Anatomy of Nucleon Self-energy from Instant to Light-Front

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Concepts and methods

- Instant form dynamics (IFD): x^0, x^3.
- Light-front dynamics (LFD): $x^\pm = (x^0 \pm x^3)/\sqrt{2}$.
- Interpolating dynamics (ID):

$$\begin{bmatrix} x^{\widehat{+}} \\ x^{\widehat{-}} \end{bmatrix} = \begin{bmatrix} \cos[\delta] & \sin[\delta] \\ \sin[\delta] & -\cos[\delta] \end{bmatrix} \begin{bmatrix} x^0 \\ x^3 \end{bmatrix} \tag{1}$$

with $0 \leq \delta \leq \pi/4$. The corresponding four momentum is

$$q^2 = \mathbb{C}(q_{\widehat{+}}^2 - q_{\widehat{-}}^2) + 2\mathbb{S} q_{\widehat{+}} q_{\widehat{-}} - q_\perp^2. \tag{2}$$

Introduction and Motivation

- **Introduction:**
  - Tree level intermediate fermion propagator can be decomposed into forward and backward moving parts[1].
  - The backward moving part in LFD features an instantaneous contribution which involves the constraint degrees of freedom of the fermion, that is unique in the LFD.
- **Motivation:**
  - To identify how much the LF instantaneous (LFI) part contributes in loop level nucleon self-energy numerically and analytically.
  - To trace the LFI from the backward moving part of nucleon self-energy by using the interpolating dynamics (ID).
  - To distinguish the LF zero mode contribution appeared at the $p^z \to -\infty$ point in LFD numerically and analytically.
  - To show the difference between the LFD and IFD.

Definition: anatomy in the tree level

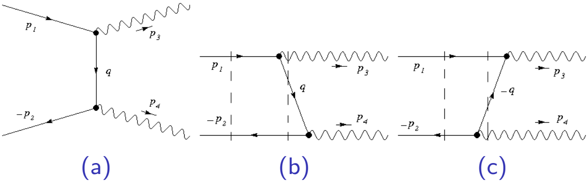


Figure 1: (a) Feynman diagram for $e^+e^- \to \gamma\gamma$ process (t-channel). Time-ordered diagrams (b) and (c) for $e^+e^- \to \gamma\gamma$ annihilation process. The u-channel amplitudes can be obtained by crossing the two final state particles.

Definition: anatomy in the tree level

- The intermediate virtual fermion propagator (Fig. 1a) and its anatomy into forward and backward parts (Fig. 1b and 1c) [1]

$$\Sigma_N = \frac{\not{q} + M}{q^2 - M^2} = \Sigma_{Na} + \Sigma_{Nb} = \frac{\not{Q}_a + M}{2Q^{\widehat{+}}(q_{\widehat{+}} - Q_{a\widehat{+}})} + \frac{-\not{Q}_b + M}{2Q^{\widehat{+}}(-q_{\widehat{+}} - Q_{b\widehat{+}})} \tag{3}$$

where capitalized symbols are "on-mass-shell" (OMS)

$$Q_a = \left(\frac{-\mathbb{S}q_{\widehat{-}} + Q^{\widehat{+}}}{\mathbb{C}},\ q_\perp,\ q_{\widehat{-}} \right)$$

$$Q_b = \left(\frac{\mathbb{S}q_{\widehat{-}} + Q^{\widehat{+}}}{\mathbb{C}},\ -q_\perp,\ -q_{\widehat{-}} \right)$$

$$Q^{\widehat{+}} = \sqrt{q_{\widehat{-}}^2 + \mathbb{C}(q_\perp^2 + M^2)}. \tag{4}$$

Definition: anatomy in the tree level

- The off-mass shell version (with prime notation) of forward and backward moving nucleon self-energy

$$\Sigma'_{Na} = \frac{\not{q} + M}{2Q^{\widehat{+}}(q_{\widehat{+}} - Q_{a\widehat{+}})} \tag{5}$$

$$\Sigma'_{Nb} = \frac{\not{q} + M}{2Q^{\widehat{+}}(-q_{\widehat{+}} - Q_{b\widehat{+}})} \tag{6}$$

- Define the remnant part to assist our calculation

$$\Sigma_{NRP} = \Sigma_{Nb} - \Sigma'_{Nb} = \frac{\gamma^{\widehat{+}}}{2Q^{\widehat{+}}}. \tag{7}$$

