Two Schrodinger-like equations for hadrons

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Physics of hadrons on the light-front

7 November 2021 to 4 December 2021

Two recent papers

Physics Letters B
Volume 823, 10 December 2021, 136754

Extending light-front holographic QCD using the 't Hooft Equation

Mohammad Ahmady [a], Harleen Dahiya [b], Satvir Kaur [b], Chandan Mondal [c, d], Ruben Sandapen [e], Neetika Sharma [f]

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Hadron spectroscopy using the light-front holographic Schrödinger equation and the 't Hooft equation

Mohammad Ahmady, Sugee Lee MacKay, Satvir Kaur, Chandan Mondal, and Ruben Sandapen
Phys. Rev. D **104**, 074013 – Published 18 October 2021

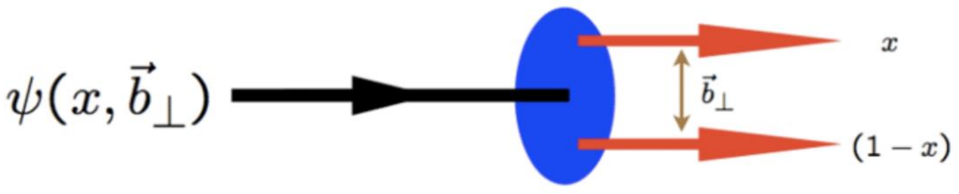
This talk is based on these two references

Meson masses in light-front QCD

$$\left[\left(-\frac{\nabla^2_{b_\perp}}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right) + U(x, b_\perp)\right]\Psi(x, \mathbf{b}_\perp) = M^2\Psi(x, \mathbf{b}_\perp)$$

Complicated QCD bound state dynamics

See also Stan Brodsky's talk at this conference



Valence meson light-front wavefunction

$$x = \frac{k^+}{P^+}$$

Light-front momentum fraction carried by quark

Transverse quark-antiquark separation at equal light-front time

Brodsky, de Teramond, Dosch, Erlich, Phys. Rep. 584, 1 (2015) (review with original references)

Light-front factorization

$$\boldsymbol{\zeta} = \sqrt{x(1-x)}\mathbf{b}_\perp$$

$$\Psi(x,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\varphi} X(x)$$

$$X(x) = \sqrt{x(1-x)}\chi(x)$$

$L = |L_z^{max}|$

LF orbital angular momentum

$$M_\perp^2 = \int d^2\boldsymbol{\zeta}\phi^*(\zeta)\left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U_\perp(\zeta)\right]\phi(\zeta) .$$

$$M_\parallel^2 = \int dx\chi^*(x)\left[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel(x)\right]\chi(x)$$

Assumption

$$U(x,\zeta) = U_\parallel(x) + U_\perp(\zeta)$$

$$M^2 = M_\perp^2 + M_\parallel^2$$

Exact derivation of confining potentials in QCD: open question

Transverse and longitudinal dynamics

$$\left[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel(x)\right]\chi(x) = M_\parallel^2 \chi(x)$$

Longitudinal dynamics

$$\left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U_\perp(\zeta)\right]\phi(\zeta) = M_\perp^2 \phi(\zeta)$$

Transverse dynamics at fixed x

$$\boldsymbol{\zeta} = \sqrt{x(1-x)}\mathbf{b}_\perp$$

Holographic Schrodinger Equation

Neglecting quark masses and longitudinal confinement

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U_\perp^{\mathrm{LFH}}(\zeta)\right)\phi(\zeta) = M_\perp^2\phi(\zeta)$$

Holographic mapping to AdS_5 $\zeta \leftrightarrow z_5$

Fixed by conformal symmetry and holographic mapping

$$U_\perp^{\mathrm{LFH}}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J - 1) .$$

