YONSEI UNIVERSITY

Institute of Physics & Applied Physics
Nuclear & Hadron Physics

December 2nd, 2021

Light Cone 2021

Doubly Heavy Tetraquarks in a Non-Relativistic Quark Model

Sungsik Noh, Woosung Park, Su Houng Lee, Doubly heavy tetraquarks in a nonrelativistic quark model with a complete set of harmonic oscillator bases, Phys. Rev. D 103, 114009 (2021)

Speaker: Sungsik Noh

Jeju Island, Korea

Outline

- Motivation

- Formalism
  - Hamiltonian
  - Spatial Wave Function

- Result & Analysis
  - Contributions from Angular Momentum Bases
  - Size and Structure of Tcc
  - Comparison with Discovery of Tcc at LHCb

- Summary

Motivation

$T_{cc}(ud\bar{c}\bar{c})$ with $I(J^P) = 0(1^+)$

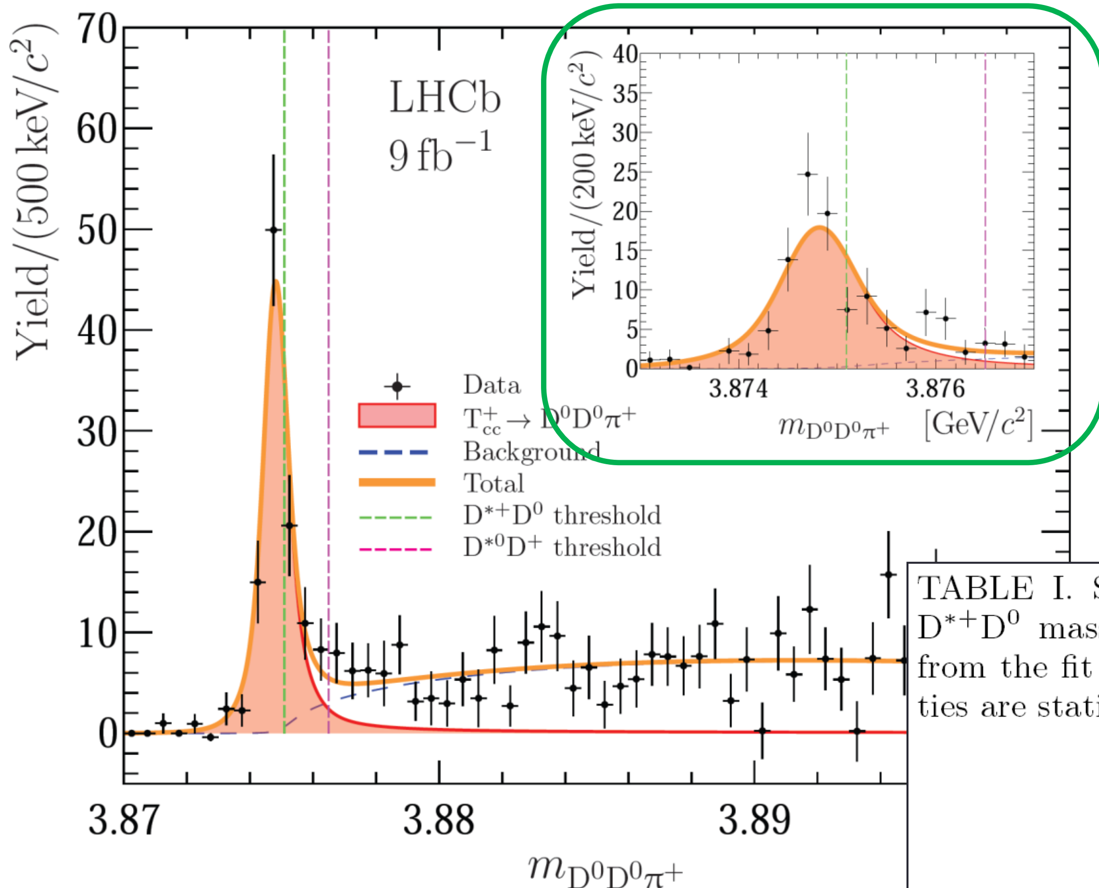

Figure 1: **The $D^0D^0\pi^+$ mass distribution.** The $D^0D^0\pi^+$ mass distribution where the contribution of the non-D^0 background has been statistically subtracted. The result of the fit described in the text is overlaid.

