Baryonic form factor of the pion

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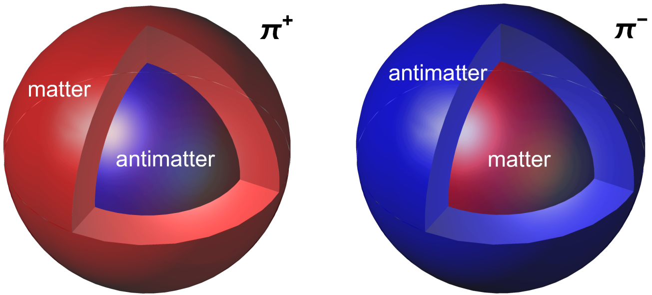
Sneak preview

$\pi^+ = u\bar{d}$, u - baryon charge (matter), $\bar{d}$ - anti-baryon charge (antimatter)



Structure of π^+

lighter u sticks out more outside, heavier $\bar{d}$ sits more inside

Symmetries and the baryon ff of the pion

Conserved vector currents

QCD:

$$\partial_\mu \left[\bar{q}_f(x)\gamma^\mu q_f(x)\right] = 0, \qquad f = u, d, s, c, b, t \quad -\text{flavor}$$

$\rightarrow$ quark number of any species conserved

$$J_B^\mu = \frac{1}{N_c}\left(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d + \dots\right), \quad J_3^\mu = \frac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right), \quad J_Q^\mu = J_3^\mu + \frac{1}{2}J_B^\mu \qquad \text{(all conserved)}$$

Baryon, isospin, and charge form factors

$$\langle \pi^a(p) \mid J^\mu_{B,3,Q}(0) \mid \pi^a(p+q)\rangle = (2p^\mu + q^\mu)F^a_{B,3,Q}(q^2), \quad a = 0, +, - \quad \text{(pion isospin)}$$

Symmetries

π^0: $I^G(J^{PC}) = 1^-(0^{-+})$, $\pi^\pm$: $I^G(J^P) = 1^-(0^-)$, $C|\pi^\pm\rangle = |\pi^\mp\rangle$, $G = Ce^{i\pi I_2}$

$J^\mu_{B,3,Q}$ are odd under C $\rightarrow$

$F^{\pi^0}_{B,3,Q}(q^2) = 0$ and $F^{\pi^+}_{B,3,Q}(q^2) = -F^{\pi^-}_{B,3,Q}(q^2)$ – always true!

e.g., $\langle\pi^0(p)|J^\mu_B(0)|\pi^0(p+q)\rangle = -\langle\pi^0(p)|CJ^\mu_B(0)C|\pi^0(p+q)\rangle = -\langle\pi^0(p)|J^\mu_B(0)|\pi^0(p+q)\rangle = 0$

Similarly, for **exact** isospin (and G) symmetry ($m_u = m_d$ and neglecting small EM effects)

J^μ_B is odd under G $\rightarrow$

$F^{\pi^\pm}_B(q^2) = 0$ ($F^{\pi^\pm}_3(q^2) \neq 0$, as J^μ_3 is even under G)

However, isospin (and G) are broken with $m_d > m_u$ and EM

$F^{\pi^\pm}_B(q^2)$ can be (and **is**) nonzero, with $F^{\pi^+}_B(q^2) = -F^{\pi^-}_B(q^2)$

Quark mass splitting

A form factor at $q = 0$ is the corresponding charge. Charges are additive. Baryon charge of π^a is 0 $\rightarrow$

$$F_B^{\pi^\pm}(0) = 0$$

(but, as said, not at $q^2 \neq 0$). On the other hand, $F_3^{\pi^\pm}(0) = \pm 1$

- If a quantity is not protected by symmetry, hence need not be zero, it usually **is nonzero**
- Magnitude is proportional to the strength of the symmetry breaking
- No probes with baryon number couple directly to the pion (except on lattice QCD) $\rightarrow$ we need indirect methods to estimate the effect

