Quantum Simulation of QFT in the Front Form

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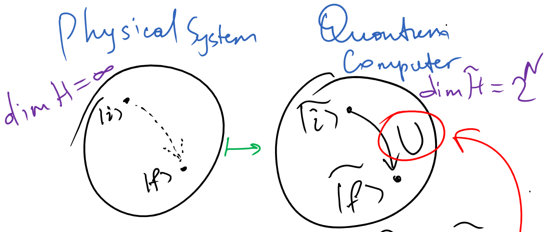
Quantum Simulation

Quantum computer: a quantum system, which can be manipulated ("controlled") and measured with high precision.

Quantum simulation: studying physical systems (i.e., calculating observables) using quantum computers.



$$\langle f | O | f \rangle \simeq \langle \tilde{f} | \tilde{O} | \tilde{f} \rangle$$

May not have a physical interpretation

Key ingredients:

* Encoding $\mapsto$
* Simulation algorithm U
* Measurement $\langle \tilde{f} | \tilde{O} | \tilde{f} \rangle$ – ?

Motivation

The currently dominant approach to digital quantum simulation of QFT is based on the equal-time lattice formulation.

A lot of progress, a lot of open questions:

- Gauge symmetry protection — highly non-trivial.
- Difficult to extract information about observables.
- Qubit number $\propto$ lattice size:

$$\mathfrak{Q}_{QCD} \sim \underbrace{(\text{internal DOFs})}_{\geq 50} \times \underbrace{L^{D-1}}_{\geq 20^3} \geq 400,000 \text{ qubits.} \tag{1}$$

Can we overcome these difficulties by using some alternative approach?

Quantum Simulation in the Front Form

Good news:

- Fact #1: Numerous techniques for the Digital Quantum Simulation of Quantum Chemistry have been developed in the last decades.
- Fact #2: QFT in the **light-front (LF)** formalism looks much like non-relativistic many-body physics![1]

[1] In what follows, we assume that the gauge-fixed second-quantized LF Hamiltonian acquires the form of

$$H = \mathrm{poly}(a, a^\dagger, b, b^\dagger)\,, \tag{2}$$

which is the case in DLCQ and, more generally, in BLFQ.

Quantum Simulation in the Front Form

	LF QFT features	Advantages for QC
Resources	No ghost fields Linear EoM	Low qubit count
	LF momentum > 0	Efficient encoding
Evolution	Sparse Hamiltonians	Using sparsity-based methods
Measurement	LF wavefunction $\rightarrow$ $\rightarrow$ static quantities; Simple form of operators in the second-quantized formalism	Simple form of measurement operators
Other	Trivial vacuum, fewer cut-offs, no fermion doubling, form invariance of H, equal treatment of matter and gauge fields in the $A^{+} = 0$ gauge	

Encoding Fock states

Consider the Fock states in the ϕ^4_{1+1} theory:

$$\{|\mathcal{F}\rangle\} \text{ at } K=5: \quad |1^5\rangle,\ |1^2,3\rangle,\ |1,2^2\rangle,\ |1,4\rangle,\ |2,3\rangle\,. \tag{3}$$

The number of $\{|\mathcal{F}\rangle\}$ scales as $p(K) = O(\exp(\sqrt{K}))$.

This implies that the **lower bound** on the number of qubits, required to encode a Fock state, scales as $\boxed{\mathfrak{Q} \sim O(\sqrt{K})}$.

$$|\mathcal{F}_1\rangle \mapsto |\ldots 000\rangle,\, |\mathcal{F}_2\rangle \mapsto |\ldots 001\rangle, \ldots \tag{4}$$

While such a mapping is impractical, we can use it to evaluate other ways of encoding Fock states in the quantum computer.

Encoding Fock states

Two ways of encoding a Fock state $|\mathcal{F}\rangle = |n_1^{w_1}, n_2^{w_2}, \ldots\rangle$.

