Light-cone expansion of the amplitudes of FCNC B-decays

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Nonfactorizable corrections induced by charm-quark loops in exclusive FCNC B-decays (i.e. B- decays induced by flavour-changing neutral currents) are discussed. We show that a consistent calculation of the appropriate QCD correlation function requires the full generic three-particle distribution amplitude (3DA) of the B-meson $\langle 0|\bar{q}(x)G_{\mu\nu}(y)b(0)|B(p)\rangle$ with *non-aligned* arguments: Expanding the latter 3DA amplitude near the light cone, one finds that the B-decay correlation function is dominated by the regions $x^2 \sim 0$ and $y^2 \sim 0$ but $(x-y)^2 \neq 0$. As the result, for a proper description of the amplitudes of FCNC B-decays, the full dependence of the 3DA on the variable $(x-y)^2$ is necessary.

A. Kozachuk, D. M., N. Nikitin, PRD97, 053007 (2018)

A. Kozachuk, D. M., PLB786, 378 (2018)

D. Melikhov, EPJ Web Conf. 222, 01007 (2019)

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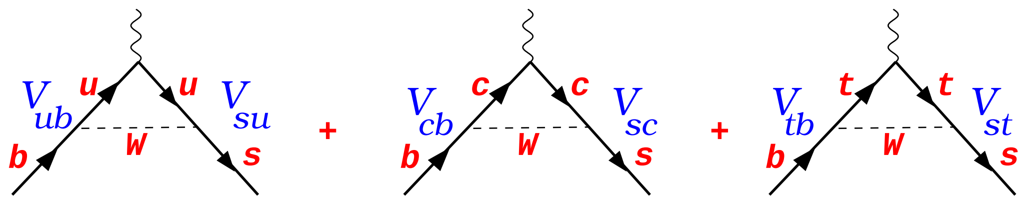
Motivation

FCNC $b \to s$ and $b \to d$ transitions do not occur at the tree level in SM and proceed via loops, where t, c and u-quarks contribute.



CKM Unitarity: $V^*_{us} V_{ub} + V^*_{cs} V_{cb} + V^*_{ts} V_{tb} = 0$

BRs of FCNC decays are small; on the other hand, new particles may show up virtually in the loops. Therefore, FCNC decays are most popular candidates for indirect search of physics BSM.

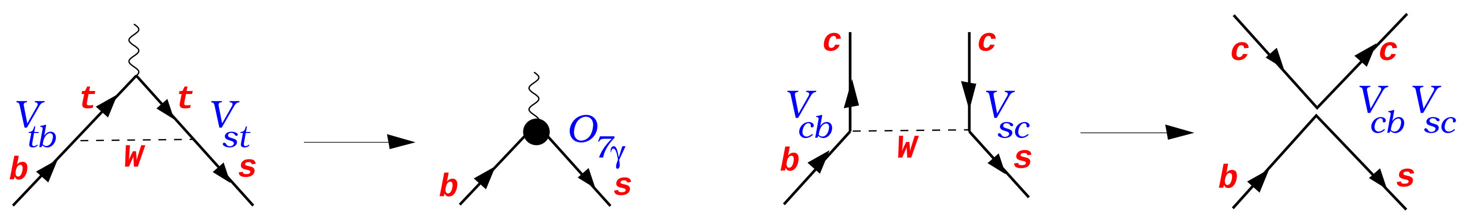
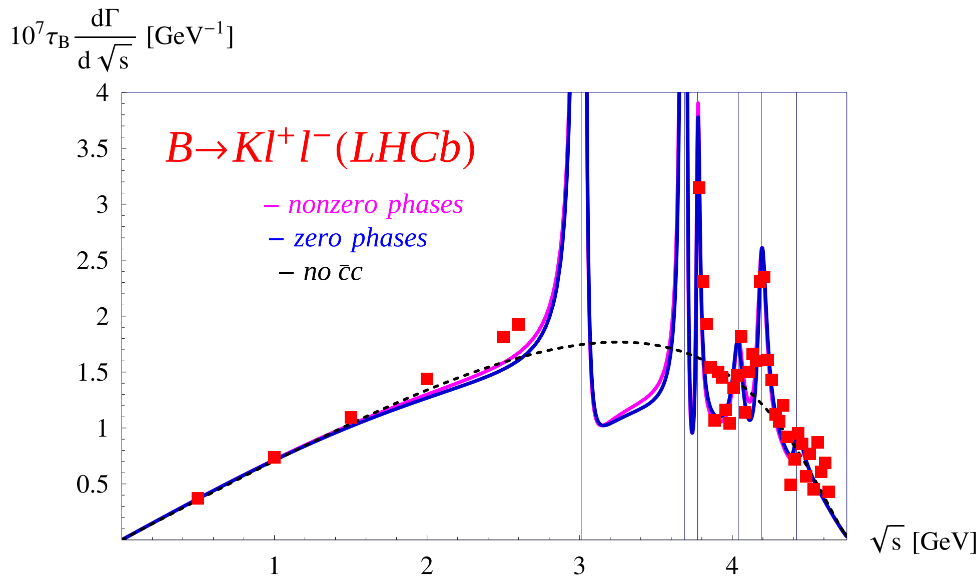


