Nucleon Electromagnetic Properties in a Light Front Quark Model

Light Cone 2021

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Main aspects

- **Effect of scalar spin coupling of constituent quarks in a light-front constituent quark model on nucleon electroweak properties.**
- **high-momentum scale component and a fit to the proton electromagnetic form factor ratio $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$**
- **Reasonable description for the static observables of the nucleon electromagnetic form factors, that moved the value of Q_0 such that $G_{Ep}(Q_0^2) = 0$.**
- **Better description of the nucleon electroweak properties can be obtained by the two-scale wave function, which is constructed by the fit of neutron magnetic moment.**

⇒ **Main references:**

- W. R. B. de Araújo, J. P. B. C de Melo, K. Tsushima, Nucl.Phys. A970 (2018) 325-352.

- W. R. B. de Araújo, T. Frederico, M. Beyer and H. J. Weber, Eur. Phys. J. A 29 (2006) 227.

- E. F. Suisso, W. R. B. de Araujo, T. Frederico, M. Beyer and H. J. Weber, Nucl. Phys. A 694 (2001) 351.

Effective Lagrangian for the N-q coupling

- **We have chosen scalar spin coupling between quark and Nucleon Fields**

$$\mathcal{L}_{N-3q} = m_N \epsilon^{lmn} \overline{\Psi}_{(l)} i\tau_2 \gamma_5 \Psi^C_{(m)} \overline{\Psi}_{(n)} \Psi_N + H.C.$$

- τ_2 is the isospin matrix,
- color indices are l, m, n
- ϵ^{lmn} is the totally antisymmetric tensor in color space
- $\Psi^C = C\overline{\Psi}^\top$, $C = i\gamma^2\gamma^0$ is the charge conjugation matrix
- m_N is the nucleon mass.

Electromagnetic Form Factors

- **In the plus component of the nucleon electromagnetic current ($J^+_N = J^0_N + J^3_N$) the Drell-Yan condition $q^+ = q^0 + q^3 = 0$**

$$
\begin{aligned}
\langle s'|J^+_N(Q^2)|s\rangle &= \bar{u}(p',s')\left(F_{1N}(Q^2)\gamma^+ + \imath\frac{\sigma^{+\mu}Q_\mu}{2m_N}F_{2N}(Q^2)\right)u(p,s) \\
&= \frac{p^+}{m_N}\langle s'|F_{1N}(Q^2) - \imath\frac{F_{2N}(Q^2)}{2m_N}\hat{n}\cdot(\vec{q}_\perp \times \vec{\sigma})|s\rangle \ ,
\end{aligned}
$$

- **F_{1N} and F_{2N} are the Dirac and Pauli form factors and $\hat{n}$ is the unit vector along the x^3 direction**

The Sachs form factors:

$$
\begin{aligned}
G_{EN}(Q^2) &= F_{1N}(Q^2) - \frac{Q^2}{4m_N^2}F_{2N}(Q^2) \\
G_{MN}(Q^2) &= F_{1N}(Q^2) + F_{2N}(Q^2)
\end{aligned}
$$

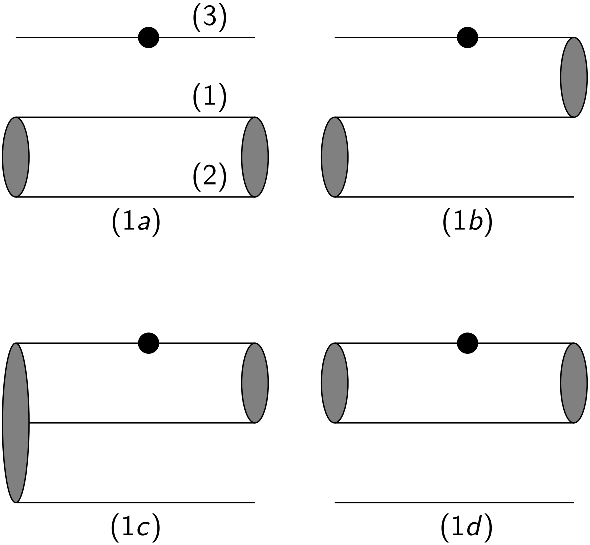
- $\mu_N = G_{MN}(0)$; $r_N^2 = -6\frac{dG_{EN}(Q^2)}{dQ^2}|_{Q^2=0}$

