A new Wilson-line based light-cone Lagrangian for Gluodynamics

Based on H. Kakkad, P. Kotko, A. Stasto, 2021- JHEP07(2021)187

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Light Cone 2021: Physics of Hadrons on the Light Front

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Introduction

Yang-Mills action on the light cone

Yang-Mills action

$$S_{\mathrm{Y-M}} = -\frac{1}{4}\int d^4x\, \mathrm{Tr} F^{\mu\nu} F_{\mu\nu}$$

- Two light like four-vectors : $\eta = \frac{1}{\sqrt{2}}(1,0,0,-1) \quad \tilde{\eta} = \frac{1}{\sqrt{2}}(1,0,0,1)$
- Two complex transverse four-vectors : $\varepsilon_{\perp}^{\pm} = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$
- The components of a four-vector v

$$v^{+} = v\cdot\eta \quad v^{-} = v\cdot\tilde{\eta} \quad v^{\bullet} = v\cdot\varepsilon_{\perp}^{+} \quad v^{\star} = v\cdot\varepsilon_{\perp}^{-}$$

- Light-cone gauge : $A\cdot\eta = A^{+} = 0$.
- Action becomes quadratic in A^{-}, can be integrated out.

Only gluon scattering amplitudes.

[J. Scherk and J.H. Schwarz, 1975]

Yang-Mills action

$$S_{\mathrm{Y-M}}^{(\mathrm{LC})}\left[A^\bullet, A^\star\right] = \int dx^+ \left(\mathcal{L}_{+-}^{(\mathrm{LC})} + \mathcal{L}_{++-}^{(\mathrm{LC})} + \mathcal{L}_{+--}^{(\mathrm{LC})} + \mathcal{L}_{++--}^{(\mathrm{LC})}\right)$$

$$\mathcal{L}_{+-}^{(\mathrm{LC})}\left[A^\bullet, A^\star\right] = -\int d^3\mathbf{x}\,\mathrm{Tr}\,\hat{A}^\bullet \Box \hat{A}^\star$$

$$\mathcal{L}_{++-}^{(\mathrm{LC})}\left[A^\bullet, A^\star\right] = -2ig' \int d^3\mathbf{x}\,\mathrm{Tr}\,\gamma_{\mathbf{x}}\hat{A}^\bullet \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right]$$

$$\mathcal{L}_{--+}^{(\mathrm{LC})}\left[A^\bullet, A^\star\right] = -2ig' \int d^3\mathbf{x}\,\mathrm{Tr}\,\overline{\gamma}_{\mathbf{x}}\hat{A}^\star \left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right]$$

$$\mathcal{L}_{++--}^{(\mathrm{LC})}\left[A^\bullet, A^\star\right] = -g^2 \int d^3\mathbf{x}\,\mathrm{Tr}\,\left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right] \partial_-^{-2} \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right]$$

$$\gamma_{\mathbf{x}} = \partial_-^{-1}\partial_\bullet, \quad \overline{\gamma}_{\mathbf{x}} = \partial_-^{-1}\partial_\star, \qquad g' = \frac{g}{\sqrt{2}}$$

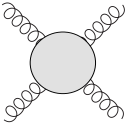
This form accommodates the so-called Coulomb instantaneous interactions.

Gluonic scattering amplitudes

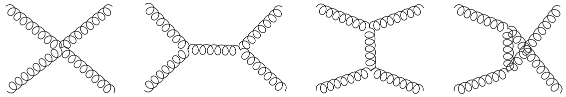
- Develop the Feynman rules from the action.
- Sum of all contributing diagrams built from the Feynman rules.

Feynman Diagram Technique

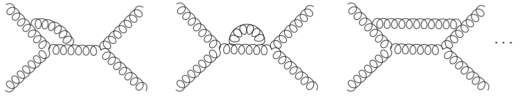
Example : Four point amplitude.



Tree Amplitude



Loop Amplitude



[M.L. Mangano, S.J. Parke, 1991]

Problem with Feynman diagram technique.