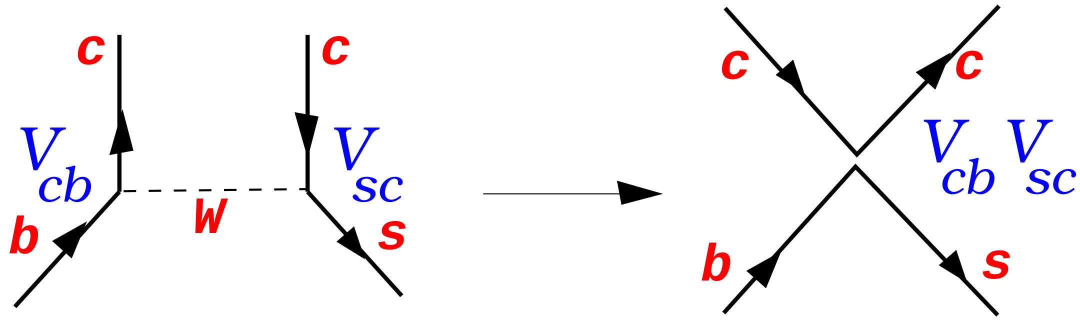
Illustration: $B \to Kl^+l^-$ **decay** $0 < \sqrt{s} < (M_B - M_K)$, s **- momentum squared of** l^+l^- **pair.**



- **In the charmonia region, charm contribution dynamically enhanced and dominates.**

- **Far from the charmonia region, top dominates (black dashed).**

Still, to study possible NP effects, need to gain theoretical control over charm contribution



• **Account of hard gluon exchanges lead to the four-quark operators**

$$H_{\rm eff}^{b\to s\bar{c}c} = -\frac{G_F}{\sqrt{2}} V_{cb}V_{cs}^{*} \, \{C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2\}$$

with

$$\mathcal{O}_1 = \bar{s}^j\gamma_\mu(1-\gamma_5)c^i \, \bar{c}^i\gamma^\mu(1-\gamma_5)b^j, \qquad \mathcal{O}_2 = \bar{s}^i\gamma_\mu(1-\gamma_5)c^i \, \bar{c}^j\gamma^\mu(1-\gamma_5)b^j,$$