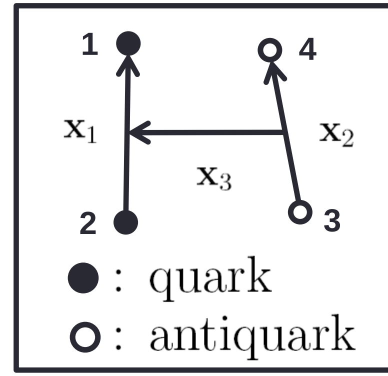

TABLE I. Signal yield, N, Breit-Wigner mass relative to $D^{*+}D^0$ mass threshold, δm_{BW}, and width, Γ_{BW}, obtained from the fit to the $D^0D^0\pi^+$ mass spectrum. The uncertainties are statistical only.

Parameter	Value
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LHCb Collaboration · Roel Aaij (Nikhef, Amsterdam) et al., Observation of an exotic narrow doubly charmed tetraquark [arXiv:2109.01038[hep-ex]]

Motivation

Reference		Year	$\delta' m$ [MeV/c^2]
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C. Semay and B. Silvestre-Brac	[38]	1994	$[-1, +13]$
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B. A. Gelman and S. Nussinov	[40]	2002	$[-25, +35]$
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	[41]	2003	-112
D. Janc and M. Rosina	[42]	2004	$[-3, -1]$
F. Navarra, M. Nielsen and S. H. Lee	[43]	2007	$+91$
J. Vijande, E. Weissman, A. Valcarce	[44]	2007	$[-16, +50]$
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	[45]	2007	$+60$
S. H. Lee and S. Yasui	[46]	2009	-79
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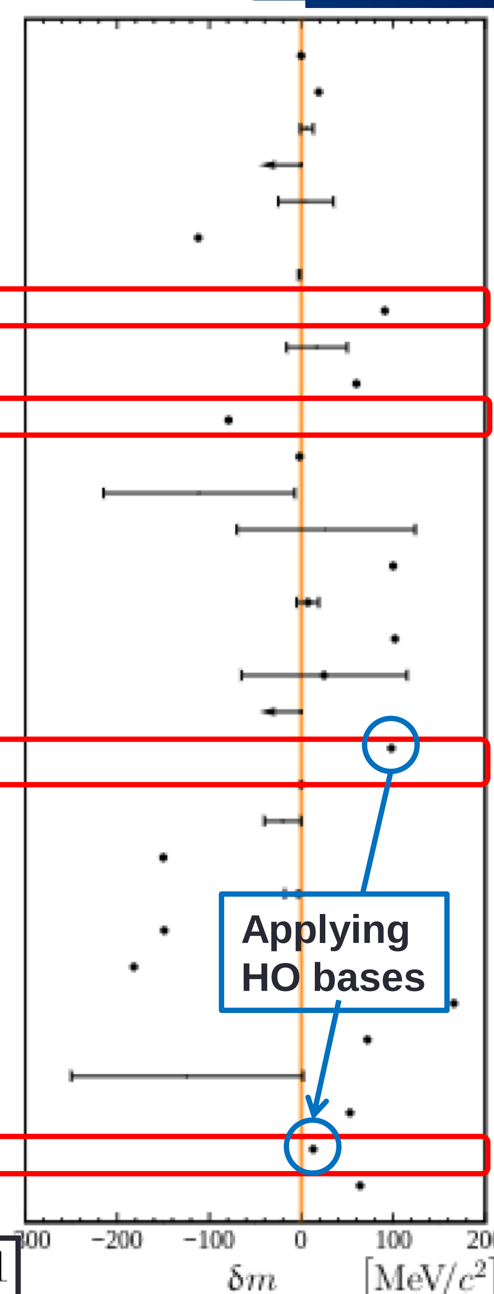
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	[49]	2013	$[-70, +124]$
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	[50]	2017	$+100$
M. Karliner and J. Rosner	[51]	2017	$7 \pm 12 \to 1$
E. J. Eichten and C. Quigg	[52]	2017	$+102$
Z. G. Wang	[53]	2017	$+25 \pm 90$
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	[54]	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	[55]	2018	$+98$
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	[56]	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	[57]	2018	$[-40, 0]$
C. Deng, H. Chen and J. Ping	[58]	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	[59]	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	[60]	2019	-149
Y. Tan, W. Lu and J. Ping	[61]	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	[62]	2020	$+166$
E. Braaten, L.-P. He and A. Mohapatra	[63]	2020	$+72$
D. Gao, D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	[64]	2020	$[-250, +2]$
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	[65]	2020	$+53$
S. Noh, W. Park and S. H. Lee	[66]	2021	$+13$
R. N. Faustov, V. O. Galkin and E. M. Savchenko	[67]	2021	$+64$