The number of Feynman diagrams contributing to the amplitude of a gluon tree level ($g + g \longrightarrow ng$) grows factorily.

n	2	3	4	5	6	7	8
# of diagrams	4	25	220	2485	34300	559405	10525900

[F.A. Berends and W.T. Giele, 1987] ; [M. Mangano, S. Parke and Z. Xu, 1988] ; [M. Mangano, 1988] ; [Z. Bern and D.A. Kosower, 1991]

Color Decomposition

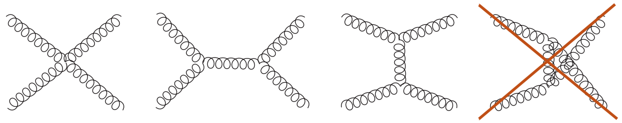
- Technique to disentangle the color and kinematical degrees of freedom in a gauge theory scattering amplitude.
- Lie Algebra structure constants in terms of generators T^a.

$$\tilde{f}^{abc} \equiv i\sqrt{2}f^{abc} = \mathrm{Tr}\left(T^aT^bT^c\right) - \mathrm{Tr}\left(T^aT^cT^b\right), \mathrm{Tr}(T^aT^b) = \delta^{ab}$$

- Fierz Identity systematically combines them into a single trace.
- n-gluon tree amplitudes :

$$\mathcal{A}_n^{tree}(\{k_i, h_i, a_i\}) = \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}}) A_n^{tree}(\sigma(1^{h_1}), \ldots, \sigma(n^{h_n}))$$

Color ordered : Planar graphs with no leg-crossings allowed



Spinor Helicity Formalism

[P. De Causmaecker et.al. 82] ; [F. A. Berends et. al. 82] ; [R. Kleiss et. al. 85] ; [Z. Xu et. al. 87] ; [R. Gastmans et. al. 90]

Helicity Spinors

- Uniform description of the on-shell degrees of freedom (momentum and polarization).
- Spinors from massless Dirac equation.
- Kinematical DOF in terms of Spinors :
  - 4-Momentum in terms of Spinors.

$$k_i^{\mu}(\sigma_{\mu})_{\alpha\dot{\alpha}} \;=\; (\not{k}_i)_{\alpha\dot{\alpha}} \;=\; \begin{pmatrix} k_i^t + k_i^z & k_i^x - ik_i^y \\ k_i^x + ik_i^y & k_i^t - k_i^z \end{pmatrix} \;=\; (\lambda_i)_{\alpha}(\tilde{\lambda}_i)_{\dot{\alpha}}\,.$$

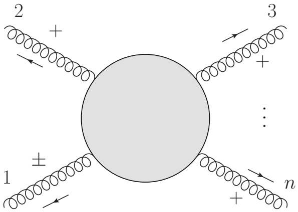
  - Polarization vectors also in terms of Spinors

-

$$\langle ij\rangle \;\equiv\; \epsilon^{\alpha\beta}(\lambda_i)_{\alpha}(\lambda_j)_{\beta}\,, [ij] \;\equiv\; \epsilon^{\dot{\alpha}\dot{\beta}}(\tilde{\lambda}_i)_{\dot{\alpha}}(\tilde{\lambda}_j)_{\dot{\beta}}\,.$$

- Renders the analytic expressions of scattering amplitudes in an often much more compact form compared to the standard four-vector notation.
- In order to uniformize the description we shall take all particles as outgoing.

Helicity Tree Amplitudes



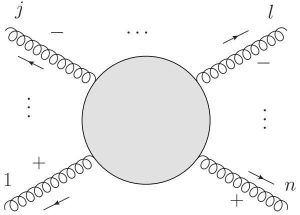
Vanishing Amplitudes

$$A_n^{tree}(1^{\pm}, 2^{+}, \ldots, n^{+}) = 0\,.$$

MHV Amplitudes

Maximally Helicity Violating

$$A_n^{tree}(\ldots, j^{-}, \ldots, l^{-}, \ldots) \;=\; \frac{\langle jl \rangle^4}{\langle 12 \rangle \cdots \langle n1 \rangle}.$$



[S.J.Parke, T.R Taylor, 1986]

Why so simple ?

Amplitudes have additional hidden symmetries/structure that constrain their form.

Lagrangian origin of MHV rules.