Definition: anatomy in the loop level

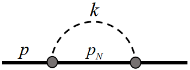


Figure 2: The nucleon-pion loop

- The nucleon self-energy is ($p_N = p - k$)

$$\hat{\Sigma} = i\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} (\not{k}\gamma^5\vec{\tau}) \frac{i(\not{p}_N + M)}{p_N^2 - M^2 + i\epsilon} (\gamma^5\not{k}\vec{\tau}) \frac{i}{k^2 - m_\pi^2 + i\epsilon}. \quad (8)$$

- Spin sum calculation: $\Sigma = \frac{1}{2}\sum_s \bar{u}(p,s)\hat{\Sigma}u(p,s)$.
- With $D_N = p_N^2 - M^2 + i\epsilon$, $D_\pi = k^2 - m_\pi^2 + i\epsilon$, the nucleon self-energy can be simplified as

$$\Sigma = -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-\left[2(p\cdot k)(k\cdot p_N) - (p\cdot p_N)k^2\right] + M^2k^2}{D_N D_\pi M}. \quad (9)$$

Definition: anatomy in the loop level

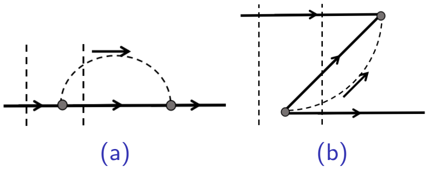


Figure 3: Time-ordered diagrams (a) and (b) for the forward moving nucleon self-energy (positive energy diagram) and the backward moving nucleon self-energy ("Z" graph).

Definition: anatomy in the loop level

- The Σ_a

$$\Sigma_a = -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\left[-2(p\cdot k)(k\cdot P_{Na}) + (p\cdot P_{Na})k^2\right] + M^2k^2}{2P_N^{\widehat{+}}(p_{N\widehat{+}} - P_{Na\widehat{+}})D_\pi M} \tag{10}$$

- The Σ_b

$$\Sigma_b = -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\left[2(p\cdot k)(k\cdot P_{Nb}) - (p\cdot P_{Nb})k^2\right] + M^2k^2}{2P_N^{\widehat{+}}(-p_{N\widehat{+}} - P_{Nb\widehat{+}})D_\pi M} \tag{11}$$

- The Σ_{RP}

$$\Sigma_{RP} = -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^2k_\perp dk_{\widehat{-}} dk_{\widehat{+}}}{(2\pi)^4} \frac{-[2(p\cdot k)k^{\widehat{+}} - p^{\widehat{+}}k^2]}{2P_N^{\widehat{+}} D_\pi M} \tag{12}$$

The backward and forward moving part in LFD

- The backward moving Σ_b in the LF limit, becomes LFI contribution

$$\Sigma_b^{LFD} = \Sigma_{LFI} = -\, i3 \left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-2(p\cdot k)k^+ + k^2 p^+}{2p_N^+ D_\pi M}, \quad (13)$$

  which is the same with Σ_{RP}^{LFD}.

- The forward moving part of the nucleon self-energy becomes, in LF limit,

$$\Sigma_a^{LFD} = -i3 \left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-\left[2(p\cdot k)(k\cdot P_{Na}^{LFD}) - (p\cdot P_{Na}^{LFD})k^2\right] + M^2 k^2}{D_N D_\pi M}, \quad (14)$$

  with $P_{Na}^{LFD} = \left(\frac{(p_\perp - k_\perp)^2 + M^2}{2(p^+ - k^+)}, p_\perp - k_\perp, p^+ - k^+\right)$.

Calculation for the LFI contribution

- The LFI in Eq.(13), with some transformations, becomes

$$\Sigma_{LFI} = -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^2k_\perp dk^- dk^+}{(2\pi)^4} \left\{ \frac{p\cdot k}{D_\pi M} + \frac{p^+[k^2 - 2(p\cdot k)]}{2(p^+ - k^+)MD_\pi} \right\}, \tag{15}$$

where the first term is zero, because of oddity in k.

