LF Transverse Momentum Distributions for $\mathcal{I} = 1/2$ Hadronic Systems in Valence Approximation.

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R. Alessandro, A. Del Dotto, E. Pace, G. Perna,G. Salmè and S. Scopetta, *Light-Front Transverse Momentum Distributions for $\mathcal{J} = 1/2$ Hadronic Systems in Valence Approximation*, To appear in **PRC** and **arXiv:2107.10187**

E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta, *EMC effect, few-nucleon systems and Poincaré covariance*, Phys. Scr. **95**, 064008 (2020)

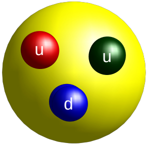
A. Del Dotto, E. Pace, G. Salmè, and S. Scopetta, *Light-Front spin-dependent Spectral Function and Nucleon Momentum Distributions for a Three-Body System*, Phys. Rev. **C 95**, 014001 (2017).

MARATHON Coll. Measurement of the Nucleon Fn2/Fp2 Structure Function Ratio by the Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment, arXiv:2104.05850.

Outline

Motivations

The quest of a reliable flavor decomposition needs sound information on the neutron dynamical observables (structure functions, polarized and non, GPDs, TMDs, etc.).



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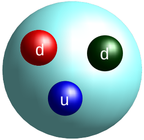


This has driven very accurate and long-lasting experimental efforts in developing effective neutron targets to carefully investigate the electromagnetic responses of the neutron

$\Rightarrow$ the polarized $^3\mathrm{He}$ target, 90% neutron target

(e.g. H. Gao et al, PR12-09-014; J.P. Chen et al, PR12-11-007, @JLAB12)

A careful theoretical description of a polarized ^{3}He is necessary for taking under control the model-dependence in the extraction of the neutron properties.

Bonus: **Nuclear Transverse-Momentum Distributions (TMDs)** for addressing in a novel way the dynamics inside the nucleus.

On the theory side, we need i) to improve the description of the Nucleon inside the nuclei, retaining as many general properties as possible, and hence ii) to validate sound procedures for *extracting* dynamical information on the Nucleon, particularly the Neutron

In our approach, the key quantity for pursuing such a program is the

Nuclear Spectral Function ($\Rightarrow$ nucleon Green's function in the medium)

$$P_{\sigma'\sigma}(k,E) = -\frac{1}{\pi}\Im m \left\{ \langle \Psi_{gr}| a^{\dagger}_{\mathbf{k},\sigma'} \; \frac{1}{E - H + i\epsilon} \; a_{\mathbf{k},\sigma} |\Psi_{gr}\rangle \right\}$$

with

$$H = \sum_{\alpha,\beta} \langle \alpha | H_1 | \beta \rangle \; a^{\dagger}(\alpha) \; a(\beta) + \frac{1}{2} \sum_{\alpha,\beta,\gamma,\eta} \langle \alpha\gamma | H_2 | \beta\eta \rangle \; a^{\dagger}(\alpha) \; a^{\dagger}(\gamma) a(\beta) \; a(\eta) + \ldots \quad \ldots$$

Probabilistic interpretation: the diagonal terms give the probability distribution to find a Nucleon with given spin, momentum and removal energy in the ground state of the interacting system, $|\Psi_{gr}\rangle$. N.B. for ^{2}H, $\rightarrow$ Nucleon Momentum Distribution.

This quantity is quite familiar in nuclear physics, less in hadron physics where the QFT framework is needed, and one introduces the correlator,

$$\Phi^{\tau}(x,y) = \langle \Psi_{gr} | \bar{\psi}^{\tau}(x) \mathcal{W}(\hat{n}\cdot A) \psi^{\tau}(y) | \Psi_{gr} \rangle$$

with $\mathcal{W}(\hat{n}\cdot A)$ the link operator, needed for the gauge invariance.

In valence approximation, one can relate $P_{\sigma'\sigma}(k,E)$ (given in a Poincaré covariant framework) and $\Phi^{\tau}(x,y) = \langle \Psi_{gr} | \bar{\psi}^{\tau}(x) \mathcal{W}(\hat{n}\cdot A) \psi^{\tau}(y) | \Psi_{gr} \rangle$ [R. Alessandro et al PRC in press and arXiv:2107.10187]

The Relativistic Hamiltonian Dynamics framework

Why a relativistic treatment ?

