Hadrons' masses in the chiral symmetry restored vacuum

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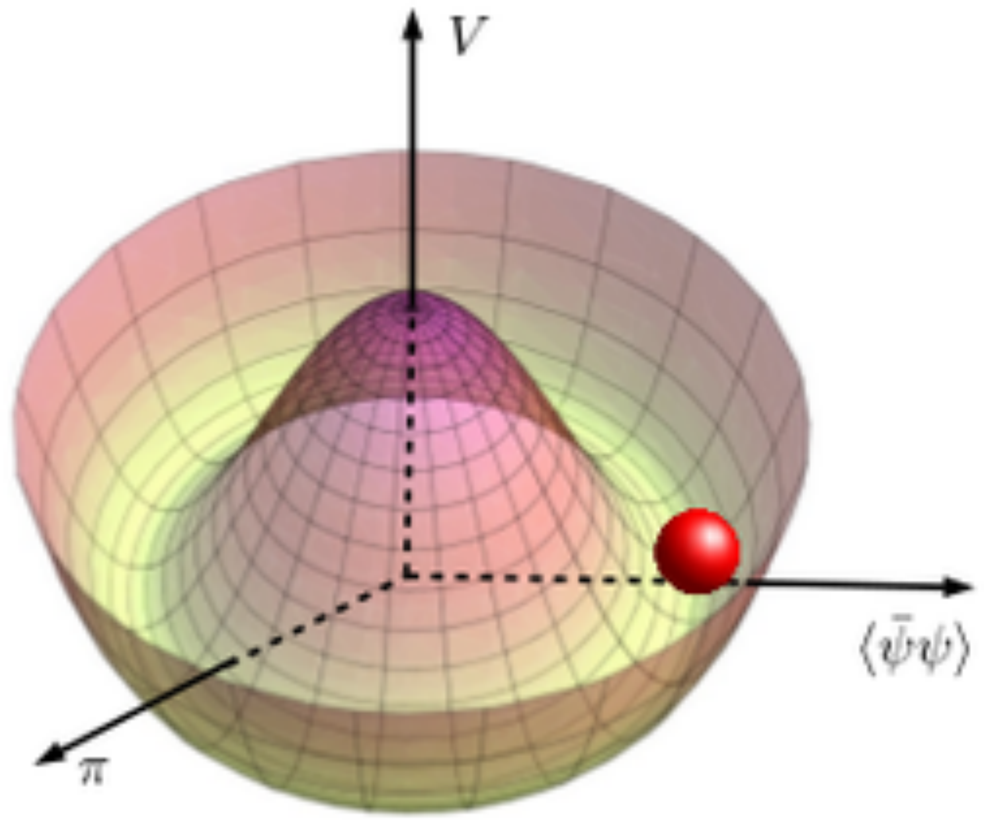
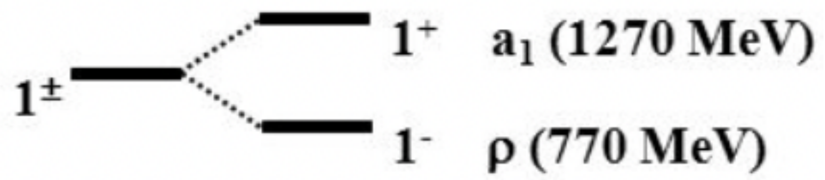
Introduction



