Two Schrodinger-like equations for hadrons

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Physics of hadrons on the light-front
7 November 2021 to 4 December 2021
Two recent papers

This talk is based on these two references
Meson masses in light-front QCD

\[
\left( -\frac{\nabla_{\vec{b}_\perp}^2}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) + U(x, b_\perp) \right) \Psi(x, b_\perp) = M^2 \Psi(x, b_\perp),
\]

Complicated QCD bound state dynamics

See also Stan Brodsky’s talk at this conference

Valence meson light-front wavefunction

Light-front momentum fraction carried by quark

Transverse quark-antiquark separation at equal light-front time

\[ \zeta = \sqrt{x(1-x)} b_\perp \]

\[ \Psi(x, \zeta, \varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi \zeta}} e^{iL\varphi} X(x) \]

\[ X(x) = \sqrt{x(1-x)} \chi(x) \]

\[ L = |L^\text{max}_z| \]

\[ M_\perp^2 = \int d^2 \zeta \phi^*(\zeta) \left[ -\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U_\perp(\zeta) \right] \phi(\zeta). \]

LF orbital angular momentum

\[ M_\parallel^2 = \int dx \chi^*(x) \left[ \frac{m_q^2}{x} + \frac{m_q^2}{1-x} + U_\parallel(x) \right] \chi(x) \]

Assumption

\[ U(x, \zeta) = U_\parallel(x) + U_\perp(\zeta). \]

\[ M^2 = M_\perp^2 + M_\parallel^2 \]

Exact derivation of confining potentials in QCD: open question
Transverse and longitudinal dynamics

\[
\left[ \frac{m_q^2}{x} + \frac{m_q^2}{1-x} + U_\parallel(x) \right] \chi(x) = M_\parallel^2 \chi(x)
\]

Longitudinal dynamics

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U_\perp(\zeta) \right] \phi(\zeta) = M_\perp^2 \phi(\zeta)
\]

Transverse dynamics at fixed \( x \)

\[\zeta = \sqrt{x(1-x)} b_\perp\]
Holographic Schrodinger Equation

Neglecting quark masses and longitudinal confinement

\[
\left( -\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U_{\perp}^{\text{LFH}}(\zeta) \right) \phi(\zeta) = M_{\perp}^2 \phi(\zeta)
\]

Holographic mapping to AdS$_5$  \( \zeta \leftrightarrow z_5 \)

\[
U_{\perp}^{\text{LFH}}(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)
\]

\( \kappa \): emerging mass scale

Underlying conformal symmetry

\[
M_{\perp,M}^2 = 4\kappa^2 \left( n_\perp + L_M + \frac{S_M}{2} \right)
\]

Massless pion as expected in chiral QCD

Fixed by conformal symmetry and holographic mapping
Non-zero quark masses were previously taken into account using the Brodsky-de Teramond ansatz

\[ X_{\text{BdT}}(x) = \sqrt{x(1-x)} \]
\[ \times \exp \left( -\frac{(1-x)m_q^2 + xm_q^2}{2\kappa^2x(1-x)} \right) \]

\[ \Delta M^2_{\text{BdT}} = \int \frac{dx}{x(1-x)} \]
\[ \times X_{\text{BdT}}^2(x) \left( \frac{m_q^2}{x} + \frac{m_q^2}{1-x} \right) \]

Spectroscopy

- Light mesons: universal $\kappa$
- Heavy mesons: $\kappa$ depends on heavy quark mass

The `t Hooft Equation

\[
\left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \chi(x) + \frac{g^2}{\pi} \mathcal{P} \int \frac{\chi(x) - \chi(y)}{(x-y)^2} \, dy = M_{\parallel}^2 \chi(x)
\]

- (1+1)-dim QCD Lagrangian
- Large $N_c$ limit: only planar diagrams contribution
- $g$ is the `t Hooft coupling with mass dimensions

Original idea to use `t Hooft Equation


with $\frac{g^2}{\pi} = m_{u\bar{u}}^2$
Other work

Several other groups are investigating longitudinal dynamics using different models:

  All in 2021
- Weller and Miller: 2111.03194[hep-ph]
Chiral symmetry

Gell-Mann-Oakes-Renner (GMOR) relation

\[ M_{\pi}^2 \propto m_u/d \]

Holographic SE

\[ M_{\pi} = 0 \]

`t Hooft Equation

\[ M_{\pi}^2 \propto g m_u/d \]

Together, hSE and `t Hooft Eq reproduce the GMOR relation provided \( g \) does not depend on \( m_u/d \)
Mass difference between vector and pseudoscalar mesons in their ground states is suppressed by heavy quark mass.