General answer: to develop a more advanced theory, appropriate for the kinematics at JLAB12 and even more so for upcoming Electron-Ion Colliders

- The Standard Model of Few-Nucleon Systems, where nucleon and pion degrees of freedom are taken into account within a non relativistic framework, has achieved a very high degree of sophistication [e.g. the NR $^3\mathrm{He}$ and $^3\mathrm{H}$ Spectral Functions in Kievsky, Pace, G.S. Viviani PRC **56**, 64 (1997)].
- Covariance wrt the Poincaré Group, $\mathcal{G}_P$, is needed for describing processes involving nucleons at large 4-momenta and pointing to high precision measurements. This is the case if one studies, e.g., i) the nucleon structure functions (unpolarized and polarized cases); ii) the nucleon GPDs and TMDs, iii) signatures of short-range correlations; iv) exotics (e.g. 6-bag quarks in ^{2}H), etc
- At least, one should carefully deal with the boosts of the nuclear states, $|\Psi_{init}\rangle$ and $|\Psi_{fin}\rangle$!

The definitely preferred framework for embedding the successful *non relativistic* phenomenology is composed by the

Light-front Relativistic Hamiltonian Dynamics (fixed dof) + Bakamjian-Thomas (BT) construction of the Poincaré generators for *an interacting theory*.

In RHD+BT, one can address both Poincaré covariance and locality

General principles to be implemented in presence of interaction

★ Poincaré covariance $\rightarrow$ The 10 generators, $P^\mu \rightarrow$ 4D displacements and $M^{\nu\mu} \rightarrow$ *Lorentz* transformations, have to fulfill

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = -\imath(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -\imath(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

Also $\mathcal{P}$ and $\mathcal{T}$ have to be taken into account !

★ ★ Macroscopic locality ($\equiv$ cluster separability (relevant in nuclear physics)): i.e. observables associated with different space-time regions must commute in the limit of large spacelike separation (i.e. causally disconnected), rather than for arbitrary (microscopic-locality) spacelike separations (Keister-Polyzou, Adv. Nucl. Phys. **20**, 225 (1991)).
This leads to a careful choice of the intrinsic relativistic coordinates.

Physical motivation: When a system is separated into disjoint subsystems by a sufficiently large spacelike separation, then the subsystems behave as independent systems.

The Light-Front framework has several advantages:

- 7 Kinematical generators: i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$, iii) Rotation around the z-axis.
- The LF boosts have a subgroup structure : then one gets a trivial separation of the intrinsic motion (as in the non-relativistic case). Separation of intrinsic and global motion is important to correctly treat the boost between initial and final states !
- $P^+ \geq 0$ leads to a meaningful Fock expansion, once massless constituents are absent
- No square root in the dynamical operator P^-, propagating the state in the LF-time.
- The infinite-momentum frame (IMF) description of DIS is easily included.

Drawback: the transverse LF-rotations are dynamical

But within the Bakamjian-Thomas construction of the generators in an interacting theory, one can construct an intrinsic angular momentum fully kinematical!

★ The Mass Operator, developed within a *non relativistic framework*, is fully acceptable for a BT construction of the Poincaré generators★

To complete the matter: the spin

- Coupling spins and orbital angular momenta is easily accomplished in the Instant Form of RHD (kinematical hyperplane t=0) through Clebsch-Gordan coefficients, since in this form the three rotation generators are independent of interaction.
- To embed the CG machinery in the LFHD one needs unitary operators, the so-called Melosh rotations that relate the LF spin wave function and the canonical one. For a particle of spin (1/2) with LF momentum $\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$

$$|\mathbf{k}; s, \sigma\rangle_c = \sum_{\sigma'} D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))\ |\tilde{\mathbf{k}}; s, \sigma'\rangle_{LF}$$

where

$D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))$ is the standard Wigner function for the $\mathcal{J} = 1/2$ case ,

$R_M(\tilde{\mathbf{k}})$ is the rotation between two rest frames of the moving particle. One reached through a LF boost and the second through a canonical boost.

$$D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'} = \chi^\dagger_\sigma \frac{m + k^+ - \imath\boldsymbol{\sigma}\cdot(\hat{z}\times\mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}}\chi_{\sigma'} = {}_{LF}\langle\tilde{\mathbf{k}}; s\sigma|\mathbf{k}; s\sigma'\rangle_c$$

χ_σ is a two-dimensional spinor.