Hadron mass generation

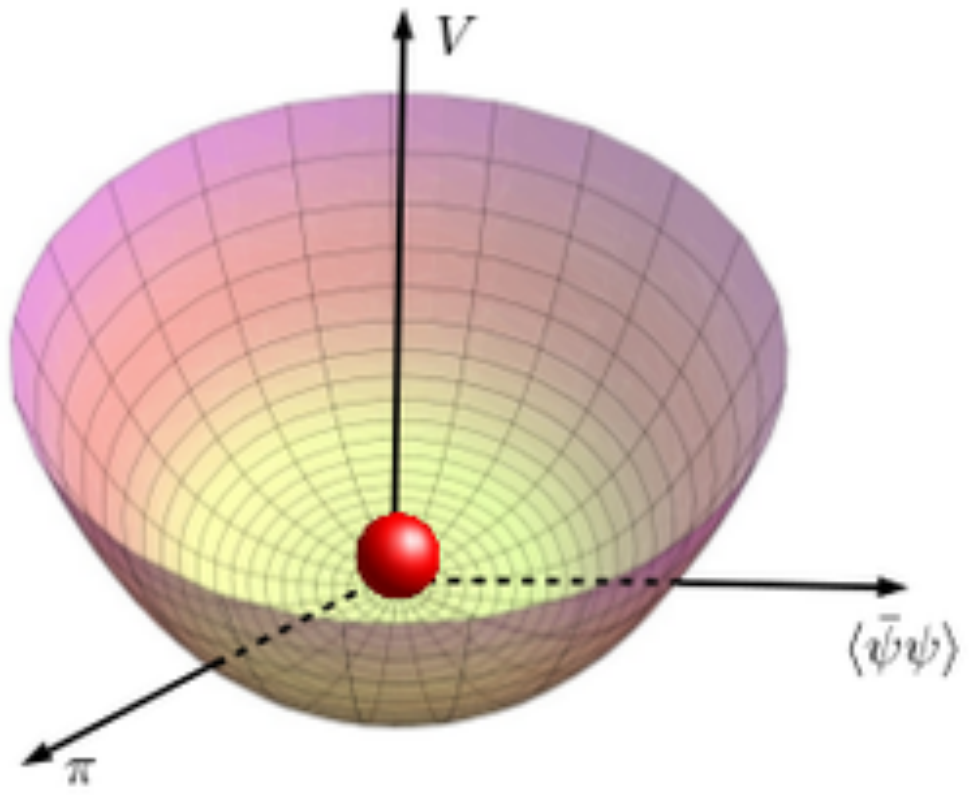
$m_q^* >> m_q$

Chiral partners' mass splitting



$\Delta m \simeq 0.5$ GeV

QCD vacuum $\Delta m = m_{a_1} - m_\rho$



Hadron mass decreases

$m_H > m_H'$

Chiral partners' mass degenerate

$m_\rho' = m_{a_1}'$

$\Delta m \rightarrow 0$

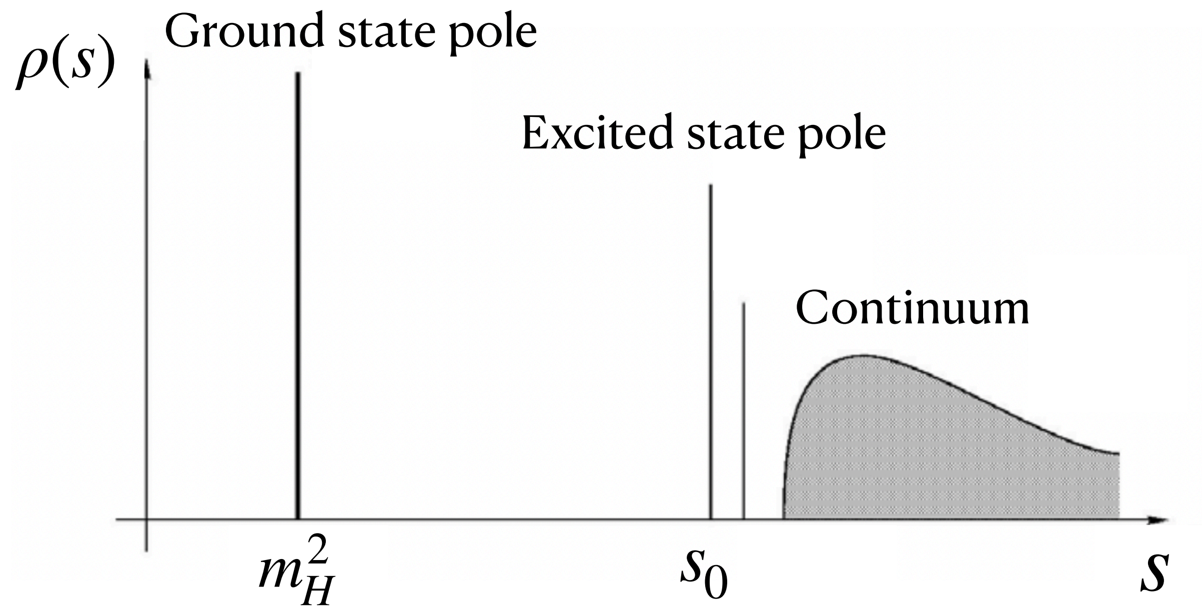
symmetry restored vacuum

- Banks-Casher Relation(Chiral order parameter $\propto \rho(0)$)
- QCD sum rules approach

We can identify the effect of chiral symmetry breaking to the hadron mass

QCD sum rules

An interpolating current $\eta(x)$ reflects the quantum #s of the hadron of interest.



