Light-front dynamic analysis of the transition form factors in 1 + 1 dimensional scalar field model

Bailing Ma

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LFD and hadron physics

- Light-Front Dynamics (LFD) is an essential theoretical tool for the JLab/EIC physics, e.g. GPDs, TMDs, 3D femtography of the hadrons, etc.
- It has the remarkable features of the maximum number (7) of kinematic operators among the 10 Poincare generators, boost invariant wave function, and simpler vacuum property.
- The advantage of LFD is maximized in 1+1 dimensional models due to the absence of transverse rotation operators which are dynamical in LFD.

Drell-Yan-West vs other frames

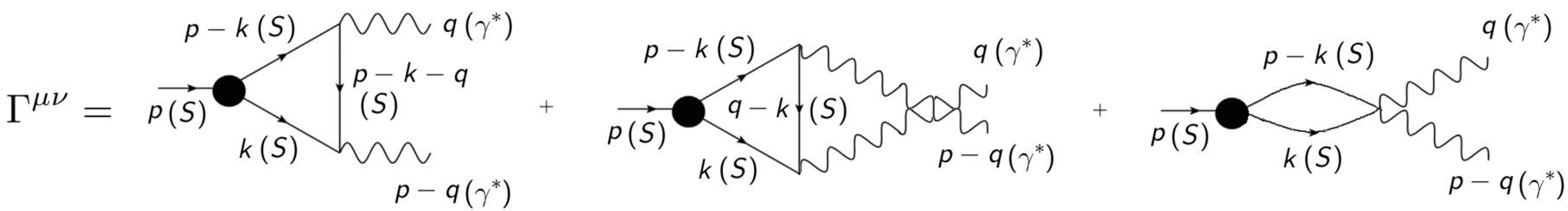
- For the study of exclusive processes, the Drell-Yan-West frame ($q^+ = 0$) is well-established, but it cannot be taken in this case of 1+1 dimensions, as it results in $q^2 = 0$.
- As one must use a $q^+ \neq 0$ frame, one must include both the valence and non-valence graphs for the calculation of the form factors.
- Choosing a $q^+ \neq 0$ frame, one can also directly access not only the space-like ($q^2 < 0$), but also the time-like ($q^2 > 0$) momentum regions.

Transition Form Factor in 1+1-dimensional simple scalar model

- We give a clear example demonstrating direct access to the time-like region of photon momenta without resorting to analytic continuation.
- We use the exactly solvable scalar field model of Sawicki and Mankiewicz.
- Even though the model is not very realistic, it serves as an initial trial for the more complicated ‘t Hooft model or the phenomenological light-front quark model.

The covariant Bethe-Salpeter (BS) model

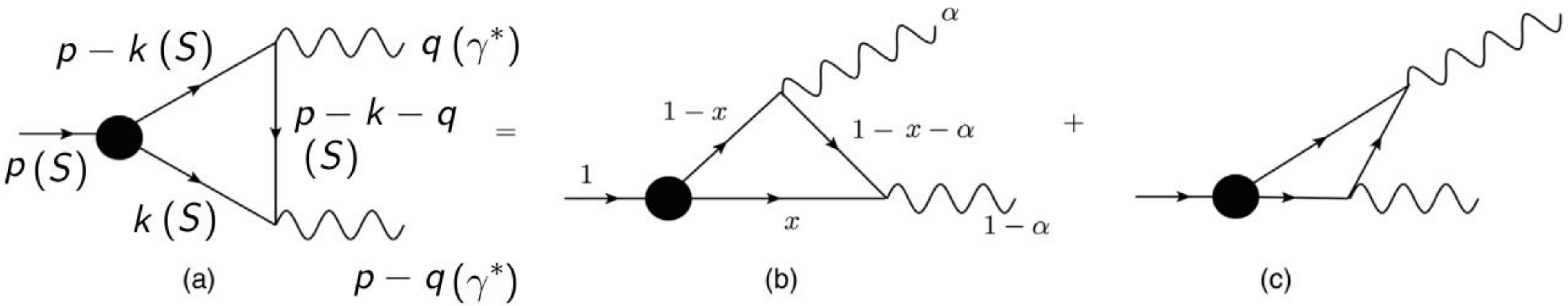
- The meson wave function is a product of two free single particle propagators
- Dirac delta function for overall momentum conservation
- And a constant vertex function



(D) (C) (S)

$$\Gamma^{\mu\nu} = F(q^2, q'^2)\,(g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu})$$

Valence and non-valence contributions : how to define them



$$f_i^{++} = \frac{\Gamma_i^{++}}{g^{++}q\cdot q' - q'^{+}q^{+}}$$

$$f_i^{+-} = \frac{\Gamma_i^{+-}}{g^{+-}q\cdot q' - q'^{+}q^{-}}$$

where i represents $D(b)$, $D(c)$, $C(b)$, $C(c)$, or S

- Problem with this definition is that $f_i^{++} \neq f_i^{+-}$

- This is because each individual $\Gamma_i^{\mu\nu}$ may not be gauge invariant by themselves, thus there may be a gauge-non-invariant component $\tilde{g}_i$ in addition to the gauge-invariant part $\tilde{f}_i$, i.e., although the total current must satisfy gauge invariance $\Gamma^{\mu\nu} = F(q^2, q'^2)\,(g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu})$ and there is only one form factor $F(q^2, q'^2)$, each individual LFTO contributions may be of the form $\Gamma_i^{\mu\nu} = \tilde{f}_i\,(g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu}) + \tilde{g}_i\,(q^{\mu} q'^{\nu})$.