Ivan Polyakov on behalf of LHCb collaboration, EPS-HEP, 29 July 2021

Formalism - Hamiltonian

Hamiltonian

$$H = \sum_{i=1}^{4} \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - \frac{3}{4} \sum_{i<j}^{4} \frac{\lambda_i^c}{2} \frac{\lambda_j^c}{2} \left(V_{ij}^C + V_{ij}^{CS} \right), \quad (1)$$

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D, \quad (2)$$

$$V_{ij}^{CS} = \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4} \frac{e^{-(r_{ij})^2/(r_{0ij})^2}}{(r_{0ij}) r_{ij}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j. \quad (3)$$

Woosung Park, Sungsik Noh, Su Houng Lee (IPAP, Seoul & Yonsei U.), Masses of the doubly heavy tetraquarks in a constituent quark model, Nucl.Phys. A983 (2019) 1-19 [arXiv:1809.05257 [nucl-th]]

$T_{cc}(ud\bar{c}\bar{c})$, $T_{bb}(ud\bar{b}\bar{b})$, $T_{cb}(ud\bar{c}\bar{b})$, $us\bar{b}\bar{b}$ with $I(J^P) = 0(1^+)$ in S-wave states

Formalism – Spatial Wave Function

Total Wave Function

$$\Psi = \boxed{\psi(spatial)} \times \psi(flavor) \times \psi(color) \times \psi(spin)$$

For Mesons,

Radial Part of Solution for 3-D Symmetric HO

$$R_{n,l}(r) = \sqrt{\frac{2\,\Gamma(n+1)}{\Gamma\left(n+l+\frac{3}{2}\right)}}\, r^l \exp\left[-\frac{r^2}{2}\right] L_n^{l+\frac{1}{2}}\left(r^2\right). \quad \text{(A.1)}$$

Rescaling by $\boxed{r \to \sqrt{2}ax}$

Variational Parameter, a

$$\psi^{Spatial}_{[n,l,m]}(\mathbf{x}) = \psi(x,\theta,\phi)^{Spatial}_{[n,l,m]} = R_{n,l}(x) Y_l^m(\theta,\phi). \quad \text{(A.2)}$$

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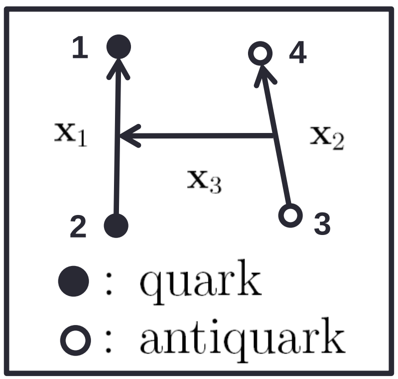
Formalism – Spatial Wave Function

For Tetraquarks,



$$
\begin{aligned}
&\psi(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)^{Spatial}_{[n_1,n_2,n_3,l_1,l_2,l_3]} \\
&= \sum_{m_1,m_2,m_3} C(l_1,m_1,l_2,m_2;L_{1,2}=l_3,m_{1,2}=-m_3) \\
&\quad \times C(L_{1,2}=l_3,m_{1,2}=-m_3,l_3,m_3;l=0,m=0) \\
&\quad \times R_{n_1,l_1}(x_1)R_{n_2,l_2}(x_2)R_{n_3,l_3}(x_3) \\
&\quad \times Y^{m_1}_{l_1}(\theta_1,\phi_1)Y^{m_2}_{l_2}(\theta_2,\phi_2)Y^{m_3}_{l_3}(\theta_3,\phi_3)\,. \qquad (11)
\end{aligned}
$$

a_1 → R_{n_1,l_1}, a_2 → R_{n_2,l_2}, a_3 → R_{n_3,l_3}

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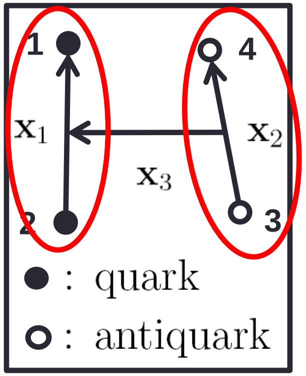
Result – Angular Momentum Contribution

TABLE I. The changes in masses of the tetraquarks when the indicated spatial bases $\psi^{Spatial}_{[n_1,n_2,n_3,l_1,l_2,l_3]}$ are included in the calculations.