The spin-dependent LF Nuclear spectral function

$$P_{\sigma'\sigma}(k,E) = -\frac{1}{\pi}\Im m\left\{\langle\Psi_{gr}|a^{\dagger}_{\mathbf{k},\sigma'}\ \frac{1}{E-H+i\epsilon}\ a_{\mathbf{k},\sigma}|\Psi_{gr}\rangle\right\}$$

$$= \sum_{f_{(A-1)}} \langle\mathbf{k},\sigma\tau;\psi_{f_{(A-1)}}|\psi^{A}_{J\mathcal{M}}\rangle\ \langle\psi^{A}_{J\mathcal{M}}|\psi_{f_{(A-1)}};\mathbf{k},\sigma'\tau\rangle\ \delta(E-E_{f_{(A-1)}}+E_A)$$

with i) $|\mathbf{k},\sigma\tau\rangle$: plane wave with momentum **k** in the system rest frame and spin along z equal to σ, and ii) $|\psi_{f_{(A-1)}}\rangle$: a state of the $(A-1)$-particle spectator system: fully interacting !

$\Rightarrow$ Key ingredients: the overlaps $\langle\mathbf{k},\sigma\tau;\psi_{f_{(A-1)}}|\psi^{A}_{J\mathcal{M}}\rangle$

The spin-dependent LF Nuclear Spectral Function can be defined through the **formal** relation between the overlaps $\langle\mathbf{k},\sigma\tau;\psi_{f_{(A-1)}}|\psi^{A}_{J\mathcal{M}}\rangle_{LF}$ in *Light-front HD* (hyperplane $x^+=0$) and the ones in *Instant-form* HD (hyperplane $t=0$)

★ Through the Bakamjian-Thomas construction, one is allowed to approximate

$$\langle\mathbf{k},\sigma\tau;\psi_{f_{(A-1)}}|\psi^{A}_{J\mathcal{M}}\rangle_{IF} \simeq \langle\mathbf{k},\sigma\tau;\psi_{f_{(A-1)}}|\psi^{A}_{J\mathcal{M}}\rangle_{NR}$$

still preserving the Poincaré covariance and taking profit of the successful NR phenomenology, in full [A. Del Dotto et al , PRC **95**, 014001 (2017)].

★ For implementing the Macro-locality, it is crucial to distinguish between the cluster reference frame, indicated by (1; 23) and the one of the whole system, (123), moving wrt the Lab frame. The LF overlaps for ^{3}He SF in terms of the IF ones are

$$\overbrace{\langle T\tau;\alpha,\epsilon;JJ_z;\tau_1\sigma,\tilde{\boldsymbol{\kappa}}}^{<\tilde{\boldsymbol{\kappa}}|\times 2N\ state}|\overbrace{j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{LF}}^{3N\ bound\ state} = \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1'}$$

$$\sqrt{(2\pi)^3\ 2E(\mathbf{k})}\sqrt{\frac{\kappa^+E_{23}}{k^+E_S}}\sum_{\sigma_2'',\sigma_3''}\sum_{\sigma_2',\sigma_3'}\mathcal{D}_{\sigma_2'',\sigma_2'}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\,\mathcal{D}_{\sigma_3'',\sigma_3'}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)$$

$$_{NR}\langle T,\tau;\alpha,\epsilon;JJ_z|\mathbf{k}_{23},\sigma_2'',\sigma_3'';\tau_2,\tau_3\rangle\ \langle\sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}|j,j_z;B_3;\frac{1}{2}T_z\rangle_{NR}$$

