Pion off-shell electromagnetic form factors with the light-front approach models

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December 1, 2021

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- Motivations
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Motivations

- Hadronic form factors: important source information for hadrons strucuture
- Off-shell effects: electroproduction $\Longrightarrow {}^1H(e, e'\pi^+)n$ (Fig. 1)
- Cross section: σ_L, σ_T, σ_{LT} e σ_{TT}
- Extraction of the pion's electromagnetic form factors: $F_1(Q^2, t)$ e $F_2(Q^2, t)$

Ref:

Ho-Meoyng Choi et al. "Pion off-shell electromagnetic form factors: Data extraction and model analysis." In: *Physical Review D* 100.11 (2019), p. 116020

Motivations

Electroproduction experiment

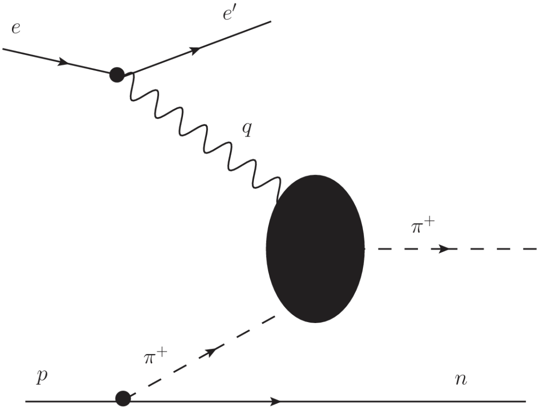


Figure 1: $e\ p \longrightarrow e'\ \pi^+\ n$ scattering

Off Shell Electromagnetic Form Factors

- Off-shell case $\Longrightarrow$ Two Electromagnetic Form Factors: F_1 and F_2
- Most simple structure:

$$< p'|\mathcal{O}|p >= (p' + p)^{\mu} F_1(Q^2, t) + (p' - p)^{\mu} F_2(Q^2, t)$$

$\Longrightarrow$ $\mathcal{O}$ is the electromagnetic current operator

- Vertex:

$$\Gamma_{\mu} = \left[(p + p')_{\mu} G_1 + (p' - p)_{\mu} G_2\right]$$

and,

$$q^{\mu}\Gamma_{\mu} = [\Delta^{-1}(p') - \Delta(p)^{-1}]$$

where $\Delta(p) = \frac{1}{p^2 - m_{\pi}^2 - \prod(p) + i\epsilon}$

Off Shell Electromagnetic Form Factors

From the Ward-Takahashi identity, we have to:

$$(p'^2 - p^2)G_1(q^2, p^2, p'^2) + q^2 G_2(q^2, p^2, p'^2) = \Delta^{-1}(p') - \Delta^{-1}(p)$$

that, with the normalization $G_1(q^2 = 0) = 1$:

$$G_2(q^2, p^2, p'^2) = \frac{(p'^2 - p^2)[G_1(0, p^2, p'^2) - G_1(q^2, p^2, p'^2)]}{q^2}$$

In the case, where $p^2 = t$ and $p'^2 = m_\pi^2$:

$$\boxed{F_2(Q^2, t) = \frac{t - m_\pi^2}{Q^2}\left[F_1(0, t) - F_1(Q^2, t)\right]}$$

when $F_i(Q^2, t) \equiv G_i(q^2, t, m_\pi^2)$ $(i = 1, 2)$ and $Q^2 = -q^2$

Off Shell Electromagnetic Form Factors

Solving, we obtain:

$$\Gamma_\mu = (p' + p)_\mu F_1(Q^2, t) + q_\mu \frac{(t - m_\pi^2)}{Q^2} \left[F_1(0, t) - F_1(Q^2, t)\right]$$

And defining

$$g(Q^2, t) \equiv \frac{F_2(Q^2, t)}{t - m_\pi^2}$$

we obtain:

$$\boxed{F_1(Q^2, t) - F_1(0, t) + Q^2 g(Q^2, t) = 0}$$

This Equation allows us to extract the form factors from the off-shell pion. The form factors $F_2(Q^2, t)$ and $g(Q^2, t)$ disappear in the on-shell limit $t = m_\pi^2$.

Ref:

Choi et al., "Pion off-shell electromagnetic form factors: Data extraction and model analysis"

Extraction of pion form factors through experimental cross section

The electromagnetic form factor of the pion is given as a function of the cross section:

$$F_\pi^2 = \frac{N}{4\hbar c}\frac{1}{(eG_{\pi NN}(t))^2}\frac{(t-m_\pi^2)^2}{-Q^2 t}\frac{d\sigma_L}{dt}$$

The cross section is given as:

$$\boxed{\frac{d\sigma_L}{dt} = \frac{16\pi}{137N}\frac{-tQ^2}{(t-m_\pi^2)}G_{\pi NN}^2(t)F_\pi^2(Q^2,t)\left(\frac{197.3}{10}\right)^2}$$