Underlying conformal symmetry

κ: emerging mass scale

$$M_{\perp,M}^2 = 4\kappa^2\left(n_\perp + L_M + \frac{S_M}{2}\right)$$

Massless pion as expected in chiral QCD

Quark masses

Non-zero quark masses were previously taken into account using the Brodsky-de Teramond ansatz

$$X_{\mathrm{BdT}}(x) = \sqrt{x(1-x)} \times \exp\left(-\frac{(1-x)m_q^2 + x m_{\bar{q}}^2}{2\kappa^2 x(1-x)}\right)$$

$$\Delta M_{\mathrm{BdT}}^2 = \int \frac{\mathrm{d}x}{x(1-x)} \times X_{\mathrm{BdT}}^2(x)\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)$$

Spectroscopy

- Light mesons: universal κ
- Heavy mesons: κ depends on heavy quark mass

H.G. Dosch, G.F. de Teramond, S.J. Brodsky, Supersymmetry across the light and heavy-light hadronic spectrum, Phys. Rev. D 92 (7) (2015) 074010, https://doi.org/10.1103/PhysRevD.92.074010, arXiv:1504.05112.

The `t Hooft Equation

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + \frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y \frac{\chi(x)-\chi(y)}{(x-y)^2} = M_\parallel^2 \chi(x)$$

`t Hooft, Nucl. Phys. B 75 461 (1974)

- (1+1)-dim QCD Lagrangian
- Large N_c limit: only planar diagrams contribution
- g is the `t Hooft coupling with mass dimensions

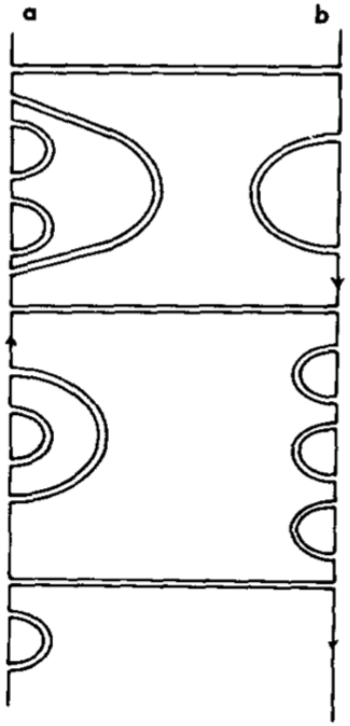
Momentum space

$$U_\parallel(x) = \frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y \frac{\chi(x)-\chi(y)}{(x-y)^2}$$

Fourier

$$U_L(x^-) = \frac{g^2}{2}P^+|x^-| = g^2 P^+ b_\parallel$$

Position space



Original idea to use `t Hooft Equation

S. S. Chabysheva and J. R. Hiller, Ann. Phys. (Amsterdam) **337**, 143 (2013).

with $g^2/\pi = m_u^2$

Other work

Several other groups are investigating longitudinal dynamics using different models

- de Teramond, Brodsky, arXiv: 2103.10930 [hep-ph]
- Yang Li, Vary. arXiv: 2103.09993 [hep-ph]
- Weller and Miller: 2111.03194[hep-ph]

All in 2021

Chiral symmetry

Gell-Mann-Oakes-Renner (GMOR) relation

$$M_\pi^2 \propto m_{u/d}$$

Pion

Holographic SE $\quad M_\pi = 0$

`t Hooft Equation $\quad M_\pi^2 \propto g m_{u/d}$

Together, hSE and `t Hooft Eq reproduce the GMOR relation provided g does not depend on $m_{u/d}$

Heavy Quark symmetry

Heavy Quark Effective Theory $M_V - M_P \sim \frac{1}{m_Q}$

Mass difference between vector and pseudoscalar mesons in their ground states is suppressed by heavy quark mass

Heavy-light mesons

Holographic SE $M_V^2 \sim \kappa^2, M_P^2 = 0$

$M_V - M_P \sim \frac{\kappa^2}{m_Q}$ if $m_Q \gg \kappa$

`t Hooft Equation $M_{V,P}^2 \sim m_Q^2$

Together, hSE and `t Hooft Eq reproduce HQET constraint provided κ does not depend on m_Q