- i.e., in our $++$ and $+-$ currents example,

$$\begin{pmatrix} \Gamma_i^{++} \\ \Gamma_i^{+-} \end{pmatrix} = \begin{pmatrix} g^{++} q \cdot q' - q'^{+} q^{+} & q^{+} q'^{+} \\ g^{+-} q \cdot q' - q'^{+} q^{-} & q^{+} q'^{-} \end{pmatrix} \cdot \begin{pmatrix} \tilde{f}_i \\ \tilde{g}_i \end{pmatrix}$$

Inverting this equation, we get

$$\begin{pmatrix} \tilde{f}_i \\ \tilde{g}_i \end{pmatrix} = \begin{pmatrix} -\frac{1}{2q^+q'^+} & \frac{1}{2q^+q'^-} \\ \frac{1}{2q^+q'^+} & \frac{1}{2q^+q'^-} \end{pmatrix} \cdot \begin{pmatrix} \Gamma_i^{++} \\ \Gamma_i^{+-} \end{pmatrix}.$$

Recall that

$$f_i^{++} = \frac{\Gamma_i^{++}}{g^{++}q \cdot q' - q'^+q^+} = -\frac{\Gamma_i^{++}}{q'^+q^+},$$

and

$$f_i^{+-} = \frac{\Gamma_i^{+-}}{g^{+-}q \cdot q' - q'^+q^-} = \frac{\Gamma_i^{+-}}{q^+q'^-}.$$

So, in fact,

$$\tilde{f}_i = \frac{f_i^{++} + f_i^{+-}}{2},$$

and

$$\tilde{g}_i = \frac{-f_i^{++} + f_i^{+-}}{2}.$$

Of course, we have

$$\sum_i \tilde{f}_i = \sum_i f_i^{++} = \sum_i f_i^{+-} = F(q^2, q'^2)$$

and

$$\sum_i \tilde{g}_i = 0$$

as it must be.

The amplitude $\Gamma^{\mu\nu}$ is calculated as such, following the Feynman rules for the scalar field theory.

$$\begin{aligned}
\Gamma^{\mu\nu} =& \Gamma_D^{\mu\nu} + \Gamma_C^{\mu\nu} + \Gamma_S^{\mu\nu} \\
=& ie^2 g_s \int \frac{d^2k}{(2\pi)^2} \times \\
& \left\{ \frac{(2p-2k-q)^\mu (p-2k-q)^\nu}{((p-k-q)^2 - m^2)((p-k)^2-m^2)(k^2-m^2)} \right. \\
& + \frac{(q-2k)^\mu (p-2k+q)^\nu}{((p-k)^2-m^2)(k^2-m^2)((q-k)^2-m^2)} \\
& \left. + \frac{-2g^{\mu\nu}}{((p-k)^2-m^2)(k^2-m^2)} \right\},
\end{aligned}$$

where the coupling constant of the simple scalar model g_s is fixed from the normalization condition. For simplicity, we take all the intermediate scalar particles' mass to be m and their charge to be e, but it can be easily generalized to unequal mass/charge cases. The initial scalar meson has mass M.

- For the manifestly covariant calculation, we use Feynman parametrization method and obtain

$$F(q^2,q'^2)=\frac{e^2 g_s}{4\pi}\int_0^1 dx\int_0^{1-x} dy(1-2y)\left(\frac{1}{\Delta_1^2}+\frac{1}{\Delta_2^2}\right),$$

where

$$\Delta_1 = x(x-1)q^2+2x(x+y-1)q\cdot q'+(x+y)(x+y-1)q'^2+m^2,$$

and

$$\Delta_2 = x(x-1)q'^2+2x(x+y-1)q\cdot q'+(x+y)(x+y-1)q^2+m^2.$$

- Doing the x and y integrations, it ends up being

$$F(q^2,q'^2)=\frac{e^2 g_s}{4\pi}\frac{(2-\omega-\gamma'-\gamma)\frac{\sqrt{\omega}}{\sqrt{1-\omega}}\tan^{-1}\left(\frac{\sqrt{\omega}}{\sqrt{1-\omega}}\right)+(\gamma-\gamma'-\omega)\frac{\sqrt{1-\gamma'}}{\sqrt{\gamma'}}\tan^{-1}\left(\frac{\sqrt{\gamma'}}{\sqrt{1-\gamma'}}\right)+(\gamma'-\gamma-\omega)\frac{\sqrt{1-\gamma}}{\sqrt{\gamma}}\tan^{-1}\left(\frac{\sqrt{\gamma}}{\sqrt{1-\gamma}}\right)}{m^4\left[4\omega\gamma'\gamma+\omega^2+(\gamma'-\gamma)^2-2\omega(\gamma'+\gamma)\right]},$$

where $\gamma=\frac{q^2}{4m^2}$, $\gamma'=\frac{q'^2}{4m^2}$, and $\omega=\frac{M^2}{4m^2}$.