where

$$\mathcal{D}_{\sigma_i'',\sigma_i'}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i)=\sum_{\sigma_i} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\pm\tilde{\mathbf{k}}_{23})]_{\sigma_i''\sigma_i}\ D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_i)]_{\sigma_i\sigma_i'}$$

and the relevant LF momenta of the emitted constituent, in the two frames are $\mathbf{k}_\perp(123)=\boldsymbol{\kappa}_\perp(1;23)$, $k^+(123)=\xi\ M_0(123)=\kappa^+(1;23)\ M_0(123)/\mathcal{M}_0(1,23)$ with

$$\mathcal{M}_0^2(1,23)=\frac{m^2+|\boldsymbol{\kappa}_\perp|^2}{\xi}+\frac{M^2(23)+|\boldsymbol{\kappa}_\perp|^2}{(1-\xi)}$$

N.B. within LFHD, normalization and momentum sum rule are automatically fulfilled !!

The LF spin-dependent spectral function for a system with polarization **S**, can be *macroscopically* decomposed in terms of the available vectors:

- the unit vector $\hat{n}$, $\perp$ to the hyperplane $n^\mu x_\mu = 0$. Our choice is $n^\mu \equiv \{1, 0, 0, 1\} \Rightarrow \hat{n} \equiv \hat{z}$
- the polarization vector **S**. Our choice: $\hat{\mathbf{S}} \equiv \hat{z}$
- the transverse (wrt the $\hat{z}$ axis) momentum component of the constituent, i.e. $\mathbf{k}_\perp(123) = \mathbf{p}_\perp(Lab) = \boldsymbol{\kappa}_\perp(1;23)$

One gets ($\tau = \pm 1/2$, isospin third component)

$$\mathcal{P}^\tau_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \frac{1}{2}\left[\mathcal{B}^\tau_{0,\mathcal{M}} \ + \ \boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathbf{S})\right]_{\sigma'\sigma}$$

The scalar $\mathcal{B}^\tau_{0,\mathcal{M}} = Tr\left[\mathcal{P}^\tau_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)\right]$ yields the unpolarized spectral function ;

the pseudovector $\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathbf{S}) = Tr\left[\hat{\boldsymbol{\mathcal{P}}}^\tau_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)\ \boldsymbol{\sigma}\right]$ is a linear combination of the available pseudovectors,

$$\boldsymbol{\mathcal{F}}^\tau_{\mathcal{M}}(x, \mathbf{k}_\perp; \epsilon, \mathbf{S}) = \mathbf{S}\mathcal{B}^\tau_{1,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^\tau_{2,\mathcal{M}}(\dots)\ +\ \hat{\mathbf{k}}_\perp\ (\mathbf{S}\cdot\hat{z})\mathcal{B}^\tau_{3,\mathcal{M}}(\dots)$$
$$+\hat{z}\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^\tau_{4,\mathcal{M}}(\dots) + \hat{z}\ (\mathbf{S}\cdot\hat{z})\mathcal{B}^\tau_{5,\mathcal{M}}(\dots)\ .$$

with $x = \kappa^+(1;23)/\mathcal{M}_0(1;23)$. N.B. The scalar functions $\mathcal{B}^\tau_{i,\mathcal{M}}(\dots)$ depend on the scalars at disposal, i.e. (N.B. for a $\mathcal{J} = 1/2$ only the first 3)

$$|\mathbf{k}_\perp|,\ x,\ \epsilon,\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2,\ (\mathbf{S}\cdot\hat{n})^2 \text{ and } (\hat{\mathbf{k}}_\perp \times \hat{n})\cdot\mathbf{S}.$$

