The flavor content of light and heavy-light pseudoscalars mesons

Light Cone 2021: Physics of Hadrons on the Light Front
Jeju Islands - South Korea
28 November 2021 to 4 December-2021

J. Pacheco B. C. de Melo[a]

[a]Laboratório de Física Téorica e Computacional-LFTC, UCS (Brazil)
Collaborators: **R. Moita, T. Frederico (ITA, Brazil) and [a]K. Tsushima**

December 1, 2021

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**
- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**
- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- Constituent Picture and Unanbiguous Partons Content of the Hadronic System

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**
- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- Constituent Picture and Unanbiguous Partons Content of the Hadronic System
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**
- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- Constituent Picture and Unanbiguous Partons Content of the Hadronic System
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**
- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- Constituent Picture and Unanbiguous Partons Content of the Hadronic System
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**
- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- Constituent Picture and Unanbiguous Partons Content of the Hadronic System
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian:** $P^2 = P^+P^- - P_\perp^2$

Quark-meson spin coupling: effective Lagrangian

- **Coupling: quark / pseudoscalar meson field: SU(4) flavor symmetry**

$$\mathcal{L}_{\mathcal{I}} = -\imath g \, \bar{\Psi} M_{\mathrm{SU}(4)} \gamma^5 \Psi \equiv -\imath \, \frac{g}{\sqrt{2}} \sum_{i=1}^{15} (\bar{\Psi} \lambda_i \gamma^5 \Psi) \varphi^i \,,$$

- g **is a coupling constant, and** $\lambda_i \, (i = 1, ..., 15)$ **are the SU(4) Gell-Mann matrices**
- φ^i **is the Cartesian components of the pseudoscalar meson fields**
- **Ref. R. M. Moita, J. P. B. C. de Melo, K. Tsushima, and T. Frederico, "Exploring the flavor content of light and heavy-light pseudoscalars", Phys. Rev.D104, 096020 (2021)**

- **Quark field is $\Psi^T = (u, d, s, c)$ (T: transposition) decomposed in its quark-flavor components**

$$M_{SU(4)} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3\eta_c}{\sqrt{12}} \end{pmatrix}$$

$\Longrightarrow$ **SU(4) pseudoscalar meson field matrix**

- **Positively charged pseudo-scalar mesons**

$$\pi^+ = Tr\left[M_{SU(4)}\,\lambda_{\pi^+}\right],\; K^+ = Tr\left[M_{SU(4)}\,\lambda_{K^+}\right],$$
$$D^+ = Tr\left[M_{SU(4)}\,\lambda_{D^+}\right],\; D_s^+ = Tr\left[M_{SU(4)}\,\lambda_{D_s^+}\right],$$

- **Flavor matrices**

$$\lambda_{\pi^+} = \frac{1}{\sqrt{2}}(\lambda_1 + \imath\lambda_2),\; \lambda_{K^+} = \frac{1}{\sqrt{2}}(\lambda_4 + \imath\lambda_5),$$
$$\lambda_{D^+} = \frac{1}{\sqrt{2}}(\lambda_{11} - \imath\lambda_{12}),\; \lambda_{D_s^+} = \frac{1}{\sqrt{2}}(\lambda_{13} - \imath\lambda_{14}),$$

- **Bethe-Salpeter amplitude model**

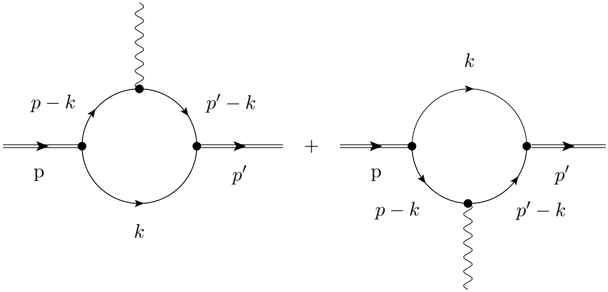
$$\Psi_M(k,p) = S_q(k)\gamma^5 g\Lambda_M(k,p)\lambda_M S_q(k-p)$$

