A study of J/ψ baryon decays beyond the leading twist accuracy

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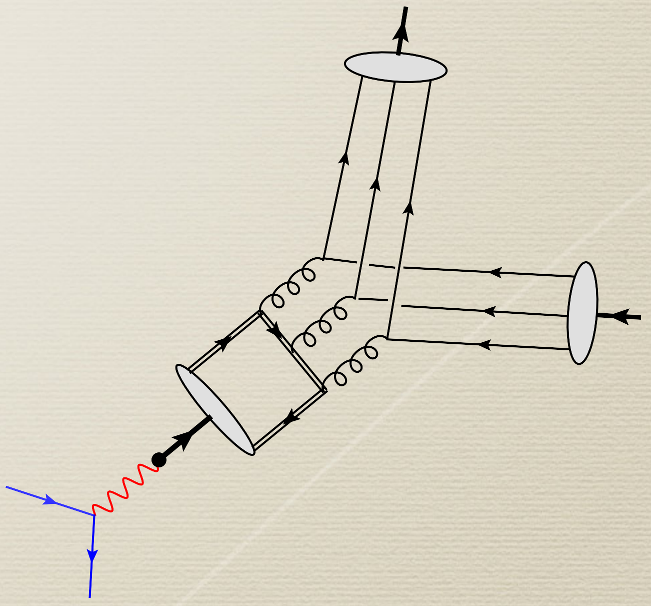
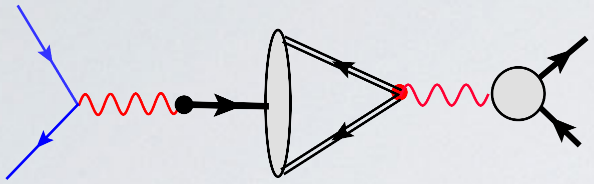


Eur.Phys.J. A 56(2020),
arXiv:2109.05847, to appear in Eur.Phys.J. A,
and new results (in preparation)

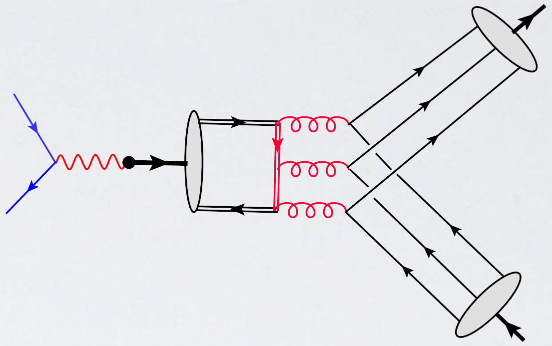
Light Cone 2021: Physics of hadrons on the light front,
November 29-December 4, 2021

$J/\psi \to B\bar{B}$ decays



e.m. FFs

$G_M,\, G_E$



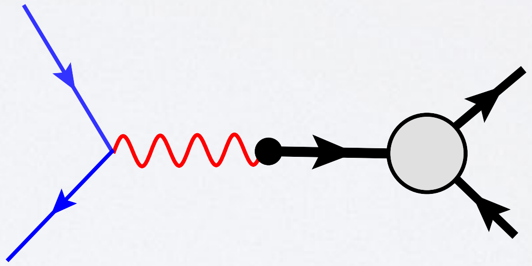
3-gluon annihilation

$G_M^g,\, G_E^g$

$$\mathrm{Br}[J/\psi \to B\bar{B}] = \frac{M_\psi \beta}{12\pi\Gamma_{tot}} \left|G_M^g + G_M\right|^2 \left(1 + \frac{2m_B^2}{M_\psi^2}\gamma_B^2\right)$$

$$\gamma_B = \left|\frac{G_E^g + G_E}{G_M^g + G_M}\right|$$

$$\frac{dN_B}{d\cos\theta} = \mathcal{N}(1 + \alpha_B \cos^2\theta)$$



$$\alpha_B = \frac{1 - 4m_B^2\gamma_B^2/M_\psi^2}{1 + 4m_B^2\gamma_B^2/M_\psi^2}$$

