Nonfactorizable contribution of charming loop to $B_s \to \gamma\gamma$ decay

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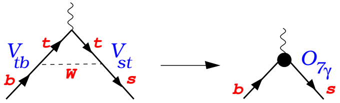
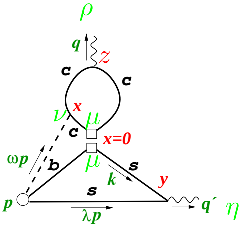
• The dominant contribution to FCNC decay is a penguin with top quark. At scale $\mu \sim m_b$ the heavy degrees of freedom might be integrated out.



Top-quark penguin for $b \to s\gamma$ transition.



Nonfactorizable contribution of charming loop to FCNC decay: the $\mathcal{O}(\alpha_s)$ diagram for soft gluon exchange.

• A penguin with charm quark: at scale $\mu \sim m_b$ W-boson is integrated out $\Longrightarrow$ factorizable charming loop.

Amplitude in a T-product form:

$$A(p,q)^{\rho\eta} = i \int dz e^{iqz} \langle 0|T\{\bar{c}(z)\gamma^\rho c(z), i\int dy\, L_{\textbf{weak}}(y), i\int dx\, L_{Gcc}(x), \bar{s}(0)\gamma^\eta s(0)\}|B_s(p)\rangle. \tag{1}$$

Amplitude in the SM:

$$A(p,q)^{\rho\eta} = \frac{-N}{(2\pi)^8}\int \frac{dk}{k^2 - m_s^2}\int dy e^{-i(k-p')y}\int\int dx d\mathbf{k} e^{-i\mathbf{k}x}$$
$$\Gamma^{\mu\nu\rho}_{cc}(\mathbf{k},q)\langle 0|\bar{s}(y)\gamma^\eta(\not{k}+m_s)\gamma^\mu(1-\gamma^5)B_\nu(x)b(0)|B_s(p)\rangle. \tag{2}$$

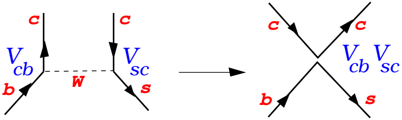
Transition $b \to s\,\bar{c}\,c$ in the SM

$$H_{\text{eff}}^{b\to s\,\bar{c}\,c} = -\frac{G_F}{\sqrt{2}}\{C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2\}, \tag{3}$$

where $\mathcal{O}_1 = \bar{s}^j\gamma_\mu(1-\gamma^5)c^i\bar{c}^i\gamma^\mu(1-\gamma^5)b^j$ and $\mathcal{O}_2 = \bar{s}^i\gamma_\mu(1-\gamma^5)c^i\bar{c}^j\gamma^\mu(1-\gamma^5)b^j$.

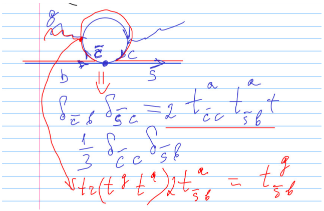
Nonfactorizable charm quark contribution:

$$H_{\text{eff}}^{b\to s\,\bar{c}\,c}(\text{octet-octet}) = \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^{*}2C_2\,(\bar{s}\gamma_\mu(1-\gamma_5)t^a b)\,(\bar{c}\gamma^\mu(1-\gamma_5)t^a c)\,, \tag{4}$$



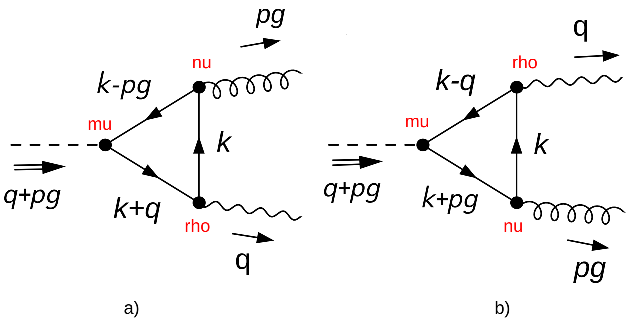
Effective vertex for $b \to s\,\bar{c}\,c$ transition.

$$\mathcal{O} = (\bar{s}_L\gamma_\mu c_L)\,(\bar{c}_L\gamma^\mu b_L) =$$
$$= -2\,(\bar{s}_L\gamma_\mu t^a b_L)\,(\bar{c}_L\gamma^\mu t^a c_L) - \frac{1}{3}\,(\bar{s}_L\gamma_\mu b_L)\,(\bar{c}_L\gamma^\mu c_L)$$



Color flow for the process $b \to s\,g\,\gamma$.

