Light Cone 2021: Physics of Hadrons on the Light Front

Nucleon quasi parton distributions in the large N_c limit

with

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Introduction

Parton distribution functions (PDFs)

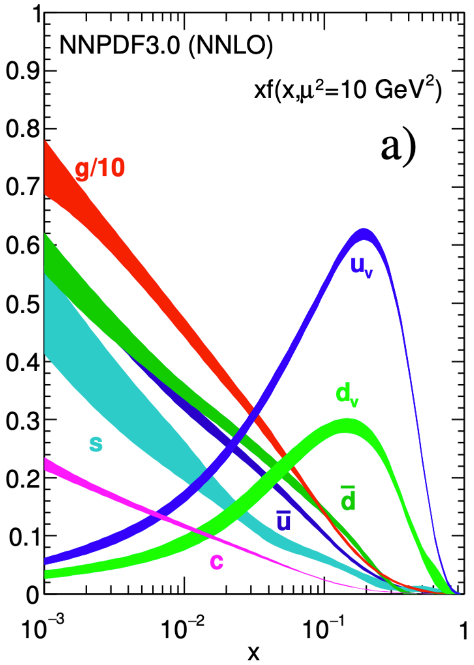
How partons (quarks and gluons) are distributed inside a hadron

Probability density on the light-cone

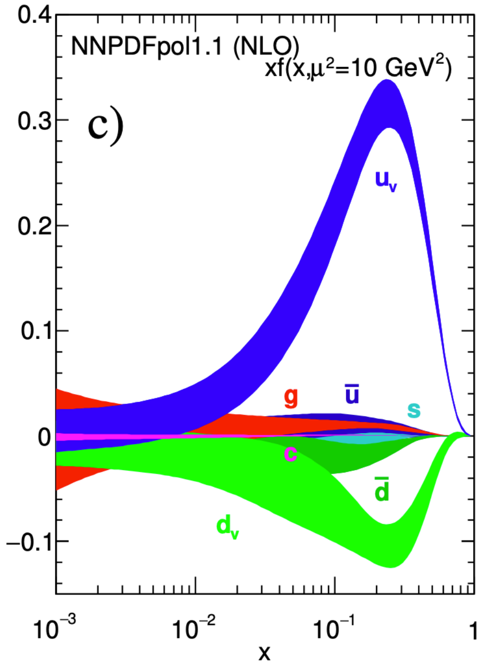
Non-perturbative QCD matrix elements

Factorization & universality

Plots from PDG 2021



R. D. Ball et al. (NNPDF), JHEP 04, 040 (2015)



E. R. Nocera et al. (NNPDF), Nucl. Phys. B887, 276 (2014)

Parton distribution functions (PDFs)

How partons (quarks and gluons) are distributed inside a hadron

Probability density on the light-cone

Non-perturbative QCD matrix elements

Factorization & universality

Theoretical understanding of PDFs

Effective models at low renormalization scale

- providing initial conditions of the QCD evolution
- to understand the properties of PDFs (qualitative and quantitative predictions)
- Chiral quark-soliton model (polarized antiquark asymmetry)

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

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Parton distribution functions (PDFs)

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Factorization & universality

Theoretical understanding of PDFs

Lattice QCD

- no direct computation is possible due to Euclidean nature

- studies on the Mellin moments of the PDFs, mostly

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2}\right)$$

$x \in (-\infty, +\infty)$

μ: renormalization scale

P_z: nucleon momentum

Large **M**omentum **E**ffective **T**heory

Jiunn-Wei Chen's talk on Monday for an overview

Spacelike equal-time matrix element → can be calculated on the Lattice

No unique definition → **$\Gamma=\gamma^3$** or **$\Gamma=\gamma^0$**

Approaches to PDFs in the limit Pz →∞, or v → 1.

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Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu_R,P^z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu_R}{\mu}, \frac{\mu}{p^z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2}\right)$$

