Nucleon quasi parton distributions in the large N_c limit

with
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Introduction
Parton distribution functions (PDFs)

How partons (quarks and gluons) are distributed inside a hadron

Probability density on the light-cone

Non-perturbative QCD matrix elements

Factorization & universality

Plots from PDG 2021

R. D. Ball et al. (NNPDF), JHEP 04, 040 (2015)

Parton distribution functions (PDFs)

How partons (quarks and gluons) are distributed inside a hadron
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Factorization & universality

Theoretical understanding of PDFs

Effective models at low renormalization scale
- providing initial conditions of the QCD evolution
- to understand the properties of PDFs (qualitative and quantitative predictions)
- Chiral quark-soliton model (polarized antiquark asymmetry)

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]
Parton distribution functions (PDFs)

- How partons (quarks and gluons) are distributed inside a hadron
- Probability density on the light-cone
- Non-perturbative QCD matrix elements
- Factorization & universality

Theoretical understanding of PDFs

Lattice QCD
- no direct computation is possible due to Euclidean nature
- studies on the Mellin moments of the PDFs, mostly
Quasi parton distribution function


\[ q(x, \mu, P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P | \bar{\psi}(0) \gamma^z \exp \left[ -ig \int_0^z dz' A^z(z') \right] \psi(z) | P \rangle + O\left( \frac{\Lambda_{QCD}^2}{(P^z)^2}, \frac{M_N^2}{(P^z)^2} \right) \]

\[ x \in (-\infty, +\infty) \]
\[ \mu: \text{renormalization scale} \]
\[ P_z: \text{nucleon momentum} \]

**Large Momentum Effective Theory**

Spacelike equal-time matrix element → can be calculated on the Lattice

No unique definition → \( \Gamma = \gamma^3 \) or \( \Gamma = \gamma^0 \)

Approaches to PDFs in the limit \( P_z \to \infty \), or \( v \to 1 \).
Quasi parton distribution function


\[ q(x, \mu_R, P^z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu_R}{\mu}, \frac{\mu}{P^z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2}\right) \]

Perturbative matching coefficients

Divergences, Renormalization, matching coefficients…

Market results \(P_z \sim 2-3\) GeV

\(N, \pi, K / PDFs, DAs, GPDs\) …

Jiunn-Wei Chen’s talk on Monday for an overview

From K. Cichy’s slide, @ Lattice 2021
Quasi parton distribution function


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\( N, \pi, K / \text{PDFs, DAs, GPDs} \ldots \)

Enough accuracy and uncertainty for actual application?

**Reliable model computations on quasi-PDFs is needed**

Community report: M. Constantinou et al, Prog.Part.Nucl.Phys. 121 (2021) 103908
and many more..
(Quasi-)PDFs in the chiral quark-soliton model

Nucleon PDFs at low renormalization scale: a successful description
[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

Nucleon matrix element in Euclidean separation

\[ \text{Lorentz boost} \rightarrow \text{PDFs} \sim \text{quasi-PDF} \]

Properties of qPDFs for quarks and antiquarks in the nucleon:

\[ \text{Sum-rules, positivity, evolution in } P_z \]

Gravitational form factor \( \bar{c}^q \) is related to the momentum sum-rule:

\[ \bar{c}^q \sim \text{non-convergence of the separate quark EMT operator} \]

Mass decomposition of the nucleon

Interaction strength between the quark- and gluon- subsystems
(Quasi-)PDFs in the chiral quark-soliton model

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[Maxim Polyakov and HDS, JHEP 09 (2018) 156]
In this talk,

Description of the chiral quark-soliton model

quasi-PDFs within the chiral quark-soliton model

Properties: sum-rules for the quasi-PDFs as their Mellin moments

Numerical results for the unpolarized and longitudinally polarized quasi-quark distributions

Comparison of the Dirac structures defining the quasi-PDFs: \( \gamma_0 \) vs \( \gamma_3 \)

Summary
quasi-PDFs in the $\chi$QSM
Quark distribution functions: large components

In general, in the large $N_c$ limit:

<table>
<thead>
<tr>
<th>Quark Configuration</th>
<th>PDF Expression</th>
<th>Large $N_c$ Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isosinglet unpolarized</td>
<td>$u(x) + d(x)$</td>
<td>$\sim N_c^2 \rho(N_c x)$</td>
</tr>
<tr>
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</table>

Quasi-PDFs acquire overall factor of $v$

$\rightarrow$ follow the same $N_c$ ordering
Chiral quark-soliton model  


\[
Z = \int D\pi^{a} d\psi^\dagger d\psi \exp \int d^{4}x \psi^\dagger(x)(i\partial + iMU\gamma_{5})\psi(x)
\]

\[
U(x) = U(x)\frac{1 + \gamma_{5}}{2} + U^\dagger(x)\frac{1 - \gamma_{5}}{2} \quad U(x) = \exp \left[ \frac{i}{F_{\pi}} \pi^{a}(x)\tau^{a} \right]
\]

From QCD to the low energy effective theory via the instantons

Parameters: $\bar{\rho} \sim 1/3 \text{ fm}$ & $\bar{R} \sim 1 \text{ fm}$

Intrinsic renormalization scale $\Lambda \sim 1/\bar{\rho} \approx 600 \text{ MeV}$

Dynamically generated quark mass $M = 350 \text{ MeV}$

Nucleon as the soliton made of $N_{c}$ valence quarks in the self-consistent mean-field at large $N_{c}$

Interplays the quark-model and (topological) soliton picture of the baryons

Fully field theoretic: successively describes a wide class of baryon properties
Quasi-PDFs in the $\chi$QSM

Isoscalar unpolarized  \( x \in (-\infty, \infty) \)

\[
\sum_f q_f(x, v) = N_c M_N v \sum_{n, occ} \int \frac{d^3 k}{(2\pi)^3} \delta(k^3 + v E_n - v M_N x) \left[ \Phi_n^+(\vec{k})(1 + v \gamma^0 \gamma^3)\gamma_0 \Gamma_n \Phi_n(\vec{k}) \right]
\]

Isovector polarized (helicity)

\[
\Delta u(x, v) - \Delta d(x, v) = -\frac{1}{3}(2T^3) N_c M_N v \sum_{n, occ} \int \frac{d^3 k}{(2\pi)^3} \delta(k^3 + v E_n - v M_N x) \left[ \Phi_n^+(\vec{k})(1 + v \gamma^0 \gamma^3)\gamma_0 \Gamma \tau^3 \gamma^5 \Phi_n(\vec{k}) \right]
\]

Dirac eq.  \( H \Phi_n(x) = E_n \Phi_n(x) \)

\( \Gamma = \gamma^0 \) or  \( \Gamma = \gamma^3 \)
Sum-rules

Baryon number

\[ \int_{-\infty}^{\infty} dx \, q(x, v) = \begin{cases} \rho N_c B, & \Gamma = \gamma^0 \\ \rho v N_c B, & \Gamma = \gamma^3 \end{cases} \]

Momentum

\[ \int_{-\infty}^{\infty} dx \, xq(x, v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases} \]

Bjorken

\[ \int_{-\infty}^{\infty} dx \, (\Delta u(x, v) - \Delta d(x, v)) = \begin{cases} \rho v g_A^{(3)}, & \Gamma = \gamma^0 \\ \rho g_A^{(3)}, & \Gamma = \gamma^3 \end{cases} \]

→ better Dirac structure for $P_N$ convergence?

→ Interpretation of the QCD symmetry currents
Sum-rules

Baryon number

\[ \int_{-\infty}^{\infty} dx \, q(x, \nu) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ vN_c B, & \Gamma = \gamma^3 \end{cases} \]

Momentum

\[ \int_{-\infty}^{\infty} dx \, xq(x, \nu) = \begin{cases} 1, & \Gamma = \gamma^0 \\ \nu, & \Gamma = \gamma^3 \end{cases} \]

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U(1): charge density ($\gamma^0$) vs flux ($\gamma^3$)
Sum-rules

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\[ \int_{-\infty}^{\infty} dx \, q(x, v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases} \]

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Momentum sum-rule is satisfied only by quarks