C-quark loop



The key properties:

- The diagrams a) and b) are equal to each other
- Transversality by gluon and photon momenta (the gauge invariance):
  $q^\rho \Gamma_{cc}^{\mu\nu\rho} = 0$ and $p_g^\nu \Gamma_{cc}^{\mu\nu\rho} = 0$
- The relevant part of effective four-vertex "μ" has an axial structure (only $\gamma^\mu\gamma^5$ contributes to)
- The $\Gamma_{cc}^{\mu\nu\rho}$ function is well defined for q^2, p_g^2 and $(p_g+q)^2$ below $4m_c^2$

C-quark loop reduction

C-quark loop might be reduced to three Lorentz structures:

$$\Gamma_{cc}^{\mu\nu\rho} = F_0\left(\mathbf{k}^\mu + q^\mu\right)\varepsilon^{\nu\rho\{\mathbf{k}\}\{q\}} + F_1\left(q^\rho\varepsilon^{\mu\nu\{\mathbf{k}\}\{q\}} + q^2\varepsilon^{\mu\nu\rho\{\mathbf{k}\}}\right) + F_2\left(\mathbf{k}^\nu\varepsilon^{\mu\rho\{\mathbf{k}\}\{q\}} + \mathbf{k}^2\varepsilon^{\mu\nu\rho\{q\}}\right),$$

where $F_0(\mathbf{k}^2;(\mathbf{k}.q);q^2) = F_0(q^2;(\mathbf{k}.q);\mathbf{k}^2)$ and $F_1(\mathbf{k}^2;(\mathbf{k}.q);q^2) = -F_2(q^2;(\mathbf{k}.q);\mathbf{k}^2)$.

Applying Shoutens identities to $\Gamma_{cc}^{\mu\nu\rho}$ allows one to reduce the convolution of $\Gamma_{cc}^{\mu\nu\rho}(\mathbf{k},q)$ with $B_\nu(x)$ to direct convolution with $G_{\alpha\nu}(x)$ as per:

$$\int d\mathbf{k}e^{-i\mathbf{k}x}\,\Gamma_{cc}^{\mu\nu\rho}(\mathbf{k},q)B_\nu(x) = \frac{-i}{2}\int d\mathbf{k}e^{-i\mathbf{k}x}\left[F_0\left(\mathbf{k}^\mu + q^\mu\right)\varepsilon^{\rho\alpha\nu\{q\}} + \right.$$
$$\left. + F_1\left(-q^\rho\varepsilon^{\mu\alpha\nu\{q\}} + q^2\varepsilon^{\mu\rho\alpha\nu}\right) + F_2\left(-\mathbf{k}^\rho\varepsilon^{\mu\alpha\nu\{q\}} + \mathbf{k}^\mu\varepsilon^{\rho\alpha\nu\{q\}} + (\mathbf{k}.q)\varepsilon^{\mu\rho\alpha\nu}\right)\right]G_{\alpha\nu}(x) =$$
$$= \frac{-i}{2}\int d\mathbf{k}e^{-i\mathbf{k}x}\,\overline{\Gamma}_{cc}^{\mu\rho\alpha\nu}(\mathbf{k},q)G_{\alpha\nu}(x). \quad (5)$$

$$A(q,p)^{\rho\eta} = \frac{-N}{(2\pi)^8}\left(\frac{-i}{2}\right)\int\frac{dk}{k^2 - m_s^2}\int dye^{-i(k-p')y}\int\int dxd\mathbf{k}e^{-i\mathbf{k}x}$$
$$\overline{\Gamma}_{cc}^{\mu\rho\alpha\nu}(\mathbf{k},q)\langle 0|\bar{s}(y)\gamma^\eta(\not{k} + m_s)\gamma^\mu(1-\gamma^5)G_{\alpha\nu}(x)b(0)|B_s(p)\rangle. \quad (6)$$

Alternative way: apply the fixed point gauge $B_\nu(x) = x_\sigma\int_0^1 uduG_{\sigma\nu}(ux)$.

C-quark loop form factors

The possible integral representation [Max Ferre]:

$$F_i\left(\mathbf{k}^2;(\mathbf{k}.q);q^2\right)=\frac{1}{\pi^2}\int\limits_0^1 dx\int\limits_0^{1-x}dz\,\frac{\Delta_i(x,z)}{m_c^2-2xz(\mathbf{k}.q)-x(1-x)q^2-z(1-z)\mathbf{k}^2},\quad i=\{0,1,2\},\qquad (7)$$

where $\Delta_0=xz$, $\Delta_1=x(z+x-1)$, $\Delta_2=z(z+x-1)$.

X-package

- Algebra of Dirac matrices in D dimensions
- Automatic calculation of one-loop integrals in dimensional regularization
- Analytic expressions in terms of Passarino-Veltman coefficient functions

$$\begin{aligned}F_0=\frac{1}{8\pi^2\left((k.q)^2-k^2q^2\right)^2}\Big[&\Big(k^2q^2k.q\left(3k^2-4\mathbf{mc}^2+3q^2\right)+4k^2q^2(k.q)^2+2\left(k^2\right)^2\left(q^2\right)^2+\\&+4\mathbf{mc}^2(k.q)^3\Big)\;C_0\left(k^2,q^2,k^2+2k.q+q^2;\mathbf{mc},\mathbf{mc},\mathbf{mc}\right)-3k^2q^2k.q\,\Lambda\left(q^2,\mathbf{mc},\mathbf{mc}\right)-\\&-2k^2\left(q^2\right)^2\Lambda\left(q^2,\mathbf{mc},\mathbf{mc}\right)-k^2\left(2k^2q^2+3q^2k.q+(k.q)^2\right)\Lambda\left(k^2,\mathbf{mc},\mathbf{mc}\right)+\\&+\left(2\left(k^2\right)^2q^2+k^2\left(6q^2k.q+(k.q)^2+2\left(q^2\right)^2\right)+q^2(k.q)^2\right)\Lambda\left(k^2+2k.q+q^2,\mathbf{mc},\mathbf{mc}\right)-\\&-q^2(k.q)^2\,\Lambda\left(q^2,\mathbf{mc},\mathbf{mc}\right)-2k^2q^2k.q+2(k.q)^3\Big].\qquad (8)\end{aligned}$$