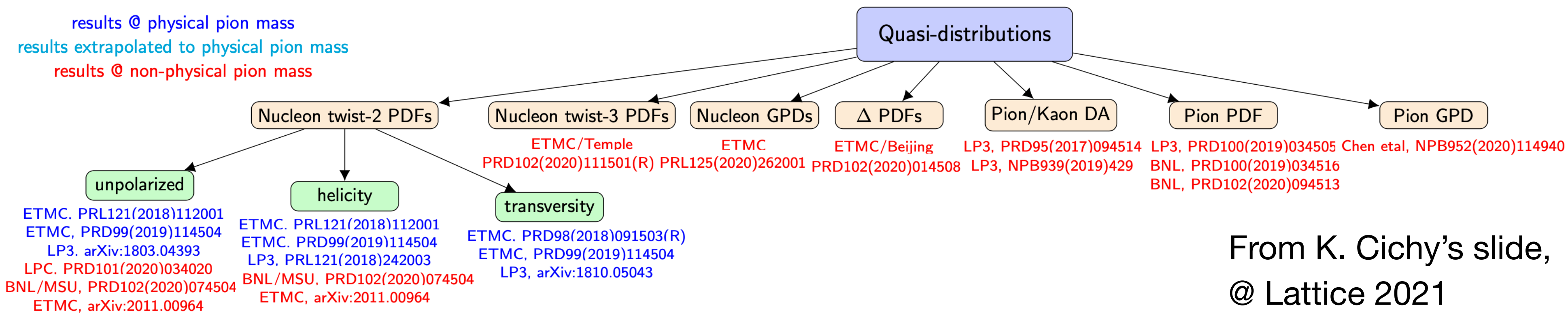
Perturbative matching coefficients

Divergences, Renormalization, matching coefficients…

Market results P_z ~ 2-3 GeV

N, π, K / PDFs, DAs, GPDs …

Jiunn-Wei Chen's talk on Monday for an overview



From K. Cichy's slide,
@ Lattice 2021

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

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Perturbative matching coefficients

Divergences, Renormalization, matching coefficients…

Market results P_z ~ 2-3 GeV

N, π, K / PDFs, DAs, GPDs …

Enough accuracy and uncertainty for actual application?

Reliable model computations on quasi-PDFs is needed

Review: K. Cichy and M. Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904
Community report: M. Constantinou et al, Prog.Part.Nucl.Phys. 121 (2021) 103908
and many more..

(Quasi-)PDFs in the chiral quark-soliton model

Nucleon PDFs at low renormalization scale: a successful description

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

Nucleon matrix element in Euclidean separation

Lorentz boost → PDFs ~ quasi-PDF

Properties of qPDFs for quarks and antiquarks in the nucleon:

Sum-rules, positivity, evolution in P_z

Gravitational form factor $\bar{c}^q$ is related to the momentum sum-rule:

$\bar{c}^q$ ~ non-convergence of the separate quark EMT operator

Mass decomposition of the nucleon

Interaction strength between the quark- and gluon- subsystems

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[Maxim Polyakov and HDS, JHEP 09 (2018) 156]

In this talk,

Description of the chiral quark-soliton model

quasi-PDFs within the chiral quark-soliton model

Properties: sum-rules for the quasi-PDFs as their Mellin moments

Numerical results for the unpolarized and longitudinally polarized quasi-quark distributions

Comparison of the Dirac structures defining the quasi-PDFs: γ_0 vs γ_3

Summary

quasi-PDFs in the χQSM

Quark distribution functions: large components

In general, in the large Nc limit:

Isosinglet unpolarized	$u(x) + d(x)$	$\sim N_c^2 \, \rho(N_c x)$
Isovector polarized	$\Delta u(x) - \Delta d(x)$	

Isovector unpolarized	$u(x) - d(x)$	$\sim N_c \, \rho(N_c x)$
Isosinglet polarized	$\Delta u(x) + \Delta d(x)$	

quasi-PDFs acquire overall factor of v

→ follow the same Nc ordering

Chiral quark-soliton model

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$Z = \int \mathcal{D}\pi^a d\psi^\dagger d\psi \ \exp \int d^4x \psi^\dagger(x)(i\partial\!\!\!/ + iMU^{\gamma_5})\psi(x)$$

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \qquad U(x) = \exp\left[\frac{i}{F_\pi}\pi^a(x)\tau^a\right]$$

From QCD to the low energy effective theory via the instantons

Parameters: $\bar{\rho} \sim 1/3 \text{ fm}$ & $\bar{R} \sim 1 \text{ fm}$

Intrinsic renormalization scale $\Lambda \sim 1/\bar{\rho} \approx 600 \text{ MeV}$

Dynamically generated quark mass $M = 350 \text{ MeV}$

Nucleon as the soliton made of Nc valence quarks in the self-consistent mean-field at large Nc

Interplays the quark-model and (topological) soliton picture of the baryons

Fully field theoretic: successively describes a wide class of baryon properties

Quasi-PDFs in the χQSM

Isoscalar unpolarized $x \in (-\infty, \infty)$

$$\sum_f q_f(x,v) = N_c M_N v \sum_{n,occ} \int \frac{d^3k}{(2\pi)^3} \delta(k^3 + vE_n - vM_N x) \left[\Phi_n^\dagger(\vec{k})(1 + v\gamma^0\gamma^3)\gamma_0 \Gamma \Phi_n(\vec{k})\right]$$

Isovector polarized (helicity)

$$\Delta u(x,v) - \Delta d(x,v) = -\frac{1}{3}(2T^3) N_c M_N v \sum_{n,occ} \int \frac{d^3k}{(2\pi)^3} \delta(k^3 + vE_n - vM_N x)$$
$$\left[\Phi_n^\dagger(\vec{k})(1 + v\gamma^0\gamma^3)\gamma_0 \Gamma \tau^3 \gamma^5 \Phi_n(\vec{k})\right]$$

Dirac eq. $H\Phi_n(\vec{x}) = E_n \Phi_n(\vec{x})$

$\Gamma = \gamma^0$ or $\Gamma = \gamma^3$

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Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\ q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\ xq(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Bjorken

$$\int_{-\infty}^{\infty} dx\ (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

→ better Dirac structure for P_N convergence ?