By integrating the LF SF on κ^-, equivalent to the integration on the $\epsilon \equiv$ internal energy of the spectator system, one straightforwardly gets the LF spin-dependent momentum distribution

$$\mathcal{N}^{\tau}_{\sigma'\sigma}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) = \frac{1}{2}\left\{b_{0,\mathcal{M}}(\dots) + \boldsymbol{\sigma}\cdot \boldsymbol{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})\right\}_{\sigma'\sigma}$$

where $\boldsymbol{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})$ is a pseudovector

$$\boldsymbol{f}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) = \mathbf{S}\, b^{\tau}_{1,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp) b^{\tau}_{2,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\ (\mathbf{S}\cdot\hat{z}) b^{\tau}_{3,\mathcal{M}}(\dots)$$
$$+\hat{z}\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp) b^{\tau}_{4,\mathcal{M}}(\dots) + \ \hat{z}\ (\mathbf{S}\cdot\hat{z}) b^{\tau}_{5,\mathcal{M}}(\dots)$$

The decomposition follows from the corresponding one of the SF, and the **scalar functions** $b^{\tau}_{i,\mathcal{M}}(\dots)$ are proper integrals over $\epsilon \equiv$the spectator energy, present in $\mathcal{B}^{\tau}_{i,\mathcal{M}}(\dots)$

The remarkable content of such a decomposition is to make explicit the interplay between transverse momentum component and spin dofs.

In turn, this can be useful for determining possible *relations* between the so-called Transverse-momentum Distributions (TMDs), in the *valence sector*, i.e. with a minimal number of on-mass-shell constituents inside the *interacting system*.

Transverse-momentum distributions for a $\mathcal{J} = 1/2$ bound system

We focus on the twist-two T-even TMDs. They can be obtained by proper traces of the Correlator (N.B. in the light-cone gauge the link operator becomes the identity) (e.g. [Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002))

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \int d\xi \; e^{ip\dot{\xi}} \left\langle P,S,A|\bar{\psi}^{\tau}_{\beta}(0)\psi^{\tau}_{\alpha}(\xi)|A,S,P\right\rangle$$

$$\Rightarrow \;\Phi(p,P,S) = \frac{1}{2}\,\not{P}\;A^{\tau}_1 + \frac{1}{2}\;\gamma_5\,\not{P}\left[A^{\tau}_2\,S_z + \frac{1}{M}\;\widetilde{A}^{\tau}_1\;\mathbf{p}_\perp\cdot\mathbf{S}_\perp\right] +$$

$$+\frac{1}{2}\,\not{P}\;\gamma_5\;\left[A^{\tau}_3\;\not{S}_\perp + \widetilde{A}^{\tau}_2\,\frac{S_z}{M}\not{p}_\perp + \frac{1}{M^2}\;\widetilde{A}^{\tau}_3\;\mathbf{p}_\perp\cdot\mathbf{S}_\perp\not{p}_\perp\right]$$

where $|A,S,P\rangle$ is the A-particle state and $\psi^{\tau}_{\alpha}(\xi)$ the fermionic field (e.g. a nucleon of isospin τ in a nucleus, or in valence approximation a quark in a nucleon). The twist-2 TMD's are combinations of both A_i and $\tilde{A}_i$.

To match the description in terms of SF, where the particles number is fixed, the particle contribution to the correlation function from on-mass-shell fermions has to be singled out through a suitable projection in the Dirac space → valence approximation [R. Alessandro et al PRC in press, arXiv:2107.10187]

Valence TDMs for a $\mathcal{J} = 1/2$ target

In a BT framework, the relations between the six T-even, twist-2 TMDs, and the six scalar functions b_i, defining the spin-dependent constituent LF momentum distribution are (recall: $\mathbf{k}_\perp(123) = \mathbf{p}_\perp(Lab) = \boldsymbol{\kappa}_\perp(1;23)$)

$$\textit{Unpolarized TMD} \quad \Rightarrow \qquad f^\tau(x, |\mathbf{p}_\perp|^2) = b_0^\tau$$

$$N \textit{ and } {}^3\mathrm{He} \textit{ spin dof's \& } k_\perp \quad \Rightarrow \begin{cases} \Delta f^\tau(x, |\mathbf{p}_\perp|^2) = b^\tau_{1,\mathcal{M}} + \ b^\tau_{5,\mathcal{M}} \\ g^\tau_{1T}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b^\tau_{4,\mathcal{M}} \\ \Delta\prime_T f^\tau(x, |\mathbf{p}_\perp|^2) = b^\tau_{1,\mathcal{M}} + \ b^\tau_{2,\mathcal{M}}/2 \\ h^{\perp\tau}_{1L}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b^\tau_{3,\mathcal{M}} \\ h^{\perp\tau}_{1T}(x, |\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2}\ b^\tau_{2,\mathcal{M}} \end{cases}$$