[H. Patel, Comput. Phys. Commun. 66 (2017) 218, arXiv:1612.00009]

Cross checks

- The axial anomaly for triangle loop:

$$(\mathbf{k}+q)^2 F_0 - q^2 F_1 + \mathbf{k}^2 F_2 = 8 m_c^2 F_5 + \frac{1}{2\pi^2}. \qquad (9)$$

The reference values:

$$F_0(0,0,0) = -F_1(0,0,0) = F_2(0,0,0) = \frac{-1}{24\pi^2 m_c^2} \quad \text{(2 diagrams are included)}, \qquad (10)$$

$$\frac{\partial F_0(0,0,0)}{\partial(\mathbf{k}.q)} = -2\frac{\partial F_1(0,0,0)}{\partial(\mathbf{k}.q)} = 2\frac{\partial F_2(0,0,0)}{\partial(\mathbf{k}.q)} = \frac{-1}{90\pi^2 m_c^4} \quad \text{(2 diagrams are included)}, \qquad (11)$$

$$\frac{\partial F_0(0,0,0)}{\partial \mathbf{k}^2} = -\frac{3}{2}\frac{\partial F_1(0,0,0)}{\partial \mathbf{k}^2} = \frac{\partial F_2(0,0,0)}{\partial \mathbf{k}^2} = \frac{-1}{120\pi^2 m_c^4} \quad \text{(2 diagrams are included)}. \qquad (12)$$

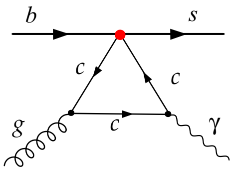


Diagram for $b \to sg\gamma$ decay The red dot represents the four-fermion weak interaction.

Case of all zero external momenta in charming loop: the linear by $\mathbf{k}$ contribution to $b \to sg\gamma$ decay is described by

$$L_{\text{eff}}^{(sg\gamma)} = \frac{e e_c}{16\pi^2}\sqrt{2} G_F V_{cs}^* V_{cb} \left(\bar{s}_L \gamma_\mu \frac{\lambda^a}{2} b_L\right) \times$$
$$\times \frac{1}{3m_c^2} g_s G^a_{\nu\lambda} \varepsilon_{\mu\nu\rho\sigma} i\partial_\lambda F_{\rho\sigma}. \qquad (13)$$

[M.B. Voloshin, Phys.Lett.B 397 (1997), arXiv:9612483]

Three particle DAs

- Parametrization with 6 independent 3DAs (the Light Cone contribution only):

$$\langle 0|\,\bar{s}^{\alpha}(y)G_{\sigma\nu}(x)b^{\beta}(0)\,|\bar{B}_s(p)\rangle = \frac{f_B m_B}{4}\int_0^\infty \mathrm{d}\lambda \int_0^\infty \mathrm{d}\omega\, e^{-i\lambda\,(v\cdot y)-i\omega\,(v\cdot x)}\Big\{(1+\not{v})\Big[$$

$$(v_\sigma\gamma_\nu - v_\nu\gamma_\sigma)[\Psi_A-\Psi_V] - i\sigma_{\sigma\nu}\Psi_V - \left(\frac{y_\sigma v_\nu - y_\nu v_\sigma}{v\cdot y}\right)Y_A + \left(\frac{y_\sigma\gamma_\nu - y_\nu\gamma_\sigma}{v\cdot y}\right)Y'_A -$$

$$-\left(\frac{x_\sigma v_\nu - x_\nu v_\sigma}{v\cdot x}\right)X_A + \left(\frac{x_\sigma\gamma_\nu - x_\nu\gamma_\sigma}{v\cdot x}\right)X'_A\Big]\gamma_5\Big\}, \qquad (14)$$

Integration by parts: $\widetilde{Y}_A = \int d\lambda Y_A(\lambda,\omega),\ \widetilde{Y}'_A = \int d\lambda Y'_A(\lambda,\omega),\ \widetilde{X}_A = \int d\omega X_A(\lambda,\omega),\ \widetilde{X}'_A = \int d\omega X'_A(\lambda,\omega)$.
One should be careful with the surface terms.

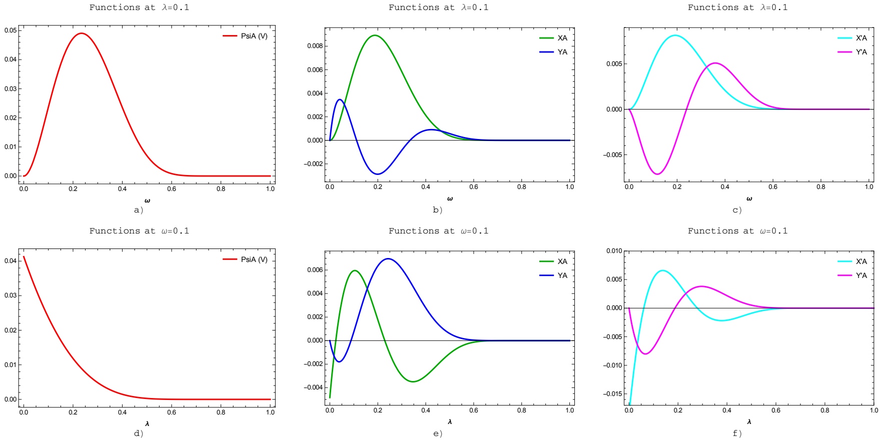
Requirements for the model:

$$\Psi_A(\omega,\lambda) \sim \Psi_V(\omega,\lambda) \sim \omega^2,$$
$$X_A(\omega,\lambda) \sim Y_A(\omega,\lambda) \sim \omega(2\lambda-\omega),$$
$$X'_A(\omega,\lambda) \sim Y'_A(\omega,\lambda) \sim \omega,$$
$$\int d\omega \int d\lambda\ \Psi_A(\omega,\lambda) = \lambda_E^2/3,$$
$$\int d\omega \int d\lambda\ \Psi_V(\omega,\lambda) = \lambda_H^2/3,$$
$$\int d\omega \int d\lambda\ \{X_A, Y_A, X'_A, Y'_A\} = 0.$$

- In [A. Khodjamirian, Th. Mannel, N. Offen, Phys.Rev.D 75 (2007) 054013, arXiv:0611193] 3DAs are defined for special case of aligned kinematics $x \sim y$.
- In [A. Khodjamirian, Th. Mannel, A Pivovarov, Y.-M. Wang, JHEP 09 (2010) 089, arXiv:1006.4945] the model is applied, The integration over interval $0 < \lambda, \omega < \infty$ is cut at close range to zero.
- In our work we define the 3DAs on the interval $0 < \lambda, \omega < 1$ and impose the normalization:

$$\int d\omega\ \{X_A, Y_A, X'_A, Y'_A\} = 0,$$
$$\int d\lambda\ \{X_A, Y_A, X'_A, Y'_A\} = 0.$$

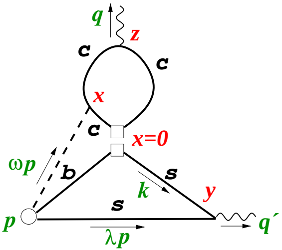
Model for 3DAs



B-meson distribution amplitudes. The top row ((a),(b),(c)): functions of ω at $\lambda = 0.1$; The bottom row ((d),(e),(f)): functions of λ at $\omega = 0.1$.

$$g^{(n)}(x) \sim x^n \exp^{-\frac{ax}{1-x}}, \qquad f^{(n)}(x) \sim (x^n + c_n x^3)(1-x) \exp^{-\frac{ax}{1-x}}. \tag{15}$$

Amplitude structure



$$p = q + q' \quad \text{and} \quad p^2 = m_B^2$$

$$\lambda,\ \omega \sim \Lambda_{\mathbf{QCD}}/m_b$$

- Lower part of the diagram:

$$\langle 0|\bar{s}(y)\gamma^{\eta}(\not{k}+m_s)\gamma^{\mu}(1-\gamma^5)G_{\sigma\nu}(x)b(0)|\bar{B}_s(p)\rangle =$$
$$= \mathrm{Tr}\left[\gamma^{\eta}(\not{k}+m_s)\gamma^{\mu}(1-\gamma_5)\left\{\dots\right\}\right]. \quad (16)$$

General amplitude has the following form:

$$A(q,q')^{\rho\eta} = iH_V\varepsilon^{\eta\rho\{q\}\{q'\}} + H_A\left(g^{\rho\eta}q.q' - q'^{\rho}q^{\eta}\right) +$$
$$+ H_3\left(q^{\rho}q.q' - q^2q'^{\rho}\right)\left(q'^{\eta}q'.q - q'^2q^{\eta}\right) + H_4\left(q^{\rho}q.q' - q^2q'^{\rho}\right)q'^{\eta}, \quad (17)$$

where $H_4\left(q^{\rho}q.q' - q^2q'^{\rho}\right)q'^{\eta}$ structure appears within $\Psi_A, \Psi_V, X'_A, Y'_A$ terms.

$$H_I(q,q') = \int\int d\lambda d\omega\left[C_I^{(\Psi_V)}\Psi_V + C_I^{(\Psi_A)}\Psi_A + C_I^{(X_A)}X_A + C_I^{(Y_A)}Y_A + C_I^{(X'_A)}X'_A + C_I^{(Y'_A)}Y'_A\right],$$
$$I = \{V, A, 3\}.$$

$B_s \to \gamma^{(*)}\gamma^{(*)}$ decay

Decay to two virtual photons:

$$\mathcal{A}^{(B\to\gamma^*\gamma^*)} = \left(i\,\overline{H}_V\;\varepsilon^{\eta\rho\{q\}\{q'\}} + \overline{H}_A\;\left(g^{\rho\eta}q.q' - q'^{\rho}q^{\eta}\right) + \overline{H}_3\;q'^{\rho}q^{\eta}\right) J(q)_{\rho}\;J(q')_{\eta}, \qquad (18)$$

where $\overline{H}_V = H_V(q,q') + H_V(q',q)$, $\overline{H}_A = H_A(q,q') + H_A(q',q)$, $\overline{H}_3 = \left(H_3(q,q') + H_3(q',q)\right) q^2 q'^2$.