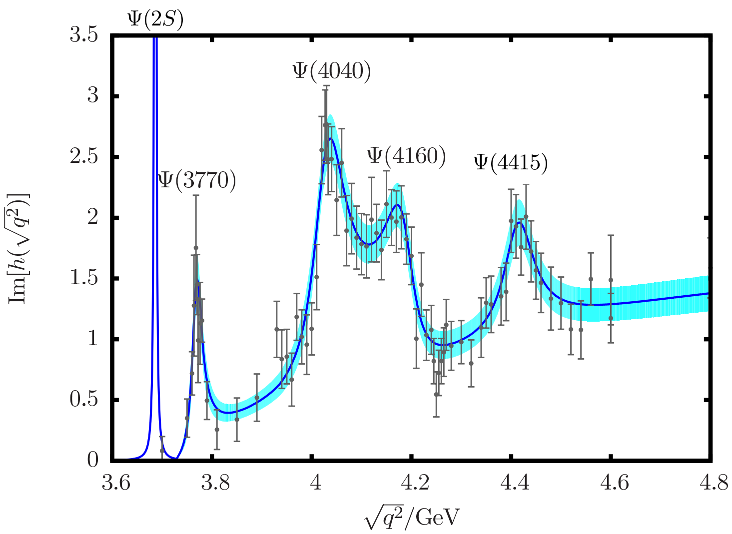
and the similar terms with $c \to u$ (i, j color indices). The SM Wilson coefficients at the scale $\mu_0 = 5$ GeV [corresponding to $C_2(M_W) = -1$]: $C_1(\mu_0) = 0.241$, $C_2(\mu_0) = -1.1$.

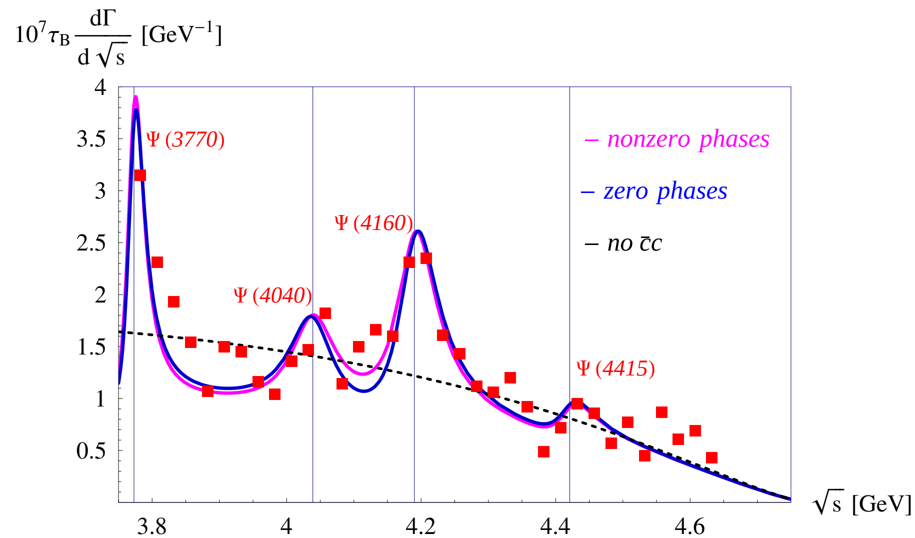
These operators lead to factorizable contributions to the amplitudes of exclusive FCNC B-decays.

• **Soft gluon exchanges between the charm-quark loop and the B-meson loop lead to nonfactorizable contributions to the amplitudes.**

- Nonfactorizable corrections are comparable with factorizable contributions

How do we know that? Compare charmonia in l^+l^--annihilation and in FCNC B-decays:



