Pion off-shell electromagnetic form factors with the light-front approach models

*Light Cone 2021*
Nov. 29 - Dec. 4, 2021 – Jeju Booyoung Hotel – South Korea

Jurandi Leão\textsuperscript{a,b}

\textsuperscript{a}Laboratório de Física Teórica e Computacional - LFTC, UCS (Brazil)
\textsuperscript{b}Instituto Federal de São Paulo - IFSP (Brazil)

Collaborators: J. P. B. C. de Melo (LFTC-UCS, Brazil), T. Frederico (ITA, Brazil), H. Choi (KNU, Korea) and Chueng R. Ji (NCSU, USA)

December 1, 2021
Motivations

Off Shell Electromagnetic Form Factors

Extraction of pion form factors through experimental cross section

Results

Conclusions
Motivations

- **Hadronic form factors**: important source of information for hadrons structure
- **Off-shell effects**: electroproduction $\rightarrow 1H(e, e'\pi^+)n$ (Fig. 1)
- **Cross section**: $\sigma_L, \sigma_T, \sigma_{LT}$ e $\sigma_{TT}$
- **Extraction of the pion’s electromagnetic form factors**: $F_1(Q^2, t)$ e $F_2(Q^2, t)$

Ref:
Motivations
Electroproduction experiment

Figure 1: $e p \rightarrow e' \pi^+ n$ scattering
Off-Shell Electromagnetic Form Factors

- Off-shell case $\implies$ Two Electromagnetic Form Factors: $F_1$ and $F_2$

- Most simple structure:

  \[
  \langle p' | O | p \rangle = (p' + p)^{\mu} F_1(Q^2, t) + (p' - p)^{\mu} F_2(Q^2, t)
  \]

  $\implies$ $O$ is the electromagnetic current operator

- Vertex:

  \[
  \Gamma_{\mu} = [(p + p')_{\mu} G_1 + (p' - p)_{\mu} G_2]
  \]

  and,

  \[
  q^{\mu} \Gamma_{\mu} = [\Delta^{-1}(p') - \Delta(p)^{-1}]
  \]

  where $\Delta(p) = \frac{1}{p^2 - m^2 + i\epsilon}$
From the Ward-Takahashi identity, we have to:

\[(p'^2 - p^2)G_1(q^2, p^2, p'^2) + q^2G_2(q^2, p^2, p'^2) = \Delta^{-1}(p') - \Delta^{-1}(p)\]

that, with the normalization \(G_1(q^2 = 0) = 1:\)

\[G_2(q^2, p^2, p'^2) = \frac{(p'^2 - p^2)[G_1(0, p^2, p'^2) - G_1(q^2, p^2, p'^2)]}{q^2}\]

In the case, where \(p^2 = t\) and \(p'^2 = m^2_\pi:\)

\[F_2(Q^2, t) = \frac{t - m^2_\pi}{Q^2} \left[ F_1(0, t) - F_1(Q^2, t) \right]\]

when \(F_i(Q^2, t) \equiv G_i(q^2, t, m^2_\pi) (i = 1, 2)\) and \(Q^2 = -q^2\).
Solving, we obtain:

\[ \Gamma_{\mu} = (p' + p)_{\mu} F_1(Q^2, t) + q_{\mu} \frac{(t - m^2_\pi)}{Q^2} \left[ F_1(0, t) - F_1(Q^2, t) \right] \]

And defining

\[ g(Q^2, t) \equiv \frac{F_2(Q^2, t)}{t - m^2_\pi} \]

we obtain:

\[ F_1(Q^2, t) - F_1(0, t) + Q^2 g(Q^2, t) = 0 \]

This Equation allows us to extract the form factors from the off-shell pion. The form factors \( F_2(Q^2, t) \) and \( g(Q^2, t) \) disappear in the on-shell limit \( t = m^2_\pi \).

**Ref:**
Choi et al., “Pion off-shell electromagnetic form factors: Data extraction and model analysis”
Extraction of pion form factors through experimental cross section

The electromagnetic form factor of the pion is given as a function of the cross section:

\[
F_\pi^2 = \frac{N}{4\hbar c} \frac{1}{(eG_{\pi NN}(t))^2} \frac{(t - m_\pi^2)^2}{-Q^2t} \frac{d\sigma_L}{dt}
\]

The cross section is given as:

\[
d\sigma_L = 16\pi \frac{137N}{t - m_\pi^2} \frac{-tQ^2}{G_{\pi NN}^2(t)} F_\pi^2(Q^2, t) \left( \frac{197.3}{10} \right)^2
\]