Spatial Bases	$M_{T_{bb}}$ (MeV)	$M_{us\bar{b}\bar{b}}$ (MeV)	$M_{T_{cb}}$ (MeV)	$M_{T_{cc}}$ (MeV)
$\psi^{Spatial}_{[0,0,0,0,0,0]}$	10577	10763	7311	3993
$\psi^{Spatial}_{[0,0,0,0,0,0]}$, $\psi^{Spatial}_{[0,0,0,1,1,0]}$	10565	10747	7287	3952
$\psi^{Spatial}_{[0,0,0,0,0,0]}$, $\psi^{Spatial}_{[0,0,0,1,1,0]}$ $\psi^{Spatial}_{[0,0,0,1,0,1]}$, $\psi^{Spatial}_{[0,0,0,0,1,1]}$	10565	10744	7280	3951
⋮	⋮	⋮	⋮	⋮
167 Spatial Bases	10517	10694	7212	3873
Total Change in Mass	-60	-69	-99	-120

-41 MeV

-1 MeV

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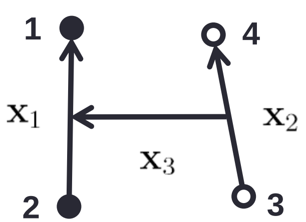
Result – Size and Structure of Tetraquarks

TABLE III. The relative distances between the quarks in the tetraquarks in fm unit. The distances are obtained with the ground state of the tetraquarks.

Quark Pair	T_{bb}	T_{cc}	T_{cb}	$us\bar{b}\bar{b}$
(1, 2)	0.676	0.830	0.753	0.644
(1, 3)	0.592	0.672	0.631	0.584
(1, 4)	0.592	0.672	0.612	0.584
(2, 3)	0.592	0.672	0.631	0.490
(2, 4)	0.592	0.672	0.612	0.490
(3, 4)	0.268	0.610	0.464	0.287
(1, 2)-(3, 4)	0.463	0.433	0.441	0.397



● : quark
○ : antiquark

$$\mathbf{x}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2),$$
$$\mathbf{x}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_3 - \mathbf{r}_4),$$
$$\mathbf{x}_3 = (\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4)$$

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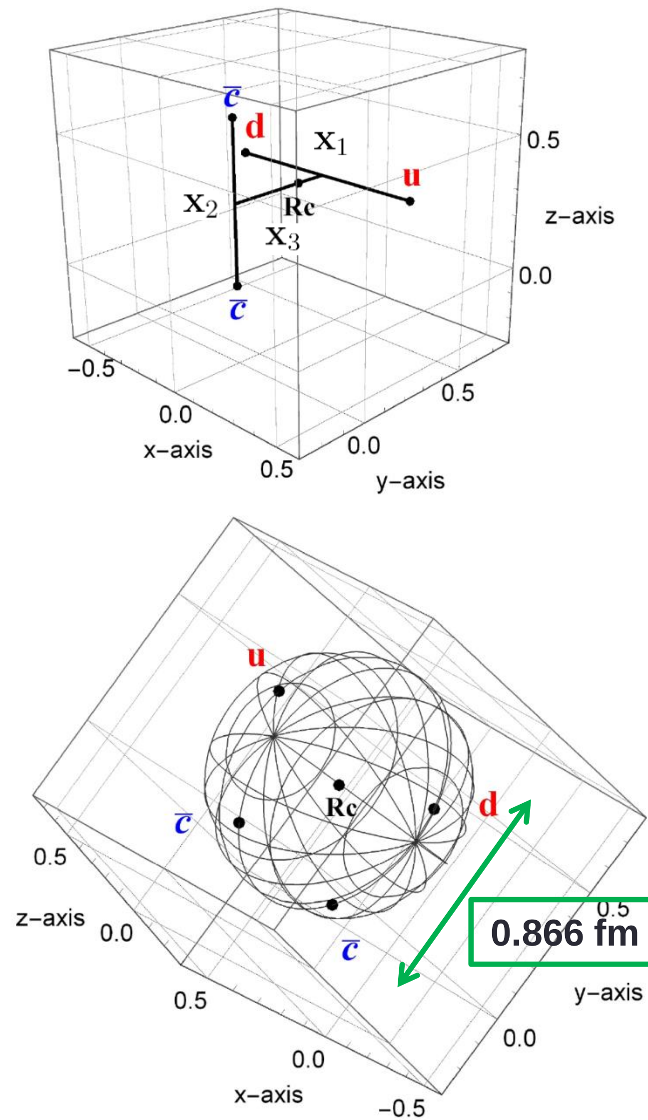

FIG. 1. The relative positions of the quarks in T_{cc} in fm unit. In the bottom figure, the diameter of the sphere is 0.866 fm.