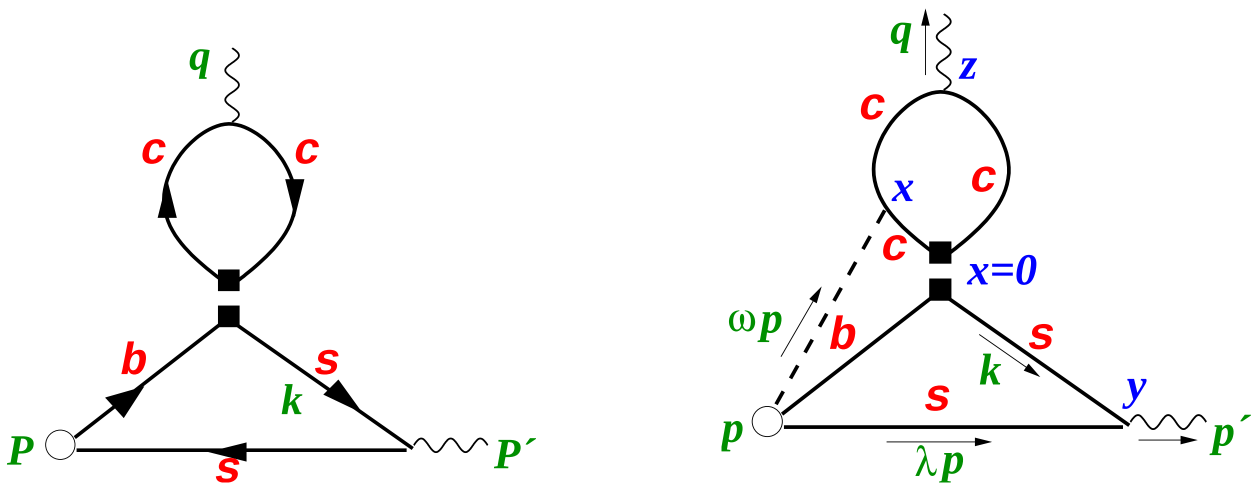


The patterns of charmonia in charm contribution to vacuum polarization (left) and in $B \to K l^+ l^-$ (right) are different. The difference is due to nonfactorizable contributions.

- **In some cases, factorizable charm contribution vanishes and thus only nonfactorizable charm contributes (e.g in $B \to \gamma\gamma$)**

We need formalism to calculate nonfactorizable charm effects in QCD.

Charming loops



At $q^2 \ll 4m_c^2$, the charm loop may be calculated in pQCD.

Factorizable part: product of $B \to M_f$ form factor and the charm polarization function.

Nonfactorizable part:

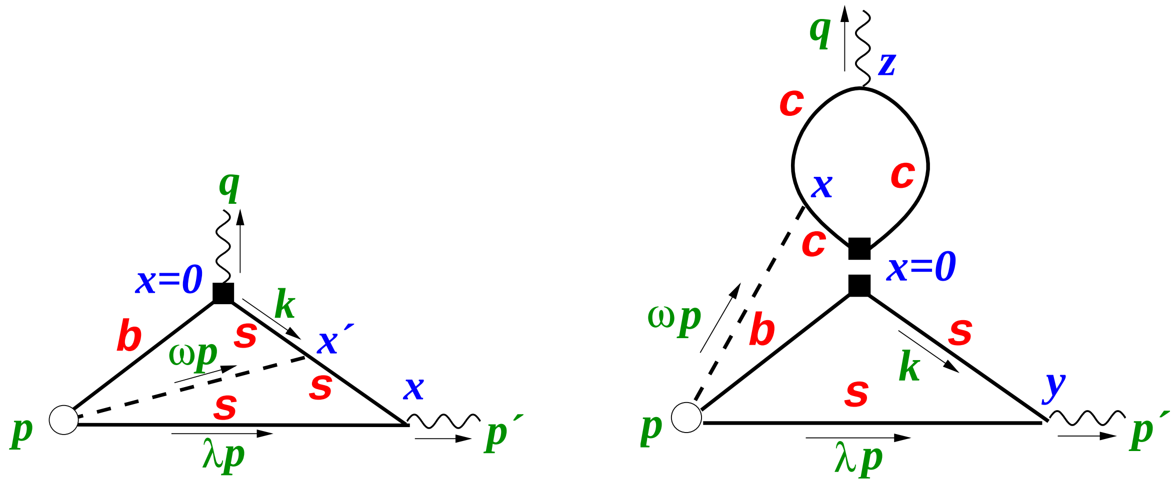
$$A(q,p) \;=\; \frac{1}{(2\pi)^8}\int \frac{dk}{m_s^2-k^2}\int dy e^{-i(k-p')y}\int dx e^{-i\kappa x} d\kappa\, \Gamma_{cc}(\kappa,q)\,\langle 0|\bar{s}(y)G(x)b(0)|B_s(p)\rangle .$$