We also have to:

\[
\frac{d\sigma_L}{dt} = 16\pi \frac{137N}{t - m_\pi^2} \frac{-tQ^2}{[H]^2} \left( \frac{197.3}{10} \right)^2
\]

where \( H \equiv F_\pi \cdot G_{\pi NN} \)
Extraction of pion form factors through experimental cross section

$H$ can be extracted from the cross section:

$$H^2 = \frac{137N}{16\pi} \frac{(t - m^2_{\pi})^2}{-tQ^2} \frac{d\sigma_L}{dt} \left( \frac{10}{197.3} \right)^2$$

Where $G_{\pi NN}(t)$ is given by:

$$G_{\pi NN}(t) = G_{\pi NN}(m^2_{\pi}) \left( \frac{\Lambda^2_{\pi} - m^2_{\pi}}{\Lambda^2_{\pi} - t} \right)^n$$

with $G_{\pi NN}(m^2_{\pi}) = 13.4$, $\Lambda_{\pi} = 0.80$ GeV and $n = 1$.

Therefore:

$$F^2_{\pi} = \frac{137N}{16\pi} \frac{1}{G_{\pi NN}(t)} \frac{(t - m^2_{\pi})^2}{-Q^2t} \frac{d\sigma_L}{dt} \left( \frac{10}{197.3} \right)^2$$
The elements of the electromagnetic current matrix:

\[
J^\mu = -i2e \frac{m^2}{f^2_\pi} N_c \int \frac{d^4k}{(2\pi)^4} Tr \left[ S(k)\gamma^5 S(k - p')\gamma^\mu S(k - p)\gamma^5 \right] \Gamma(k, p')\Gamma(k, p)
\]

where \( S(p) = \frac{1}{p - m + i\epsilon} \) and \( N_c = 3 \).

For the non-symmetric vertex model:

\[
\Gamma^{NSV}(k, p) = \frac{N}{(p - k)^2 - m^2_R + i\epsilon}
\]

For the symmetric vertex model:

\[
\Gamma^{SV}(k, p) = \frac{C}{k^2 - m^2_R + i\epsilon} + \frac{C}{(p - k)^2 - m^2_R + i\epsilon}
\]
First results

Figure 2: (a) Form factor $F^1_\pi(Q^2, t)$ for the non-symmetric model; (b) Form factor $F^1_\pi(Q^2, t)$ for the symmetric model

Ref:
HP Blok et al. “Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV2. I. Measurements of the cross section for the 1 H (e, e’$\pi^+$) n reaction.” In: Physical Review C 78.4 (2008), p. 045202
GM Huber et al. “Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV2. II. Determination of, and results for, the pion form factor.” In: Physical Review C 78.4 (2008), p. 045203
Choi et al., “Pion off-shell electromagnetic form factors: Data extraction and model analysis”
Figure 3: Comparison between the form factors for the two models symmetric and non-symmetric.
First results

Figure 4: Cross section

Ref:
Blok et al., “Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV$^2$. I. Measurements of the cross section for the $^1\!H$ $(e, e'\pi^+)$ n reaction”
Huber et al., “Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV$^2$. II. Determination of, and results for, the pion form factor”
Choi et al., “Pion off-shell electromagnetic form factors: Data extraction and model analysis”
First results

Figure 5: Cross section

Ref:
Blok et al., “Charged pion form factor between $Q^2 = 0.60$ and $2.45$ GeV$^2$. I. Measurements of the cross section for the $1 \ H \ (e, e' \pi^+) \ n$ reaction”
Huber et al., “Charged pion form factor between $Q^2 = 0.60$ and $2.45$ GeV$^2$. II. Determination of, and results for, the pion form factor”
Choi et al., “Pion off-shell electromagnetic form factors: Data extraction and model analysis”
First results

Figure 6: (a) Form factor $F^1(Q^2, t)$ and (b) $g(Q^2, t)$ for the non-symmetric model
Figure 7: (a) Form factor $F^1(Q^2, t)$ and (b) $g(Q^2, t)$ for the symmetric model.
Figure 8: The master equation - symmetric model
Conclusions

- We reviewed some of the calculations for (Choi et. al., 2019);
- We verified that our calculations indicate that they do not depend on models;
- The graphs show that our result agrees with the references;
- We will continue our study for other models and parameters;
- The present approach will also be applied to other pseudoscalar mesons, for example: the kaon.

Choi et al., “Pion off-shell electromagnetic form factors: Data extraction and model analysis”
Blok et al., “Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV$^2$. I. Measurements of the cross section for the $1\, H (e, e'\pi^+) n$ reaction”
Huber et al., “Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV$^2$. II. Determination of, and results for, the pion form factor”
Thanks to the Organizers
Light-Cone 2021 - Juju Island, Korea

Support LFTC, IFSP and Brazilian Agencies: FAPESP, CNPq and CAPES

Thank You!