[P. Mansfield, 2006]

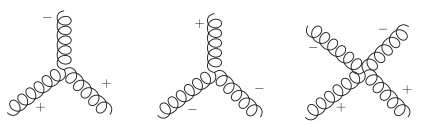
Basic Idea

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[A^{\bullet}, A^{\star}\right] = \left(\mathcal{L}^{(\mathrm{LC})}_{+-} + \mathcal{L}^{(\mathrm{LC})}_{++-} + \mathcal{L}^{(\mathrm{LC})}_{+--} + \mathcal{L}^{(\mathrm{LC})}_{++--}\right) .$$

- Only plus-helicity and minus-helicity gluon fields.

$$\{A^{\bullet}, A^{\star}\} \to \{B^{\bullet}, B^{\star}\}$$

Interaction vertices



Transformation

$$\mathcal{L}^{(\mathrm{LC})}_{+-} + \mathcal{L}^{(\mathrm{LC})}_{++-} \longrightarrow \mathcal{L}^{(\mathrm{LC})}_{+-}$$

MHV action : Action with MHV vertices

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[B^{\bullet}, B^{\star}\right] = \left(\mathcal{L}^{(\mathrm{LC})}_{+-} + \mathcal{L}^{(\mathrm{LC})}_{--+} + \cdots + \mathcal{L}^{(\mathrm{LC})}_{--+\cdots+} + \dots\right)$$

MHV Action

MHV Vertices (color stripped)

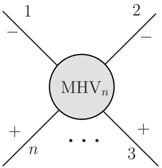
$$\mathcal{V}\left(1^-,2^-,3^+,\ldots,n^+\right) \equiv \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{v}_{21}^{*4}}{\widetilde{v}_{1n}^{*}\widetilde{v}_{n(n-1)}^{*}\widetilde{v}_{(n-1)(n-2)}^{*}\ldots\widetilde{v}_{21}^{*}}$$

The $\widetilde{v}_{ij}$, $\widetilde{v}_{ij}^{\star}$ are directly related to spinor products $\langle ij\rangle$, $[ij]$.

$$\widetilde{v}_{ij}^{*} = p_i^+\left(\frac{p_j^\bullet}{p_j^+} - \frac{p_i^\bullet}{p_i^+}\right) .$$

Important points

- The transformation results in only MHV vertices.
- The presence of one triple gluon vertex $(--+)$.
- Interpretation of B-Fields ?



B - Fields

Wilson Line

$$\mathcal{W}[A]\,(x,y) = \mathbb{P}\exp\left[ig\int_C dz_\mu\,\hat{A}^\mu\,(z)\right]$$

B - Fields as Wilson lines

[P. Kotko, 2014], [P. Kotko, A. Stasto, 2017]

$B^\bullet[A^\bullet]$

$$B^\bullet_a[A]\,(x) = \int_{-\infty}^{\infty} d\alpha\,\mathrm{Tr}\left\{\frac{1}{2\pi g}t^a\partial_-\,\mathbb{P}\exp\left[ig\int_{-\infty}^{\infty} ds\,\varepsilon^+_\alpha\cdot\hat{A}\left(x+s\varepsilon^+_\alpha\right)\right]\right\}$$

$$\varepsilon^+_\alpha = \varepsilon^+_\perp - \alpha\eta, \quad \hat{A} = A_a t^a$$

[H. Kakkad, P. Kotko, A. Stasto, 2020]

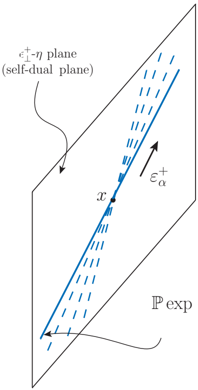
$B^\star[A^\bullet, A^\star]$

$$B^\star_a(x) = \int d^3\mathbf{y}\left[\frac{\partial^2_-(y)}{\partial^2_-(x)}\,\frac{\delta B^\bullet_a(x^+;\mathbf{x})}{\delta A^\bullet_c(x^+;\mathbf{y})}\right]A^\star_c(x^+;\mathbf{y})$$

B - Fields

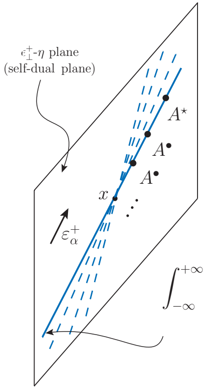
Geometrical Representation.