Energy-momentum tensor: momentum flux \((T^{30} \sim \partial_3 \gamma^0)\) vs pressure \((T^{33} \sim \partial_3 \gamma^3)\)

In general, \(M_2^q(\Gamma = \gamma^3) = v \left( A^q(0) - \frac{1 - v^2}{v^2} \tilde{c}^q(0) \right)\)

S. Bhattacharya et al, Phys.Rev.D 102 (2020) 5, 054021
Maxim Polyakov and HDS, JHEP 09 (2018) 156
Sum-rules

Baryon number

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Axial current: \( \gamma^3 \sim S^3 g_A^{(3)} \) vs \( \gamma^0 \sim \vec{S} \cdot \vec{v} g_A^{(3)} \)
Numerical results
$u + d$

\[ \bar{q}(x) = -q(-x) \text{ (LC PDF)} \]

Strong $v$ dependence at small $x$: due to smearing of the quark and antiquark parts

Antiquark part (negative $x$) breaks the positivity

Baryon number and Momentum sum-rules are satisfied in good accuracy

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808, 135665 (2020)]
Strong $v$ dependence at small $x$: due to smearing of the quark and antiquark parts.

Antiquark part (negative $x$) breaks the positivity.

Baryon number and Momentum sum-rules are satisfied in good accuracy.

\[ \bar{q}(x) = - q(-x) \]

\[ u + d \]

\[ \Gamma = \gamma^0 \]

\[ P_N/M_N = \infty \]

\[ 22.3 \quad 7.0 \quad 2.1 \quad 1.0 \]

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808, 135665 (2020)]
Quarks and antiquarks smear outside the LC support: 
\( x < 0 \) for quarks and \( x > 0 \) for antiquarks

\[ \bar{q}(x) = - q(-x) \]
Due to the smearing, at $|x|<0.2$ the quasi-PDFs are significantly different than the light-cone PDFs, even at $P_N = 7M_N$.
Strong $v$ dependence at small $x$: due to smearing of the quark and antiquark parts

Antiquark part (negative $x$) breaks the positivity

Baryon number and Momentum sum-rules are satisfied in good accuracy

$$\bar{q}(x) = - q(-x)$$

Negative antiquarks at $P_N = 7M_N$ and smaller

\[ \Delta u - \Delta d \]

\[ \Delta \bar{q}(x) = \Delta q(-x) \]

Antiquark asymmetry \( \Delta \bar{u} - \Delta \bar{d} \neq 0 \)

At \( v=0.9 \) (\( P \sim 2 \text{ GeV} \)), qPDF \( \sim \) PDF

\[ \Gamma = \gamma^3 \text{ qPDF approaches faster to the lightcone PDF} \]

\[ \int_{-\infty}^{\infty} dx \left( \Delta u(x, v) - \Delta d(x, v) \right) = \begin{cases} vg_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}, & \Gamma = \gamma^3 \end{cases} \]
vs. Lattice results

\[ \Delta u - \Delta d \]

\[ \Delta \bar{u} - \Delta \bar{d} \]

\[ (m_g, P_z, \mu) = (0.37, 1.4, 2.0) \] [ETMC'17 Alexandrou et al. Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017]

\[ (0.13, 1.4, 2.0) \] [ETMC'18 Alexandrou et al. Phys.Rev.Lett. 121 (2018) 11, 112001, 2018]

\[ (0.135, 3.0, 3.0) \] [LP3'18 Lin et al.Phys. Rev. Lett., vol. 121, no. 24, p. 242003, 2018]
Summary

\(\chi\text{QSM: a working framework for understanding the (quasi-)PDFs}\)

Sum-rules: \(\bar{c}q\), 'better' \(\Gamma\) for the convergence to the PDFs

Good convergence of the isovector polarized

vs. poor \(P_{\perp} \to \infty\) convergence for the isoscalar unpolarized quasi-PDF

Antiquark flavor asymmetry is predicted

Future tasks

Transversity PDF and other small components in the large \(N_c\)

Quantitative comparison vs Lattice: scale evolution, large \(x, m_\pi \neq 0\)

gluon PDFs (the EIC physics)
Thank you very much!