Rotational symmetry

Heavy-heavy meson

Holographic $U_\perp \sim (\kappa^4/4) b_\perp^2$

`t Hooft $U_\parallel \sim 2 m_Q b_\parallel$

Light-front potentials

Relation between light-front and instant-form CM potentials

$$U_{\rm LF} = V_{\rm IF}^2 + 4 m V_{\rm IF}$$

A.P. Trawiński, S.D. Głazek, S.J. Brodsky, G.F. de Téramond, H.G. Dosch, Phys. Rev. D 90 (7) (2014) 074017,

$$V_\perp \sim (\kappa^2/2) b_\perp \text{ and } V_\parallel \sim (g^2/2) b_\parallel .$$

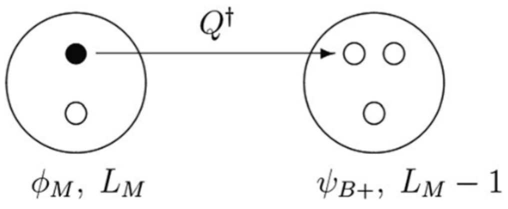
Instant form potentials

Rotational symmetry is restored in the non-relativistic limit if $g = \kappa$

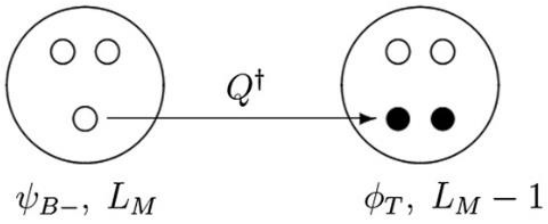
Supersymmetry

Brodsky, de Teramond, Dosch, Lorce, Phys. Lett. B 759 (2016)
Dosch, de Teramond, Brodsky, Phys. Rev. D 95 034016 (2017)
Neilson, Brodsky, Phys. Rev. D 97 114001 (2018)

Color $SU(3)_{Nc}$: a cluster of $N_C - 1$ constituents can be in the same color representation as the anticonstituent



quark-antiquark quark-diquark



quark-diquark diquark-antidiquark

A baryon has two superpartners : a meson and a tetraquark

Same color confinement scales (κ, g) within a family of superpartners

Computing the hadron spectrum

$$M^2_{\perp,M} = 4\kappa^2\left(n_\perp + L_M + \frac{S_M}{2}\right)$$

$$M^2_{\perp,B} = 4\kappa^2\left(n_\perp + L_B + \frac{S_D}{2} + 1\right)$$

$$M^2_{\perp,T} = 4\kappa^2\left(n_\perp + L_T + \frac{S_T}{2} + 1\right)$$

SuSy Holography | `t Hooft

$$M^2_M = M^2_{\perp,M}(n_\perp, L_M, S_M; \kappa) + M^2_{\parallel,M}(n_\parallel; m_q, m_{\bar{q}}, g) \ ,$$

$$M^2_B = M^2_{\perp,B}(n_\perp, L_B, S_D; \kappa) + M^2_{\parallel,B}(n_\parallel; m_q, m_{[qq]}, g) \ ,$$

$$M^2_T = M^2_{\perp,B}(n_\perp, L_T, S_T; \kappa) + M^2_{\parallel,T}(n_\parallel; m_{[\bar{q}\bar{q}]}, m_{[qq]}, g)$$