The 3DA depends on 5 variables xp, yp, x^2, y^2, xy ($p^2=M_B^2$) and may be parametrized as follows:

$$\langle 0|s^\dagger(y)G(x)b(0)|B_s(p)\rangle = \int d\lambda e^{-i\lambda yp}\int d\omega e^{-i\omega xp}\Phi(\omega,\lambda)\left[1+O\left(\Lambda_{\rm QCD}^2 x^2, \Lambda_{\rm QCD}^2 y^2, \Lambda_{\rm QCD}^2 (x-y)^2\right)\right].$$

$\Phi(\omega,\lambda)$ is peaked at $\lambda,\omega \sim \Lambda_{\rm QCD}/m_b$.

3DA contributions in ffs vs charming loops



• **Ffs** 3Da contribution to the heavy-to-light ffs (left panel): if the light s-quark is energetic, a soft gluon emission cannot change its direction so that the points $x = 0$, x' and x are on the same line. The diagram is dominated by the LC configuration $x^2 = x'^2 = 0$, and $x' = ux$, $0 < u < 1$. Due to this property the leading contribution may be calculated via $\langle 0|\bar{s}(x)G_{\mu\nu}(ux)b(0)|B_s(p)\rangle$.

• **FCNC** The situation is more difficult in FCNC (right panel): We show that the dominant contribution comes from $x^2 = 0$, $y^2 = 0$, i.e. both are on the LC, but $xy \neq 0$. I.e., if x is along the "+" direction, then y along the "−" direction.

Unpleasant consequence - we need to know $\langle 0|\bar{s}(y)G_{\mu\nu}(x)b(0)|B_s(p)\rangle$, where points 0, x and y are not located along the same line, $y \neq ux$.

$$A(q,p) \;=\; \frac{1}{(2\pi)^8}\int \frac{dk}{m_s^2-k^2}\int dy e^{-i(k-p')y}\int dx e^{-i\kappa x} d\kappa\, \Gamma_{cc}(\kappa,q)\,\langle 0|\bar{s}(y)G(x)b(0)|B_s(p)\rangle .$$

$$\langle 0|s^\dagger(y)G(x)b(0)|B_s(p)\rangle = \int d\lambda e^{-i\lambda y p}\int d\omega e^{-i\omega x p}\Phi(\omega,\lambda)\left[1+O\left(\Lambda_{\rm QCD}^2 x^2, \Lambda_{\rm QCD}^2 y^2, \Lambda_{\rm QCD}^2 (x-y)^2\right)\right].$$

• LC contribution of 3DA

The contribution of the LC term in 3DA, $\Phi(\omega,\lambda)$, to $A(q,p)$ is easy to calculate.

$$A(q,p) = \int_0^\infty d\lambda \int_0^\infty d\omega\, \Phi(\lambda,\omega)\Gamma_{cc}\left(-\omega p, q\right)\frac{1}{m_s^2-(\lambda p - p')^2}.$$

• The triangle charming loop is easily calculable

$$\Gamma_{cc}(\kappa,q) = \frac{1}{8\pi^2}\int_0^1 du \int_0^1 dv \frac{\theta(u+v<1)}{m_c^2+2uv\kappa q - u(1-u)\kappa^2 - v(1-v)q^2}.$$

The ω-integral is peaked at $\omega \sim \Lambda_{\rm QCD}/m_b$ so the gluon is soft:
$\kappa = -\omega p$ and $\kappa^2 \sim O(\Lambda_{\rm QCD}^2) \ll m_c^2$.