- For the LFD calculation, we define the light-front momentum fraction parameter of the emitted photon with respect to the initial scalar meson $\alpha = q^+/p^+$
- So that the 2-momenta of the external particles are

$$p = (p^+, p^-) = \left(p^+, \frac{M^2}{2p^+}\right)$$

$$q = (q^+, q^-) = \left(\alpha p^+, \frac{M^2}{2p^+} - \frac{q'^2}{2(1-\alpha)p^+}\right)$$

$$q' = p - q = (q'^+, q'^-) = \left((1-\alpha)p^+, \frac{q'^2}{2(1-\alpha)p^+}\right),$$

where α satisfies

$$\alpha^2 M^2 - \alpha M^2 + \alpha q'^2 - \alpha q^2 + q^2 = 0,$$

for which we get the solutions

$$\alpha_\pm = \frac{(M^2 - q'^2 + q^2) \pm \sqrt{(-M^2 + q'^2 - q^2)^2 - 4M^2 q^2}}{2M^2}.$$

- We ensure $q^+ > 0$ by taking the α_+ root in our calculation

- For the LFD calculation, we pick the ++ component of the current and the +− one to show their difference
- We use the Cauchy integration method to do the energy integration enclosing the poles, then there is only the kinematic momentum fraction integration left over
- We get

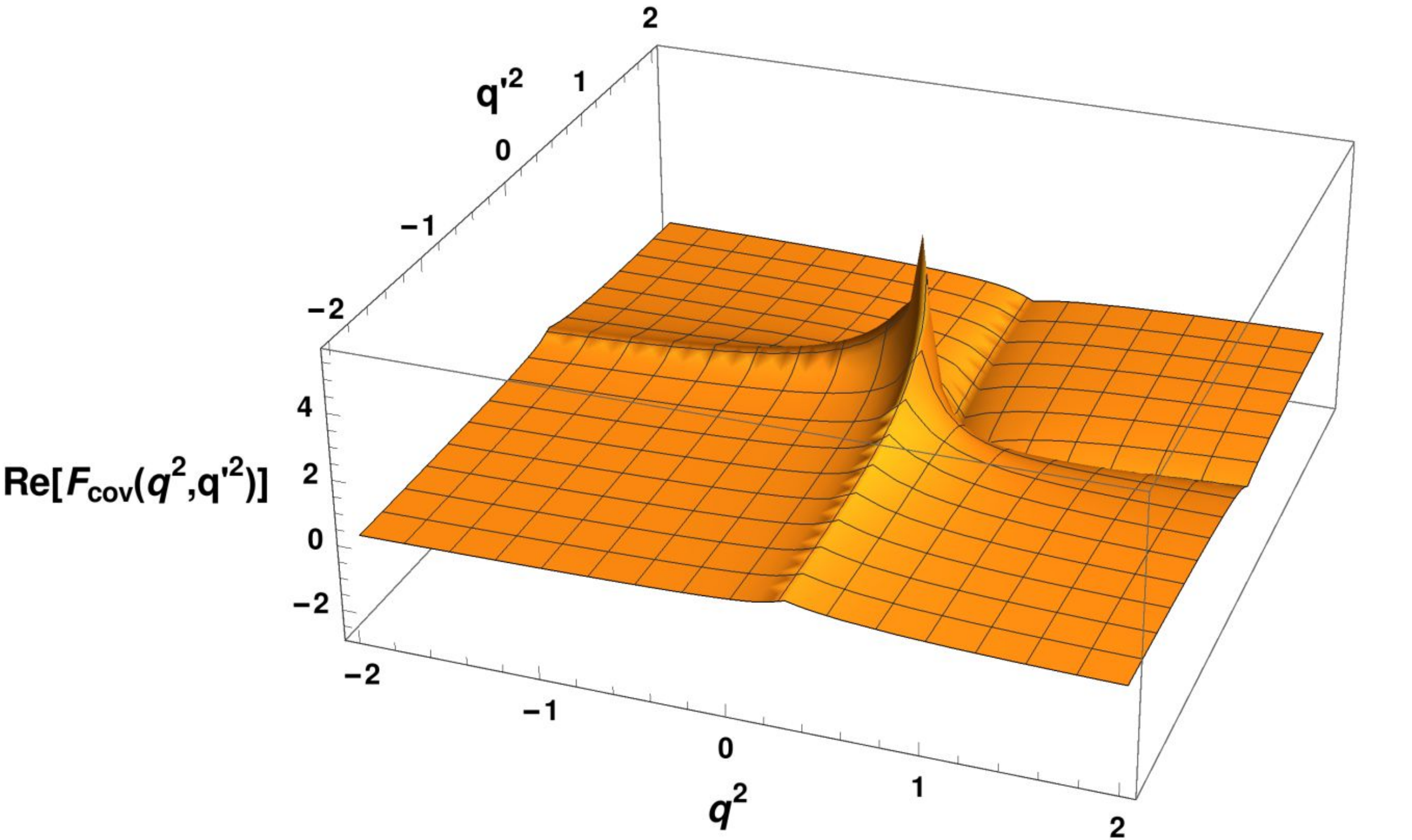
$$f^{++}_{D(b)} = \frac{e^2 g_s}{4\pi} \int_0^{1-\alpha} dx\,(2-2x-\alpha)\,(1-2x-\alpha)$$
$$\cdot \left[\alpha(\alpha-1)(1-x-\alpha)(1-x)x\left(\frac{m^2}{x}+\frac{m^2}{1-x-\alpha}-\frac{q'^2}{1-\alpha}\right)\left(\frac{m^2}{x}+\frac{m^2}{1-x}-M^2\right)\right]^{-1},$$

$$f^{++}_{D(c)} = \frac{e^2 g_s}{4\pi} \int_{1-\alpha}^{1} dx\,(2-2x-\alpha)\,(1-2x-\alpha)$$
$$\cdot \left[\alpha(\alpha-1)\,(1-x-\alpha)\,(1-x)\,x\left(\frac{m^2}{1-x-\alpha}-\frac{m^2}{1-x}+M^2-\frac{q'^2}{1-\alpha}\right)\left(\frac{m^2}{1-x}+\frac{m^2}{x}-M^2\right)\right]^{-1},$$