L and T in the subscript refer to the target polarization.
In the case of ${}^3\mathrm{He}$ the Nuclear TMDs could be obtained through measurements of appropriate spin asymmetries in ${}^3\vec{\mathrm{He}}(\vec{e}, e'p)$ experiments at high momentum transfer (theoretical framework in progress).

To mention: for hadrons $\rightarrow$ SiDIS reactions

From the general principles implemented in the SF, TMDs receive contributions from both $L = 0$ and $L = 2$ orbital angular momenta. The relative weight depends upon the TMD.

Interestingly, Jacob, Mulders, Rodrigues, [NPA 626, 937 (1997)] and B. Pasquini, S. Cazzaniga and S. Boffi [PRD 78, 034025 (2008)] suggested approximate relations between TMDs, viz

$$\Delta f(x, |\mathbf{p}_\perp|^2) = \Delta'_T f(x, |\mathbf{p}_\perp|^2) \ + \ \frac{|\mathbf{p}_\perp|^2}{2M^2} \ h^\perp_{1T}(x, |\mathbf{p}_\perp|^2)$$

$$g_{1T}(x, |\mathbf{p}_\perp|^2) = \ - \ h^\perp_{1L}(x, |\mathbf{p}_\perp|^2)$$

In our approach,

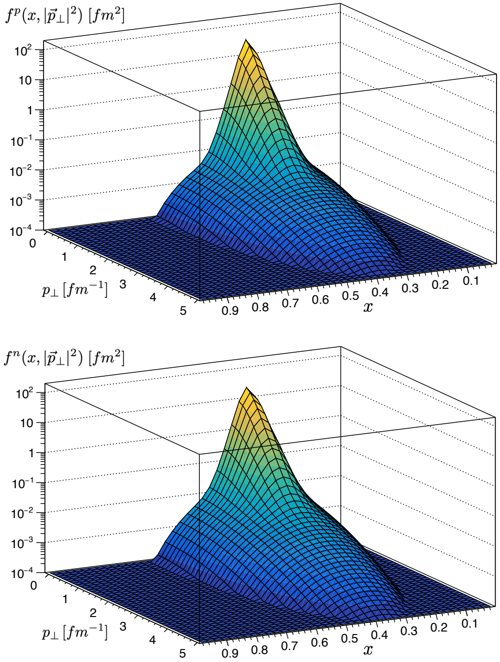
- the first relation is recovered retaining only the $L = 0$ contribution. Taking into account both contributions, $L = 0, \ 2$, the quantitative difference between the lhs and rhs is quite small for the neutron, while not negligible for the proton;
- the second relation certainly holds in modulus, since if the $L = 0$ component, tiny for those TMDs, is retained the minus sign works, while the dominant $L = 2$ contribution leads to a plus sign.

A quadratic relation is also discussed in the above papers

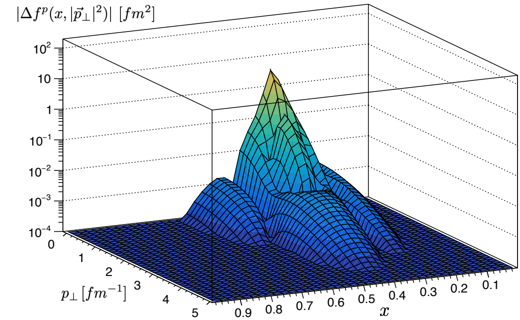
$$(g_{1T})^2 \ + \ 2 \ \Delta'_T f \ \ h^\perp_{1T} = 0$$