• The s-quark propagator takes the form

$$m_s^2-(\lambda p - p')^2 = m_s^2 - \lambda q^2 - (1-\lambda)p'^2 + (1-\lambda)\lambda M_B^2.$$

In the bulk of λ-integration the virtuality of the s-quark propagator is $O(M_B)$.

• off-LC contribution of 3DA

The difficulty of the problem is the existence of two heavy quark scales, one of which is much heavier than the other:

$$\boxed{\Lambda_{\rm QCD} \ll m_c \ll m_b, \text{ and } \Lambda_{\rm QCD} m_b/m_c^2 \simeq 1}$$

We need to sum all powers of the parameter $\lambda_{\rm QCD} m_b/m_c^2$.

Contributions of other terms to the amplitude $A(q,p)$ relative to the $\Phi(\omega,\lambda)$ term:

$$\Lambda_{\rm QCD}^2 y^2 \to \frac{\Lambda_{\rm QCD}}{m_b}, \qquad \Lambda_{\rm QCD}^2 x^2 \to \frac{\Lambda_{\rm QCD}^3 m_b}{m_c^4}, \qquad \Lambda_{\rm QCD}^2 xy \to \frac{\Lambda_{\rm QCD} m_b}{m_c^2}.$$

Nonfactorizable corrections are expressed via

$$\langle 0|s^\dagger(y)G(x)b(0)|B_s(p)\rangle = \int d\lambda e^{-i\lambda yp} \int d\omega e^{-i\omega xp} \Phi(\omega,\lambda) \left[1 + O\left(\Lambda_{\rm QCD}^2 x^2, \Lambda_{\rm QCD}^2 y^2, \Lambda_{\rm QCD}^2 (x-y)^2\right)\right],$$

The new result is that the knowledge of its functional dependence on $(x-y)^2$ is essential for a proper resummation of large $\Lambda_{\rm QCD} m_b/m_c^2$ corrections.

Previosuly, it way asserted that the 3DA with aligned arguments
$x_\mu = u y_\mu$, on the LC $x^2 = 0$, $y^2 = 0$ and $(x-y)^2 = 0$
is sufficient to calculate nonfactorizabe contributions.

One needs the off-LC contributions. A challenge for future calculations

Conclusions and outlook

• **A serious open theoretical problem in FCNC B-decays is the contribution of charming loops which "pollute" the differential distributions.**

• **At $q^2 \ll 4m_c^2$, a consistent description of charming loops requires the knowledge of off-LC 3DAa.**

• **In QCD, B-meson 3DAs with non-aligned arguments involve new Lorentz structures compared to LC 3DAs. Respectively, new invariant amplitudes arise.**

$$\langle 0|\bar{s}(y)G_{\alpha\beta}(uy)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda yv}\int d\omega e^{-i\omega uyv}\left[\frac{y_\alpha v_\beta}{yv}-\frac{y_\beta v_\alpha}{yv}\right]\Phi(\lambda,\omega).$$

For non-aligned case, new structures and new amplitudes arise:

$$\langle 0|\bar{s}(y)G_{\alpha\beta}(x)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda xv}\int d\omega e^{-i\omega yv}$$
$$\times\frac{1}{2}\left[\left(\frac{x_\alpha v_\beta}{xv}-\frac{x_\beta v_\alpha}{xv}+\frac{y_\alpha v_\beta}{yv}-\frac{y_\beta v_\alpha}{yv}\right)\Phi_S(\lambda,\omega)+\left(\frac{x_\alpha v_\beta}{xv}-\frac{x_\beta v_\alpha}{xv}-\frac{y_\alpha v_\beta}{yv}+\frac{y_\beta v_\alpha}{yv}\right)\Phi_A(\lambda,\omega)\right].$$

$\Phi_S = \Phi$ from (1), whereas Φ_A is new. If the contributions induced by Φ_A are not suppressed, a consistent calculation of the decay amplitude A needs further inputs.

Detailed investigations are underway.