$B^\bullet[A^\bullet]$



$$\mathbb{P}\exp\left\{ig\int_{-\infty}^{\infty} ds\, \varepsilon_\alpha^+\cdot\hat{A}\left(x+s\varepsilon_\alpha^+\right)\right\}$$

$B^\star[A^\bullet, A^\star]$



$$\int_{-\infty}^{+\infty} ds\ \mathbb{P}\,\overline{\exp}\left\{ig\int_{-\infty}^{s} ds'\, \varepsilon_\alpha^+\cdot\hat{A}\left(x+s'\varepsilon_\alpha^+\right)\right\} \times \varepsilon_\alpha^-\cdot\hat{A}(x+s\varepsilon_\alpha^+)$$

$B^\star[A^\bullet, A^\star]$

Cut through a bigger structure spanning two planes ?

New Results

Towards a new action

Reminder

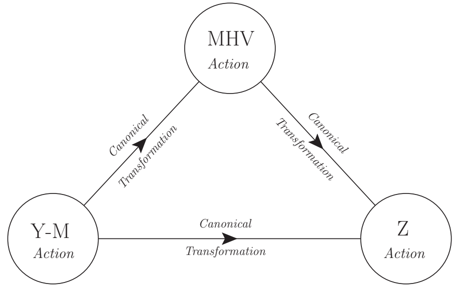
The presence of one triple gluon vertex $(--+)$ in the MHV action.

Motivation

- New classical action which does not involve any triple-gluon vertices.
- Extension of the geometric structure.

[H. Kakkad, P. Kotko, A. Stasto, 2021] JHEP07(2021)187

Z Action



$$\{B^{\bullet}, B^{\star}\} \to \{Z^{\bullet}, Z^{\star}\}$$

Z-field Action

Structure of the new action

$$S_{\mathrm{Y-M}}^{(\mathrm{LC})}[Z^{\bullet}, Z^{\star}] = \Big\{ \mathcal{L}_{-+}^{(\mathrm{LC})} + \mathcal{L}_{--++}^{(\mathrm{LC})} + \mathcal{L}_{--+++}^{(\mathrm{LC})} + \mathcal{L}_{--++++}^{(\mathrm{LC})} + \dots$$
$$+ \mathcal{L}_{---++}^{(\mathrm{LC})} + \mathcal{L}_{---+++}^{(\mathrm{LC})} + \mathcal{L}_{---++++}^{(\mathrm{LC})} + \dots$$
$$\vdots$$
$$+ \mathcal{L}_{---\dots-++}^{(\mathrm{LC})} + \mathcal{L}_{---\dots-+++}^{(\mathrm{LC})} + \mathcal{L}_{---\dots-++++}^{(\mathrm{LC})} + \dots \Big\}$$

Example : $\mathcal{L}_{----++}^{(\mathrm{LC})}$

$$= \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{v}_{21}^{*4}}{\widetilde{v}_{16}^{*}\,\widetilde{v}_{6(345)}^{*}\,\widetilde{v}_{(345)2}^{*}\,\widetilde{v}_{21}^{*}} \times \left(\frac{p_5^+}{p_{345}^+}\right)^2 \frac{\widetilde{v}_{(345)3}}{\widetilde{v}_{54}\,\widetilde{v}_{43}\,\widetilde{v}_{3(345)}}$$
$$+ \left(\frac{p_3^+}{p_4^+}\right)^2 \frac{\widetilde{v}_{43}^{*4}}{\widetilde{v}_{3(612)}^{*}\,\widetilde{v}_{(612)5}^{*}\,\widetilde{v}_{54}^{*}\,\widetilde{v}_{43}^{*}} \times \left(\frac{p_6^+}{p_{612}^+}\right)^2 \frac{\widetilde{v}_{(612)6}}{\widetilde{v}_{21}\,\widetilde{v}_{16}\,\widetilde{v}_{6(612)}} + \dots$$

Z-field Action

Important features

- There are no three point interaction vertices.
- At the classical level there are no all-plus, all-minus, as well as $(- + \cdots +)$, $(- \cdots - +)$ vertices.
- There are MHV vertices, $(- - + \cdots +)$, corresponding to MHV amplitudes in the on-shell limit.

$$\mathcal{A}\left(1^-, 2^-, 3^+, \ldots, n^+\right) \equiv \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{v}_{21}^{*4}}{\widetilde{v}_{1n}^{*}\widetilde{v}_{n(n-1)}^{*}\widetilde{v}_{(n-1)(n-2)}^{*}\cdots\widetilde{v}_{21}^{*}}$$