We also have to:

$$\frac{d\sigma_L}{dt} = \frac{16\pi}{137N}\frac{-tQ^2}{(t-m_\pi^2)}[H]^2\left(\frac{197.3}{10}\right)^2$$

where $H \equiv F_\pi \cdot G_{\pi NN}$

Extraction of pion form factors through experimental cross section

H can be extracted from the cross section:

$$H^2 = \frac{137N}{16\pi}\frac{(t-m_\pi^2)^2}{-tQ^2}\frac{d\sigma_L}{dt}\left(\frac{10}{197.3}\right)^2$$

Where $G_{\pi NN}(t)$ is given by:

$$G_{\pi NN}(t) = G_{\pi NN}(m_\pi^2)\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t}\right)^n$$

with $G_{\pi NN}(m_\pi^2) = 13.4$, $\Lambda_\pi = 0.80$ GeV and $n = 1$.
Therefore:

$$\boxed{F_\pi^2 = \frac{137N}{16\pi}\frac{1}{G_{\pi NN}(t)}\frac{(t-m_\pi^2)^2}{-Q^2 t}\frac{d\sigma_L}{dt}\left(\frac{10}{197.3}\right)^2}$$

The elements of the electromagnetic current matrix:

$$J^{\mu} = -i2e\frac{m^2}{f_\pi^2}N_c\int\frac{d^4k}{(2\pi)^4}Tr\left[S(k)\gamma^5 S(k-p')\gamma^{\mu}S(k-p)\gamma^5\right]\Gamma(k,p')\Gamma(k,p)$$

where $S(p) = \dfrac{1}{\not{p} - m + i\epsilon}$ and $N_c = 3$.

For the non-symmetric vertex model:

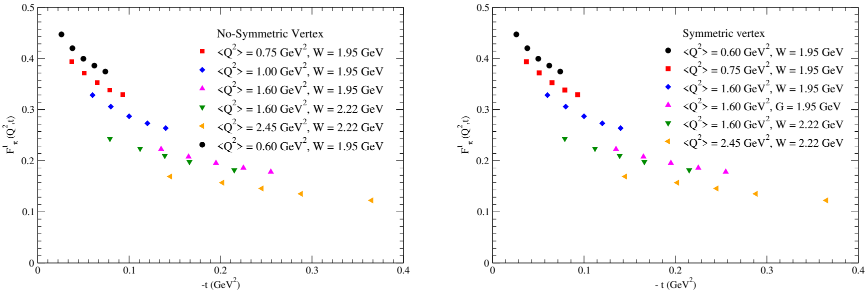
$$\boxed{\Gamma^{NSV}(k,p) = \frac{N}{(p-k)^2 - m_R^2 + i\epsilon}}$$

For the symmetric vertex model:

$$\boxed{\Gamma^{SV}(k,p) = \frac{C}{k^2 - m_R^2 + i\epsilon} + \frac{C}{(p-k)^2 - m_R^2 + i\epsilon}}$$

JPBC de Melo et al. "Pair term in the electromagnetic current within the Front-Form dynamics: spin-0 case." In: *Nuclear Physics A* 707.3-4 (2002), pp. 399–424
JPBC de Melo. "Covariância na frente de luz." PhD thesis. 1998

First results



(a) (b)

Figure 2: (a) Form factor $F^1_\pi(Q^2,t)$ for the non-symmetric model; (b) Form factor $F^1_\pi(Q^2,t)$ for the symmetric model

Ref:

HP Blok et al. “Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. I. Measurements of the cross section for the 1 H (e, e'π+) n reaction.” In: *Physical Review C* 78.4 (2008), p. 045202
GM Huber et al. “Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. II. Determination of, and results for, the pion form factor.” In: *Physical Review C* 78.4 (2008), p. 045203
Choi et al., “Pion off-shell electromagnetic form factors: Data extraction and model analysis”

First results

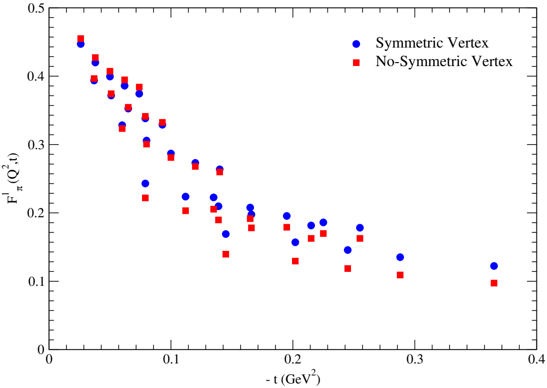
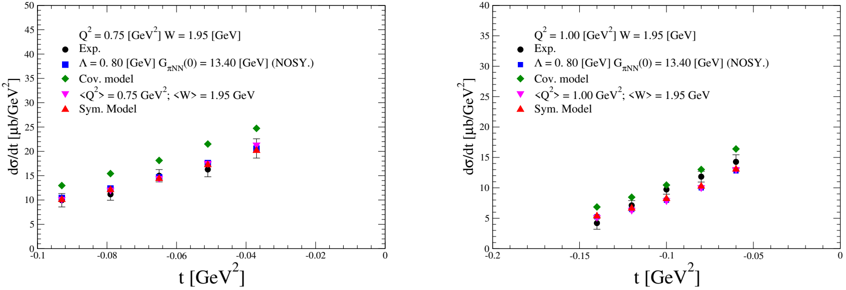