$m_Q \to \infty \quad \alpha = 1$

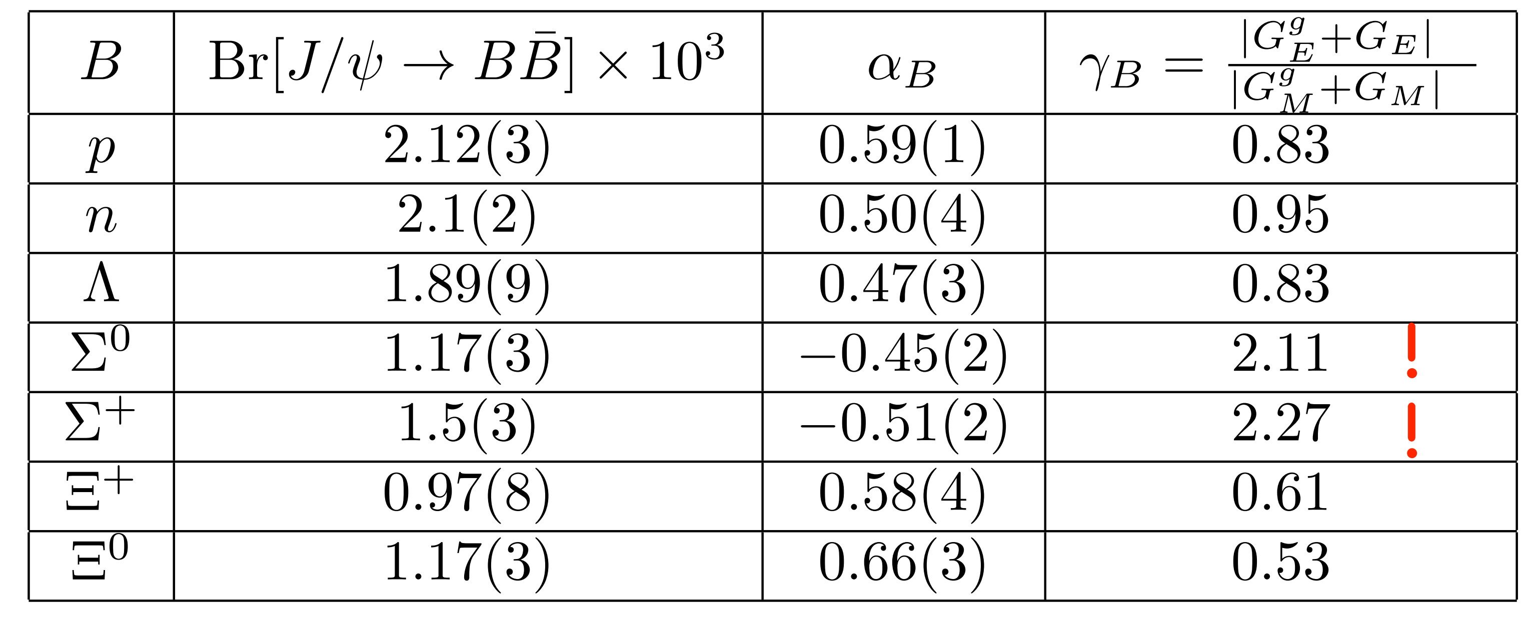
Brodsky, Lepage 1981

$$\frac{4m_N^2}{M_\psi^2} \simeq 0.37$$

$$\frac{4m_N^2}{M_\Upsilon^2} \simeq 0.04$$

J/ψ baryonic decays

Data: BESIII/2008/2012/2016/2017/2019/2020/ & PDG



B	$\mathrm{Br}[J/\psi \to B\bar{B}] \times 10^3$	α_B	$\gamma_B = \frac{\lvert G_E^g+G_E\rvert}{\lvert G_M^g+G_M\rvert}$
p	2.12(3)	0.59(1)	0.83
n	2.1(2)	0.50(4)	0.95
Λ	1.89(9)	0.47(3)	0.83
Σ^0	1.17(3)	−0.45(2)	2.11 !
Σ^+	1.5(3)	−0.51(2)	2.27 !
Ξ^+	0.97(8)	0.58(4)	0.61
Ξ^0	1.17(3)	0.66(3)	0.53