•**Breit-frame momentum**

$q = (0, \vec{q}_\perp, 0)$ **such that** $q^+ = q^0 + q^3 = 0$ $\vec{q}_\perp = (q^1, q^2)$

$$Q^2 = -q^2 = q_\perp^2 \Longrightarrow p = (\sqrt{\frac{q_\perp^2}{4} + m_N^2}, -\frac{\vec{q}_\perp}{2}, 0)$$

. **and** $p' = (\sqrt{\frac{q_\perp^2}{4} + m_N^2}, \frac{\vec{q}_\perp}{2}, 0)$



Feymann diagrams for the electroweak current

- **Microscopic nucleon electromagnetic current operator**

$$J^+_N(Q^2) = J^+_{aN}(Q^2) + 4J^+_{bN}(Q^2) + 2J^+_{cN}(Q^2) + 2J^+_{dN}(Q^2)$$

- **Electromagnetic current matrix elements**

$$\begin{aligned}\langle s'|J^+_{aN}(Q^2)|s\rangle &= -m_N^2\langle N|\hat{Q}_q|N\rangle Tr[\imath\tau_2(-\imath)\tau_2] \\ &\times\int\frac{d^4k_1 d^4k_2}{(2\pi)^8}\Lambda(k_i,p')\Lambda(k_i,p) \\ &\times\bar{u}(p',s')S(k_3')\gamma^+S(k_3)u(p,s)\,Tr\left[S(k_2)\gamma^5S_c(k_1)\gamma^5\right]\ ,\end{aligned}$$

- $k_3' = k_3 + q$
- **In the same way we have obtained J^+_{bN}, J^+_{cN} and J^+_{dN}**
- $\Lambda_N(k_i,p)$ is chosen to introduce a part of the three quark momentum space function, after integrations over energies in light front, k^-
- $S(p) = \frac{1}{\not{p}-m_q+\imath\epsilon}$, $S_c(p) = \left[\gamma^0\gamma^2\frac{1}{\not{p}-m_q+\imath\epsilon}\gamma^0\gamma^2\right]^\top$, $\hat{Q}_q = \frac{1}{6}+\frac{\tau_3}{2}$

- **Momentum part of the wave function and microscopic current, after integrations over the light-front energies**

$$\frac{1}{2(2\pi)^3}\frac{\Lambda(k_i,p)}{m_N^2-M_0^2}\rightarrow\Psi(M_0^2).$$

- **After analytic integration for k^- in electromagnetic microscopic currents:**

$$
\begin{aligned}
&< s' \mid J^+_{aN} \mid s >= 2p^+ m_N^2 < N \mid \hat{Q}_q \mid N > \\
&\times \int \frac{dk_1^+ d^2\vec{k}_{1\perp} dk_2^+ d^2\vec{k}_{2\perp}}{k_1^+ k_2^+ k_3^{+2}} \theta(p^+ - k_1^+)\theta(p^+ - k_1^+ - k_2^+) \\
&\times Tr\left[(\not{k}_2 + m_q)(\not{k}_1 + m_q)\right] \\
&u(p',s')(\not{k}'_3 + m_q)\gamma^+(\not{k}_3 + m_q)u(p,s)\Psi(M_0'^2)\Psi(M_0^2)
\end{aligned}
$$

- $k_1^2 = m_q^2$ **and** $k_2^2 = m_q^2$

- **Squared mass of the free-three quarks**

$$M_0^2 = p^+\left(\frac{k_{1\perp}^2 + m_q^2}{k_1^+} + \frac{k_{2\perp}^2 + m_q^2}{k_2^+} + \frac{k_{3\perp}^2 + m_q^2}{k_3^+}\right) - p_\perp^2 \ ,$$

- $M_0'^2 = M_0^2(k_3 \to k_3' \ , \vec{p}_\perp \to \vec{p}'_\perp)$

- **Following the same steps for the nucleon current after integration over k^- we have obtained $J^+_{bN}(Q^2)$, $J^+_{cN}(Q^2)$ and $J^+_{dN}(Q^2)$**

- **The light-front spinor:**

$$u(p,s) = \frac{\not{p} + m_N}{\sqrt{2m_N 2p^+}} \gamma^+ \gamma^0 \begin{pmatrix} \chi_s^{Pauli} \\ 0 \end{pmatrix},$$

- **Dirac spinor in the instant form**

$$u_D(p,s) = \frac{\not{p} + m_N}{\sqrt{2m(p^0 + m_N)}} \begin{pmatrix} \chi_s^{Pauli} \\ 0 \end{pmatrix}.$$

- **Melosh rotation:**

$$\begin{aligned} < s' \mid R_M(p) \mid s > &= \bar{u}_D(\vec{p}, s') u(p^+, \vec{p}_\perp, s) \\ &= < s' \mid \frac{p^+ + m_N - \vec{n} \cdot (\vec{\sigma} \times \vec{p}_\perp)}{\sqrt{(p^+ + m_N)^2 - \vec{p}_\perp^{\,2}}} \mid s >, \end{aligned}$$

where $\vec{n}$ is the vector along the p^3 direction.