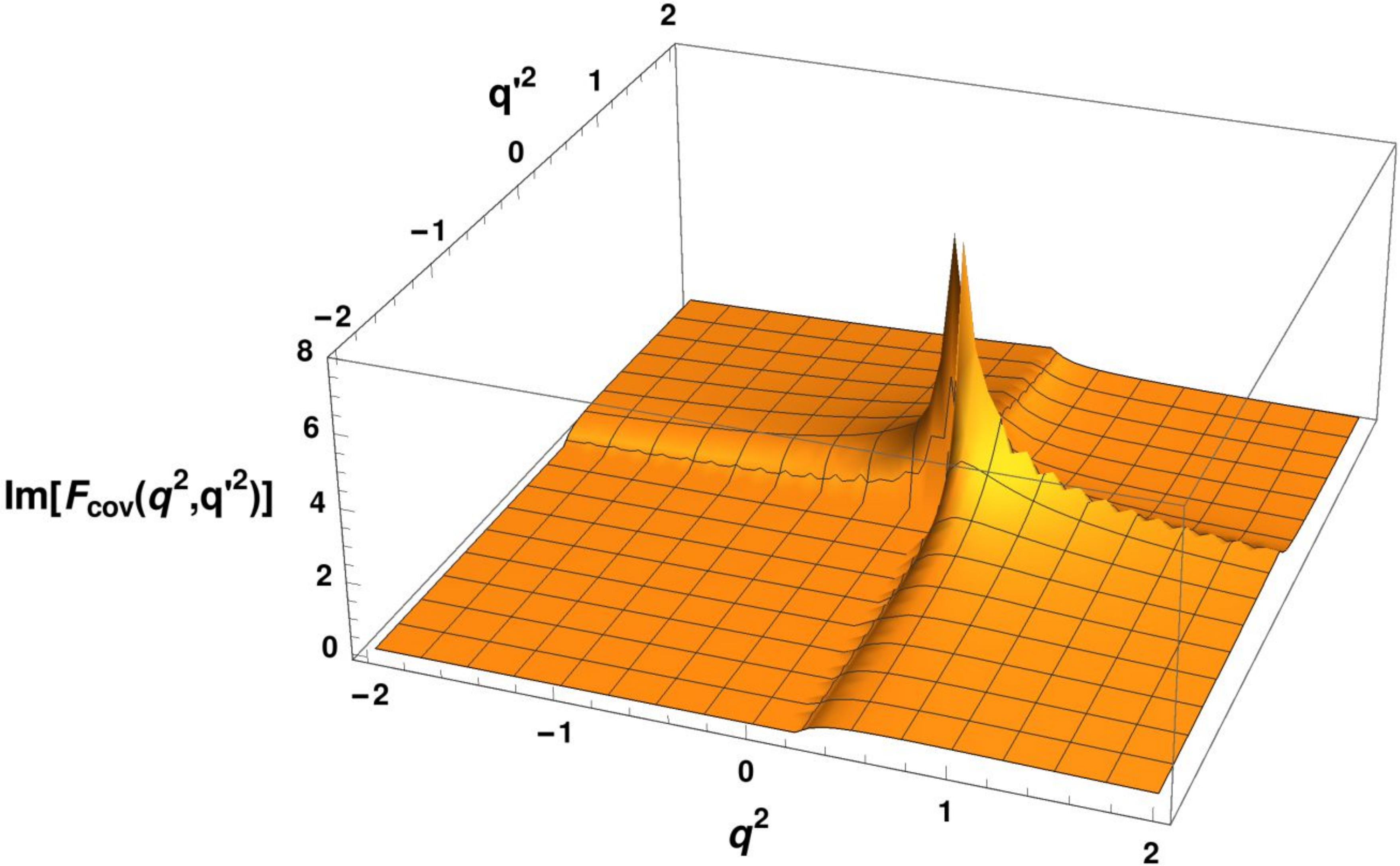
$$f^{++}_{C(b)} = f^{++}_{D(c)}, \text{ and } f^{++}_{C(c)} = f^{++}_{D(b)} .$$

- The x integration can be computed analytically to confirm that $f^{++}_{D(b)} + f^{++}_{D(c)} + f^{++}_{C(b)} + f^{++}_{C(c)} = F_{cov}.$
- Similarly, one can also calculate the +− component of the current