- $S_q(k) \implies$ **Constituent quark propagator**
- **Vertex function** for $M = (\pi, K^+, D^+, D_s^+)$

$$g\Lambda_M(k,p) = \frac{C_M}{k^2 - \mu_M^2 + \imath\epsilon} + [k \to p-k],$$

- **Covariant normalization of the Bethe-Salpeter amplitude**

$$2ip^\mu = N_c \, \mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} g^2\Lambda_M^2(k,p)$$
$$\times \Big[\gamma^5\lambda_M S_q(k-p)\gamma^\mu S_q(k-p)\gamma^5\lambda_M^\dagger S_q(k)$$
$$+\gamma^5\lambda_M^\dagger S_q(k+p)\gamma^\mu S_q(k+p)\gamma^5\lambda_M S_q(k) \Big]$$



Feynman diagrams representing the electromagnetic interactions with pseudoscalar mesons for calculating the elastic electromagnetic form factors

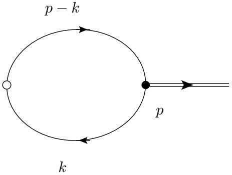
Weak decay constant

$$\Gamma(M \to l\nu_l) = \frac{G_F^2}{8\pi} f_M^2 m_l^2 m_M \left(1 - \frac{m_l^2}{m_M^2}\right)^2 |V_{q_1 q_2}|^2$$

- G_F **Fermi coupling constant,** m_l **lepton mass,** m_M **is the pseudoscalar meson mass, and** $V_{q_1 q_2}$ **CKM matrix element**
- **Pseudoscalar meson decay constant (axial-vector current)**

$$\langle 0 | A_\mu^j | M^k \rangle = \imath p_\mu f_{M^k} \delta^{jk}$$

- $A_\mu^j = \bar{q}(0)\gamma_\mu \gamma^5 \frac{\lambda_j}{2} q(0)$ **axial-vector current (**j **and** k **identify the isospin (flavor) components of the current operator and pseudoscalar meson)**



Diagrammatic representation of the pseudoscalar meson weak decay amplitude

$$\imath\, p^{\mu} f_M = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2} \mathrm{Tr}\left[\gamma^{\mu}\gamma^{5}\lambda_M^{\dagger}\Psi(k,p)\right]$$

$$f_M = \frac{N_c}{4\pi^3} \int d^2 k_{\perp} \int_0^1 dx\, \psi_M(x, \vec{k}_{\perp}; m_M, \vec{0}_{\perp}),$$

- **Valence wave function**

$$\psi_M(x,\vec{k}_\perp;p^+,\vec{p}_\perp) = \frac{p^+}{m_M}\frac{g\,C_M}{m_M^2-\mathcal{M}^2(m_q,m_{\bar{q}})} \times\left[\frac{1}{(1-x)(m_M^2-\mathcal{M}^2(m_q,\mu_M))} + \frac{1}{x(m_M^2-\mathcal{M}^2(\mu_M,m_{\bar{q}})}\right]+[m_q\leftrightarrow m_{\bar{q}}]$$

$$\mathcal{M}^2(m_1,m_2) = \frac{|\vec{k}_\perp|^2+m_1^2}{x} + \frac{|\vec{p}_\perp-\vec{k}_\perp|^2+m_2^2}{1-x} - |\vec{p}_\perp|^2$$

$$j^\mu = \frac{2}{3}\bar{u}\gamma^\mu u + \frac{2}{3}\bar{c}\gamma^\mu c - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s$$

- **$\Longrightarrow$ u, d, s and c are the quark fields**
- **Flavor space, diagonal charge operador**

$$\mathrm{diag}\ [\widehat{Q}] = [e_u,\ e_d,\ e_s,\ e_c] = [2/3,\ -1/3,\ -1/3,\ 2/3]$$

- **Matrix elements of the electromagnetic current for the mesons: Mandelstam formula**

$$\langle p'; M | j_1^\mu | p; M\rangle = -iN_c \int \frac{d^4k}{(2\pi)^4} g\Lambda_M(k,p') g\Lambda_M(k,p)$$
$$\times \mathrm{Tr}\left[\gamma^5 \lambda_M S_q(k-p)\widehat{Q}\gamma^\mu S_q(k-p')\gamma^5\lambda_M^\dagger S_q(k)\right],$$
$$\langle p'; M | j_2^\mu | p; M\rangle = -iN_c \int \frac{d^4k}{(2\pi)^4} g\Lambda_M(k,p') g\Lambda_M(k,p)$$
$$\times \mathrm{Tr}\left[\gamma^5 \lambda_M^\dagger S_q(k+p')\widehat{Q}\gamma^\mu S_q(k+p)\gamma^5\lambda_M S_q(k)\right]$$