Diquark mass=2 x quark mass

$$P = (-1)^{L_M+1} = (-1)^{L_B} = (-1)^{L_T} \ .$$

$$C = (-1)^{n_\parallel + L_M + S_M} = (-1)^{n_\parallel + L_T + S_T - 1}$$

$$n_\parallel \geq n_\perp + L$$

A radial and/or orbital transverse excitation ⟺ a longitudinal excitation

Confinement scales and quark masses

Effective quark masses

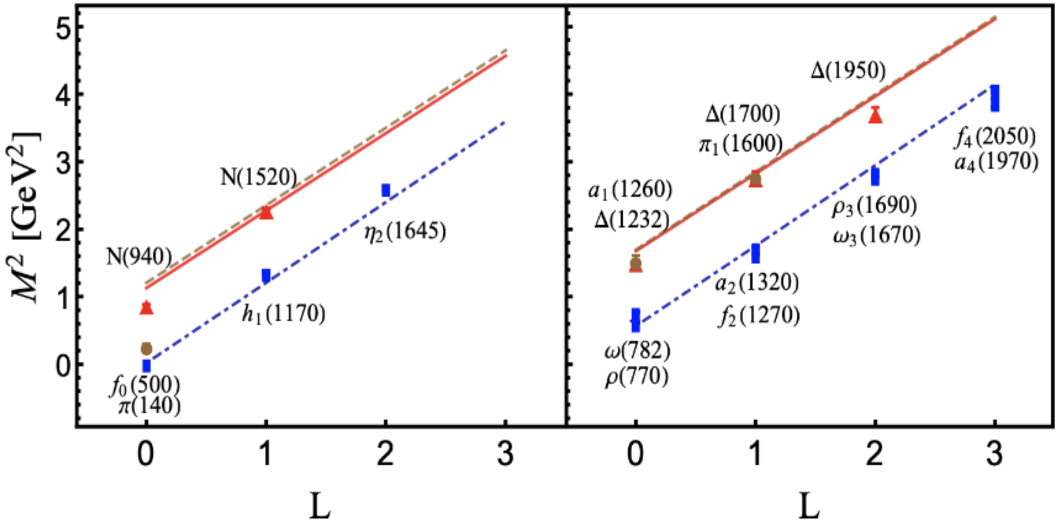
Hadron	g	$m_{u/d}$	m_s	m_c	m_b
Light	0.128	0.046	0.357	-	-
Heavy-light	0.410	0.330	0.500	1.370	4.640
Heavy-heavy	0.523	-	-	1.370	4.640