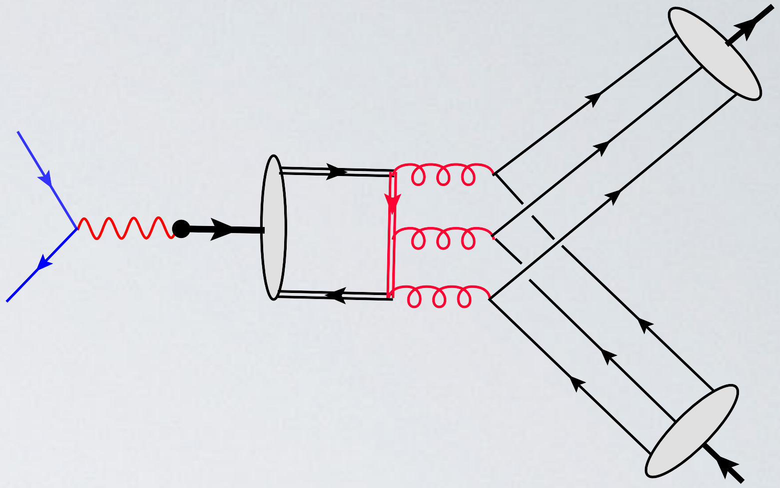
Theoretical estimates

Largest amount of papers are dedicated to Br's and constrains of baryon DAs

Carimalo, 1987 $\alpha_N = 0.70$ constituent quarks, non-relativistic nucleon WF

Murgia, Melis, 1995 $\alpha_N = 0.561 - 0.963$ constituent quark mass $m_i = x_i m_N$
& the LT factorisation formula

J/ψ baryonic decays



Theory: NRQCD + QCD collinear factorisation

$$G_M^g \simeq R_{10}(0)\ \{\underbrace{\phi^{\rm tw3} * T_{33} * \phi^{\rm tw3}}_{\text{LP}} + \underbrace{\phi^{\rm tw3} * T_{35} * \phi^{\rm tw5} + \phi^{\rm tw4} * T_{44} * \phi^{\rm tw4}}_{\text{NLP}\quad \sim \Lambda^2/m_c^2}\}$$

$$G_E^g \simeq R_{10}(0)\, \phi^{\rm tw3} * T_{34} * \phi^{\rm tw4}$$

$G_M^g(\mathrm{LP})$ Brodsky, Lepage 1981/ Chernyak et al, 1984

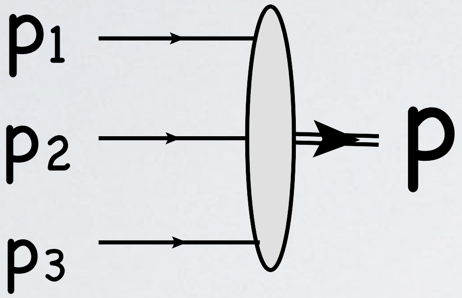
$G_M^g(\mathrm{NLP})$, G_E^g NK, 2020/2021 & 2021 (in preparation)

$\gamma_B^g = \left|\dfrac{G_E^g}{G_M^g}\right|$ dominates & sensitive to a baryon structure !

Baryon Light-Cone Distribution Amplitudes

Long distance physics associated with the nucleon WFs

Light-cone Distribution Amplitude $\phi(x_i)$ describes how the long. momentum is shared between the constituents



$$\phi^{\text{tw3}} \equiv \phi_3(x_i,\mu) \sim \int_{|p_{i\perp}|\lesssim\mu} dp_{i\perp}\,\Psi(x_i,p_{i\perp})$$

$p_i = x_i\,p + p_{\perp i}$ $\quad p_{\perp i} \sim \Lambda$

Defined as a light-cone matrix element

$$\langle 0|\varepsilon^{abc} q^a(z_1) q^b(z_2) q^c(z_3)|B(p)\rangle_{z_i^2=0} \sim \mathrm{FT}[\phi_3(x_i)]$$

$$\phi_3(x_i,\mu) = f_B(\mu) x_1 x_2 x_3 (1 + \phi_{10}^{(3)}(\mu) P_{10}(x_i) + \phi_{11}^{(3)}(\mu) P_{11}(x_i) + \ldots)$$