- Microscopic Current: $\langle p'; M | j^\mu | p; M\rangle = \langle p'; M | j_1^\mu | p; M\rangle + \langle p'; M | j_2^\mu | p; M\rangle$

- **Form factor**

$$\langle p'; M | j^\mu | p; M \rangle = (p'^\mu + p^\mu) F_M(q^2)$$

- **Flavor decomposition: Two traces sum**

$$\mathrm{Tr}[\ ] = 2\left(\frac{2}{3}\Delta^\mu_{a\bar{b}a} + \frac{1}{3}\Delta^\mu_{\bar{b}a\bar{b}}\right),$$

here:

$$\Delta^\mu_{aba} = \mathrm{Tr}\left[\gamma^5 S^a_q(k-p)\gamma^\mu S^a_q(k-p')S^b_q(k)\gamma^5\right],$$

- **Current with the quark flavor content**

$$\langle p'; M | j^\mu | p; M \rangle = -2iN_c \int \frac{d^4k}{(2\pi)^4} g\Lambda_M(k,p') g\Lambda_M(k,p) \times \left(\frac{2}{3}\Delta^\mu_{a\bar{b}a} + \frac{1}{3}\Delta^\mu_{\bar{b}a\bar{b}}\right)$$

Flavor Decomposition: Electromagnetic Form Factor

- **Photon interacts with the quark a in the first term and with the antiquark $\bar{b}$ in the second one**

$$
\begin{aligned}
F_{\pi^+}(q^2) &= \frac{2}{3} F_{u\bar{d}u}(q^2) + \frac{1}{3} F_{\bar{d}u\bar{d}}(q^2)\,, \\
F_{K^+}(q^2) &= \frac{2}{3} F_{u\bar{s}u}(q^2) + \frac{1}{3} F_{\bar{s}u\bar{s}}(q^2)\,, \\
F_{D^+}(q^2) &= \frac{2}{3} F_{c\bar{d}c}(q^2) + \frac{1}{3} F_{\bar{d}c\bar{d}}(q^2)\,, \\
F_{D_s^+}(q^2) &= \frac{2}{3} F_{c\bar{s}c}(q^2) + \frac{1}{3} F_{\bar{s}c\bar{s}}(q^2)\,.
\end{aligned}
$$

- **SU(2) symmetry** $\implies m_u = m_d = m \implies$ **Equal quarks masses**
- Implies

$$F_{\pi^+}\left(q^2\right) = F_{u\bar{d}u}\left(q^2\right) = F_{\bar{d}u\bar{d}}\left(q^2\right)$$

- K^+, D^+**and** $D_s^+ \implies \mathrm{SU}(2), \mathrm{S}(3)$ **and** $\mathrm{SU}(4)$ **Symmetries are broken**
- **By Charge conservation**

$$F_{u\bar{s}u}(0) = F_{\bar{s}u\bar{s}}(0) = 1$$
$$F_{c\bar{d}c}(0) = F_{\bar{d}c\bar{d}}(0) = 1$$
$$F_{c\bar{s}c}(0) = F_{\bar{s}c\bar{s}}(0) = 1$$

- **LF technique**
- **Elastic photo-absorption transition amplitude**
- **Breit frame**

$$p^\mu = (\sqrt{m_M^2 + \frac{1}{4}q_x^2}, -\frac{1}{2}q_x, 0, 0) \text{ and } p'^\mu = (\sqrt{m_M^2 + \frac{1}{4}q_x^2}, \frac{1}{2}q_x, 0, 0)$$

- **Meson light-front momentum components**

$$p^+ = p'^+ = p^- = p'^-, \quad p'_\perp = (q_x/2, 0) \text{ and } p_\perp = (-q_x/2, 0)$$

$\Longrightarrow \quad q^+ = 0$ fullfilling the **Drell-Yan condition**

- **Plus component Electromagnetic current:** $J^+ = J^0 + J^3$
- **Current associated with** $\gamma^+ = \gamma^0 + \gamma^3$
- **Electromagnetic form factor**: $F_M(q^2) = \frac{1}{2p^+}\langle p'|j_M^+|p\rangle$
- VIP!! **Binding energy** $\Longrightarrow$ $\epsilon_M = m_a + m_{\bar{a}} - m_M > 0$

- **Results: Static Electroweak Observables**

Table: **The masses $m_q, m_{\bar{q}}$, μ_M, binding energy (ϵ_M) and decay constant (f_M) are given in [MeV]. The charge radius (r_M) is given in [fm].**