Together, hSE and 't Hooft Eq reproduce HQET constraint provided \( \kappa \) does not depend on \( m_Q \).
Rotational symmetry

Heavy-heavy meson

Holographic

\[ U_\perp \sim (\kappa^4/4)b_\perp^2 \]

Light-front potentials

\[ U_\parallel \sim 2m_O b_\parallel \]

`t Hooft

Relation between light-front and instant-form CM potentials

\[ U_{\text{LF}} = V_{\text{IF}}^2 + 4m V_{\text{IF}} \]

: \[ V_\perp \sim (\kappa^2/2)b_\perp \text{ and } V_\parallel \sim (g^2/2)b_\parallel \]

Instant form potentials

Rotational symmetry is restored in the non-relativistic limit if \( g = \kappa \)

Supersymmetry

Dosch, de Teramond, Brodsky, Phys. Rev. D 95 034016 (2017)

Color $SU(3)_{N_C}$: a cluster of $N_C - 1$ constituents can be in the same color representation as the anticonstituent

\[ \phi_M, L_M \quad \psi_{B+}, L_M - 1 \]

quark-antiquark \quad quark-diquark

\[ \psi_{B-}, L_M \quad \phi_T, L_M - 1 \]

quark-diquark \quad diquark-antidiquark

A baryon has two superpartners: a meson and a tetraquark

Same color confinement scales $(\kappa, g)$ within a family of superpartners
Computing the hadron spectrum

\[ M_{1,M}^2 = 4\kappa^2 \left( n_\perp + L_M + \frac{S_M}{2} \right) \]
\[ M_{1,B}^2 = 4\kappa^2 \left( n_\perp + L_B + \frac{S_D}{2} + 1 \right) \]
\[ M_{1,T}^2 = 4\kappa^2 \left( n_\perp + L_T + \frac{S_T}{2} + 1 \right) \]

SuSy Holography

\[ M_M^2 = M_{1,M}(n_\perp, L_M, S_M; \kappa) + M_{||,M}(n_\parallel; m_q, m_{\bar{q}}, g) \]
\[ M_B^2 = M_{1,B}(n_\perp, L_B, S_D; \kappa) + M_{||,B}(n_\parallel; m_q, m_{[qq]}; g) \]
\[ M_T^2 = M_{1,T}(n_\perp, L_T, S_T; \kappa) + M_{||,T}(n_\parallel; m_{\bar{q}q}, m_{[qq]}; g) \]

\[ P = (-1)^{L_M+1} = (-1)^{L_B} = (-1)^{L_T} \]
\[ C = (-1)^{n_\parallel + L_M + S_M} = (-1)^{n_\parallel + L_T + S_T - 1} \]

\[ n_\parallel \geq n_\perp + L \]

A radial and/or orbital transverse excitation \(\Leftrightarrow\) a longitudinal excitation
Confinement scales and quark masses

Effective quark masses

<table>
<thead>
<tr>
<th>Hadron</th>
<th>$g$</th>
<th>$m_{u/d}$</th>
<th>$m_s$</th>
<th>$m_c$</th>
<th>$m_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.128</td>
<td>0.046</td>
<td>0.357</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Heavy-light</td>
<td>0.410</td>
<td>0.330</td>
<td>0.500</td>
<td>1.370</td>
<td>4.640</td>
</tr>
<tr>
<td>Heavy-heavy</td>
<td>0.523</td>
<td>-</td>
<td>-</td>
<td>1.370</td>
<td>4.640</td>
</tr>
</tbody>
</table>

$\kappa = 0.523 \text{ GeV}$ \quad \text{Universal (same for all hadrons)}

$g$
- Same in a family of superpartners
- Equal to $\kappa$ in superpartners with two heavy quarks
- Varies with number of heavy quarks

In GeV
Blue: mesons
Red: baryons
Brown: tetraquarks
Heavy-light hadrons

Blue: mesons
Red: baryons
Brown: tetraquarks
Heavy-heavy hadrons

Blue: mesons
Red: baryons
Brown: tetraquarks
Results

- Agreement is within 13% for all mesons and baryons (except for $\eta'(958)$ where it is 21%)
- Notable disagreement for tetraquark candidates $f_0(500), f_0(980)$ (already present without longitudinal dynamics)
- Precise slopes of Regge trajectories sensitive to both $\kappa$ and $g$ across the full spectrum
Conclusions and Acknowledgements

The holographic Schrodinger Equation and the `t Hooft Equation seem to be complementary in describing hadron spectroscopy.

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