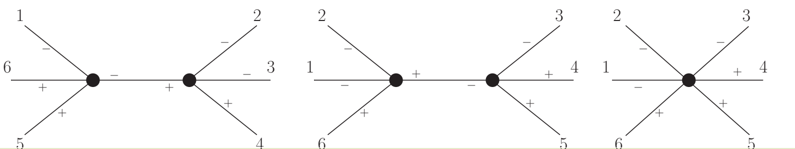
- There are $\overline{\mathrm{MHV}}$ vertices, $(- \cdots - ++)$, corresponding to $\overline{\mathrm{MHV}}$ amplitudes in the on-shell limit.

$$\mathcal{A}\left(1^-, \ldots, n-2^-, n-1^+, n^+\right) \equiv \left(\frac{p_{n-1}^+}{p_n^+}\right)^2 \frac{\widetilde{v}_{n(n-1)}^{4}}{\widetilde{v}_{1n}\widetilde{v}_{n(n-1)}\widetilde{v}_{(n-1)(n-2)}\cdots\widetilde{v}_{21}}$$

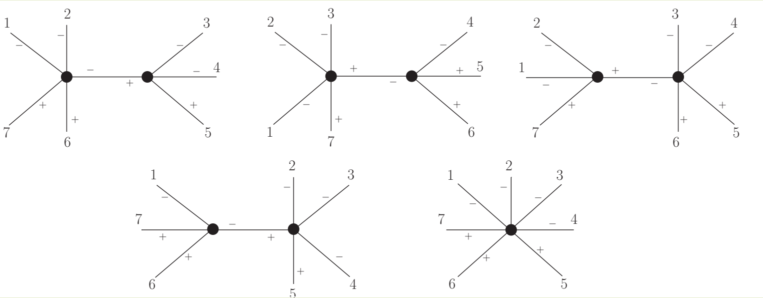
- All vertices have the form which can be easily calculated.

