Anatomy of Nucleon Self-energy from Instant to Light-Front

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Introduction

Anatomy of the Nucleon self-energy

Anatomy of the reduced nucleon self-energy

Discussions and Conclusions
Concepts and methods

- Instant form dynamics (IFD): $x^0, x^3$.
- Light-front dynamics (LFD): $x^\pm = (x^0 \pm x^3)/\sqrt{2}$.
- Interpolating dynamics (ID):

$$
\begin{bmatrix}
x_+ \\
x_-
\end{bmatrix} =
\begin{bmatrix}
\cos[\delta] & \sin[\delta] \\
\sin[\delta] & -\cos[\delta]
\end{bmatrix}
\begin{bmatrix}
x^0 \\
x^3
\end{bmatrix}
$$

(1)

with $0 \leq \delta \leq \pi/4$. The corresponding four momentum is

$$
q^2 = C(q_+^2 - q_-^2) + 2Sq_+q_- - q_\perp^2.
$$

(2)
Introduction and Motivation

▶ Introduction:
  ▶ Tree level intermediate fermion propagator can be decomposed into forward and backward moving parts[1].
  ▶ The backward moving part in LFD features an instantaneous contribution which involves the constraint degrees of freedom of the fermion, that is unique in the LFD.

▶ Motivation:
  ▶ To identify how much the LF instantaneous (LFI) part contributes in loop level nucleon self-energy numerically and analytically.
  ▶ To trace the LFI from the backward moving part of nucleon self-energy by using the interpolating dynamics (ID).
  ▶ To distinguish the LF zero mode contribution appeared at the $p^z \to -\infty$ point in LFD numerically and analytically.
  ▶ To show the difference between the LFD and IFD.
Introduction

Anatomy of the Nucleon self-energy

Anatomy of the reduced nucleon self-energy

Discussions and Conclusions
Definition: anatomy in the tree level

Figure 1: (a) Feynman diagram for $e^+e^- \rightarrow \gamma\gamma$ process (t-channel). Time-ordered diagrams (b) and (c) for $e^+e^- \rightarrow \gamma\gamma$ annihilation process. The u-channel amplitudes can be obtained by crossing the two final state particles.
Definition: anatomy in the tree level

- The intermediate virtual fermion propagator (Fig. 1a) and its anatomy into forward and backward parts (Fig. 1b and 1c) [1]

\[
\Sigma_N = \frac{q + M}{q^2 - M^2} = \Sigma_{Na} + \Sigma_{Nb} = \frac{Q_a + M}{2Q^+(q^+ - Q_{a^+})} + \frac{-Q_b + M}{2Q^+(-q^+ - Q_{b^+})}
\]

(3)

where capitalized symbols are "on-mass-shell" (OMS)

\[
Q_a = \left( \frac{-S q^- + Q^+}{C}, \; q^\perp, \; q^- \right)
\]

\[
Q_b = \left( \frac{S q^- + Q^+}{C}, \; -q^\perp, \; -q^- \right)
\]

\[
Q^+ = \sqrt{q^-^2 + C(q^\perp^2 + M^2)}.
\]

(4)
Definition: anatomy in the tree level

- The off-mass shell version (with prime notation) of forward and backward moving nucleon self-energy

\[ \Sigma'_{Na} = \frac{\not{q} + M}{2Q^+(q_+^\uparrow - Q_{a+}^\uparrow)} \]  
\[ \Sigma'_{Nb} = \frac{\not{q} + M}{2Q^+(-q_+^\uparrow - Q_{b+}^\uparrow)} \]  

- Define the remnant part to assist our calculation

\[ \Sigma_{NRP} = \Sigma_{Nb} - \Sigma'_{Nb} = \frac{\gamma^\uparrow}{2Q^\uparrow} \]
Definition: anatomy in the loop level

Figure 2: The nucleon-pion loop

- The nucleon self-energy is \( p_N = p - k \)

\[
\hat{\Sigma} = i \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \left( k \gamma^5 \tau \right) \frac{i(p_N + M)}{p_N^2 - M^2 + i\epsilon} \left( \gamma^5 k \tau \right) \frac{i}{k^2 - m^2_\pi + i\epsilon}.
\]  

(8)

- Spin sum calculation: \( \Sigma = \frac{1}{2} \sum_s \bar{u}(p, s) \hat{\Sigma} u(p, s) \).