$P_{kn}(x_i)$ homogeneous orthogonal polynomials of degree k

$f_B, \phi_{ij}^{(3)}$ non-perturbative moments, m.e.'s of local operators

Baryon Light-Cone DAs: 3-quark operators only

$B = N, \Lambda, \Sigma, \Xi$

$\langle 0 | [q(z_1) C \Gamma q(z_2)] q(z_3) | B(p) \rangle_{z_i^2 = 0}$

	Γ chiral even	Γ chiral odd		
twist 3	$\Phi^B_{3\pm}(x_i)$	$\Pi^B_3(x_i)$		
twist 4	$\Phi^B_{4\pm}(x_i)$	$\Xi^B_{4\pm}(x_i)$	$\Pi^B_4(x_i)$	$\Upsilon^B_4(x_i)$
twist 5	$\Phi^B_{5\pm}(x_i)$	$\Xi^B_{5\pm}(x_i)$	$\Pi^B_{5\pm}(x_i)$	$\Upsilon^B_{5\pm}(x_i)$

Definitions/ SU(3) relations/ Conformal expansions/ Evolution/ Wandzura-Wilzeck decompositions are known

Braun et al, 2000, 2008, 2013

Manashov, Anikin 2013

Shäfer, Wein 2015

Anikin 2015

Baryon Light-Cone DAs

Models: first few term of the conformal expansion:

Twist-3

$$\phi_{3+}^B(x_i) = f_B x_1 x_2 x_3 (1 + \phi_{11}^B 21(x_1 - 2x_2 + x_3))$$

$$\phi_{3-}^B(x_i) = f_B x_1 x_2 x_3 \, \phi_{10}^B 21(x_1 - x_3)$$

$$\phi^{\rm tw4}(x_i, \mu) = \mathcal{P}[x_i; \phi_{ij}^{(3)}] + \bar{\phi}^{\rm \,tw4}(x_i, \mu)$$

Twist-4

$$\bar{\Phi}_{4+}^B(x_i) = \lambda_1^B \; 24 x_1 x_2 \left(\; - \eta_{11}^B (2x_1 - x_2 - 2x_3)\right)$$

$$\bar{\Phi}_{4-}^B(x_i) = \lambda_1^B \; 24 x_1 x_2 \left(1 + \eta_{10}^B (4 - 10 x_2)\right)$$

quark-gluon moments contribute starting from

η_{2i}^B and therefore can be neglected

Twist-5

$$\phi^{\rm tw5}(x_i, \mu) = \mathcal{P}[x_i; \phi_{ij}^{(3)}, \phi_{ij}^{(4)}] + \cancel{\bar{\phi}^{\rm \,tw5}}(x_i, \mu)$$

Baryon Light-Cone DAs

What do we know about the moments?

QCD Sum Rules: twist-3,4 moments

Chernyak et al, 1989
Braun et al, 2000

QCD Light-Cone SR: twist-3,4 moments
(data fit, ABOI model for N)

Anikin et al, 2013

Lattice: twist-3,4 moments

Bali et al, 2019

relations in the SU(3)$_f$ limit + data fit

The moments
used
in calculations

$\mu^2 = 1.5\ \mathrm{GeV}^2$

B	f_B, GeV^2	ϕ_{10}	ϕ_{11}	ϕ_{20}	ϕ_{21}	ϕ_{22}	$f_\perp^B$, GeV^2	π_{10}^B	π_{11}^B
N	4.94×10^{-3}	0.051	0.051	0.078	-0.028	0.179	–	–	–
Λ	5.60×10^{-3}	0.120	0.048	0	0	0	–	0.042	–
Σ	4.71×10^{-3}	0.021	0.047	0	0	0	4.61×10^{-3}	–	-0.022
Ξ	4.94×10^{-3}	0.078	-0.003	0	0	0	4.83×10^{-3}	–	0.093

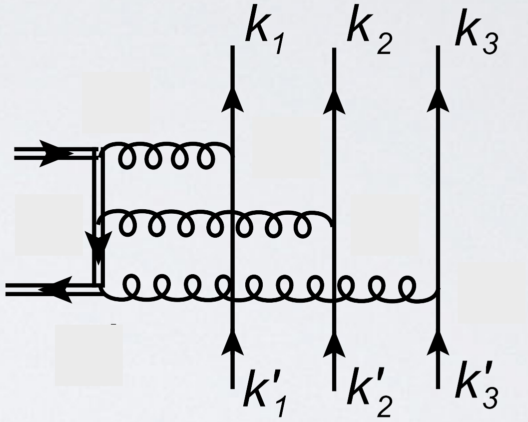
B	λ_1^B, GeV^2	η_{10}^B	η_{11}^B	$\lambda_\perp^B$, GeV^2	ζ_{10}^B	ζ_{11}^B	λ_2, GeV^2	ξ_{10}
N	-28×10^{-3}	-0.040	0.163	–	–	0.127	53×10^{-3}	-0.27
Λ	-39×10^{-3}	-0.040	0.147	-49×10^{-3}	-0.040	–	92×10^{-3}	–
Σ	-43×10^{-3}	-0.051	0.262	–	–	0.262	93×10^{-3}	-0.169
Ξ	-46×10^{-3}	-0.040	0.147	–	–	0.147	92.6×10^{-3}	-0.27