→ Interpretation of the QCD symmetry currents

Sum-rules

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Momentum

$$\int_{-\infty}^{\infty} dx\; x q(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Bjorken

$$\int_{-\infty}^{\infty} dx\; (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

U(1): charge density (γ^0) vs flux (γ^3)

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Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\, q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

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Momentum sum-rule is satisfied only by quarks

Energy-momentum tensor: momentum flux ($T^{30} \sim \partial_3 \gamma^0$) vs pressure ($T^{33} \sim \partial_3 \gamma^3$)

In general, $M_2^q(\Gamma = \gamma^3) = v \left(A^q(0) - \frac{1-v^2}{v^2} \bar{c}^q(0) \right)$

S. Bhattacharya et al, Phys.Rev.D 102 (2020) 5, 054021

Maxim Polyakov and HDS, JHEP 09 (2018) 156

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\; q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

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Bjorken

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Axial current: $\gamma^3 \sim S^3 g_A^{(3)}$ vs $\gamma^0 \sim \vec{S} \cdot \vec{v}\, g_A^{(3)}$

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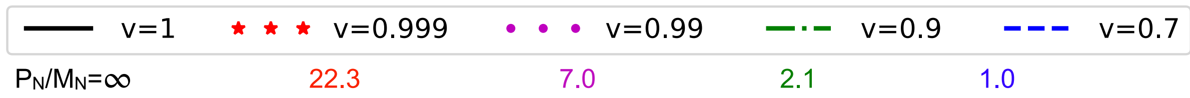
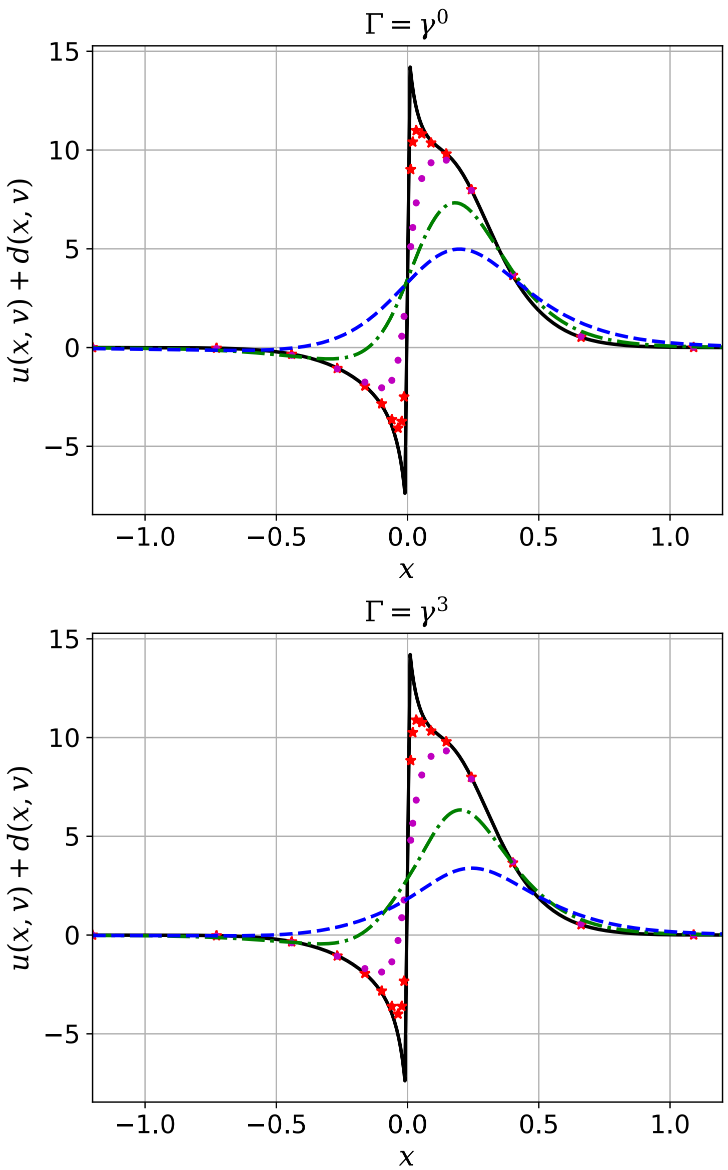
Numerical results

$u + d$





$\bar{q}(x) = -q(-x)$ (LC PDF)