- The Pauli-Villars (PV) regularization

$$\frac{1}{D_\pi} \to \frac{-\Lambda^2}{D_\pi D_\Lambda} = \frac{-\Lambda^2}{D_\Lambda - D_\pi}\left(\frac{1}{D_\pi} - \frac{1}{D_\Lambda}\right) = \frac{-\Lambda^2}{m_\pi^2 - \Lambda^2}\left(\frac{1}{D_\pi} - \frac{1}{D_\Lambda}\right). \tag{16}$$

where the $D_\Lambda = k^2 - \Lambda^2 + i\epsilon$.

- Then the LFI becomes ($x = k^+/p^+$ is momentum fraction)

$$\Sigma_{LFI:1F} = \frac{-\Lambda^2}{D_\Lambda}\Sigma_{LFI} = -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^2k_\perp dk^- dxp^+}{(2\pi)^4} \frac{-\Lambda^2}{2M}\left[\frac{1}{(1-x)D_\Lambda} + \frac{m_\pi^2}{(1-x)D_\pi D_\Lambda} - \frac{2p^+k^- + 2p^-p^+x}{(1-x)D_\pi D_\Lambda}\right]. \tag{17}$$

Calculation for the LFI contribution

- The point like theory:

$$\int dk^- \frac{1}{D_\pi} = \pi i \log\left[\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right] \frac{\delta[x]}{p^+} \tag{18}$$

where the μ is the regularization mass parameter.

- The first term in Eq.(17) as an example becomes

$$\int dxp^+ \int dk^- \frac{1}{(1-x)D_\Lambda} = \int dx \log\left[\frac{k_\perp^2 + \Lambda^2}{\mu^2}\right] \frac{\pi i \delta[x]}{(1-x)} = \pi i \log\left[\frac{k_\perp^2 + \Lambda^2}{\mu^2}\right] \tag{19}$$

where all the p^+s are canceled and only the $x = 0$ contribute.

- The LF zero mode contribution where $p^+ = 0$ is included during the x integration with $0 < x < 1$.

Calculation for the LFI contribution

- The derivative technique

$$\Sigma_{x:nF} = \frac{(-\Lambda^2)^n (m-1)!}{(n-1)!} \frac{\partial^{n-1}}{\partial(\Lambda^2)^{n-1}} \frac{\Sigma_{x:mF}}{(-\Lambda^2)^m} \qquad (20)$$

  with $n > m$, where the nF and mF in the subscript represent the number of PV Form factor (F) multiplied to the subject Σ_x.

- With m=1 and n=4 being chosen for the integrations, the final result for LFI reads

$$\Sigma_{LFI:4F} = -\frac{g_A^2 \Lambda^8}{256 M f_\pi^2 \pi^2} \left(-\frac{\Lambda^4 + \frac{2m_\pi^6}{\Lambda^2} - 6\Lambda^2 m_\pi^2 + 6 m_\pi^4 \log\left[\frac{\Lambda^2}{m_\pi^2}\right] + 3 m_\pi^4}{2\left(\Lambda^2 - m_\pi^2\right)^4} \right). \qquad (21)$$

Calculation for the LFI contribution

- The leading non-analytic behavior for LFI is

$$\Sigma_{LFI}^{LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(\frac{m_\pi^4 \log[m_\pi^2]}{8M\pi} + \mathcal{O}(m_\pi^5)\right). \qquad (22)$$

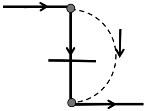


Figure 4: Time-ordered diagram for the LF instantaneous contribution Σ_{LFI}.

Calculation for the forward moving part in LFD

- The forward moving part of the nucleon self-energy in LFD, after some simplifications yields

$$\Sigma_a^{LFD} = -\, i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{(P_{Na}^{LFD}\cdot k)}{D_\pi M} + \frac{(M^2 + p_N\cdot P_{Na}^{LFD})k^2}{D_N D_\pi M} \tag{23}$$

which can be decomposed as $\Sigma_a^{LFD} = \Sigma - \Sigma_{LFI}$.