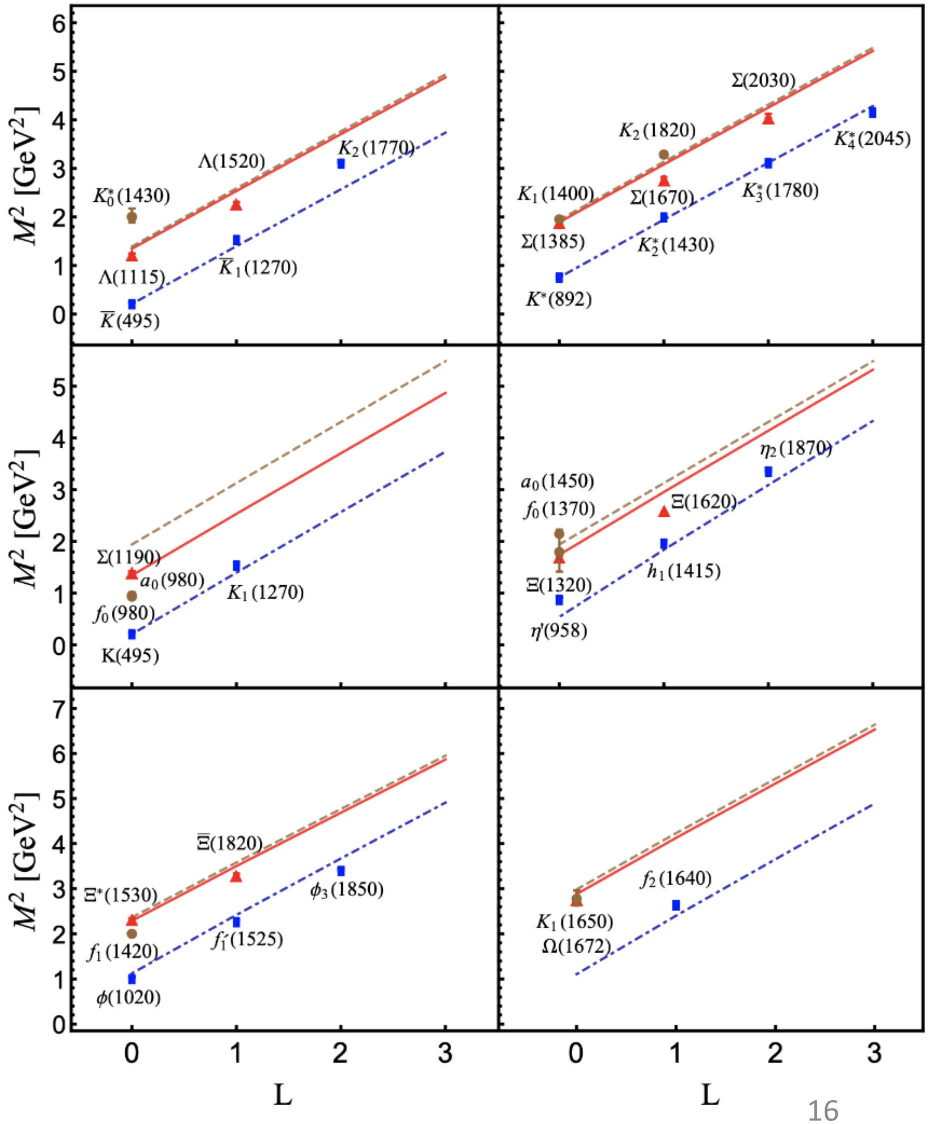
In GeV

- $\kappa = 0.523$ GeV   Universal (same for all hadrons)

- g
  - Same in a family of superpartners
  - Equal to κ in superpartners with two heavy quarks
  - Varies with number of heavy quarks