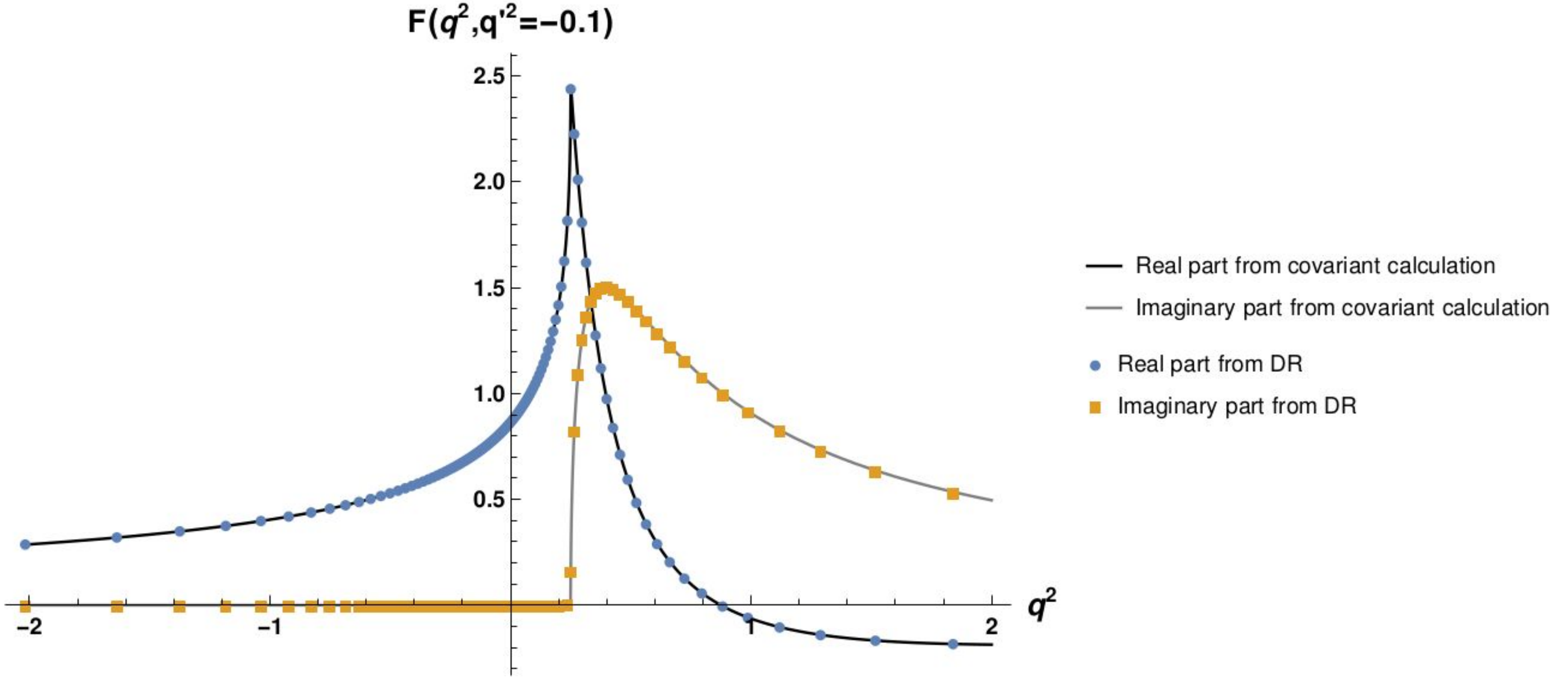
Numerical Results



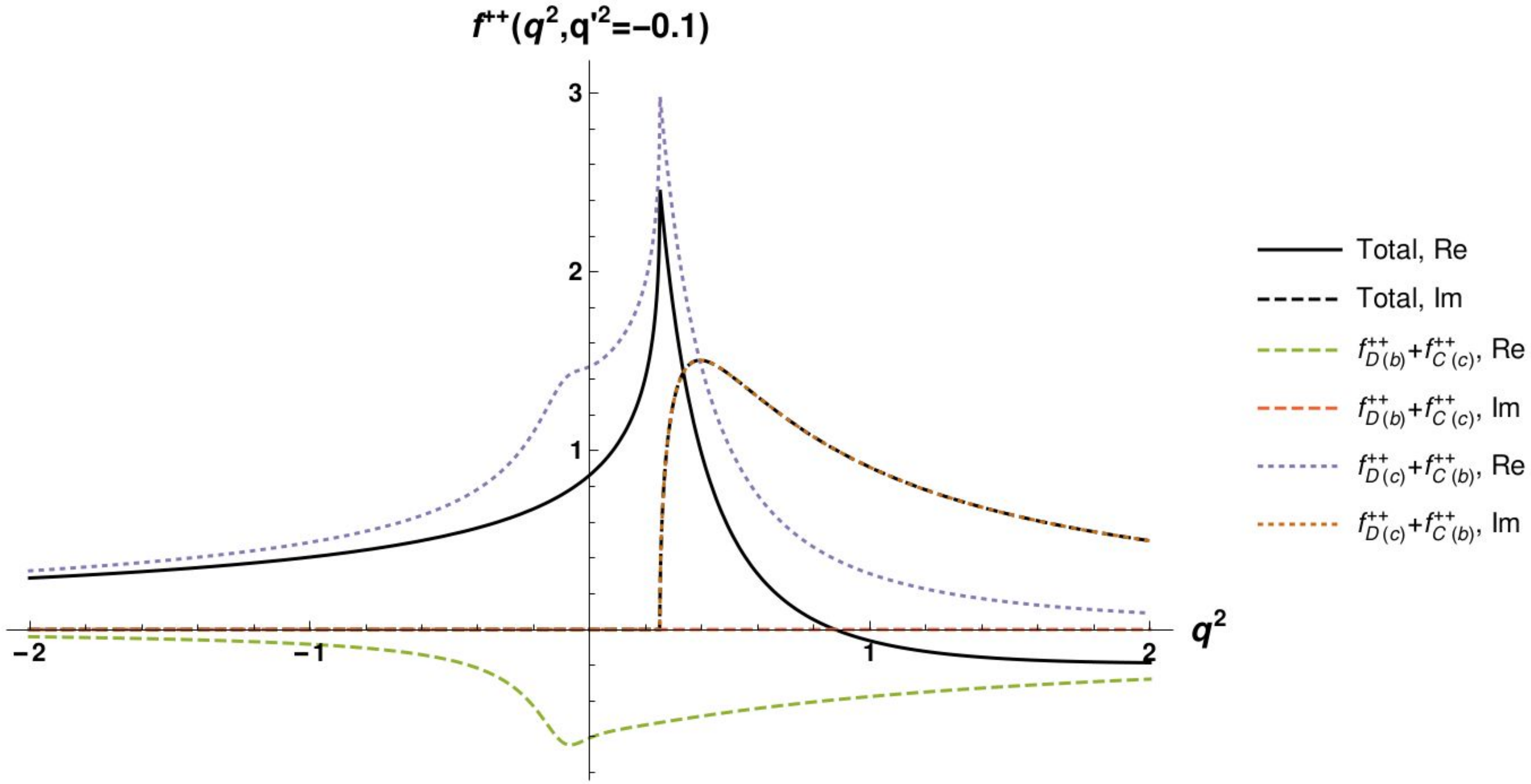
The 3D plot of the real part of the form factor for the case of $m = 0.25\ GeV$, $M = 0.14\ GeV$, normalized to $F(q^2 = 0, q'^2 = 0) = 1$, with $-2GeV^2 < q^2 < 2GeV^2$ and $-2GeV^2 < q'^2 < 2GeV^2$.