Strong v dependence at small x: due to smearing of the quark and antiquark parts

Antiquark part (negative x) breaks the positivity

Baryon number and Momentum sum-rules are satisfied in good accuracy

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

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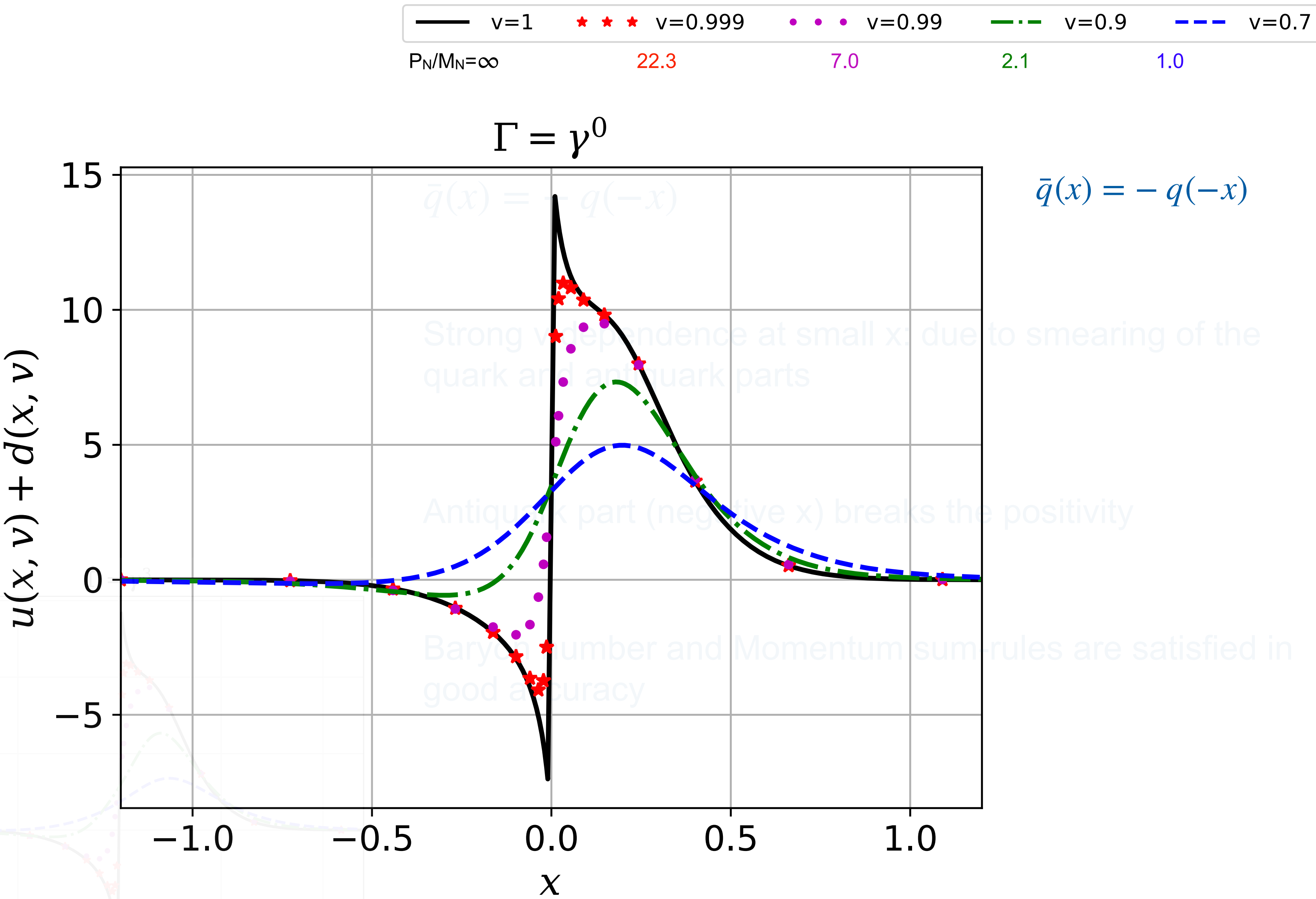
$u + d$