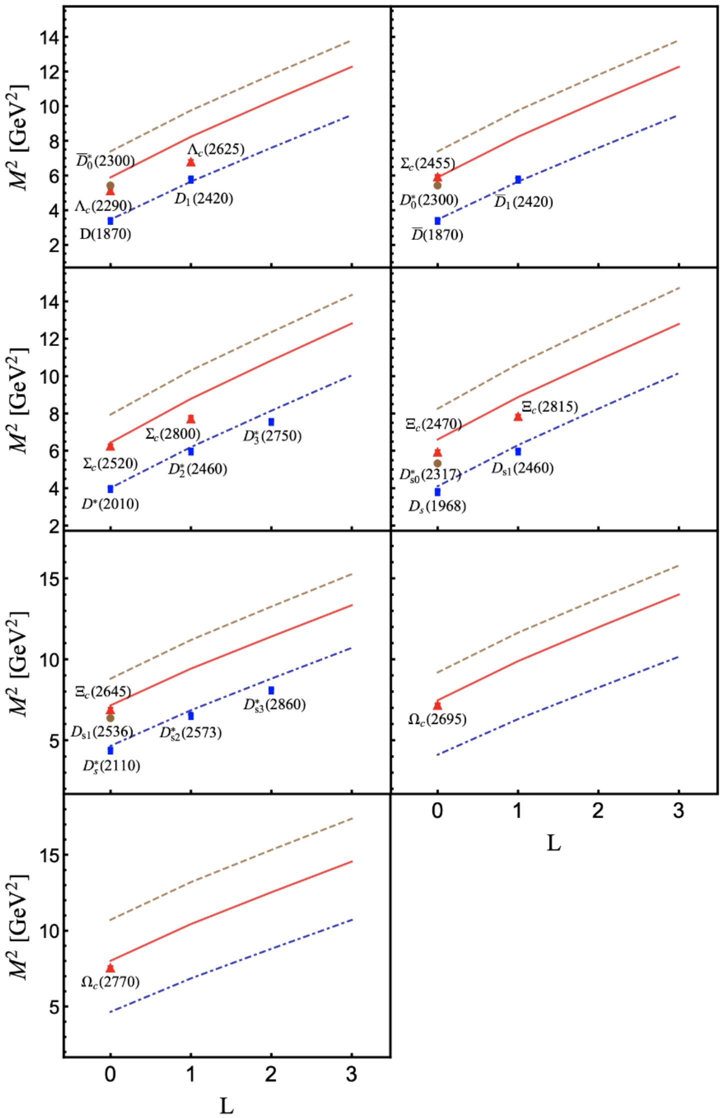
Light hadrons





Blue: mesons
Red: baryons
Brown: tetraquarks

Heavy-light hadrons