- **Scalar spin coupling between quarks and Nucleon Fields:**

$$\chi_{Sc}(s_1, s_2, s_3; s_N) = \bar{u}_1 \gamma^5 u_2^c \bar{u}_3 u_N .$$

- **Matrix element of the spin zero coupled pair**

$$\begin{aligned} I_{Sc}(s_1, s_2, 0) &= \bar{u}(k_1, s_1)\gamma^5 u^c(k_2, s_2) \\ &= i \sum_{\bar{s}_1 \bar{s}_2} < s_1 \mid R_M^\dagger(k_1^{cm}) \mid \bar{s}_1 >< \bar{s}_1 \mid \sigma_2 \mid \bar{s}_2 > \\ &\quad \times < \bar{s}_2 \mid [R_M^\dagger(k_2^{cm})]^\top \mid s_2 > \end{aligned}$$

-

$$\bar{u}_3 u_N = \sqrt{\frac{k_3^{0cm_N} + m}{2m}} \sum_{\bar{s}_3} < s_3 \mid R_M^\dagger(k_3^{cm_N}) \mid \bar{s}_3 > \delta_{\bar{s}_3 s_N} .$$

• From our model Coupling coefficients of the Melosh rotation of the quark spin have the arguments defined by the momentum of quarks in pair, and the nucleon rest frame constrained by the total momentum.

• W. R. B. de Araújo, T. Frederico, M. Beyer and H. J. Weber, J. Phys. G 25 (1999) 1589.

• Bakamjian and Thomas (BT) construction the arguments of the Melosh rotation are defined in the rest frame of the three free constituent quarks.

• B. Bakamjian and L. H. Thomas,"Relativistic particle dynamics. 2," Phys. Rev. 92 (1953) 1300.

Two Scale Wave Function:

$$\begin{aligned}\Psi_{\mathrm{Power}} &= N_{\mathrm{Power}}\left[(1+M_0^2/\beta^2)^{-p}+\lambda(1+M_0^2/\beta_1^2)^{-p}\right] \ , \\ \lambda &= \left[(1+M_H^2/\beta_1^2)/(1+M_H^2/\beta^2)\right]^p \ ,\end{aligned}$$

$$\beta_H = \beta\beta_1\left(\frac{1-|\lambda|^{\frac{1}{3}}}{\beta_1^2|\lambda|^{\frac{1}{3}}-\beta^2}\right)^{\frac{1}{2}} \ .$$

- **W. R. B. de Araújo, T. Frederico, M. Beyer and H. J. Weber, Eur. Phys. J. A 29 (2006) 227.**

- **W. R. B. de Araújo, J. P. B. C de Melo, K. Tsushima, Nucl.Phys. A970 (2018) 325-352.**

Numerical Results

- **Wave function set I: Parameters chosen to reproduce better the proton magnetic moment μ_p**

- **Wave function set II: Parameters chosen to reproduce better the neutron magnetic moment μ_n**

- **Wave function one scale: β parameter chosen to fit μ_n^{expt}**

Table: Parameters of one and two scales wave functions

Reference	β (GeV)	β_1 (GeV)	M_H (GeV)
Set I	0.676	5.72	4.79
Set II	0.396	10.56	5.92
one scale	0.477	-	-

Table: Electromagnetic properties of nucleons, and the value of zero Q_0^2 of $G_{Ep}(Q_0^2) = 0$ obtained with the two-scale (set I and set II) and one-scale wave functions.