$\bar{q}(x) = -q(-x)$

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

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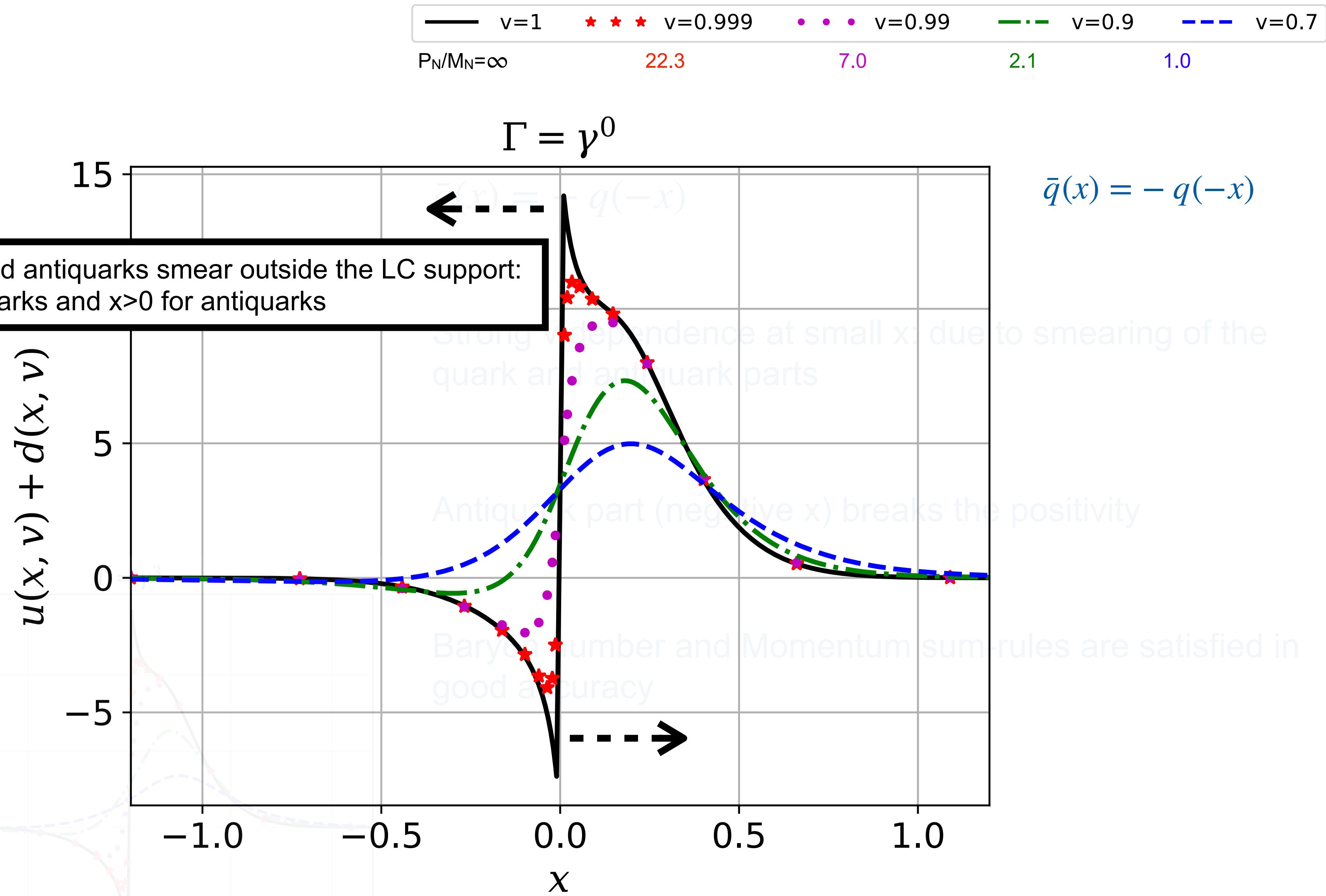


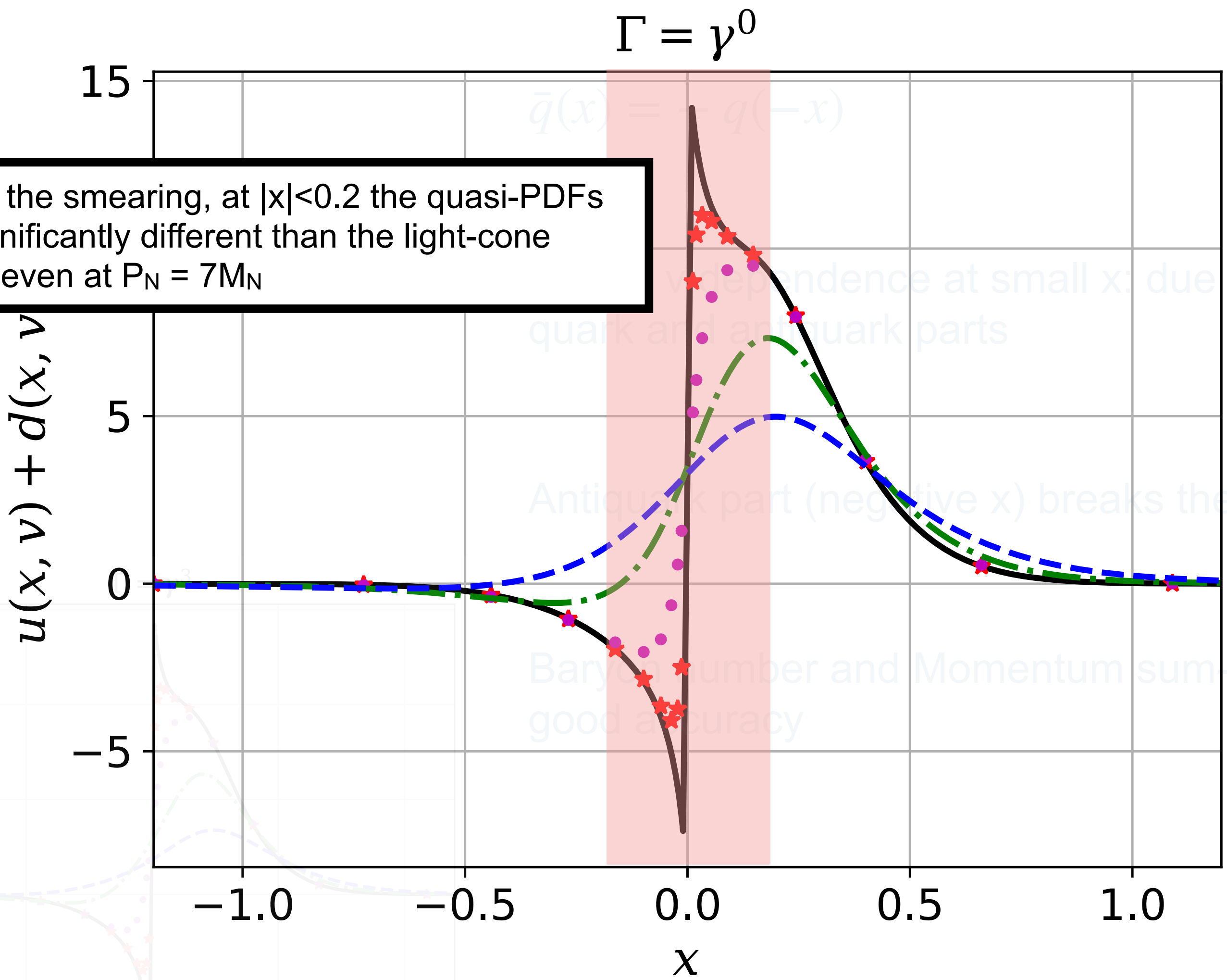
Quarks and antiquarks smear outside the LC support:
x<0 for quarks and x>0 for antiquarks