Hard kernels

Power corrections:

contributions with derivatives with respect to ⊥ components of quark momenta:

$$\sum_{i,j=1,2}\left\{A(x_i)+B(x_i)\frac{\partial}{\partial k_{\perp i}}+C(x_i)\frac{\partial}{\partial k_{\perp i}}\frac{\partial}{\partial k_{\perp j}}+\dots\right\}$$



$k_i = x_i k + k_{\perp i}$

$k'_i = y_i k' + k'_{\perp i}$

this usually produces the singular terms $\sim 1/x_i^n$ and often yields IR-divergencies, that usually indicates about a violation of the collinear factorization

however a soft gluon emission is suppressed by power of the small heavy quark velocity v and therefore such region is subleading, hence factorization holds and for the power corrections associated with higher twist baryon DAs

a separation of the 3-quark and 3-quark+gluon contributions for the twist-5 matrix elements, is not simple and, probably, not unique more details in the paper

NK, 2021 (in preparation)

Amplitudes

analytical expressions are not complicated but somewhat lengthy,

Nucleon amplitude:

$$G_E^g = \frac{f_\psi}{m_c^6}(\pi\alpha_s)^3\,\frac{20}{81}\int Dy_i\,\frac{1}{y_1y_2y_3}\int Dx_i\,\frac{1}{x_1x_2x_3}\,\frac{1}{D_1D_2D_3}\{\dots\}$$

$$D_i = x_i(1-y_i)+(1-x_i)y_i$$

tw.4 tw.3

$$\begin{aligned}\{\dots\} = {} & (\mathcal{A}_1-\mathcal{V}_1)\,(x_{123})\,\phi_3(x_{213})\;x_1(x_2(y_2-y_3)-\bar{y}_1y_2)\\ & +(\mathcal{A}_1+\mathcal{V}_1)\,(x_{123})\phi_3(y_{123})\,x_2(x_2-y_2)(y_1-y_3)\\ & +(\mathcal{T}_{21}-\mathcal{T}_{41})\,(x_{123})(\phi_3(y_{132})+\phi_3(y_{231}))x_3(x_2(y_1-y_2)+y_2\bar{y}_3)\end{aligned}$$

$\mathcal{V}_1\,,\mathcal{A}_1\,,\mathcal{T}_{21}-\mathcal{T}_{41}$ convenient auxiliary tw.4 DAs $\mathcal{V}_1(x_i)\equiv\mathcal{V}_1[\Phi_{4\pm}(x_i)],\dots$

$$\phi_3(y_i)\sim(y_1y_2y_3)\times\text{polynomials in } y_i$$

$$\{\mathcal{V}_1\,,\mathcal{A}_1\,,\mathcal{T}_{21}-\mathcal{T}_{41}\}\sim(x_1x_2x_3)\times\text{polynomials in } x_i$$

$$\Rightarrow\quad\int Dy_i\,\frac{1}{\underline{y_1y_2y_3}}\int Dx_i\,\frac{1}{\underline{x_1x_2x_3}}\,\frac{1}{D_1D_2D_3}\{\dots\}$$

well defined

Phenomenological analysis

Power corrections

$$G_M^g \simeq R_{10}(0)\ \{\underbrace{\phi^{\mathrm{tw3}} * T_{33} * \phi^{\mathrm{tw3}}}_{\text{LP}} + \underbrace{\phi^{\mathrm{tw3}} * T_{35} * \phi^{\mathrm{tw5}} + \phi^{\mathrm{tw4}} * T_{44} * \phi^{\mathrm{tw4}}}_{\text{NLP} \quad \sim \Lambda^2/m_c^2}\}$$

	N	Λ	Σ	Ξ
$\frac{G_M^g(\mathrm{NLP})}{G_M^g(\mathrm{LP})} \times 100\%$	8%	4.5%	13.3%	35.6%
$\frac{m_B^2}{M_\psi^2}$	9.2%	13%	14.8%	17.5%

Not very large!