- Decay to two real photons:

$$\mathcal{A}^{(B\to\gamma\gamma)} = \left(i\,\overline{H}_V(0,0)\;\varepsilon^{\eta\rho\{q\}\{q'\}} + \overline{H}_A(0,0)\;\left(g^{\rho\eta}(m_B^2/2) - q'^{\rho}q^{\eta}\right)\right) \varepsilon(q)_{\rho}\;\varepsilon(q')_{\eta}, \qquad (19)$$

The charming loop effect may be introduced as $C_{7\gamma} \longrightarrow C_{7\gamma} + \Delta C_{7\gamma}$:

$$\overline{H}_I^{tot}(0,0) = \underbrace{2C_{7\gamma}(\mu)m_B F_{T,I}(0,0)}_{\text{top quark}} + \frac{16\pi^2}{3}\left(\cancel{\overline{H}_I^{fact}(0,0)} + \overline{H}_I^{nonfact}(0,0)\right). \qquad (20)$$

Nonfactorizable part of $\overline{H}_I$ is expanded as per:

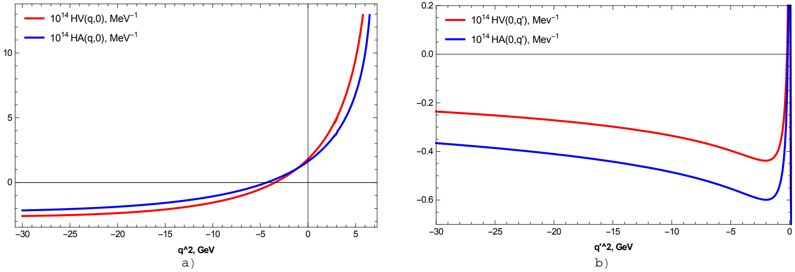
$$\overline{H}_I(0,0) = \int\int \frac{d\lambda\; d\omega}{\omega\left((\lambda-1)\lambda m_B^2 - m_s^2\right)}\left[\Psi_A(\ldots)+\Psi_V(\ldots)+X_A(\ldots)+Y_A(\ldots)+X'_A(\ldots)+Y'_A(\ldots)\right],$$

$I = \{V, A\}$.

Dependence on $q^2,\ q'^2$

Parameters

$m_B = 5.3$ GeV $\lambda_B = 0.46$ GeV $f_B = 0.18$ GeV $m_s = 0.1$ GeV $m_c = 1.3$ GeV
$\alpha = 1/137$ $\alpha_s = 0.2$



H_I dependence on kinematics: a) $H_I(q^2, 0)$, b) $H_I(0, q'^2)$.

$$\overline{H}_A(0,0) \approx 1.62 \cdot 10^{-14} \text{ MeV}^{-1}, \qquad \overline{H}_V(0,0) \approx 1.81 \cdot 10^{-14} \text{ MeV}^{-1}. \tag{21}$$

- Sharp rise near the threshold $q^2 \sim m^2_{J/\psi}$ and $q'^2 \sim m^2_\phi$
- Amplitude (18) for $B_s \to \gamma^*\gamma^*$ can be employed in the range $q'^2 < m_s^2$ and $q^2 < 4m_c^2$

Conclusions

To do list for $B_s \to \gamma\gamma$ challenge:

- Reproduce the analytic results in a fixed point gauge
- Set the model for three particle DAs
- Study the impact of X_A, X'_A, Y_A, Y'_A shape on the amplitude
- Perform the result as a correction to Wilson coefficient: $C_7^{\text{eff}} = C_7 + \Delta C_7$

- Nonfactorizable contribution of charming loop to B-meson decays is considered: the soft gluon exchange between B-meson and charming loop
- The process kinematics requires the $\langle 0| \, \bar{s}^\alpha(y) G_{\sigma\nu}(x) b^\beta(0) \, |\bar{B}_s(p)\rangle$ with three independent coordinates. The correction magnitude is sensitive to 3DAs model.
- $B_s \to \gamma\gamma$ decay: preliminary estimations for correction to $C_{7\gamma}$ amount to several per cent:
$$\Delta C_{7\gamma}/C_{7\gamma} \approx -0.02$$
- The developed analytic approach might be expanded on more complicated decays: $B \to \gamma\, l\, l,\; l\, l\, l\, l,\; K^*\gamma,\; K^*\, l\, l.$

Thank you for attention !

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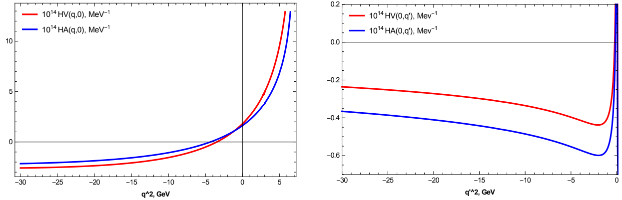
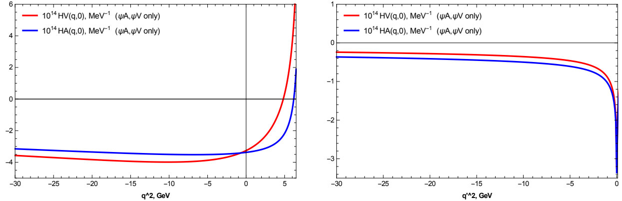
Backup slides

Dependence on q^2, q'^2



a) b)

H_I dependence on kinematics: a) $H_I(q^2, 0)$, b) $H_I(0, q'^2)$.



a) b)

H_I dependence on kinematics: a) $H_I(q^2, 0)$, b) $H_I(0, q'^2)$. Only ψ_A, ψ_V terms are included.