$\bar{q}(x) = -q(-x)$

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

$u + d$

v=1 | v=0.999 | v=0.99 | v=0.9 | v=0.7
$P_N/M_N=\infty$ | 22.3 | 7.0 | 2.1 | 1.0

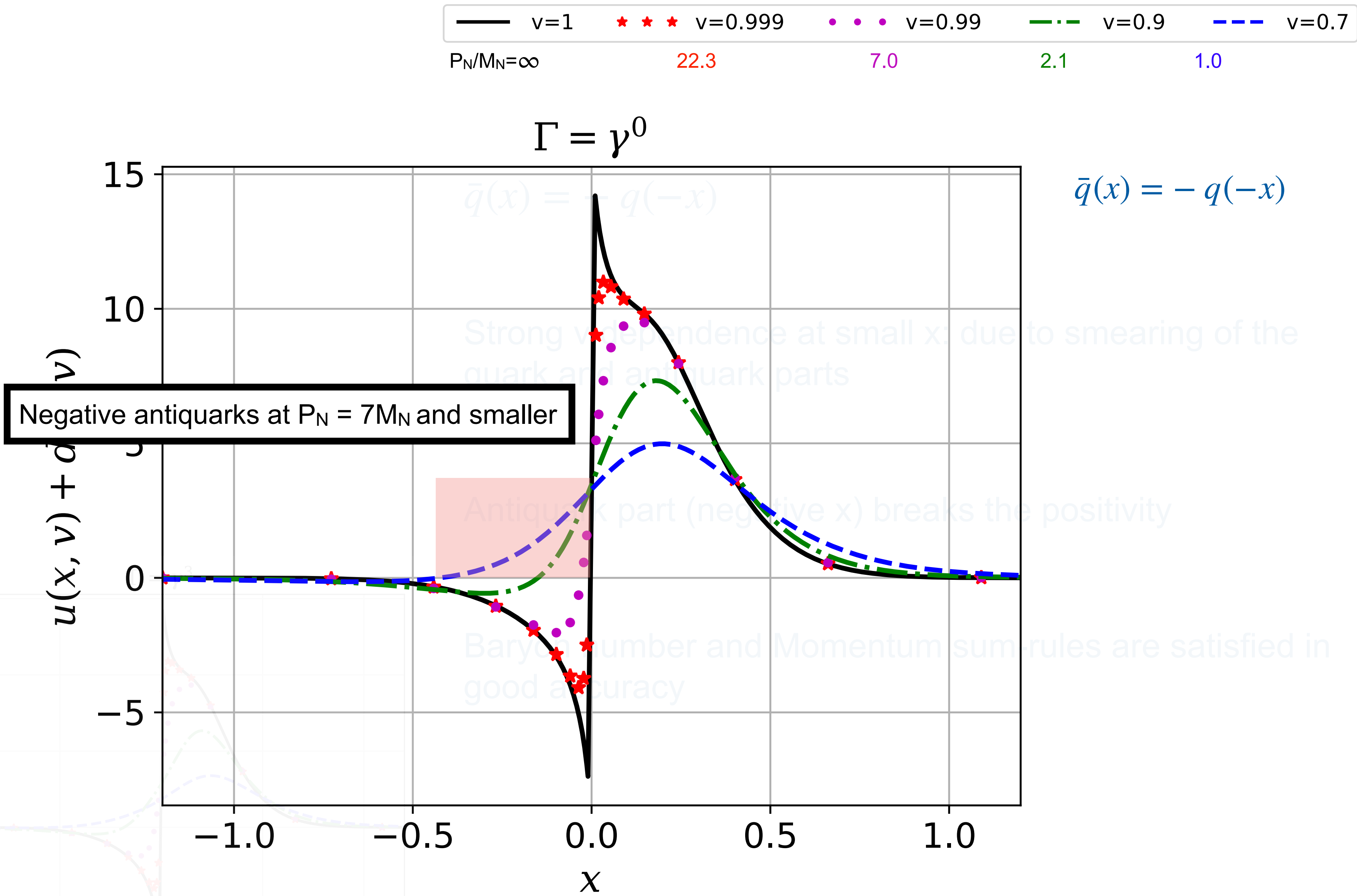


Due to the smearing, at |x|<0.2 the quasi-PDFs are significantly different than the light-cone PDFs, even at $P_N = 7M_N$

$\bar{q}(x) = -q(-x)$

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

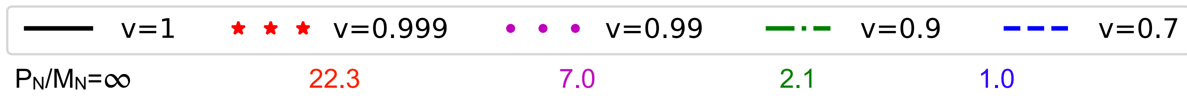