Phenomenological analysis

$$\mathrm{Br}[J/\psi \to B\bar{B}] = \frac{M_\psi \beta}{12\pi\Gamma_{tot}} \left|G_M^g + G_M\right|^2 \left(1 + \frac{2m_B^2}{M_\psi^2}\gamma_B^2\right)$$

$$\gamma_B = \left|\frac{G_E^g + G_E}{G_M^g + G_M}\right|$$

e.m. FFs estimate $|G_M| \approx |G_{eff}|$ $e^+e^- \to B\bar{B}$ BABAR 2007

BESIII 2017, 2019, 2020, 2021

$G_M = |G_M|e^{i\phi_M}$ $\cos[\phi_M]$ is estimated from Br data

$G_E = |G_E|e^{i\phi_E}$ $0 < |G_E^B| \cos\varphi_E \leq \ 1.5 G_{\mathrm{eff}}^B$ assumption

Phenomenological analysis

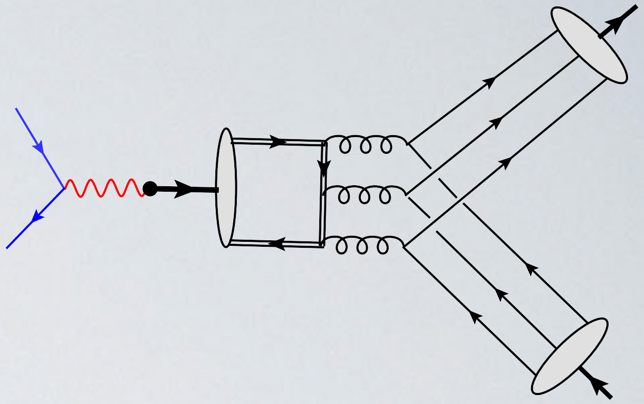
normalization $\mu^2 = 1.5\ \mathrm{GeV}^2$ $\alpha_s(\mu^2) = 0.35$

charmonium WF $|R_{10}(0)|^2 \simeq 0.81\mathrm{GeV}^3$ $m_c = 1.48\,\mathrm{GeV}$ Eichten Quigg, 1995

	$\lvert G_M^B \rvert$	$\cos\phi_M$	$Br[J/\psi \to B\bar{B}]$	$Br[\mathrm{data}]$	γ_B	$\gamma_B[\mathrm{data}]$
p	3.47	0.45	2.13	2.12(3)	$0.67^{+0.16}_{-0.08}$	0.83(2)
n	2.10	0.65	2.10	2.09(2)	$0.67^{+0.13}_{-0.06}$	0.95(6)
Λ	2.29	0.50	1.71	1.89(9)	$0.67^{+0.17}_{-0.06}$	0.83(4)
Σ^0	2.00	0.50	1.23	1.17(3)	$1.75^{+0.10}_{-0.19}$	2.11(5)
Σ^+				1.50(3)		2.27(5)
Ξ^+	1.60	0.50	1.14	0.97(8)	$0.47^{+0.11}_{-0.003}$	0.61(5)
Ξ^0				1.16(44)		0.66(3)

The numerical estimates are in agreement with the exp. data within 10-30% accuracy

Conclusions



All decay amplitudes associated with the 3g annihilation, are computed within the QCD EFT framework.

The considered models for baryon DAs provide reliable description of Br's for the relatively low norm. scale only $\mu^2 = 1.5\ \mathrm{GeV}^2$

The interference of hadronic amplitudes and e.m. baryon FF's is important.

The numerical ratios of hadronic amplitudes are in agreement with the exp. data within 10-30% accuracy

A description of the Σ-channels requires a strong SU(3) violation for parameters of twist-4 DAs

The power corrections of order Λ^2/m_c^2 are also computed (higher twist 3q DAs). They are about 4-15% for N/Λ/Σ and 35% for Ξ, that is not very large.

Thanks!