- The LNA behavior for OMS is

$$\Sigma_a^{LFD:LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{3m_\pi^4 \log[m_\pi^2]}{8M\pi} + \mathcal{O}(m_\pi^5)\right). \tag{24}$$

The ID: to relate LFD with IFD

- The off-mass-shell forward and backward part

$$\begin{aligned} \Sigma'_a =& \Sigma_a + \Sigma_{RP} \\ \Sigma'_b =& \Sigma_b - \Sigma_{RP} \end{aligned} \tag{25}$$

$$\begin{aligned} \Sigma'_a &= -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-2(p\cdot k)^2 + k^2(p\cdot k) + 2k^2M^2}{2P_N^{\widehat{+}} M(p_{N\widehat{+}} - P_{Na\widehat{+}})D_\pi} \\ \Sigma'_b &= -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-2(p\cdot k)^2 + k^2(p\cdot k) + 2k^2M^2}{2P_N^{\widehat{+}} M(-p_{N\widehat{+}} - P_{Nb\widehat{+}})D_\pi} \end{aligned} \tag{26}$$

which can be further decomposed into four terms $\Sigma'_{a\pm}$, $\Sigma'_{b\pm}$ by decomposing the pion poles in D_π.

Rescale the variables

- Rescale: $k_{\widehat{-}} = k'_{\widehat{-}}\sqrt{\mathbb{C}}$, and $k_{\widehat{+}} = k'_{\widehat{+}}/\sqrt{\mathbb{C}}$ (to not alter the area spanned by the 4-momentum).
- Inference: $k'^{\widehat{+}} = k'_{\widehat{+}} + \mathbb{S}k'_{\widehat{-}}$, and $dk'^{\widehat{+}} = dk'_{\widehat{+}}$.
- The rescaled Σ'_{a+} becomes "independent" of interpolation angles

$$\Sigma'_{a+} = -i3\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^2k_\perp\, dk'_{\widehat{-}}\, dk'^{\widehat{+}}}{(2\pi)^4} \frac{-2(p\cdot k)^2 + k^2(p\cdot k) + 2k^2M^2}{-2P_N'^{\widehat{+}} M 2\omega'_{\widehat{k}}(k'^{\widehat{+}} - \kappa'^{0+})(k'^{\widehat{+}} - \kappa'^{1-})} \tag{27}$$

with rescaled poles

$$\kappa'^{0\pm} = \pm\sqrt{k'^2_{\widehat{-}} + k^2_\perp + m^2_\pi - i\epsilon} = \pm\omega'_{\widehat{k}} \tag{28}$$

$$\kappa'^{1\pm} = \sqrt{p'^2_{\widehat{-}} + p^2_\perp + M^2} \pm \sqrt{p'^2_{N\widehat{-}} + p^2_{N\perp} + M^2 - i\epsilon} = p'^{\widehat{+}} \pm P_N'^{\widehat{+}}. \tag{29}$$

Definition: the reference frames

- Frame X

$$\vec{p}' = \left(p'_{\hat{-}}, p_\perp\right) = 0, \tag{30}$$

with

$$p'_{\hat{-}} = p_{\hat{-}}/\sqrt{\mathbb{C}} = \frac{\sqrt{M^2 + p^{z2}}\sin\delta + p^z\cos\delta}{\sqrt{\cos 2\delta}}. \tag{31}$$

In IFD ($\delta = 0$): $p_{\hat{-}}/\sqrt{\mathcal{C}} = p^z = 0$ is the rest frame.

- Frame Y

$$k'_{\hat{-}} = \pm(1-y)p'_{\hat{-}},\ dk'_{\hat{-}} = \mp p'_{\hat{-}}dy, \tag{32}$$

with $p'_{\hat{-}} \to \infty$ and $p_\perp = 0$. The $\pm$ corresponds to the forward and backward moving parts.

In IFD ($\delta = 0$): $p_{\hat{-}}/\sqrt{\mathcal{C}} = p^z \to \infty$ is the infinite momentum frame (IMF).

In ID: the LNA for the forward and backward parts

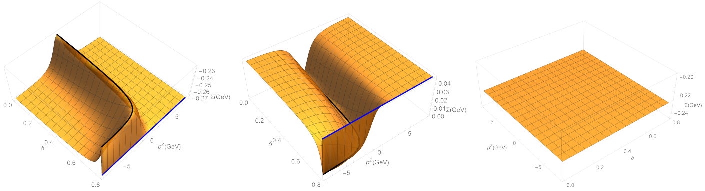
LNA (in the unit of $-\frac{3g_A^2}{32\pi f_\pi^2}$)	Σ'_a	Σ'_b	Σ_a	Σ_b	Σ
ID(Frame X) =IFD(rest) =LFD($p^z \to -\infty$)	$m_\pi^3 + \frac{11}{16\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$-\frac{3}{16\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$m_\pi^3 + \frac{11}{16\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$-\frac{3}{16\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$
ID(Frame Y) =IFD($p^z \to \infty$) =LFD(p^z independent)	$m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	0	$m_\pi^3 + \frac{3}{8\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$\frac{1}{8\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$

Table 1: Summary of the LNA terms of the forward/backward moving on/off mass shell nucleon self energy and the remnant part in different forms and frames.