$u + d$

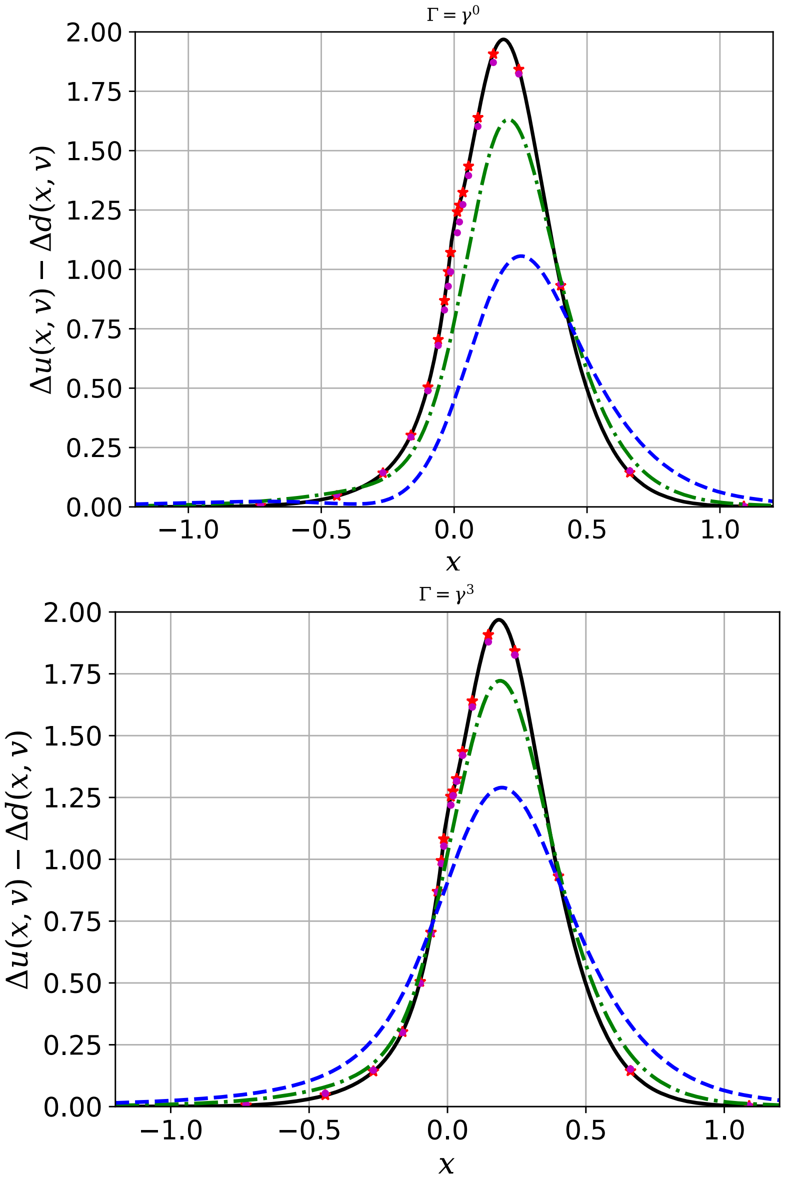


$\bar{q}(x) = -q(-x)$

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

$\Delta u - \Delta d$





$\Delta\bar{q}(x) = \Delta q(-x)$

Antiquark asymmetry $\Delta\bar{u} - \Delta\bar{d} \neq 0$

At v=0.9 (P ~ 2 GeV), qPDF ~ PDF

$\Gamma = \gamma^3$ qPDF approaches faster to the lightcone PDF

$$\int_{-\infty}^{\infty} dx\ (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