Figure 3: Comparison between the form factors for the two models symmetric and non-symmetric



(a) (b)

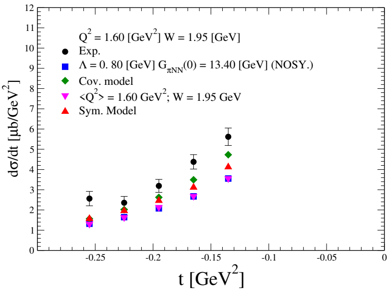
Figure 4: Cross section

Ref:

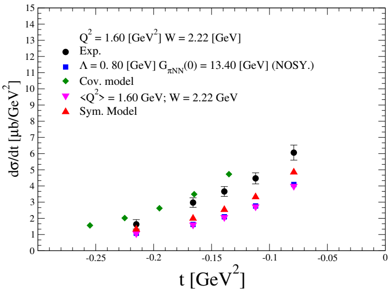
Blok et al., "Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. I. Measurements of the cross section for the 1 H (e, e'π+) n reaction"

Huber et al., "Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. II. Determination of, and results for, the pion form factor"

Choi et al., "Pion off-shell electromagnetic form factors: Data extraction and model analysis"



(a)



(b)

Figure 5: Cross section

Ref:

Blok et al., “Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. I. Measurements of the cross section for the 1 H (e, e'π+) n reaction”
Huber et al., “Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. II. Determination of, and results for, the pion form factor”
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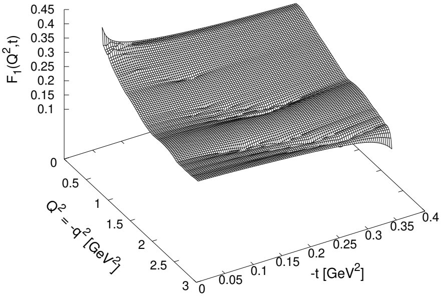
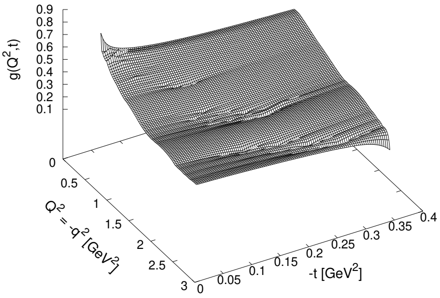
First results



(a)



(b)

Figure 6: (a) Form factor $F^1(Q^2, t)$ and (b) $g(Q^2, t)$ for the non-symmetric model

First results

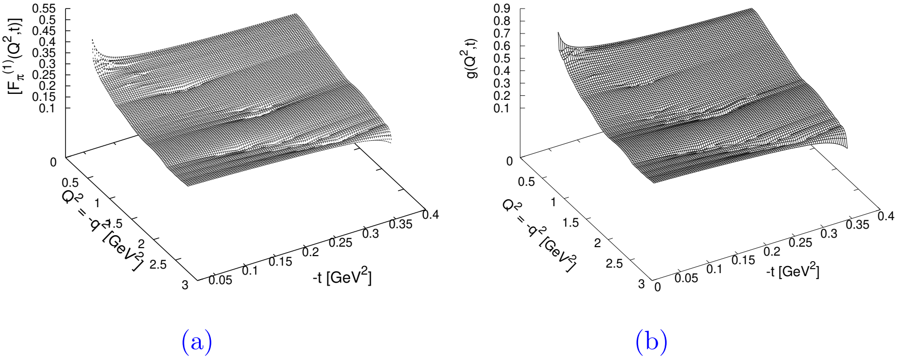


Figure 7: (a) Form factor $F^1(Q^2, t)$ and (b) $g(Q^2, t)$ for the symmetric model

First results

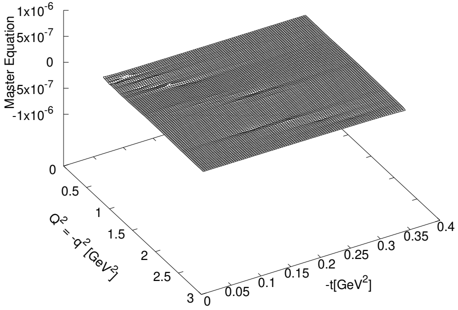


Figure 8: The master equation - symmetric model

Conlusions

- We reviewed some of the calculations for (Choi et. al., 2019);
- We verified that our calculations indicate that they do not depend on models;
- The graphs show that our result agrees with the references;
- We will continue our study for other models and parameters;
- The present approach will also be applied to other pseudoscalar mesons, for example: the kaon.

Choi et al., "Pion off-shell electromagnetic form factors: Data extraction and model analysis"
Blok et al., "Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. I. Measurements of the cross section for the 1 H (e, e'π+) n reaction"
Huber et al., "Charged pion form factor between Q2 = 0.60 and 2.45 GeV2. II. Determination of, and results for, the pion form factor"

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