- The LNA for the LF zero mode contributions $p^+ = 0$ lost in frame X: $\mp\frac{5}{16\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$ (in unit of $-\frac{3g_A^2}{32\pi f_\pi^2}$) to the Σ_a and Σ_b; and $\mp\frac{3}{16\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$ (in unit of $-\frac{3g_A^2}{32\pi f_\pi^2}$) to the Σ'_a and Σ'_b.

In ID: the numerical results

- $\Sigma_{a:4F}$, $\Sigma_{b:4F}$ and Σ_{4F} with δ, p^z

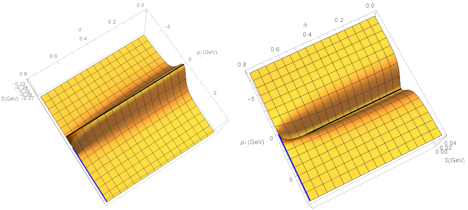


(a) (b) (c)

Figure 5: Numerical calculations for interpolating (a) forward moving part of nucleon self-energy: $\Sigma_{a:4F}$; (b) backward moving part of nucleon self-energy: $\Sigma_{b:4F}$; (c) the entire nucleon self-energy: $\Sigma_{a:4F} + \Sigma_{b:4F}$. Frame X: black lines; frame Y: blue lines.

In ID: the numerical results

- $\Sigma_{a:4F}$ and $\Sigma_{b:4F}$ with δ, $p_{\frown}$: squeezed "I" form with δ function at the LFD end

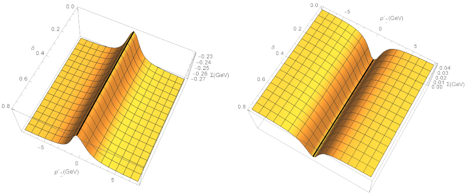


(a) (b)

Figure 6: Numerical calculations (δ versus $p_{\frown}$) for interpolating (a) forward moving part of nucleon self-energy: $\Sigma_{a:4F}$; (b) backward moving part of nucleon self-energy: $\Sigma_{b:4F}$; Frame X: black lines; frame Y: blue lines.

In ID: the numerical results

- $\Sigma_{a:4F}$ and $\Sigma_{b:4F}$ with δ, $p'_{\underline{\wedge}}$: "I" form showing dynamical form invariance



(a) (b)

Figure 7: Numerical calculations (δ versus $p'_{\underline{\wedge}}$) for interpolating (a) forward moving part of nucleon self-energy: $\Sigma_{a:4F}$; (b) backward moving part of nucleon self-energy: $\Sigma_{b:4F}$; Frame X: black lines; and frame Y is in the infinite $p'_{\underline{\wedge}}$ which is out of the range.

In ID: the numerical results

Numerical results (GeV)	$\Sigma'_{a:4F}$	$\Sigma'_{b:4F}$	$\Sigma_{a:4F}$	$\Sigma_{b:4F}$	Σ_{4F}
ID(Frame X) =IFD(rest) =LFD($p^z \to -\infty$)	-0.202	-0.030	-0.228	-0.004	-0.232
ID(Frame Y) =IFD($p^z \to \infty$) =LFD(p^z independent)	-0.232	0	-0.275	0.043	-0.232

Table 2: Summary of numerical results of $\Sigma'_{a:4F}$, $\Sigma'_{b:4F}$, $\Sigma_{a:4F}$, $\Sigma_{b:4F}$, and Σ_{4F} in frame X and Y.

- The LF zero mode contributions at $p^z = -\infty$ to the Σ'_a and Σ'_b are ∓ 0.03 GeV, and are ∓ 0.047 GeV to the Σ_a and Σ_b .