I. *Direct encoding* — qubits store w_j (qubit register per mode):

$$|\Psi\rangle = |\underbrace{0101}_{w_1}\underbrace{1001}_{w_2}\ldots\rangle\,, \tag{5}$$

$$\mathfrak{Q}_{\mathrm{Direct}} = O(K\log K)\,. \tag{6}$$

II. *Compact encoding* — qubits store n_j and w_j, only for $w_j > 0$:

$$|\Psi\rangle = |\overbrace{\underbrace{0111}_{n_1}\underbrace{0101}_{w_1}\underbrace{1100}_{n_2}\underbrace{1001}_{w_2}\ldots}^{\text{at most } O(\sqrt{K}) \text{ modes}}\rangle\,, \tag{7}$$

$$\mathfrak{Q}_{\mathrm{Compact}} = \boxed{O(\sqrt{K}\log K)}\,. \tag{8}$$

In the presence of transverse dimensions:

$$\mathfrak{Q}_{\mathrm{Direct}} = \widetilde{O}(K\Lambda_\perp^{d-1}) \text{ vs. } \mathfrak{Q}_{\mathrm{Compact}} = \widetilde{O}(K)\,. \tag{9}$$

Quantum Simulation Algorithms

Should we always use compact mapping? No, because the choice of encoding restricts the choice of simulation algorithms.

Near-term, phenomenology | Resource requirements ⟷ Low — High | Far-future, ab initio

$\underbrace{\text{VQE} \quad \text{Adiabatic}}_{\text{spectroscopy}} \quad \text{Scattering}$

Resources	Paradigm	Circuits	Convergence
Near-term	Variational	Short	Not provable (heuristic)
Far-future	Time/adiabatic evolution	Long	Provable

Quantum Simulation Algorithms

Most existing algorithms are based either on product formulas ("trotterization") or on sparsity-based "oracle" routines.[2]

	Trotter (product formulas)	Sparsity (more advanced)
Direct	✓	✓
Compact	✗	✓

Using compact mapping results in longer circuits.

Near-term $\rightarrow$ Variational $\rightarrow$ Direct+Trotter

Far-future $\rightarrow$ Hamiltonian evolution $\rightarrow$ Tight on gates?

$$\rightarrow \begin{cases} \text{Yes} \rightarrow \text{Direct+Sparse} \\ \text{No} \rightarrow \text{Compact+Sparse} \end{cases}$$

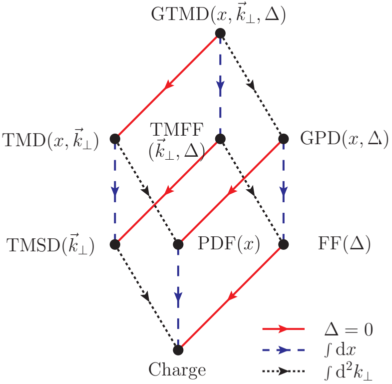
[2] One can also use heuristic algorithms, see the talk by Wenyang Qian.

Measurement

Using QCs for simulating spectroscopy is particularly natural, as most of the LF observables have the form of

$$\mathcal{O} = \mathrm{poly}(a, a^\dagger, b, b^\dagger)\,, \qquad (10)$$

which can be easily measured in the quantum computer, once the final state is prepared.



(Pasquini, Lorce, 2012)

Takeaways

- Numerous advantages of the second-quantized LF Hamiltonian formulation come in handy at the stage of quantum simulation.
- Various LF models (phenomenology, ab initio) and quantum simulation algorithms (heuristic, Hamiltonian evolution) can be employed, depending on available resources.
- Results:
  - ⋆ 2002.04016 — adiabatic preparation of interacting eigenstates. Qubit counts and observables for Yukawa$_{1+1}$ and QCD$_{3+1}$.
  - ⋆ 2105.10941 — details of sparsity-based simulation in the compact encoding.
  - ⋆ 2011.13443, 2009.07885 — variational algorithms, unitary coupled cluster, BLFQ-NJL model of light mesons.
- Several approaches to the simulation of scattering are currently under development.

THANK YOU!!!