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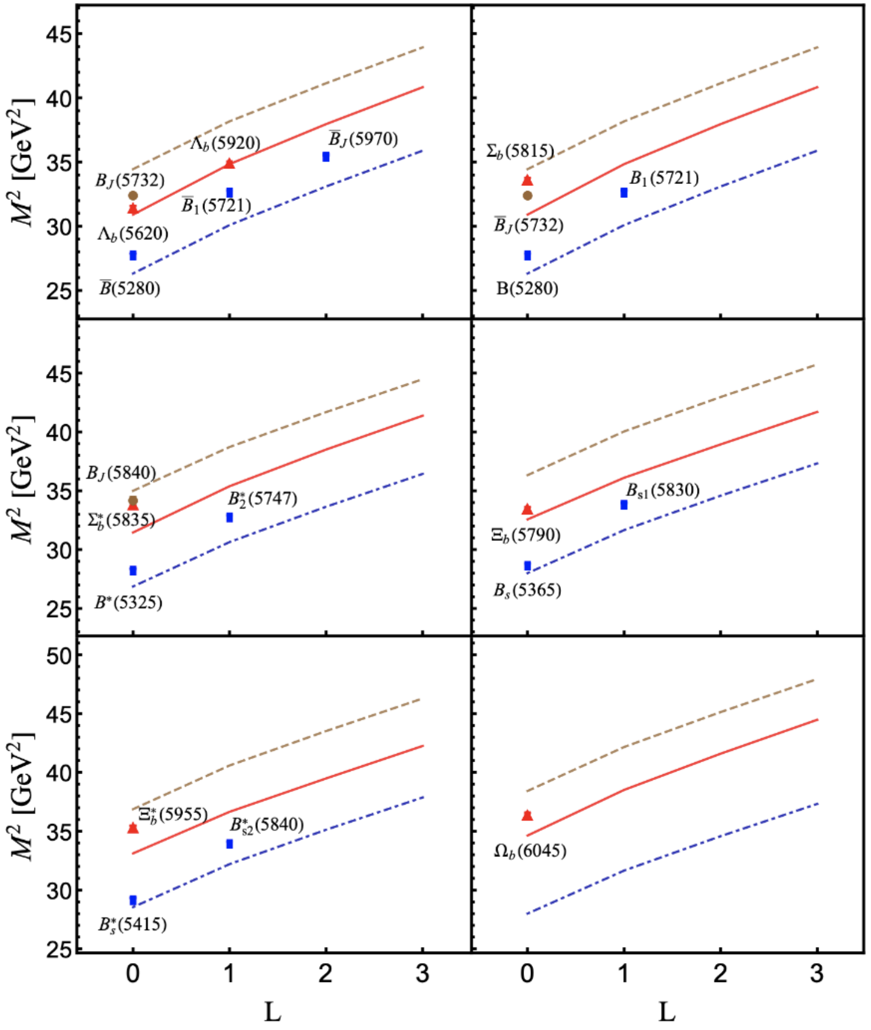
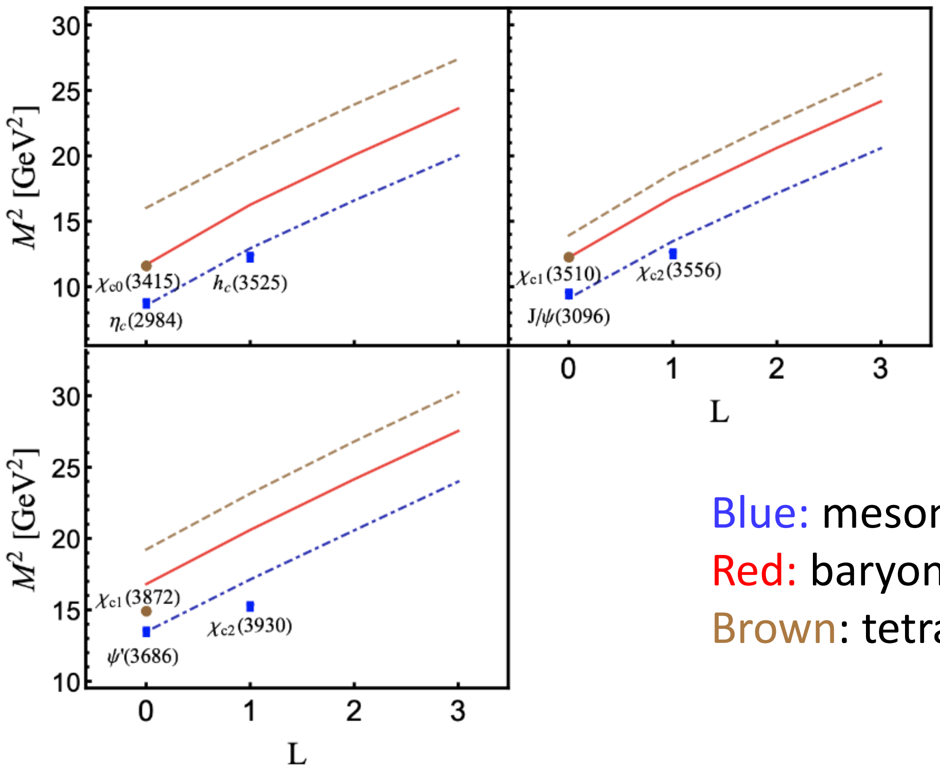
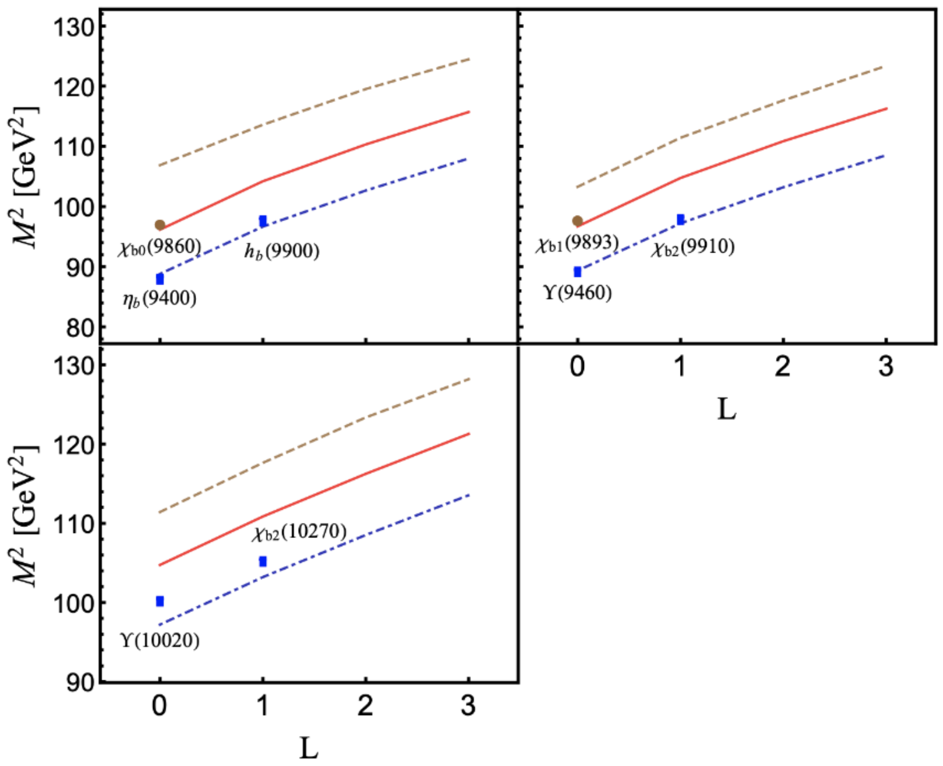