Reducing the nucleon self-energy

- The nucleon self-energy in Eq.(9), after applying PV regularization, reads

$$\Sigma_{2F} = -i3\Lambda^4 \left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-\left[2(p\cdot k)(k\cdot p_N) - (p\cdot p_N)k^2\right] + M^2k^2}{D_N D_\pi D_\Lambda^2}. \quad (33)$$

- Substitution

$$k^2 = D_\pi + m_\pi^2,\ p\cdot k = \frac{1}{2}(D_\pi - D_N + m_\pi^2) \quad (34)$$

- Reduced nucleon self-energy

$$\Sigma_{2F} = -\frac{3ig_A^2 M}{32f_\pi^2\pi^4}(I'_{2F} + m_\pi^2 I_{2F}). \quad (35)$$

with nucleon-pion and nucleon propagating parts

$$I_{2F} = \int d^4k \frac{\Lambda^4}{D_N D_\pi D_\Lambda^2},\ I'_{2F} = \int d^4k \frac{\Lambda^4}{D_N D_\Lambda^2} \quad (36)$$

Distinguish forward and backward parts

- Take I'_{1F} as an example, which can be split into four poles (rescaled)

$$I'_{1F} = \int d^4k \frac{-\Lambda^2}{D_\Lambda D_N} = \int d^2k_\perp \int dk'_{\widehat{-}} \int \frac{-\Lambda^2 dk'^{\widehat{+}}}{(k'^{\widehat{+}} - \kappa'^{1+})(k'^{\widehat{+}} - \kappa'^{1-})(k'^{\widehat{+}} - \kappa'^{2+})(k'^{\widehat{+}} - \kappa'^{2-})}, \tag{38}$$

with the form factor poles

$$\kappa'^{2\pm} = \pm\sqrt{k'^2_{\widehat{-}} + k_\perp^2 + \Lambda^2 - i\epsilon} = \pm\omega'_{\widehat{k}\Lambda}. \tag{39}$$

The poles with $+/-$ are in the lower/upper half plane (L/UHP).

- Transform the integrand

$$I'_{1F} = -\Lambda^2 \int d^2k_\perp \int dk'_{\widehat{-}} \int dk'^{\widehat{+}} \left(\frac{1}{k'^{\widehat{+}} - \kappa'^{1+}} - \frac{1}{k'^{\widehat{+}} - \kappa'^{1-}}\right)\frac{1}{\kappa'^{1+} - \kappa'^{1-}}$$
$$\times \left(\frac{1}{k'^{\widehat{+}} - \kappa'^{2+}} - \frac{1}{k'^{\widehat{+}} - \kappa'^{2-}}\right)\frac{1}{\kappa'^{2+} - \kappa'^{2-}}. \tag{40}$$

Distinguish the forward and backward parts

- Only the combination of the different sides poles contribute:

$$-\frac{3ig_A^2M}{32f_\pi^2\pi^4}I'_{1F} = -\frac{3ig_A^2M(-\Lambda^2)}{32f_\pi^2\pi^4}\int d^2k_\perp \int dk'_{\widehat{-}} \int dk'^{\widehat{+}} \frac{-1}{\kappa'^{1+}-\kappa'^{1-}}\frac{1}{\kappa'^{2+}-\kappa'^{2+}} \times \left[\frac{1}{(k'^{\widehat{+}}-\kappa'^{1-})(k'^{\widehat{+}}-\kappa'^{2+})}+\frac{1}{(k'^{\widehat{+}}-\kappa'^{1+})(k'^{\widehat{+}}-\kappa'^{2-})}\right] = \Sigma_{1F}^{NF:-+}+\Sigma_{1F}^{NF:+-} \quad (41)$$

The superscript "$NF : -+$" means that the nucleon pole (N) and the form factor pole (F) are chosen as κ'^{1-} $(-)$ and κ'^{2+} $(+)$.

- The I_{1F} can be similarly decomposed into

$$-\frac{3ig_A^2M}{32f_\pi^2\pi^4}m_\pi^2 I_{1F} = \frac{m_\pi^2}{D_\Lambda - D_\pi}\left(\Sigma_{1F}^{NP:-+}+\Sigma_{1F}^{NP:+-}-\Sigma_{1F}^{NF:-+}-\Sigma_{1F}^{NF:+-}\right) = \Sigma_{1F}^{N(P-F):-+}+\Sigma_{1F}^{N(P-F):+-} \quad (42)$$

- $-+$: the normal diagram (Fig. 3a); $+-$: the "Z" graph (Fig. 3b).