Model/meson	Flavors	$I(J^P)$	m_q	$m_{\bar{q}}$	m_M	ϵ_M	μ_M	r_M	f_M
(A) π^+	$u\bar{d}$	$1(0^-)$	384	384	140	628	225	0.665	92.55
(B)			220	220		300	600	0.736	92.12
(C) K^+	$u\bar{s}$	$\frac{1}{2}(0^-)$	384	508	494	398	420	0.551	110.8
(D)			220	440		166	600	0.754	110.8
(E) D^+	$c\bar{d}$	$\frac{1}{2}(0^-)$	1623	384	1869	138	1607	0.505	144.5
(F) D_s^+	$c\bar{s}$	$0(0^-)$	1623	508	1968	163	1685	0.377	182.7

Table: **Experimental dada, decay constant (f_M) are given in [MeV]. The charge radius (r_M) is given in [fm].**

Meson	Flavors	$I(J^P)$	$r_M^{\text{Expt.}}$	$f_M^{\text{Expt.}}$
π^+	$u\bar{d}$	$1(0^-)$	0.672(8)	92.28(7) [*]
K^+	$u\bar{s}$	$\frac{1}{2}(0^-)$	0.560(3)	110(1) [*]
D^+	$c\bar{d}$	$\frac{1}{2}(0^-)$	-	144(3) [*]
D_s^+	$c\bar{s}$	$0(0^-)$	-	182(3) [*],[**]

[*] P. A. Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020)
[**] M. Ablikim et al. (BESIII), Phys. Rev. Lett. 122, 071802 (2019)

Table: **Decay constants and electromagnetic radii of π^+ (model A) and K^+ (model C) in the present model, compared with the other works in the literature, as well as the experimental data in particle data group (PDG) [*]. The decay constants are in [MeV], and the charge radius are in [fm].**

Reference	f_π^+	f_K^+	r_{π^+}	r_{K^+}	f_{K^+}/f_{π^+}
Present work	92.55	110.8	0.665	0.551	1.196
Maris / Tandy	92.62	109.60	0.671	0.615	1.182
Faessler et al.	92.62	113.83	0.65		1.23
Ebert et al.	109.60	165.45	0.66	0.57	1.24
Bashir et al.	101				
Chen & Chang	93	111			1.192
Hutauruk et al.	93	97	0.629	0.586	1.043
Ivanov et al.	92.14	111.0			1.20
Silva et al.	101	129	0.672	0.710	1.276
Jia & Vary	142.8	166.7	0.68(5)	0.54(3)	1.166
PDG [*]	92.28(7)	110(1)	0.672(8)	0.560(3)	1.192(14)

- **References / Table**

- P. Maris and P. C. Tandy, Phys. Rev. C62, 055204(2000).
- A. Faessler, T. Gutsche, M. A. Ivanov, V. E. Lyubovitskij, and P. Wang, Physical Review D68 (2003), 10.1103
- D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Lett. B635, 93 (2006).
- D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C47,745 (2006).
- A. Bashir, L. Chang, I. C. Cloët, B. El-Bennich, Y.-X. Liu, C. D. Roberts, and P. C. Tandy, Commun. Theor. Phys.58, 79 (2012).
- M. Chen and L. Chang. Chin. Phys. C43. 114103 (2019).
- P. T. P. Hutauruk, I. C. Cloët, and A. W. Thomas, Phys. Rev. C94, 035201 (2016).
- M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, N. R. Soni, and C.-T. Tran, Front. Phys. (Beijing) 14, 64401(2019).
- E. O. da Silva, J. de Melo, B. El-Bennich, and V. S. Filho, Phys. Rev. C86,038202 (2012).
- S. Jia and J. P. Vary, Phys. Rev. C99, 035206 (2019).