Heavy-heavy hadrons

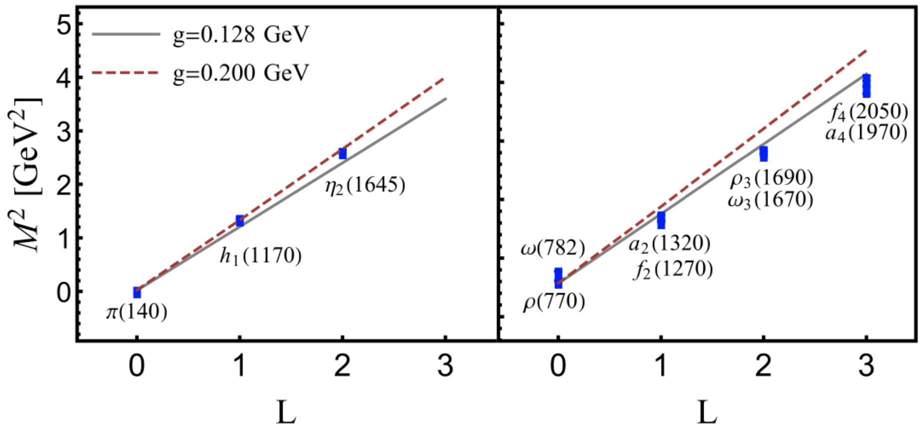


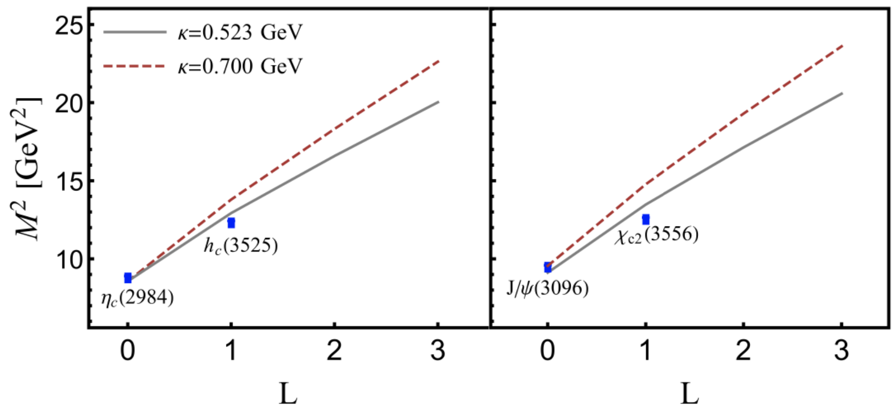


Blue: mesons
Red: baryons
Brown: tetraquarks

Results

- Agreement is within 13% for all mesons and baryons (except for $\eta'(958)$ where it is 21%)
- Notable disagreement for tetraquark candidates $f_0(500), f_0(980)$ (already present without longitudinal dynamics)
- Precise slopes of Regge trajectories sensitive to both κ and g across the full spectrum





Conclusions and Acknowledgements

The holographic Schrodinger Equation and the `t Hooft Equation seem to be complementary in describing hadron spectroscopy

- I thank my co-authors (M. Ahmady, H. Dahiya, S. Lee Mackay, C. Mondal S. Kaur, N. Sharma) of the two papers reported here
- I also thank S. Brodsky, G. de Teramond and G. Miller for useful discussions
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