Calculating scattering amplitudes in Z-field theory

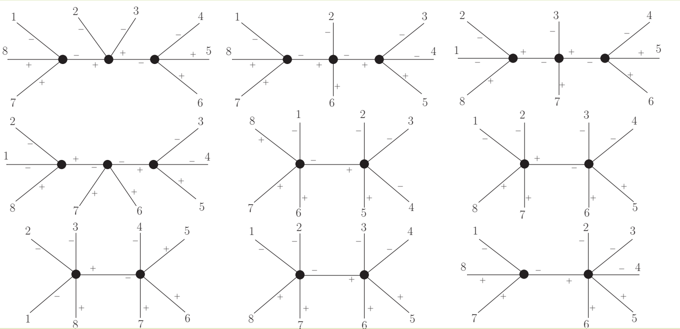
6 point NMHV $(- - - + + +)$

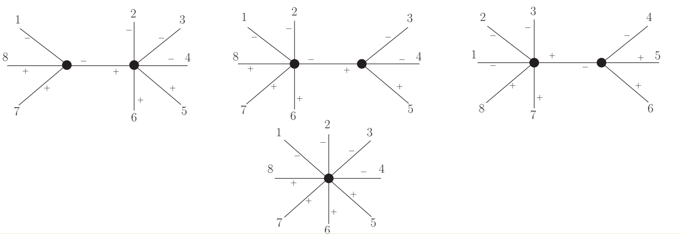


7 point NNMHV $(- - - - + + +)$



8 point NNMHV $(-\,-\,-\,-\,+\,+\,+\,+)$





Z field Wilson Line

Z - Fields as Wilson Line functionals

$Z^{\star}[B^{\star}]$

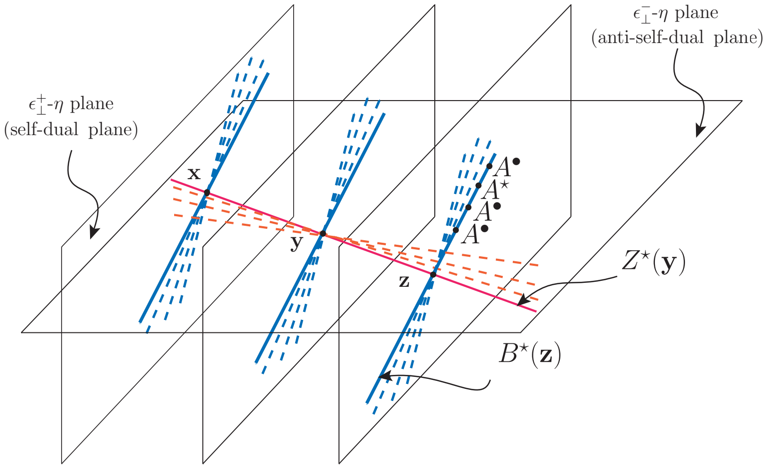
$$Z_a^{\star}[B^{\star}](x) = \int_{-\infty}^{\infty} d\alpha\, \mathrm{Tr}\left\{ \frac{1}{2\pi g} t^a \partial_- \, \mathbb{P} \exp\left[ig \int_{-\infty}^{\infty} ds\, \varepsilon_{\alpha}^{-} \cdot \hat{B}\left(x + s\varepsilon_{\alpha}^{-}\right)\right]\right\}$$

$$\varepsilon_{\alpha}^{-} = \varepsilon_{\perp}^{-} - \alpha\eta, \quad \hat{B} = B_a t^a$$

$Z^{\bullet}[B^{\bullet}, B^{\star}]$

$$Z_a^{\bullet}(x) = \int d^3\mathbf{y} \left[\frac{\partial_-^2(y)}{\partial_-^2(x)} \frac{\delta Z_a^{\star}(x^+;\mathbf{x})}{\delta B_c^{\star}(x^+;\mathbf{y})}\right] B_c^{\bullet}(x^+;\mathbf{y})$$

Geometrical Representation $Z^\star[B^\star]$



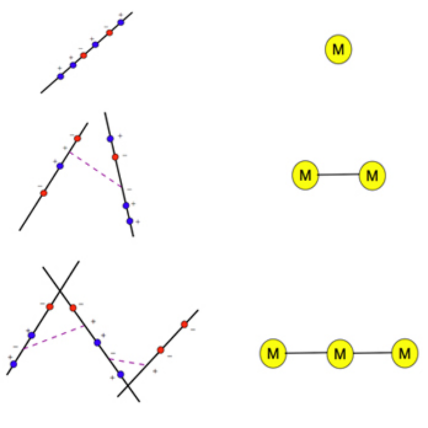
Summary

- Feynman diagram technique is not the best way to calculate amplitudes.
- Simplicity of MHV amplitudes led to the development of different techniques.
- The CSW action can be derived using field transformation whose solutions are given by certain Wilson Lines.
- Z-field action has no triple-gluon vertices. The starting point is 4-point MHV.
- Vertices in Z-field action have easy calculable form.
- Pure gluonic scattering amplitudes can be calculated conviniently with very few number of diagrams using the Z-field action.
- The Z-field are geometrically rich and intriguing.

Thank You for your Time !

Back-up

Geometrical Origin



Correspondence

Lines in Twistor space $\Longleftrightarrow$ Points in Minkowski Space

BCFW

BCFW Technique

- Complex shift the momentum of two consecutive legs such that the total momentum remains conserved.

$$\begin{aligned} \tilde{\lambda}_n &\rightarrow \hat{\tilde{\lambda}}_n = \tilde{\lambda}_n - z\tilde{\lambda}_1\,, \qquad \lambda_n \rightarrow \lambda_n\,, \\ \lambda_1 &\rightarrow \hat{\lambda}_1 = \lambda_1 + z\lambda_n\,, \qquad \tilde{\lambda}_1 \rightarrow \tilde{\lambda}_1 \end{aligned}$$

- Identify the location of complex poles.
- At poles the amplitude factorizes and the off-shell particle goes on-shell.
- Express amplitude in terms of residues.
- Apply Cauchy's residue theorem under the limit that the residue for large 'z' vanishes'.

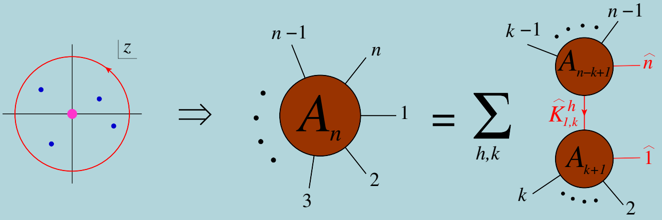


Figure – Illustration of how Cauchy's theorem leads to the BCFW recursion relation

THE END!