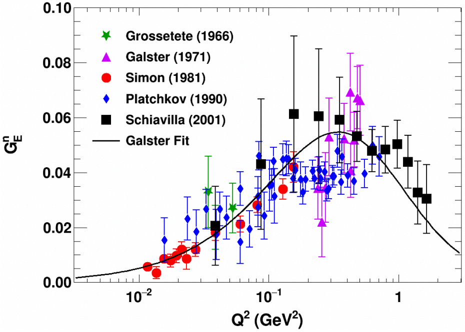
Current quark mass splitting at 2 GeV (PDG)

$m_u/m_d = 0.47^{+0.06}_{-0.07}$, $m = \frac{1}{2}(m_u + m_d) = 3.45^{+0.55}_{-0.15}$ MeV $\rightarrow$ $m_d - m_u = 2.5(1)$MeV

- $m_d \neq m_u$ –a.k.a. charge symmetry breaking
- EM violating effects more tricky, of the order $\alpha_{\mathrm{QED}}/(2\pi) \sim 0.001$

Neutron

Reminiscent of the neutron, which has no electric charge, but has a non zero ff (for $q^2 \neq 0$):



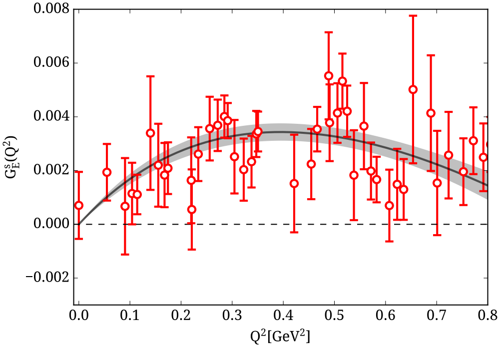
unpolarized elastic ed scattering

[Obrecht 2019]

Strangeness in the nucleon

Another example: strange ff's of the nucleon, $G^s_{E,M}$

[Jaffe 1989, Musolf, Burkardt 1993, Forkel, Cohen, Forkel, Nielsen 1993,...]



Alexandrou et. al (ETM Coll.) 2020

Effective Lagrangian estimate

Order of magnitude from effective Lagrangian

At leading order in the pion momenta and the quark mass splitting

$$J_B^\mu = -2i\frac{c\Delta m}{\Lambda^3}\partial_\nu\left(\partial^\mu\pi^+\partial^\nu\pi^- - \partial^\nu\pi^+\partial^\mu\pi^-\right) + \ldots$$

(odd under C, trivially conserved) c – dimensionless number, Λ – typical hadronic scale
$c/\Lambda^3 = \frac{8B_0}{N_c F^4}(2C_{63} - C_{65})$ with coefficients from the $\mathcal{O}(p^6)$ χPT Lagrangian [Bijnens 1999]

Baryonic ms radius

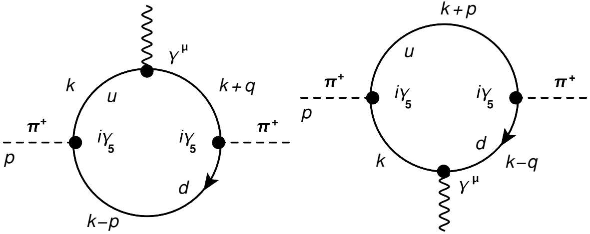
$\langle r^2\rangle_B^{\pi^+} = 6c\Delta m/m_\rho^3 \simeq c\ 0.002\mathrm{fm}^2 \simeq c(0.04\mathrm{fm})^2$, $\quad c \sim 1$, sign undetermined

– as expected, small compared to the charge radius $\langle r^2\rangle_Q^{\pi^+} = 0.434(5)\mathrm{fm}^2 = (0.659(4)\mathrm{fm})^2$

Chiral quark model estimate

Nambu–Jona-Lasinio (NJL) model

Covariant field-theoretic model. Dynamical chiral symmetry breaking, point-like interaction, large-N_c (one-loop), regularization. Generally successful for soft matrix elements of the pion in various processes (including PDF, GPD, TDA, quasi/pseudo PDF, dPDF, ...)