	μ_p (μ_N)	r_p (fm)	μ_n (μ_N)	r_n^2 (fm^2)	Q_0^2 (GeV^2)
Set I	2.74	0.80	-1.52	-0.07	8.27
Set II	3.05	0.94	-1.88	-0.06	15.12
One scale	3.11	1.03	-1.91	-0.08	3.28

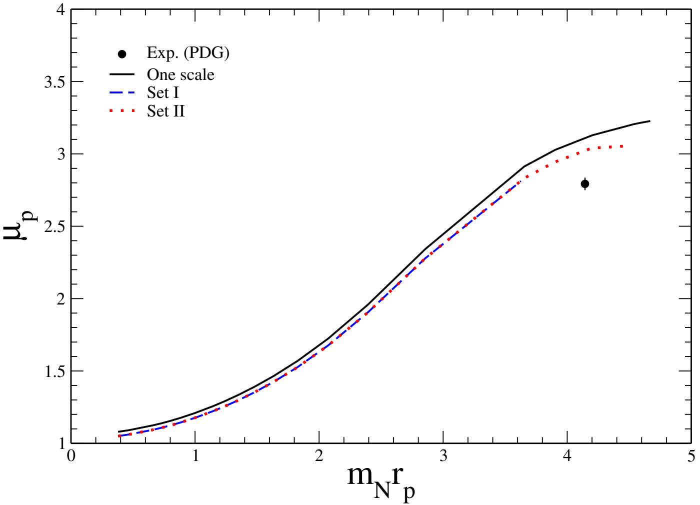


Figure: Correlation between μ_p and $m_N r_p$ calculated by the two-scale (set I and set II) and one-scale wave functions. Experimental data point is shown by the dot.

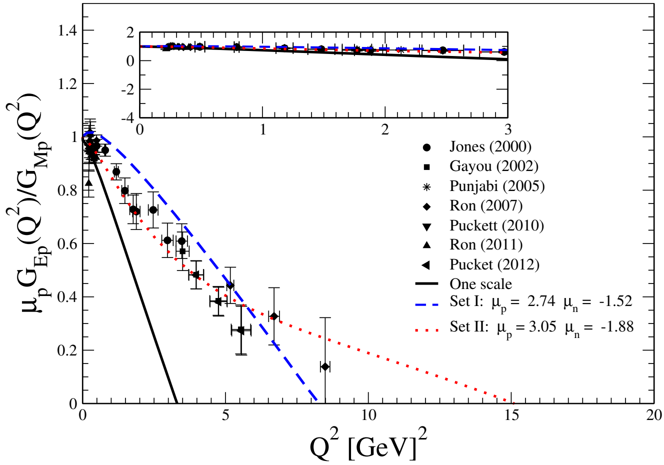


Figure: Proton electromagnetic form factor ratio $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, calculated by the two-scale (set I and set II) and one-scale wave functions.

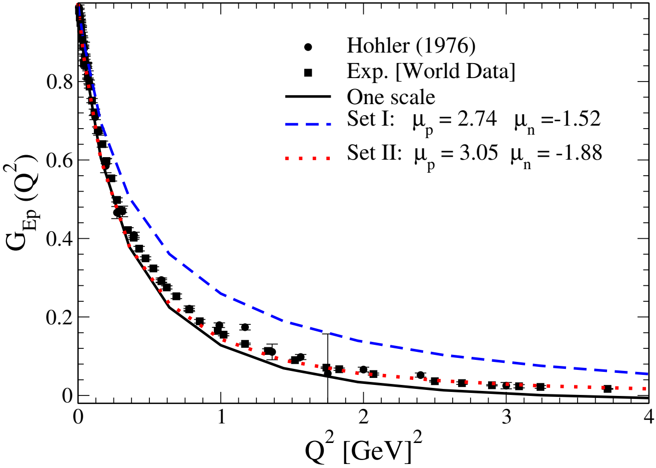


Figure: Proton electric form factor $G_{Ep}(Q^2)$ calculated with the two-scale (set I and set II) and one-scale wave functions.

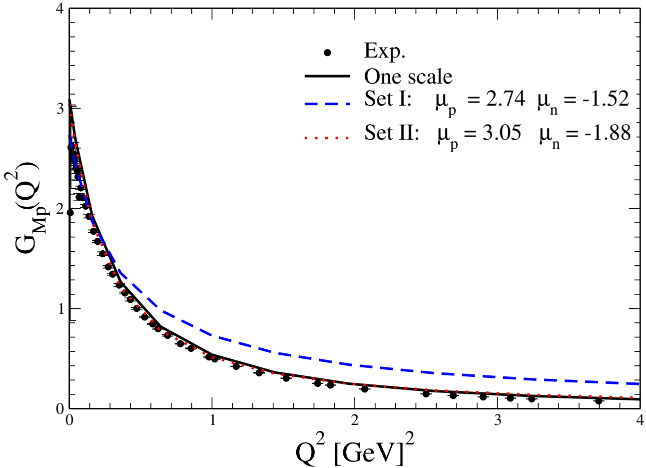


Figure: Proton magnetic form factor $G_{Mp}(Q^2)$ calculated with the two-scale (set I and set II) and one-scale wave functions.