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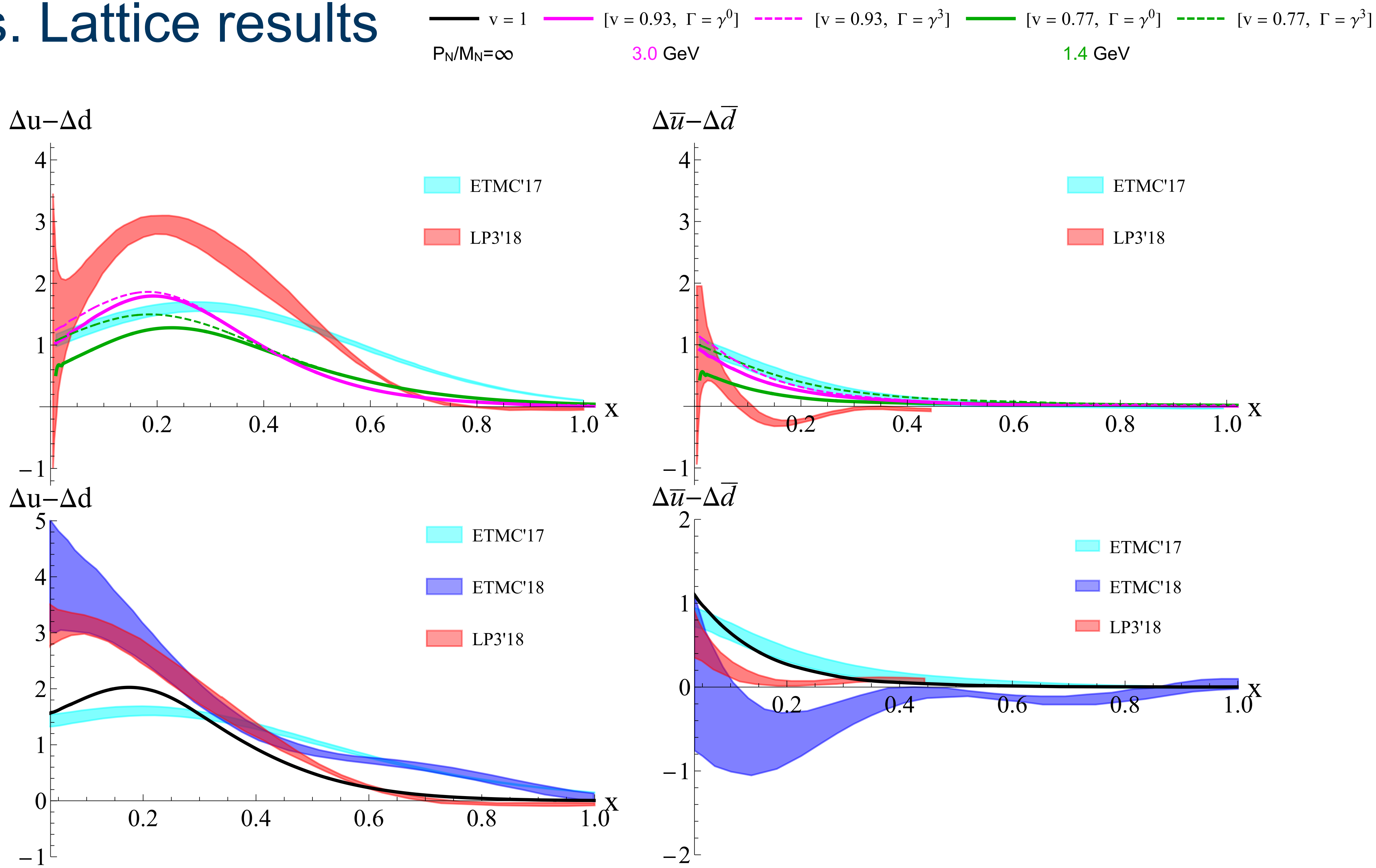
vs. Lattice results



(m_π, P_z, μ) = (0.37, 1.4, 2.0) **[ETMC'17 Alexandrou et al. Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017]**

(0.13,1.4,2.0) **[ETMC'18 Alexandrou et al. Phys.Rev.Lett. 121 (2018) 11, 112001, 2018]**

(0.135, 3.0, 3.0) **[LP3'18 Lin et al.Phys. Rev. Lett., vol. 121, no. 24, p. 242003, 2018]**

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Closing remarks

Summary

χQSM: a working framework for understanding the (quasi-)PDFs

Sum-rules: $\bar{c}^q$, 'better' Γ for the convergence to the PDFs

Good convergence of the isovector polarized

vs. poor $\mathbf{P_z} \to \infty$ convergence for the isoscalar unpolarized quasi-PDF

Antiquark flavor asymmetry is predicted

Future tasks

Transversity PDF and other small components in the large Nc

Quantitative comparison vs Lattice: scale evolution, large x, $m_\pi \neq 0$

gluon PDFs (the EIC physics)

Thank you very much!