Quarks acquire a large masses $M_{d,u} \sim 300$ MeV, pion is a pseudo-Goldstone boson
$\Delta = M_d - M_u$, $\quad M = \frac{1}{2}(M_d + M_u)$, $\quad f$ - pion decay constant [Pauli-Villars regularization]

NJL 2

Result very simple in the small Δ and chiral limits:

$$F_B^{\pi^+}(t) = \frac{\Delta M^3}{2\pi^2 f^2 t}\left[\log^2\left(\frac{1+s}{1-s}\right) - 2s\log\left(\frac{1+s}{1-s}\right)\right]\Bigg|_{\mathrm{reg}}, \quad s = 1/\sqrt{1-\frac{4M^2}{t}}$$

With finite m_π

$$F_B^{\pi^+}(t) = t\frac{\Delta}{24\pi^2 f^2 M}\left[1 - \frac{3M^4}{\Lambda^4} + \frac{4m_\pi^2}{15M^2} - \frac{N_c m_\pi^2}{12\pi^2 f M} + \mathcal{O}(m_\pi^4, \Lambda^{-6})\right]$$

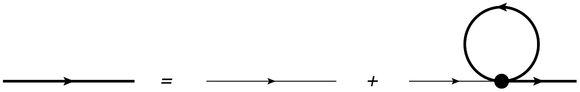
$$\langle r^2\rangle_B^{\pi^+} = \frac{\Delta}{4\pi^2 f^2 M} + \mathcal{O}(m_\pi^2, \Lambda^{-4})$$

The ratio of the baryon to charge ms radii is

$$\frac{\langle r^2\rangle_B^{\pi^+}}{\langle r^2\rangle_Q^{\pi^+}} = \frac{\Delta}{N_c M} + \mathcal{O}(m_\pi^2, \Lambda^{-2})$$

Obviously, estimates need the value of the constituent mass splitting Δ

Estimating $\Delta = M_d - M_u$

In models with the gap equation 

$$M_f(m_f) = m_f - G\langle \bar{q}_f q_f\rangle(m_f) = m_f - M_f(0)\frac{\langle \bar{q}_f q_f\rangle(m_f)}{\langle \bar{q}_f q_f\rangle(0)}$$

$$= M_f(0) + \left[1 + M_f(0)\frac{d}{dm_f}\log(-\langle \bar{q}_f q_f\rangle)\Big|_{m_f=0}\right] m_f + \mathcal{O}(m_f^2) = M_f(0) + \alpha m_f, \quad \alpha \simeq 2 - 2.4$$

(in NJL and also on the lattice). Another enhancement comes from QCD running of the constituent masses. At the (low) quark model scale μ_0

$$m_f(\mu_0) = \left[\frac{\alpha_S(\mu_0)}{\alpha_S(2\mathrm{GeV})}\right]^{\frac{4}{9}} m_f(2\mathrm{GeV}) \simeq 2\, m_f(2\mathrm{GeV})$$

NJL

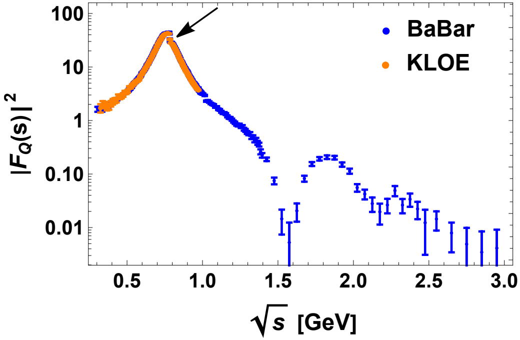
NJL: $M = 300$ MeV, $\Delta = 9 - 13$ MeV $\quad \rightarrow \quad \langle r^2\rangle_B^{\pi^+} \simeq (0.06(1)\ \mathrm{fm})^2$

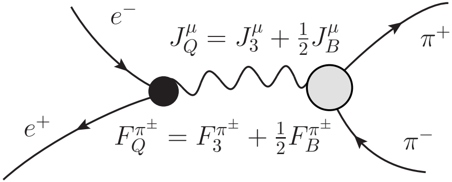
A comparable estimate can be extracted from a similar approach of [Hutauruk et al. 2018]

Determination from exp. data (!)