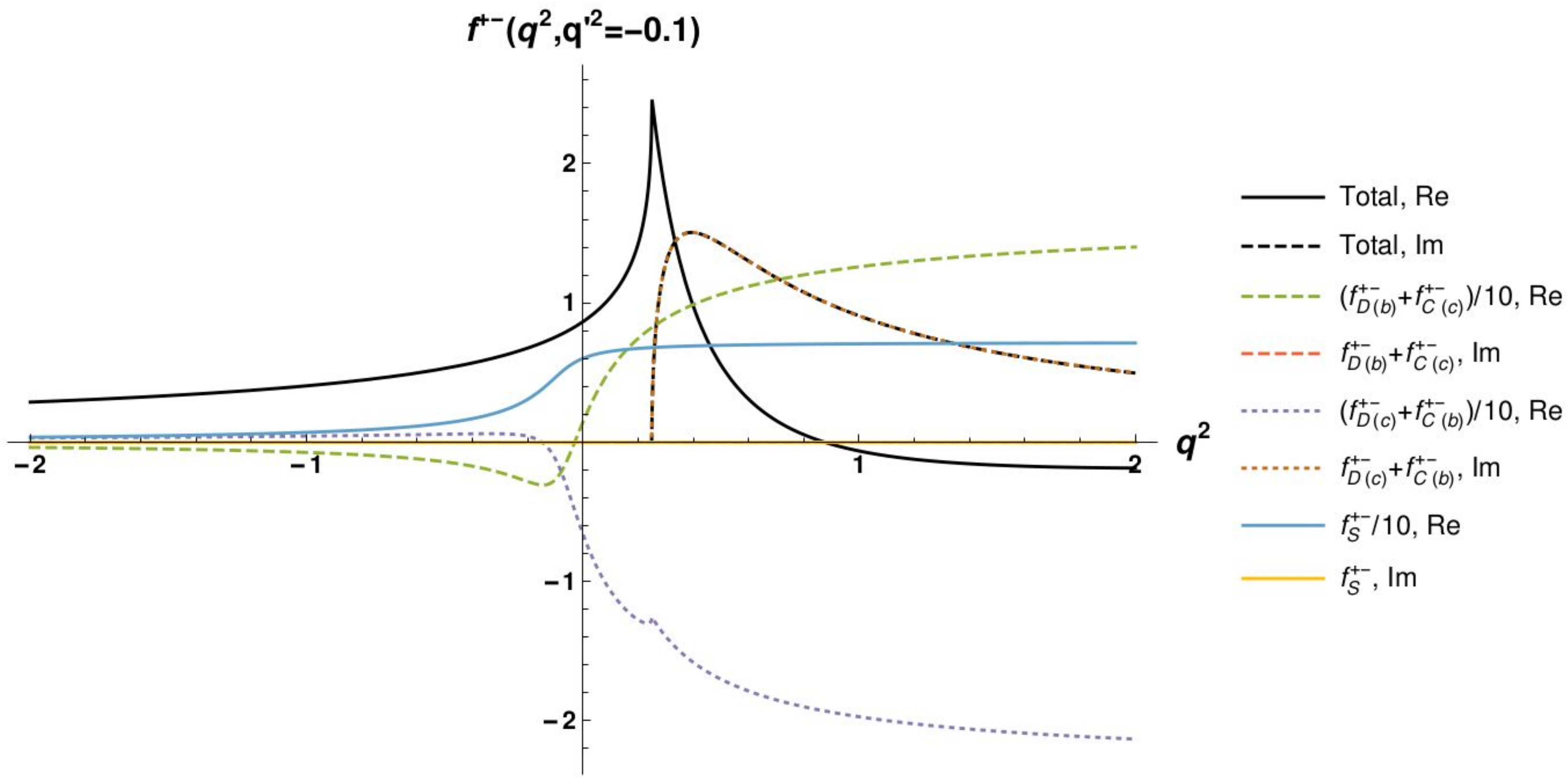
The 3D plot of the imaginary part of the form factor for the case of $m = 0.25\ GeV$, $M = 0.14\ GeV$, normalized to $F(q^2 = 0, q'^2 = 0) = 1$, with $-2GeV^2 < q^2 < 2GeV^2$ and $-2GeV^2 < q'^2 < 2GeV^2$.