In ID: LNA results

- Using Eq.(20), we can update Σ_{1F} with

$$\Sigma_{2F} = \Sigma_{2F}^{NF:-+} + \Sigma_{2F}^{NF:+-} + \Sigma_{2F}^{NPF:-+} + \Sigma_{2F}^{NPF:+-}. \tag{43}$$

- LNA results

LNA (in the unit of $-\frac{3g_A^2}{32\pi f_\pi^2}$)	$\Sigma_{2F}^{NF:-+}$	$\Sigma_{2F}^{NF:+-}$	$\Sigma_{2F}^{N(P-F):-+}$	$\Sigma_{2F}^{N(P-F):+-}$	Σ_{2F}
ID(Frame X) =IFD(rest) =LFD($p^z \to -\infty$)	0	0	$m_\pi^3 + \frac{3}{4\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$-\frac{1}{4\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	$m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$
ID(Frame Y) =IFD($p^z \to \infty$) =LFD(p^z independent)	0	0	$m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$	0	$m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$

Table 3: Summary of the LNA terms of the $\Sigma_{2F}^{NF:\mp\pm}$, $\Sigma_{2F}^{N(P-F):\mp\pm}$ and their summation in frame X and Y.

- The zero mode contributes $\mp\frac{1}{4\pi}\frac{m_\pi^4 \log[m_\pi^2]}{M}$ (in unit of $\frac{3g_A^2}{32\pi f_\pi^2}$) to the $\Sigma_{2F}^{N(P-F):\mp\pm}$ in the frame X with $p^z \to -\infty$.

In ID: numerical results

Numerical results (GeV)	$\Sigma_{2F}^{NF:-+}$	$\Sigma_{2F}^{NF:+-}$	$\Sigma_{2F}^{N(P-F):-+}$	$\Sigma_{2F}^{N(P-F):+-}$	Σ_{2F}
ID(Frame X) =IFD(rest) =LFD($p^z \to -\infty$)	-0.763	-0.342	0.026	0.005	-1.074
ID(Frame Y) =IFD($p^z \to \infty$) =LFD(p^z independent)	-1.105	0	0.031	0	-1.074

Table 4: Summary of numerical results of $\Sigma_{2F}^{NF:\mp\pm}$, $\Sigma_{2F}^{N(P-F):\mp\pm}$ and their summation in frame X and Y.

- The LF zero mode contributions at $p^z = -\infty$ to the $\Sigma_{2F}^{NF:\mp\pm}$ and $\Sigma_{2F}^{N(P-F):\mp\pm}$ are ∓ 0.342 GeV and ± 0.005 GeV respectively.

Discussions: Contrast between the LFD and IFD

- LFD has frame invariance ($p'_{\hat{-}} = p_{\hat{-}}/\mathcal{C} = p^+/0$)
  - In frame X: i.e., $p'_{\hat{-}} = 0 \Rightarrow$ light-front zero mode $p^+ = 0 \Rightarrow$ $p^0 = -p^z \Rightarrow \sqrt{(p^z)^2 + M^2} = -p^z \Rightarrow p^z = -\infty$.
  - In frame Y: i.e., $p'_{\hat{-}} \to \infty \Rightarrow p^+ > 0 \Rightarrow \sqrt{(p^z)^2 + M^2} + p^z > 0 \Rightarrow$ $p^z \neq -\infty$ or $p^z > -\infty$.
  - LFD frame invariance in $-\infty \leq p^z \leq +\infty \Rightarrow \Sigma_{Frame\ Y} = \Sigma_{Frame\ X}$.
- IFD has no frame invariance: $\Sigma_{Frame\ Y} \neq \Sigma_{Frame\ X}$ for each anatomy part.
- The IMF in IFD is literally taking a particular frame, i.e. $p^z \to +\infty$, and is not equivalent with LFD.

Conclusions

- Figure out how much the LFI contribute and how it is related with backward moving part of nucleon self-energy in IFD, numerically and analytically, as well as the corresponding results for forward moving part.
- Identify the LF zero mode contribution missed at $p^z \to -\infty$ in the LFD for each anatomic part of the (reduced) nucleon self-energy numerically and analytically.

Thanks for your attention.

[1] Ji, Chueng-Ryong, et al. "Interpolating quantum electrodynamics between instant and front forms." Physical Review D 98.3 (2018): 036017.

[2] Ji, Chueng-Ryong, W. Melnitchouk, and A. W. Thomas. "Equivalence of pion loops in equal-time and light-front dynamics." Physical Review D 80.5 (2009): 054018.