Reference	f_{D^+}	$f_{D_s^+}$	r_{D^+}	$r_{D_s^+}$	$f_{D_s^+}/f_{D^+}$
This work	144.50	182.70	0.505	0.377	1.26
Faessler et al.	149.20	156.98			1.05
Bashir et al.	155.4	205.1			1.32
Ivanov et al.	145.7	182.2			1.25
Choi	149.2	179.6			1.20
Hwang	145.7(6.3)	189(13)	$0.406^{-0.012}_{+0.014}$	$0.300^{-0.018}_{+0.023}$	1.30(4)
Das et al.			0.510	0.465	
Dhiman & Dahiya	147.8	167.6			1.13
Tang et al.	295(63)	313(67)			1.06(32)
LQCD					
Aubin et al.	142(2)(12)	176(2)(11)			1.24(1)(7)
Follana et al.	147(3)	170(2)			1.16(3)
Chen et al.	143.1(1.6)(1.8)	182.9(0.8)(2.0)			1.28(3)
Carrasco et al.	146.6(2.6)(0.6)	174.8(2.8)(1.0)			1.19(3)(1)
Can et al.			0.371(17)		
			0.390(33)		
Li & Wu			0.402(61)	0.286(19)	
			0.420(82)	0.354(18)	
PDG	144(3)	182(3)			1.26(5)
Ablikin et al.		178.8(2.6)			

Table: Pseudoscalar meson static observables with the masses for D and D_s mesons from Lattice QCD ensembles (B1), (C1) used in Refs. [*] to compute the EM form factors; and the **present model (E,F) parameters:** $m_c = 1623$ **MeV,** $m_u = 384$ **MeV,** $m_s = 508$ **MeV,** $\mu_{D^+} = 1607$ **MeV, and** $\mu_{D_s^+} = 1685$ **MeV**. The mass and decay constant are given in MeV, while the charge radius is given in fm.

Inputs	(B1)	(C1)	(E,F)
m_D^+	1737	1824	1869
$m_{D_s^+}$	1801	1880	1968
Static observ.	**(B1)**	**(C1)**	**(E,F)**
r_{D^+} [*]	0.402(61)	0.420(82)	
r_{D^+}	0.347	0.422	0.505
f_{D^+}	205.5	170.3	144.5
$r_{D_s^+}$ [*]	0.286(19)	0.354(18)	
$r_{D_s^+}$	0.281	0.312	0.377
$f_{D_s^+}$	243.3	219.4	182.7

- **Ref. [*] N. Li and Y.-J. Wu, Eur. Phys. J. A53, 56 (2017).**

Radius (fm)	LQCD [1]	LQCD [2]	This work
r_{D^+}	0.371(17) (L) 0.390(33) (Q)	0.402(61) (B1) 0.420(82) (C1)	0.505
$r_{D^+,c}$	0.226(24) (L) 0.272(29) (Q)	0.17(15) (B1) 0.20(19) (C1)	0.233
$r_{D^+,\bar{d}}$	0.585(57) (L) 0.566(104) (Q)	0.692(61) (B1) 0.718(82) (C1)	0.810
$r_{D_s^+}$		0.286(19) (B1) 0.354(18) (C1)	0.377
$r_{D_s^+,c}$		0.119(50) (B1) 0.222(33) (C1)	0.218
$r_{D_s^+,\bar{s}}$		0.461(12) (B1) 0.545(15) (C1)	0.576

****Ref.**

[1] K. U. Can, G. Erkol, M. Oka, A. Ozpineci, and T. T. Takahashi, Phys. Lett. B 719, 103 (2013), arXiv: 1210.0869 **[hep-lat].**
[2] N. Li and Y.-J. Wu, Eur. Phys. J. A 53, 56 (2017).

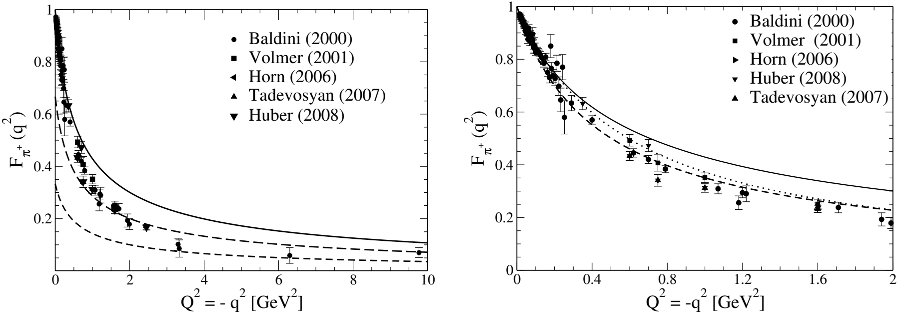
Table: Ratio of the electromagnetic form factors for pion (A) and kaon (C).