- Sharp rise near the threshold $q^2 \sim m^2_{J/\psi}$ and $q'^2 \sim m^2_\phi$
- Amplitude (18) for $B_s \to \gamma^* \gamma^*$ can be employed in the range $q'^2 < m_s^2$ and $q^2 < 4m_c^2$

$$F_1 = \frac{1}{8\pi^2\left((k.q)^2 - k^2q^2\right)^2}\left[\left(\left(\left(k^2\right)^2\left(5q^2k.q + 2(k.q)^2 + q^2\left(q^2 - 4\mathbf{mc}^2\right)\right) + k^2k.q\left(4k.q\left(\mathbf{mc}^2 + 2q^2\right) + \right.\right.\right.\right.$$

$$\left. +4(k.q)^2 + q^2\left(3q^2 - 4\mathbf{mc}^2\right)\right) + \left(k^2\right)^3q^2 + 4\mathbf{mc}^2(k.q)^3\Big) C_0\left(k^2, q^2, k^2 + 2k.q + q^2; \mathbf{mc}, \mathbf{mc}, \mathbf{mc}\right) +$$

$$+\left(-2k^2(k.q)^2\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) - 4k^2q^2k.q\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) - 2k^2\left(q^2\right)^2\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) - \right.$$

$$-\left(k^2\right)^2q^2\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) - k^2\left(k^2\left(3k.q + q^2\right) + k.q\left(5k.q + 3q^2\right)\right)\Lambda\left(k^2, \mathbf{mc}, \mathbf{mc}\right) +$$

$$+\left(\left(k^2\right)^2\left(3k.q + 2q^2\right) + k^2\left(7q^2k.q + 7(k.q)^2 + 2\left(q^2\right)^2\right) + (k.q)^2\left(2k.q + q^2\right)\right)\cdot$$

$$\cdot\Lambda\left(k^2 + 2k.q + q^2, \mathbf{mc}, \mathbf{mc}\right) - 2(k.q)^3\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) - q^2(k.q)^2\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) +$$

$$\left. + \; 2\left(k^2 + k.q\right)\left((k.q)^2 - k^2q^2\right)\right)\Big]. \qquad \textbf{(22)}$$

$$F_2 = \frac{-1}{8\pi^2\left((k.q)^2 - k^2q^2\right)^2}\left[2q^2(k.q)^2\left(4k^2 + 2\mathbf{mc}^2 + q^2\right) + k^2q^2k.q\left(3k^2 - 4\mathbf{mc}^2 + 5q^2\right) + \right.$$

$$+k^2\left(q^2\right)^2\left(k^2 - 4\mathbf{mc}^2 + q^2\right) + 4(k.q)^3\left(\mathbf{mc}^2 + q^2\right)\Big) C_0\left(k^2, q^2, k^2 + 2k.q + q^2; \mathbf{mc}, \mathbf{mc}, \mathbf{mc}\right) -$$

$$-\frac{2}{\left((k.q)^2 - k^2q^2\right)^2}\left(3k^2q^2k.q\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) + k^2\left(q^2\right)^2\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) + \left(2\left(k^2\right)^2q^2 + \right.\right.$$

$$+k^2\left(4q^2k.q + (k.q)^2 + \left(q^2\right)^2\right) + 2(k.q)^2\left(k.q + q^2\right)\Big)\Lambda\left(k^2, \mathbf{mc}, \mathbf{mc}\right) - \left(2\left(k^2\right)^2q^2 + \right.$$

$$+k^2\left(7q^2k.q + (k.q)^2 + 2\left(q^2\right)^2\right) + k.q\left(7q^2k.q + 2(k.q)^2 + 3\left(q^2\right)^2\right)\Big)\Lambda\left(k^2 + 2k.q + q^2, \mathbf{mc}, \mathbf{mc}\right) +$$

$$\left. +5q^2(k.q)^2\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) + 3\left(q^2\right)^2k.q\Lambda\left(q^2, \mathbf{mc}, \mathbf{mc}\right) - 2\left(k.q + q^2\right)\left((k.q)^2 - k^2q^2\right)\right]. \qquad \textbf{(23)}$$

Coefficients I

$$C_V^{(\Psi V)} = 2L\left(F_0\left(m^2 - q'^2\right)(m(\lambda+\omega-1) - m_s) + q^2(m(F_0(\lambda+\omega-1) + 2F_1) - m_s F_0)\right),$$

$$C_V^{(\Psi A)} = -2L\left(m^2(m(F_0(2\lambda\omega - \lambda - \omega + 1) + (2\lambda-3)\omega F_2) + m_s((2\omega-1)F_0 + 2\omega F_2)) - \right.$$
$$\left. -q^2(m(\lambda F_0 - \omega F_0 + \omega F_2 + F_0 - 4F_1) + m_s F_0) + q'^2(m(F_0(\lambda-\omega-1) + \omega F_2) + m_s F_0)\right),$$

$$C_V^{(XA)} = L\left(m(2\lambda m - m - 2m_s) + q^2 - q'^2\right)\left(m^2\omega F_2' + q^2(\omega F_2' - 2F_1') - q'^2\omega F_2' - 2F_0 - 6F_2\right)$$

$$C_V^{(YA)} = 2L^2\left(m(2\lambda m - m - 2m_s) + q^2 - q'^2\right)\left(\omega F_2\left(m^2 - q'^2\right) + q^2(\omega F_2 - 2F_1)\right),$$