$$\Pi(q) = i\int d^4x\, e^{iqx}\,\langle 0|T\{\eta(x)\bar{\eta}(0)\}|0\rangle$$

Wilson's Operator Product Expansion

$$\widehat{\Pi}^{\text{phen.}}(M^2) = \int_0^\infty ds\, e^{-s/M^2}\left(\rho^{\text{pole}}(s) + \rho^{\text{cont}}(s)\right)$$

$$\widehat{\Pi}^{\text{OPE}}(M^2) = \sum_d C_d(M^2)\langle O^d\rangle$$

M	: Borel mass
d	: Mass dimension
C_d	: Wilson coefficient
$\langle O^d\rangle_{vac}$	: Vacuum condensates

Phenomenological (Hadronic) Side = OPE (Quark-Gluon) Side

From the sum rule, we can obtain information about the ground state hadron mass

Chiral symmetry restoration changes the non-perturbative contribution(Condensates) in OPE.
If OPE in the chiral symmetry restored vacuum can be obtained, the mass of the hadron can be extracted.

Condensates

J. Kim, S. H. Lee, arXiv:2109.12791

$$\widehat{\Pi}_H^{OPE} = C_0(M^2) + C_4(M^2)\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle + C_4'(M^2)\langle m_u\bar{u}u\rangle + C_6(M^2)\langle(\bar{u}\Gamma^\alpha\lambda^a u)(\bar{u}\Gamma_\alpha\lambda_a u)\rangle + \cdots$$

<**In QCD vacuum**>

Chiral symmetric : $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 12\left\langle\sum_\lambda \rho(\lambda)\right\rangle$

$i\not{D}\psi = \lambda\psi$

S. H. Lee and S. Cho, IJMPE(2013)

Chiral order parameter :
(Chiral symmetry breaking)

$\langle\bar{u}u\rangle = -\pi\langle\rho(\lambda = 0)\rangle$

Banks-Casher relation

T. Banks, A. Casher, NPB(1980)

Every four-quark operator can be separated into the breaking and symmetric parts.

$$\langle(\bar{q}\gamma^\alpha\lambda^a q)(\bar{q}\gamma_\alpha\lambda_a q)\rangle_{tot} = \langle(\bar{q}\gamma^\alpha\lambda^a q)(\bar{q}\gamma_\alpha\lambda_a q)\rangle_S + \langle(\bar{q}\gamma^\alpha\lambda^a q)(\bar{q}\gamma_\alpha\lambda_a q)\rangle_B \propto \rho(0)$$

Symmetry restoration

$\rho(\lambda = 0) \to 0$

<**In chiral symmetry restored vacuum**>

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 12\left\langle\sum_\lambda \rho(\lambda)\right\rangle$$

$$\langle\bar{u}u\rangle = 0$$

$$\langle(\bar{q}\gamma^\alpha\lambda^a q)(\bar{q}\gamma_\alpha\lambda_a q)\rangle_{tot} = \langle(\bar{q}\gamma^\alpha\lambda^a q)(\bar{q}\gamma_\alpha\lambda_a q)\rangle_S$$

Example $(\rho - a_1)$

<**In symmetry broken phase**>

$$\Pi_\rho^{OPE} = \left(1+\frac{\alpha_s}{\pi}\right)M^2 + \frac{\pi^2}{3M^2}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{8\pi^2 m_q}{M^2}\langle\bar{u}u\rangle - \frac{2\pi\alpha_s}{M^4}\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a\tau^3 q)^2\rangle - \frac{4\pi\alpha_s}{9M^4}\langle(\bar{q}\gamma_\mu\lambda^a q)(\sum_{f=u,d,s}\bar{q}_f\gamma_\mu\lambda^a q_f)\rangle$$

$$\Pi_{a_1}^{OPE} = \left(1+\frac{\alpha_s}{\pi}\right)M^2 + \frac{\pi^2}{3M^2}\langle\frac{\alpha_s}{\pi}G^2\rangle - \frac{8\pi^2 m_q}{M^2}\langle\bar{u}u\rangle - \frac{2\pi\alpha_s}{M^4}\langle(\bar{q}\gamma_\mu\lambda^a\tau^3 q)^2\rangle - \frac{4\pi\alpha_s}{9M^4}\langle(\bar{q}\gamma_\mu\lambda^a q)(\sum_{f=u,d,s}\bar{q}_f\gamma_\mu\lambda^a q_f)\rangle$$