Result – Discovery of Tcc at LHCb

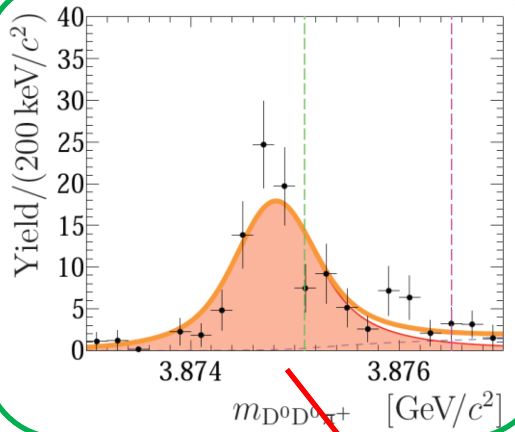

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TABLE IV. The masses of T_{cc} and the corresponding lowest threshold mesons D and D^*.

Type	T_{cc}	D		D^*	
		Theory	Exp.	Theory	Exp.
Mass (MeV)	3873	1854	1865	2006	2007

$$\text{Binding Energy } B_T \equiv M_{Tetraquark} - M_{Meson1} - M_{Meson2}$$

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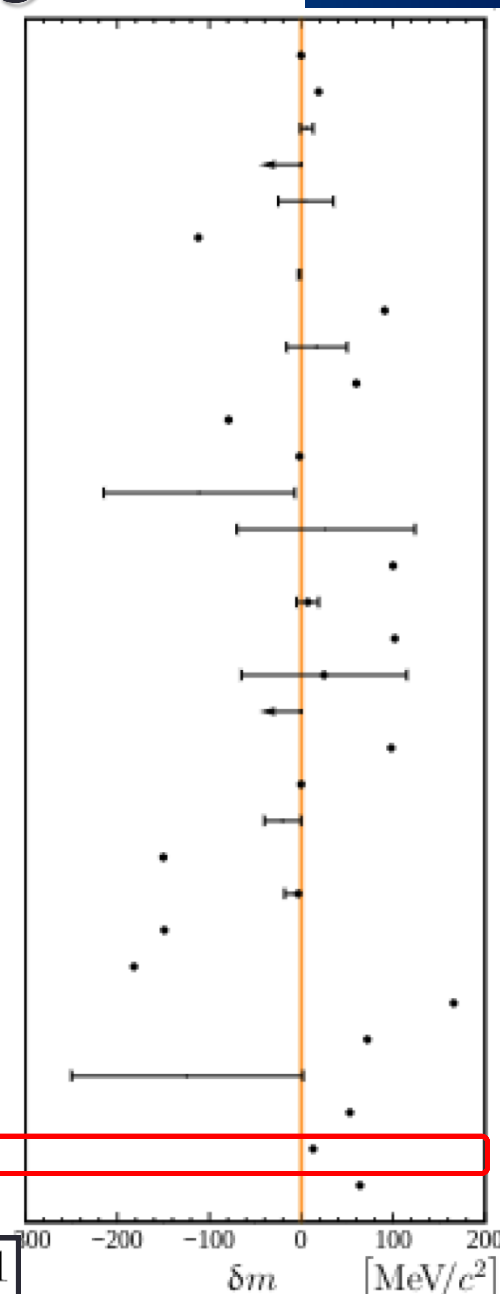

Ivan Polyakov on behalf of LHCb collaboration, EPS-HEP, 29 July 2021

Summary

- Harmonic Oscillator Bases as Spatial Functions

  - We constructed the spatial wave functions by introducing a complete set of re-scaled 3-D harmonic oscillator bases.

- Large Contributions from Internal Excited Orbital States

  - Internal orbital states are necessary to obtain the exact ground state masses for the tetraquarks.

- Model Calculations with a Compete Set of HO Bases

  - Our model provides result with highly precise accuracy.
  - Our prediction for Tcc mass is very close to the measurements at LHCb.
  - We need to improve our quark model by better fit for the threshold mesons.