$$C_V^{(X'A)} = 4mL\left(m\left(2(\lambda-1)(F_0 + 3F_2) + m^2\omega(F_0''(2\lambda\omega - \lambda - \omega + 1) + (2\lambda-3)\omega F_2'' - \lambda F_2' + F_2') + \right.\right.$$
$$\left. +mm_s\omega((2\omega-1)F_0'' + 2\omega F_2'')\right) + q^2\left(m\left(-\omega(\lambda F_0'' + (\lambda-1)F_2' + F_0'' - 4F_1'') + 2(\lambda-1)F_1' + \right.\right.$$
$$\left.\left. +\omega^2(F_0'' - F_2'')\right) - m_s\omega F_0''\right) + q'^2\omega(m(F_0''(\lambda-\omega-1) + (\lambda-1)F_2' + \omega F_2'') + m_s F_0'')\Big),$$

$$C_V^{(Y'A)} = -4mL^2(\lambda-1)\left(m^2(m(F_0(2\lambda\omega - \lambda - \omega + 1) + (2\lambda-1)\omega F_2) + m_s((2\omega-1)F_0 + 2\omega F_2)) - \right.$$
$$\left. -q^2(m(\lambda F_0 - \omega F_0 - \omega F_2 + F_0) + m_s F_0) + q'^2(m(F_0(\lambda-\omega-1) - \omega F_2) + m_s F_0)\right),$$

where $L = \frac{1}{\lambda^2 m^2 + \lambda(q^2 - q'^2 - m^2) + q'^2 - m_s^2}$; $F_i' = \frac{\partial F_i}{\partial(\mathbf{k}.q)}$; $F_i'' = \frac{\partial F_i}{\partial \mathbf{k}^2}$.

Coefficients II

$$
\begin{aligned}
C_A^{(\Psi_A)} = 2LL_q \Big(2q^2 \Big(& m^2(-(m(F_0(\lambda+\omega-1)+2\lambda\omega F_2-4\lambda F_1+\omega F_2+2F_1)+ \\
& + m_s(2\omega F_2+F_0-4F_1))) - q'^2(m(F_0(\lambda+\omega-1)-\omega F_2+2F_1)+m_sF_0)\Big) + \Big(m^2-q'^2\Big)\cdot \\
& \Big(m^2(F_0(m(\lambda+\omega-1)+m_s)+\omega F_2(-4\lambda m+3m-4m_s)) - q'^2(F_0(m(\lambda+\omega-1)+m_s)-m\omega F_2)\Big) \\
& + \left(q^2\right)^2 (m(F_0(\lambda+\omega-1)-\omega F_2+4F_1)+m_sF_0)\Big),
\end{aligned}
$$

$$
\begin{aligned}
C_A^{(\Psi_V)} = -2LL_q \Big(2q^2 \Big(& m^2((\omega-1)F_0(\lambda m-m_s)-F_1(-2\lambda m+m+2m_s)) - q'^2(m(-\lambda F_0+\omega F_0+F_1)+ \\
& +m_sF_0)) + F_0\Big(m^2-q'^2\Big)\Big(m^2(-\lambda m-\omega(-2\lambda m+m+2m_s)+m+m_s)-q'^2(m(-\lambda+\omega+1)+ \\
& +m_s)) + \left(q^2\right)^2 (mF_0(-\lambda+\omega-1)+2mF_1+m_sF_0)\Big),
\end{aligned}
$$

$$
C_A^{(X_A)} = LL_q\left(2q^2\left(m^2+q'^2\right)-\left(m^2-q'^2\right)^2-\left(q^2\right)^2\right)\left(\omega F_2'(m^2+q^2-q'^2)-2q^2F_1'-2F_0-6F_2\right),
$$

$$
C_A^{(Y_A)} = 2L^2L_q\left(2q^2\left(m^2+q'^2\right)-\left(m^2-q'^2\right)^2-\left(q^2\right)^2\right)\left(\omega F_2\left(m^2-q'^2\right)+q^2(\omega F_2-2F_1)\right),
$$

where $L=\frac{1}{\lambda^2m^2+\lambda(q^2-q'^2-m^2)+q'^2-m_s^2}$; $L_q=\frac{1}{m^2-q^2-q'^2}$; $F_i'=\frac{\partial F_i}{\partial(\mathbf{k}.q)}$; $F_i''=\frac{\partial F_i}{\partial\mathbf{k}^2}$.

Coefficients II

$$C_A^{(X'_A)} = -4mL_qL\left(-2q^2\left(m^3\left(\omega^2(2\lambda F_2'' + F_0'' + F_2'') + \omega((\lambda-1)F_0'' - \lambda(4F_1'' + F_2') + 2F_1'')+\right.\right.\right.$$
$$\left.+(\lambda-1)F_1'\right) + m^2m_s(\omega(2\omega F_2'' + F_0'' - 4F_1'' - F_2') + F_1') + (\lambda+1)m(F_0 + 3F_2)+$$
$$+q'^2\left(m\left(\omega((\lambda-1)F_0'' + \lambda F_2' + 2F_1'') - \lambda F_1' + \omega^2(F_0'' - F_2'') + F_1'\right) + m_s(\omega(F_0'' + F_2') - F_1')\right)+$$
$$+m_s(F_0 + 3F_2)) + \left(m^2 - q'^2\right)\left(m^3\omega(F_0''(\lambda+\omega-1) + (3-4\lambda)\omega F_2'' + (\lambda-1)F_2')+\right.$$
$$+m^2m_s\omega(-4\omega F_2'' + F_0'' + F_2') - 2(\lambda-1)m(F_0 + 3F_2) - q'^2\omega(m(F_0''(\lambda+\omega-1) + (\lambda-1)F_2'-$$
$$\left.-\omega F_2'') + m_s(F_0'' + F_2')) - 2m_s(F_0 + 3F_2)\right) + \left(q^2\right)^2\left(m\left(\omega((\lambda-1)F_0'' + \lambda F_2' + 4F_1'' + F_2')-\right.\right.$$
$$\left.\left.\left.-2(\lambda+1)F_1' + \omega^2(F_0'' - F_2'')\right) + m_s(\omega(F_0'' + F_2') - 2F_1')\right)\right),$$