$e^+e^- \to \pi^+\pi^-$

Long tradition of $e^+e^- \to \pi^+\pi^-$ measurements

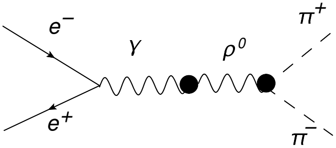




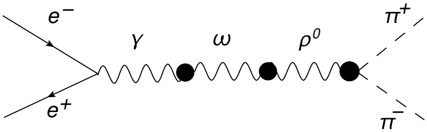
Arrow indicates the wiggle due to $F_B^{\pi^\pm} \neq 0$!

Vector meson dominance

isospin part



baryonic part



$\rho_0 - \omega$ mixing

$$F_3^{\pi^+}(s) = \frac{1}{1+c'+c''+c'''}\left[D_{\rho^0}(s) + c' D_{\rho'^0}(s) + c'' D_{\rho''^0}(s) + c''' D_{\rho'''^0}(s)\right]$$

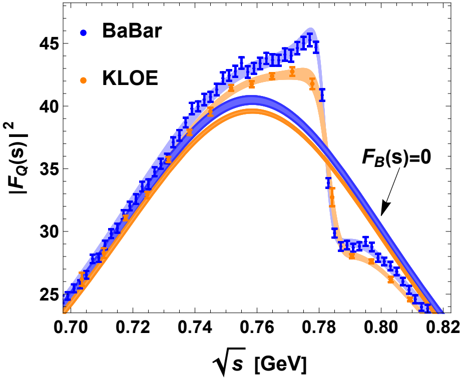
$$\frac{1}{2} F_B^{\pi^+}(s) = c_{\rho^0\omega}\, s\, D_{\rho^0}(s) D_\omega(s), \qquad D_V(s) = \frac{1}{m_V^2 - s - i\, m_V \Gamma_V(s)}$$

[Gounaris-Sakurai 1968] – largely used by exp. groups

We make sure that $F_B^{\pi^\pm}(0) = 0$

Our fit to KLOE and BaBar

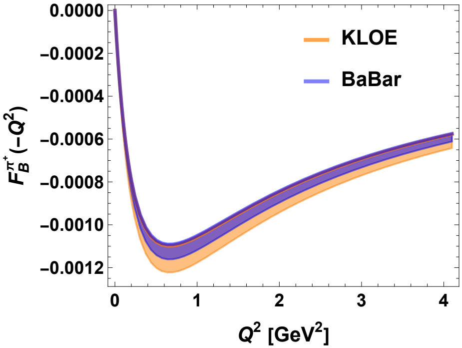
... shown in the relevant range of s

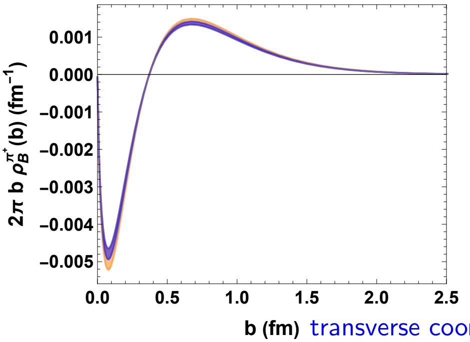


Necessity of $F_B^{\pi^\pm} \neq 0$ ($\rho - \omega$ mixing)

Continuation space-like Q^2 with the dispersion relation

$$F_B^{\pi^\pm}(-Q^2) = \frac{1}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im}F_B^{\pi^\pm}(s)}{s+Q^2} = -\frac{Q^2}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im}F_B^{\pi^\pm}(s)}{s(s+Q^2)}$$





transverse coordinate

$e^+e^- \to \pi^+\pi^-$:

BaBar: $\langle r^2\rangle_B^{\pi^+} \simeq (0.0411(7)\ \mathrm{fm})^2$, KLOE: $\langle r^2\rangle_B^{\pi^+} \simeq (0.0412(12)\ \mathrm{fm})^2$ (stat. errors only)