T. Hatsuda and S. H. Lee, PRC(1992)

Separating their 4-quark condensates into breaking and symmetric parts,

$$\rho : -\frac{2\pi\alpha_s}{M^4}\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a\tau^3 q)^2\rangle - \frac{4\pi\alpha_s}{9M^4}\langle(\bar{q}\gamma_\mu\lambda^a q)(\sum_{f=u,d,s}\bar{q}_f\gamma_\mu\lambda^a q_f)\rangle = -\frac{28\pi\alpha_s}{9M^4}\langle B_{uu}\rangle_B + \frac{\pi\alpha_s}{M^4}\langle S_{\rho-a_1}\rangle_S$$

$$a_1 : -\frac{2\pi\alpha_s}{M^4}\langle(\bar{q}\gamma_\mu\lambda^a\tau^3 q)^2\rangle - \frac{4\pi\alpha_s}{9M^4}\langle(\bar{q}\gamma_\mu\lambda^a q)(\sum_{f=u,d,s}\bar{q}_f\gamma_\mu\lambda^a q_f)\rangle = \frac{44\pi\alpha_s}{9M^4}\langle B_{uu}\rangle_B + \frac{\pi\alpha_s}{M^4}\langle S_{\rho-a_1}\rangle_S$$

J. Kim and S. H. Lee, PRD(2021)

Example ($\rho - a_1$)

$$\rho : -\frac{2\pi\alpha_s}{M^4}\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a\tau^3 q)^2\rangle - \frac{4\pi\alpha_s}{9M^4}\langle(\bar{q}\gamma_\mu\lambda^a q)(\sum_{f=u,d,s}\bar{q}_f\gamma_\mu\lambda^a q_f)\rangle = -\frac{28\pi\alpha_s}{9M^4}\langle B_{uu}\rangle_B + \frac{\pi\alpha_s}{M^4}\langle S_{\rho-a_1}\rangle_S$$

$$a_1 : -\frac{2\pi\alpha_s}{M^4}\langle(\bar{q}\gamma_\mu\lambda^a\tau^3 q)^2\rangle - \frac{4\pi\alpha_s}{9M^4}\langle(\bar{q}\gamma_\mu\lambda^a q)(\sum_{f=u,d,s}\bar{q}_f\gamma_\mu\lambda^a q_f)\rangle = \frac{44\pi\alpha_s}{9M^4}\langle B_{uu}\rangle_B + \frac{\pi\alpha_s}{M^4}\langle S_{\rho-a_1}\rangle_S$$

J. Kim and S. H. Lee, PRD(2021)

<**In symmetry restored phase**>

$$\Pi_\rho^{OPE} = \left(1 + \frac{\alpha_s}{\pi}\right)M^2 + \frac{\pi^2}{3M^2}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{\pi\alpha_s}{M^4}\langle S_{\rho-a_1}\rangle_S$$

$$\Pi_{a_1}^{OPE} = \left(1 + \frac{\alpha_s}{\pi}\right)M^2 + \frac{\pi^2}{3M^2}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{\pi\alpha_s}{M^4}\langle S_{\rho-a_1}\rangle_S$$

J. Kim and S. H. Lee, PRD(2021)

Results

Particle	$\kappa(\sqrt{s_0}(\text{GeV}))$	S/B	$\bar{m}_{sym}(\text{MeV})$
ρ	2.60(1.17)	0.760	572.5 ± 27.5
a_1	0.76(1.58)	-0.485	
ω	3.20(1.16)	1.165	655 ± 15
f_1	1.85(1.58)	0.253	1060 ± 30
K^*	2.097(1.33)	2.831	545 ∓ 5
K_1	0.39(1.56)	-0.227	

Table 2. meson sum rule results
J. Kim, S. H. Lee, arXiv:2109.12791

$$\kappa = \frac{\text{sum rule value}}{\text{vacuum sturation value}}$$

$$S/B = \frac{\text{Symmetric contirubiton}}{\text{Breaking contribution}}$$

$\bar{m}_{sym}$: expected mass in the symmetry restored vacuum

- From κ values, our sum rule method and the hypothesis give different values of the total 4-quark condensates.
- This difference mainly comes from the symmetric parts.
- From S/B values, it is shown that the symmetric and breaking parts have similar magnitudes.

Summary

- From QCD sum rule approach, the masses of hadrons in chiral($SU(2) \times SU(2)$) symmetry restored vacuum are extracted.
- All the hadrons masses in this study tend to decrease.
- The chiral partners have the degenerate mass states.
- The parity partners($\omega - f_1$) have non-degenerate(655 MeV, 1060 MeV) mass states.
- Chiral symmetry breaking is responsible not only for the mass difference of chiral partners but also for a fraction of common mass(for mesons, 1/3, for baryons, 1/2).
- Despite their strangeness, $K^* - K_1$(540-550 MeV) mass is expected to be slightly smaller than that of $\rho - a_1$(545-600 MeV).