Reference	Q^2 [(GeV/c)^2]	F_{π^+}/F_{K^+}
This work	10.0	1.10
	13.48	1.14
	14.2	1.13
	17.4	1.16
Hutauruk et al. [1]	10.0	0.87
Bakulev et al. [2]	13.48	0.53
Shi et al. [3]	17.4	0.81
Pedlar et al. [4]	13.48	1.19(17)
Seth et al. [5]	14.2	1.21(3)
Seth et al. [5]	17.4	1.09(4)

• Refs. **[1] P. T. P. Hutauruk, I. C. Cloët, and A. W. Thomas, Phys. Rev. C 94, 035201 (2016).**

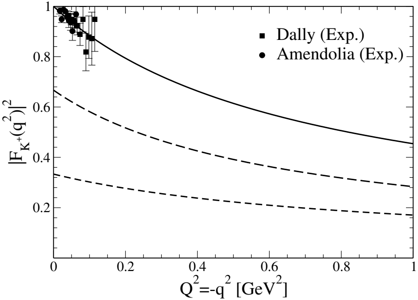
[2] A. P.Bakulev, S.V. Mikhailov, and N.G. Stefanis, Phys.Lett.B508,279 (2001), [Erratum: Phys.Lett.B 590, 309-310 (2004)]
[3] C. Shi, L. Chang, C. D. Roberts, S. M. Schmidt, P. C. Tandy, and H.-S. Zong, Phys. Lett. B738,512(2014)
[4] T. K. Pedlar et al. (CLEO), Phys. Rev. Lett. 95, 261803 (2005)¿

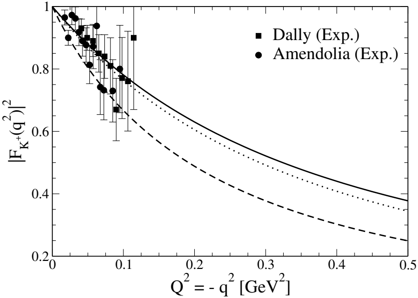
[5] K. K. Seth, S. Dobbs, Z. Metreveli, A. Tomaradze, T. Xiao, and G. Bonvicini, Phys. Rev. Lett. 110, 022002 (2013).



Pion electromagnetic form factor as a function of $q^2 < 0$. Left panel: Flavor decomposition of $F_\pi(q^2)$ for the parameter set (A) . Pion form factor (solid line), u contribution - $e_u F_{u\bar{d}u}$ (dashed line) and $\bar{d}$ contribution - $e_{\bar{d}} F_{\bar{d}u\bar{d}}$ (short-dashed line). Right panel: Comparison between the parameter sets (A) (solid line) and (B) (dashed line) with the VMD model (dotted line) Experimental data.

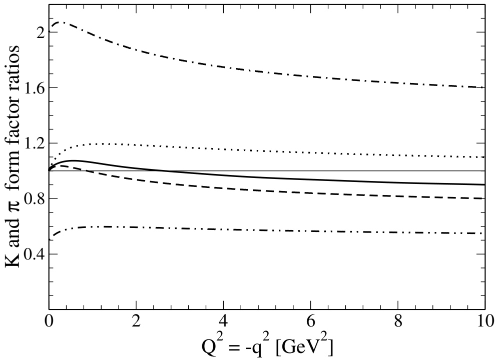
- Ref. Moita, de Melo, Tsushima, Frederico, Phy. Rev. D 104, 096020 (2021) ,e-Print: 2104.02787 [hep-ph]





Kaon electromagnetic form factor as a function $q^2 < 0$. Left panel: kaon form factor (full line), u contribution - $e_u F_{u\bar{s}u}$ (dashed line) and $\bar{s}$ contribution - $e_{\bar{s}} F_{\bar{s}u\bar{s}}$ (short-dashed line), computed with the parameter set (C). Right panel: $|F_{K^+}|^2$, comparison between the results from parameter sets (C) (full line), (D) (dashed line) and the VMD model (dotted line). Experimental [*]

- Ref. Moita, de Melo, Tsushima, Frederico, Phy. Rev. D 104, 096020 (2021) ,e-Print: 2104.02787 [hep-ph]



