturbatively computable

P. Caucal, FS, and R. Venugopalan (2021)