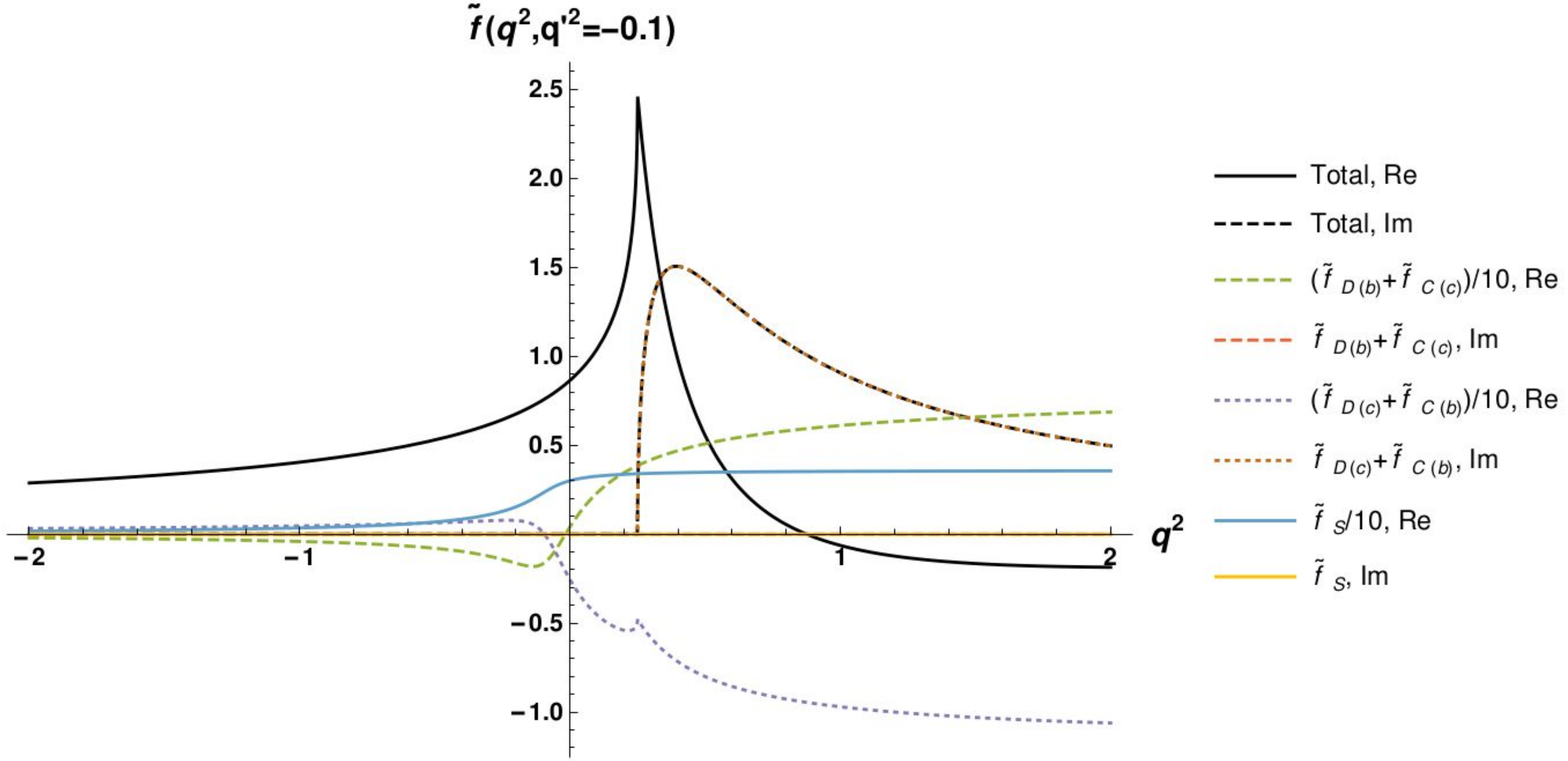
The numerical results of the form factor for the case of $m = 0.25\ GeV$, $M = 0.14\ GeV$, and $q'^2 = -0.1\ GeV^2$, normalized to $F(q^2 = 0, q'^2 = 0) = 1$, from the manifestly covariant calculation as well as the light-front one, and their agreements with the Dispersion Relation.