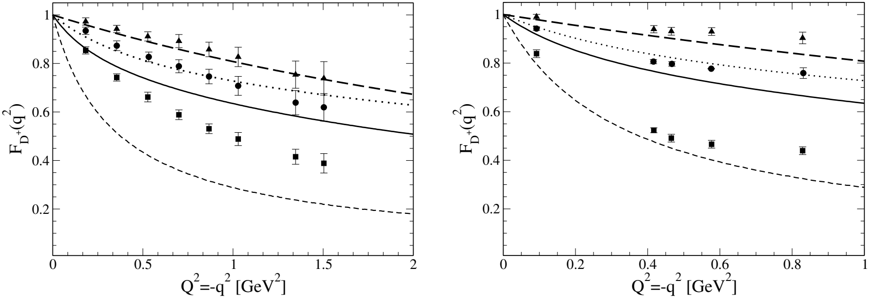
The electromagnetic form factor ratios for the pion and kaon using sets (A) and (C), respectively. Ratios of flavor contributions to the pion and kaon form factors: F_{K^+}/F_{π^+} (solid line); $e_u F_{u\bar{s}u}/(e_{\bar{d}} F_{\bar{d}u\bar{d}})$ (dot-dashed line), $F_{u\bar{s}u}/F_{u\bar{d}u}$ (dashed line); $F_{\bar{s}u\bar{s}}/F_{\bar{d}u\bar{d}}$ (dotted line); $e_{\bar{s}} F_{\bar{s}u\bar{s}}/(e_u F_{u\bar{s}u})$ (dot-dot-dashed line). A thin solid line is the reference for the SU(3) flavor symmetry.

- **Flavored Vector Meson Dominance**

$$F_{D^+}\left(q^2\right) = \frac{2}{3}\frac{m^2_{J/\psi}}{m^2_{J/\psi} - q^2} + \frac{1}{3}\frac{m^2_{\rho}}{m^2_{\rho} - q^2}$$

$$F_{D_s^+}\left(q^2\right) = \frac{2}{3}\frac{m^2_{J/\psi}}{m^2_{J/\psi} - q^2} + \frac{1}{3}\frac{m^2_{\phi}}{m^2_{\phi} - q^2}$$

- **Quarks contents**
- $|j/\psi >= |c\bar{c}\rangle, \;\; m_{J/\psi} = 3.096916 \; [GeV]$
- $|\phi\rangle = |s\bar{s}\rangle, \;\; m_{\phi} = 1.019445 \pm 0,020 [Gev]$
- Ref. **P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)**

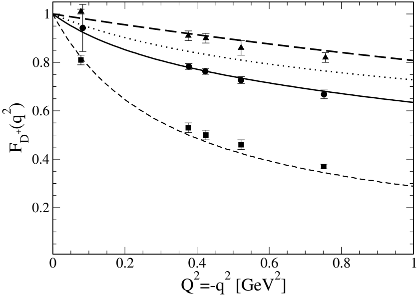


D^+ electromagnetic form factor with the corresponding quark contributions and comparison with lattice calculations. Our results: D^+ form factor (full line), $\bar{d}$ contribution - $e_{\bar{d}} F_{\bar{d}c\bar{d}}$ (short-dashed line) and c contribution - $e_c F_{c\bar{d}c}$ (dashed line). VMD (dotted line). LQCD results for D^+ form factor (circles), $\bar{d}$ contribution (squares) and c contribution (triangles).

Left panel:comparison with LQCD results [*]

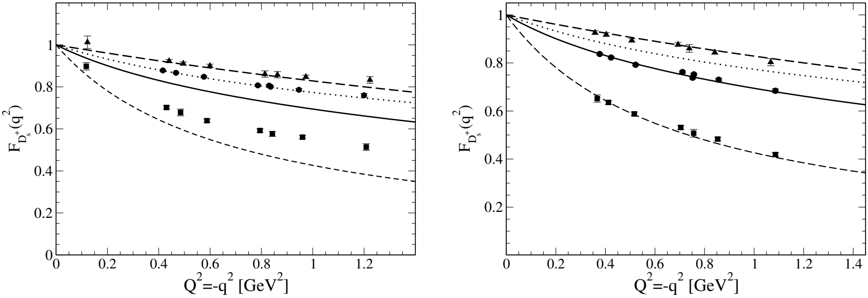
Right panel:comparison with LQCD results from ensemble (C1) [**].