$$C_A^{(Y'_A)} = 4mL_qL^2\left(-2q^2\left(m^2\left(\left(2\lambda^2 + \lambda - 1\right)m\omega F_2 + (\lambda-1)m(F_0(\lambda+\omega-1) - 4\lambda F_1)+\right.\right.\right.$$
$$+m_s(2(\lambda\omega F_2 - 2\lambda F_1 + F_1) + (\lambda-1)F_0)) + q'^2((\lambda-1)m(F_0(\lambda+\omega-1) + 4F_1) + (1-3\lambda)m\omega F_2+$$
$$+m_s(\lambda F_0 - 2\omega F_2 - F_0 + 2F_1))) + \left(m^2 - q'^2\right)\left(m^2((\lambda-1)F_0(m(\lambda+\omega-1) + m_s)-\right.$$
$$\left.\left.-\omega F_2\left(4\lambda^2m - 5\lambda m + m + 4\lambda m_s - 2m_s\right)\right)\right) - q'^2((\lambda-1)F_0(m(\lambda+\omega-1) + m_s)+$$
$$+\omega F_2(-3(\lambda-1)m - 2m_s))) + \left(q^2\right)^2(m((\lambda-1)F_0(\lambda+\omega-1) - (3\lambda+1)\omega F_2 + 8\lambda F_1)+$$
$$+m_s((\lambda-1)F_0 - 2\omega F_2 + 4F_1))),$$

where $L = \frac{1}{\lambda^2m^2 + \lambda(q^2 - q'^2 - m^2) + q'^2 - m_s^2}$; $L_q = \frac{1}{m^2 - q^2 - q'^2}$; $F_i' = \frac{\partial F_i}{\partial(\mathbf{k}.q)}$; $F_i'' = \frac{\partial F_i}{\partial \mathbf{k}^2}$.

Coefficients III

$$C_3^{(\Psi A)} = 8LL_q\left(\frac{1}{q'^2}\right)\left((\lambda m + m_s)\left(m^2(F_1 - 2\omega F_2) + q^2 F_1\right) + q'^2(m(F_0(-(\lambda+\omega-1)) + (\lambda-2)F_1 + \right.$$
$$\left. + \omega F_2) + m_s(F_1 - F_0))\right),$$

$$C_3^{(\Psi V)} = -8LL_q\left(\frac{1}{q'^2}\right)\left(m^2(\lambda m(\omega F_0 - F_1) + m_s(-\omega F_0 + F_0 - 2F_1)) + q^2(m_s F_1 - \lambda m(F_0 - 2F_1)) - \right.$$
$$\left. - q'^2(m(-2\lambda F_1 + \omega F_0 + F_1) + m_s(F_0 - F_1))\right),$$

$$C_3^{(XA)} = 4LL_q\left(m^2\omega F_2' + q^2(\omega F_2' - 2F_1') - q'^2\omega F_2' - 2F_0 - 6F_2\right),$$

$$C_3^{(YA)} = 8L^2L_q\left(\omega F_2\left(m^2 - q'^2\right) + q^2(\omega F_2 - 2F_1)\right),$$

$$C_3^{(X'A)} = 8mLL_q\left(\frac{1}{q'^2}\right)\left((\lambda m + m_s)\left(m^2\omega(4\omega F_2'' - 2F_1'' - F_2') + q^2(2F_1' - \omega(2F_1'' + F_2')) + 2(F_0 + 3F_2)\right) + \right.$$
$$\left. + q'^2\omega(m(2F_0''(\lambda+\omega-1) - 2\lambda F_1'' + \lambda F_2' - 2\omega F_2'' + 4F_1'') + m_s(2F_0'' - 2F_1'' + F_2'))\right),$$

$$C_3^{(Y'A)} = 16mL^2L_q\left(\frac{1}{q'^2}\right)\left((\lambda m + m_s)\left(m^2((1-2\lambda)\omega F_2 + (\lambda-1)F_1) + q^2(\lambda F_1 - \omega F_2 + F_1)\right) + \right.$$
$$\left. + q'^2((\lambda-1)(F_1((\lambda-2)m + m_s) - F_0(m(\lambda+\omega-1) + m_s)) + \omega F_2((2\lambda-1)m + m_s))\right),$$

where $L = \frac{1}{\lambda^2 m^2 + \lambda(q^2 - q'^2 - m^2) + q'^2 - m_s^2}$; $L_q = \frac{1}{m^2 - q^2 - q'^2}$; $F_i' = \frac{\partial F_i}{\partial(\mathbf{k}.q)}$; $F_i'' = \frac{\partial F_i}{\partial \mathbf{k}^2}$.