Comparison of our various estimates

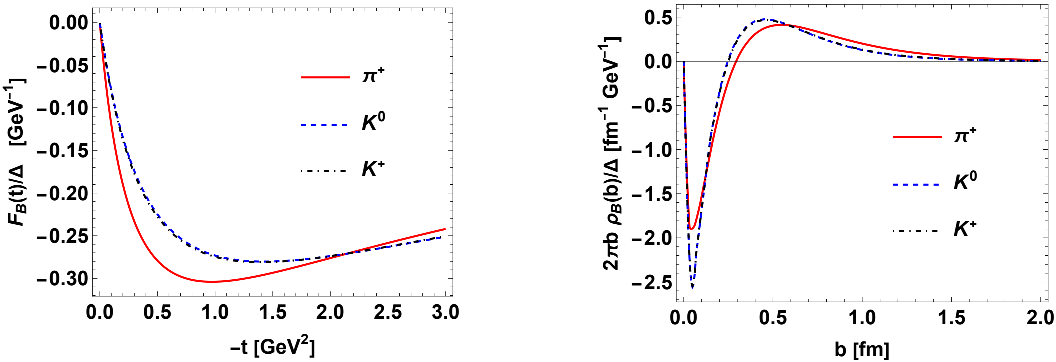
approach	$\langle r^2\rangle_B^{\pi^+}$	comment
effective Lagrangian	$c(0.04\ \mathrm{fm})^2$	c - number of order 1, any sign
NJL with PV reg.	$(0.06(1)\ \mathrm{fm})^2$	controlled by Δ/M
BaBar	$(0.041(1)\ \mathrm{fm})^2$	VMD, statistical error only
KLOE	$(0.041(1)\ \mathrm{fm})^2$	—

- Agreement within a factor of ~ 2 between very different methods
- NJL at leading N_c (no pion loops) and without EM
- BaBar and KLOE involve both $m_d - m_u$ and EM

Kaon

Kaon in NJL

Full analogy to π^+: for $K^+ = u\bar{s}$ replace $d \to s$, for $K^0 = d\bar{s}$ replace $u \to d$ and $d \to s$
NJL: $m_s/m = 26$ (fits m_K), PDG: $m_s/m = 27.3^{+0.7}_{-1.3}$



(for π^+, K^0, $K+$, correspondingly, $\Delta = M_d - M_u$, $\Delta = M_s - M_d$, $\Delta = M_s - M_u$)

Kaon baryonic radius

NJL:

$$\langle r^2 \rangle_B^{K^+} = (0.24(1)\ \mathrm{fm})^2, \quad \langle r^2 \rangle_B^{K^0} = (0.23(1)\ \mathrm{fm})^2$$

In NJL, $\langle r^2 \rangle_B^{K^0} = -\langle r^2 \rangle_Q^{K^0}$, since the baryon number and electric charge of d and $\bar{s}$ quarks are equal and opposite

PDG: $\langle r^2 \rangle_Q^{K^0} = -(0.28(2)\ \mathrm{fm})^2$, of the same sign and close in magnitude to NJL

Within the reach of the lattice

Conclusions

Outlook

1. Fundamental feature of the pion, eventually should end up in PDG Tables (!)
2. Any approach with conserved currents can provide an estimate
3. Small, but as shown, possible to extract from the present experimental data – could be elevated to a strict determination after some experimental and theoretical systematic issues are resolved
4. Our estimates from very different approaches yield $\langle r^2\rangle_B^{\pi^+} = (0.04 - 0.06 \text{ fm})^2 = 0.002 - 0.004 \text{ fm}^2$
5. Sign follows the "mechanistic" interpretation: heavier particle more inside
6. Lattice QCD: $\langle r^2\rangle_Q^{\pi} = (0.648(15) \text{ fm})^2 = 0.42(2) \text{ fm}^2$ – our signal for the baryon ff is too small (0.002 vs 0.02) to be currently detected on the lattice (but still could be tried with extrapolation in mass splitting)
7. Good lattice prospects for the kaon or heavy-light mesons

감사합니다