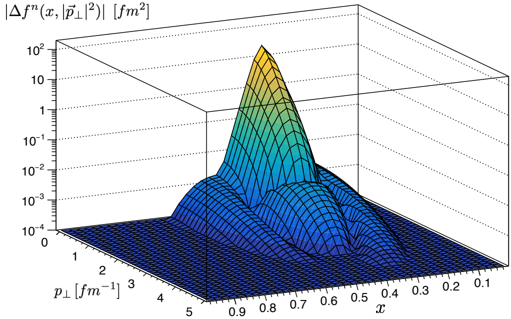
In our approach it does not hold, even if the $L = 2$ contribution is vanishing. Noteworthy, the integration on k_{23}, imposed by **Macro-locality**, spoils the relation:

$\Rightarrow$ its effect becomes measurable **!**



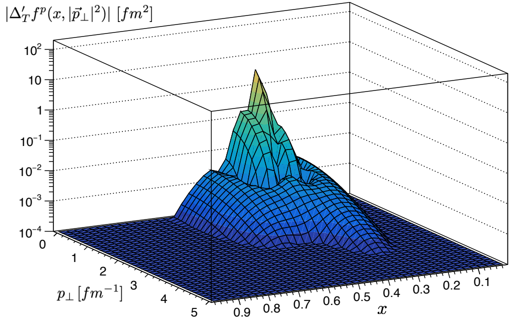
Unpolarized TMD $f^\tau(x, |\mathbf{p}_\perp|^2)$ in an unpolarized ^{3}He. Upper panel: Proton. Lower panel: Neutron. Notice the peak around $x = 1/3$. The integral over $\mathbf{p}_\perp$ yields the longitudinal momentum $f_1^\tau(x)$

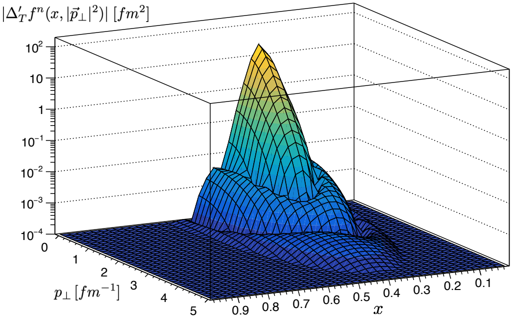




Absolute value of the nucleon longitudinal-polarization distribution, $\Delta f^\tau(x, |\mathbf{p}_\perp|^2)$, in a longitudinally polarized ^{3}He. Upper panel: Proton. Lower panel: Neutron. N.B. longitudinal wrt the virtual-photon axis.

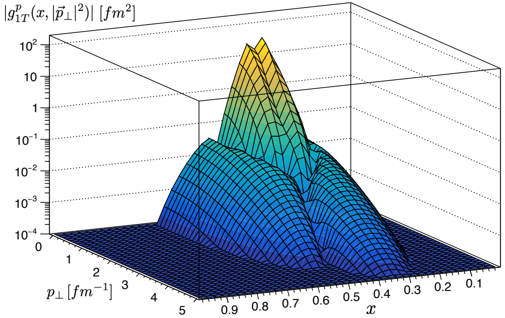
$\Delta f^\tau(x, |\mathbf{p}_\perp|^2) \to N \Uparrow \ \ ^3\mathrm{He} \Uparrow$

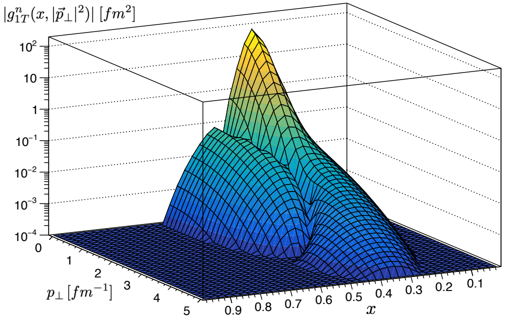




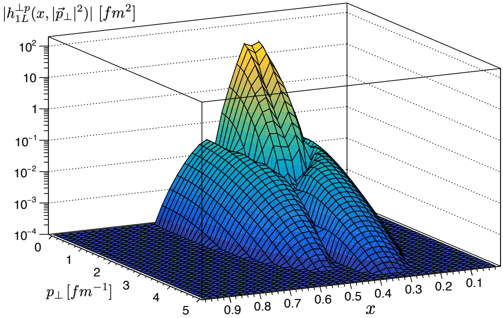
Absolute value of the nucleon transverse-polarization distribution, $\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$, in a ^{3}He transversely polarized in the same direction of the nucleon polarization. Upper panel: Proton. Lower panel: Neutron. N.B. transverse wrt the virtual-photon axis.