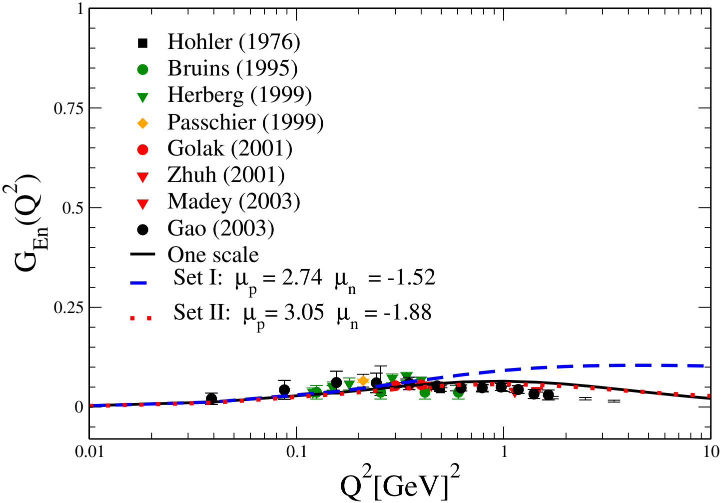


Figure: Neutron electric form factor $G_{En}(Q^2)$ calculated by the two-scale (set I and set II) and one-scale wave functions.

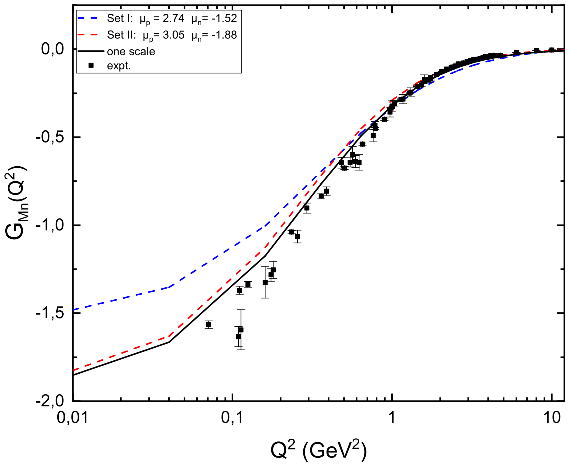


Figure: Neutron magnetic form factor $G_{Mn}(Q^2)$, calculated by the two-scale (set I and set II) and one-scale wave functions.

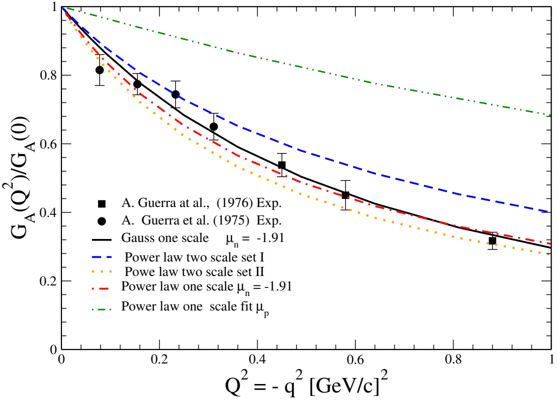


Figure: Normalized axial-vector form factor, $G_A(Q^2)/G_A(0)$, calculated by the two-scale (set I and set II), and one-scale wave functions. While the power-law form is used for both the two-scale and one scale wave functions, the gaussian form is also used for the latter.

- g_A= 0.98 for Set II, g_A=0.68 for set I and g_A =1.15 for one scale (fit μ_n^{expt}); g_A^{expt}=1.27

1. By the introduction of high-momentum scale in the two-scale in the nucleon wave function, we can describe better the zero, the Q^2 point that the proton electric form factor becomes zero, for a reasonable parameter set without destroying the good feature already achieved one-scale wave function in describing the nucleon static properties.
2. The best description is achieved by the two-scale wave function with the parameters (set II) that are chosen to reproduce better μ_n.
3. For $G_{En}(Q^2)$ both the two-scale set II and the one-scale wave functions can describe reasonably well this quantities.
4. A similar behaviour can be seen by using two scale wave functions in our model for proton and neutron magnetic form factors .
5. Some perspectives and New results for this work: mixture of Gaussian and Power Law form in the Wave function; We also have some results of axial properties; Study this possibilities showed here in a nuclear medium by using QMC model

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