Ref. [*] K. U. Can, G. Erkol, M. Oka, A. Ozpineci, and T. T. Takahashi, Phys. Lett. B 719, 103 (2013)



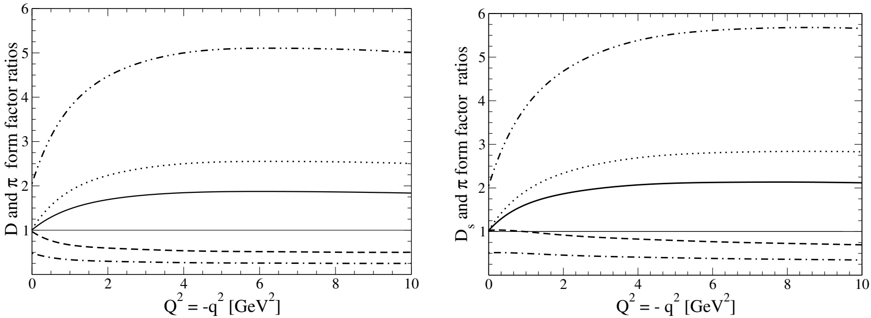
D^+ electromagnetic. LQCD results from ensemble (C1) [**]

Ref. [**] N. Li and Y.-J. Wu, Eur. Phys. J. A 53, 56 (2017)



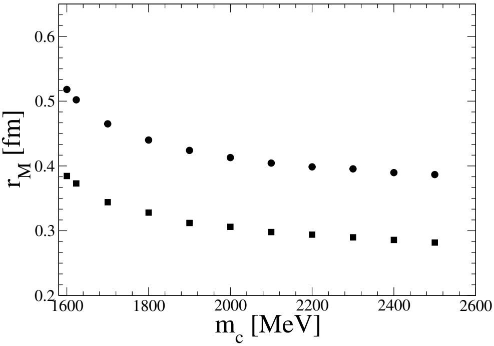
D_s^+ electromagnetic form factors with the corresponding quark contributions and comparison with lattice calculations. Our results: D_s^+ form factor (full line), $\bar{d}$ contribution (short-dashed line) and c contribution (dashed line), VMD (dotted line). LQCD results for D_s^+ form factor (circles), $\bar{s}$ contribution (squares) and c contribution (triangles). Comparison with LQCD results from ensemble (C1) [**].

Ref. [**] N. Li and Y.-J. Wu, Eur. Phys. J. A 53, 56 (2017)



The electromagnetic form factor ratios for the pion (A) and D mesons.
Left panel: F_{D^+}/F_{π^+} **(solid line);** $F_{\bar{d}c\bar{d}}/F_{\bar{d}u\bar{d}}$ **(dashed line);** $e_{\bar{d}}F_{\bar{d}c\bar{d}}/(e_u F_{u\bar{d}u})$ **(dot-dashed line);** $F_{c\bar{d}c}/F_{u\bar{d}u}$ **(dotted line);** $e_c F_{c\bar{d}c}/(e_{\bar{d}}F_{\bar{d}u\bar{d}})$ **(dot-dot-dashed line). Right panel:** $F_{D_s^+}/F_{\pi^+}$ **(solid line);** $F_{\bar{s}c\bar{s}}/F_{\bar{d}u\bar{d}}$ **(dashed line);** $e_{\bar{s}}F_{\bar{s}c\bar{s}}/(e_u F_{u\bar{d}u})$ **(dot-dashed line);** $F_{c\bar{s}c}/F_{u\bar{d}u}$ **(dotted line);** $e_c F_{c\bar{s}c}/(e_{\bar{d}}F_{\bar{d}u\bar{d}})$ **(dot-dot-dashed line).**

The thin solid line is the reference for the SU(4) flavor symmetry.



Charge radii of D^+ and D_s^+ mesons as a function of the charm quark mass, m_c, having all other parameters fixed to set (E) for D^+ (circles) and to set (F) for D_s^+ (squares) .

Summary and Conclusions

- $\Longrightarrow$ **Computation of Form-Factors and Decay Constants**
- $\Longrightarrow$ **Easy to test different analytical models**
- $\Longrightarrow$ **Correct Asymptotic Form-Factors**
- $\Longrightarrow$ **Scalar mesons proprieties**
- $\Longrightarrow$ **Agreement LQCD and others models**

$\Longrightarrow$ **Next**

$\star$ **B mesons**
$\star$ **Heavy mesons**
$\star$ η_c, η_b, ,J/ψ, Υ
$\star$ **Vector mesons**
$\star$ **Partons distributions amplitudes, GPD's, TMD's ...**

Thanks to the Organizers
Light-Cone 2021

Support LFTC and Brazilian Agencies

- **FAPESP** , **CNPq** and **CAPES**

Thanks (Obrigado)!!