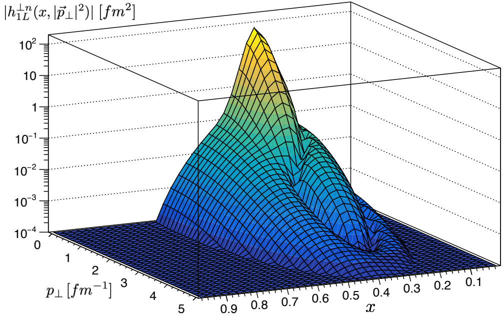
$\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2) \rightarrow N_\perp \ {}^3\mathrm{He}_\perp$





Absolute value of the nucleon longitudinal-polarization distribution, $g_{1T}^{\tau}(x,|\mathbf{p}_\perp|^2)$, in a transversely polarized ^{3}He. Upper panel: Proton. Lower panel: Neutron. Notice $|g_{1T}^{\tau}(x,|\mathbf{p}_\perp|^2)| \sim |h_{1L}^{\perp\tau}(x,|\mathbf{p}_\perp|^2)|$, next slide.
$g_{1T}^{\tau}(x,|\mathbf{p}_\perp|^2) \rightarrow N \Uparrow \ {}^3\mathrm{He}_\perp$





Absolute value of the nucleon transverse-polarization distribution, $h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)$ in a longitudinally polarized ^{3}He. Upper panel: Proton. Lower panel: Neutron. Notice $|h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)| \sim |g_{1T}^{\tau}(x, |\mathbf{p}_\perp|^2)|$, previous slide.

$h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2) \to N_\perp \ \ {}^3\mathrm{He} \Uparrow$

Effective polarizations

Key role in the extraction of neutron polarized structure functions and neutron Collins and Sivers single spin asymmetries, from the corresponding quantities measured for ^{3}He.

Effective longitudinal polarization

$$p^\tau_{||} = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta f^\tau(x, |\mathbf{p}_\perp|^2)$$

Effective transverse polarization

$$p^\tau_\perp = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2) \; .$$

Effective polarizations	proton	neutron
LF longitudinal polarization	-0.02299	0.87261
LF transverse polarization	-0.02446	0.87314
non relativistic polarization	-0.02118	0.89337

Comments:

- The difference between the LF polarizations and the non relativistic results are up to 2% in the neutron case (larger for the proton ones, but it has an overall small contribution), and should be ascribed to the intrinsic coordinates, implementing the Macro-locality, and not to the Melosh rotations involving the spins.
- N.B. Within a NR framework: $p^\tau_{||}(NR) = p^\tau_\perp(NR)$

Conclusions & Perspectives

- **A Poincaré covariant description of nuclei, based on the light-front Hamiltonian dynamics, has been proposed.** The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework. N.B. Normalization and momentum sum rule are both automatically fulfilled.
- ★ Macro-locality can be implemented, as it must be and plays a role in precision experiments (see also TMD's relations).
- ★ Notably, the Spectral Function is related to the valence contribution to the correlator introduced for a QFT description of SiDIS reactions involving the nucleon , and applied for the first time to ^{3}He.

  ★★ General principles implemented in the LF Spectral function entail relations among T-even twist-2 (and also twist-3) valence TMDs, with interesting angular momentum dependence.
- Preliminary calculation of $^3\mathrm{He}$ EMC encourages the application of our approach, shedding light on the role of a reliable description of the nucleus. Also the LF spin-depedent momentum distributions are available, for both longitudinal and transverse polarizations of the nucleon.
- Analyses of exclusive reactions, with polarized initial and final states, for accessing nuclear TMD's in ^{3}He are in progress