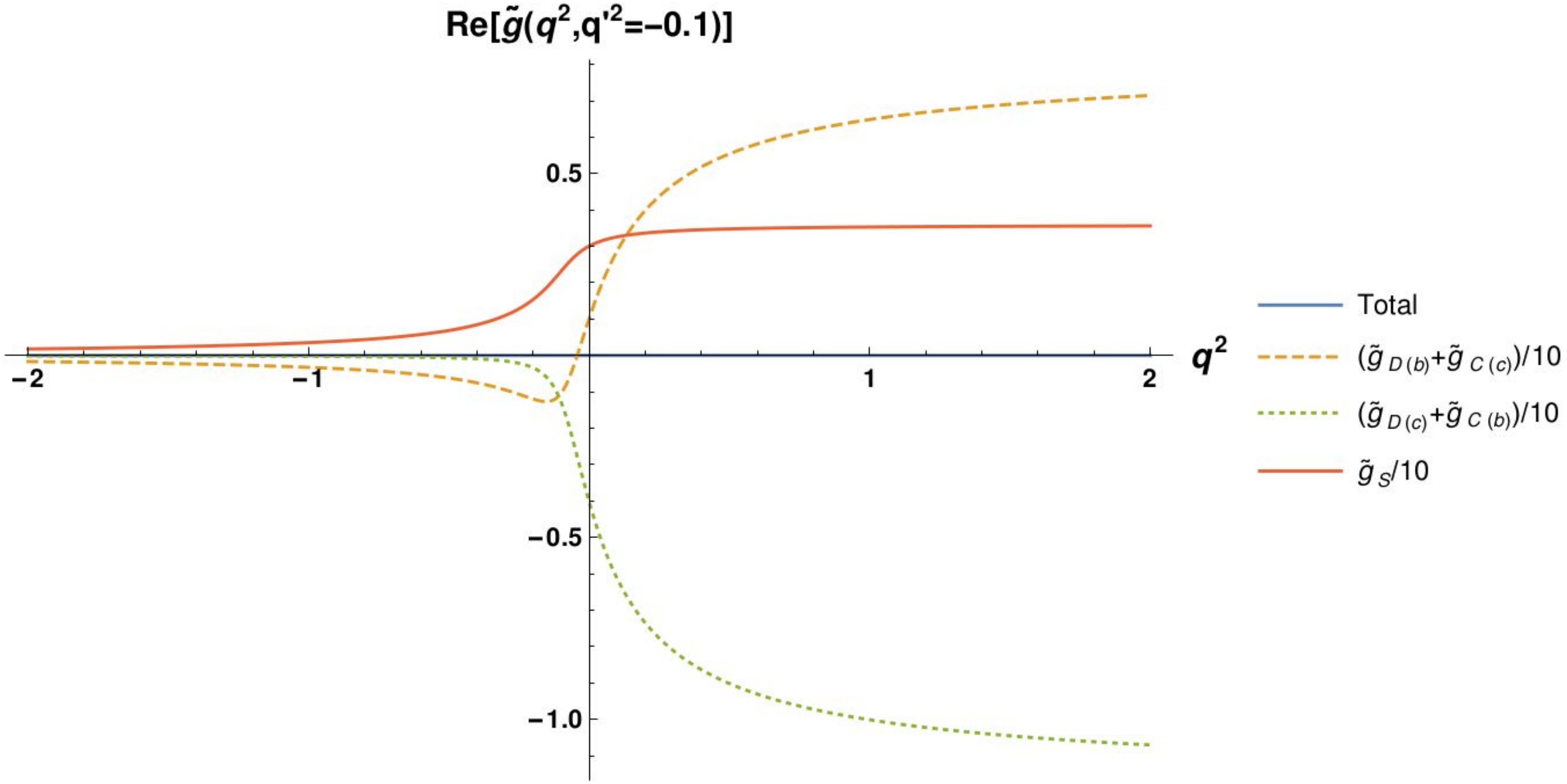
The numerical results of individual x^+-ordered contributions to the form factor for the case of $m = 0.25\ GeV$, $M = 0.14\ GeV$, and $q'^2 = -0.1\ GeV^2$, normalized to $F(q^2 = 0, q'^2 = 0) = 1$, by picking the $++$ component of the current. Note that $f^{++}_{D(b)} = f^{++}_{C(c)}$ and $f^{++}_{D(c)} = f^{++}_{C(b)}$.



The numerical results of individual x^+-ordered contributions to the form factor for the case of $m = 0.25\ GeV$, $M = 0.14\ GeV$, and $q'^2 = -0.1\ GeV^2$, normalized to $F(q^2 = 0, q'^2 = 0) = 1$, by picking the $+-$ component of the current. Note that $f^{+-}_{D(b)} = f^{+-}_{C(c)}$ and $f^{+-}_{D(c)} = f^{+-}_{C(b)}$.



The numerical results of the gauge-invariant part of the individual x^+-ordered contributions to the form factor for the case of $m = 0.25\ GeV$, $M = 0.14\ GeV$, and $q'^2 = -0.1\ GeV^2$, normalized to $F(q^2 = 0, q'^2 = 0) = 1$, taking the tilde definition. Note that $\tilde{f}_{D(b)} = \tilde{f}_{C(c)}$ and $\tilde{f}_{D(c)} = \tilde{f}_{C(b)}$.



The numerical results of the gauge-non-invariant part of the individual x^+-ordered contributions to the form factor for the case of $m = 0.25\ GeV$, $M = 0.14\ GeV$, and $q'^2 = -0.1\ GeV^2$ by taking the tilde definition. Imaginary parts are all zero here. Note that $\tilde{g}_{D(b)} = \tilde{g}_{C(c)}$ and $\tilde{g}_{D(c)} = \tilde{g}_{C(b)}$.

Thank you !