- With \( D_N = p_N^2 - M^2 + i\epsilon, D_\pi = k^2 - m^2_\pi + i\epsilon \), the nucleon self-energy can be simplified as

\[
\Sigma = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} - \frac{[2(p \cdot k)(k \cdot p_N) - (p \cdot p_N)k^2]}{D_N D_\pi M} + M^2 k^2.
\]

(9)
Definition: anatomy in the loop level

Figure 3: Time-ordered diagrams (a) and (b) for the forward moving nucleon self-energy (positive energy diagram) and the backward moving nucleon self-energy ("Z" graph).
Definition: anatomy in the loop level

- The $\Sigma_a$

$$\Sigma_a = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \left[ -2(p \cdot k)(k \cdot P_{Na}) + (p \cdot P_{Na})k^2 \right] + M^2k^2 \frac{2P_N^+(p_{N^+} - P_{Na^+})D_{\pi}M}{2P_N^+(p_{N^+} - P_{Na^+})D_{\pi}M}$$ (10)

- The $\Sigma_b$

$$\Sigma_b = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \left[ 2(p \cdot k)(k \cdot P_{Nb}) - (p \cdot P_{Nb})k^2 \right] + M^2k^2 \frac{2P_N^+(p_{N^+} - P_{Nb^+})D_{\pi}M}{2P_N^+(p_{N^+} - P_{Nb^+})D_{\pi}M}$$ (11)

- The $\Sigma_{RP}$

$$\Sigma_{RP} = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^2k_{\perp} dk^- dk^+}{(2\pi)^4} \left[ -2(p \cdot k)k^+ - p^+k^2 \right] \frac{2P_N^+D_{\pi}M}{2P_N^+D_{\pi}M}$$ (12)
The backward and forward moving part in LFD

- The backward moving $\Sigma_b$ in the LF limit, becomes LFI contribution

$$\Sigma_{LFD}^b = \Sigma_{LFI} = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-2(p \cdot k)k^+ + k^2p^+}{2p_N^+D_\pi M},$$ (13)

which is the same with $\Sigma_{RP}^{LFD}$.

- The forward moving part of the nucleon self-energy becomes, in LF limit,

$$\Sigma_{LFD}^a = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\left[2(p \cdot k)(k \cdot P_{Na}^{LFD}) - (p \cdot P_{Na}^{LFD})k^2 \right] + M^2k^2}{D_ND_\pi M},$$ (14)

with $P_{Na}^{LFD} = \left( \frac{(p_\perp - k_\perp)^2 + M^2}{2(p^+ - k^+)} \right), p_\perp - k_\perp, p^+ - k^+).$
Calculation for the LFI contribution

- The LFI in Eq.(13), with some transformations, becomes

\[ \Sigma_{LFI} = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^2k_\perp dk^- dk^+}{(2\pi)^4} \left\{ \frac{p \cdot k}{D_\pi M} + \frac{p^+[k^2 - 2(p \cdot k)]}{2(p^+ - k^+)MD_\pi} \right\}, \]

where the first term is zero, because of oddity in \( k \).

- The Pauli-Villars (PV) regularization

\[ \frac{1}{D_\pi} \rightarrow \frac{-\Lambda^2}{D_\pi D_\Lambda} = \frac{-\Lambda^2}{D_\Lambda - D_\pi} \left( \frac{1}{D_\pi} - \frac{1}{D_\Lambda} \right) = \frac{-\Lambda^2}{m_\pi^2 - \Lambda^2} \left( \frac{1}{D_\pi} - \frac{1}{D_\Lambda} \right). \]

where the \( D_\Lambda = k^2 - \Lambda^2 + i\epsilon \).

- Then the LFI becomes (\( x = k^+/p^+ \) is momentum fraction)

\[ \Sigma_{LFI:1F} = \frac{-\Lambda^2}{D_\Lambda} \Sigma_{LFI} = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^2k_\perp dk^- dx p^+}{(2\pi)^4} \left[ \frac{1}{(1 - x)D_\Lambda} + \frac{m_\pi^2}{(1 - x)D_\pi D_\Lambda} - \frac{2p^+ k^- + 2p^- p^+ x}{(1 - x)D_\pi D_\Lambda} \right]. \]
Calculation for the LFI contribution

► The point like theory:

\[ \int \frac{dk^-}{D_\pi} \frac{1}{\Gamma} = \pi i \log \left[ \frac{k_{\perp}^2 + m_{\pi}^2}{\mu^2} \right] \frac{\delta[x]}{p^+} \]

where the \( \mu \) is the regularization mass parameter.

► The first term in Eq.(17) as an example becomes

\[ \int dx p^+ \int \frac{dk^-}{(1-x)D_\Lambda} = \int dx \log \left[ \frac{k_{\perp}^2 + \Lambda^2}{\mu^2} \right] \frac{\pi i \delta[x]}{1-x} = \pi i \log \left[ \frac{k_{\perp}^2 + \Lambda^2}{\mu^2} \right] \]

where all the \( p^+ \)'s are canceled and only the \( x = 0 \) contribute.

► The LF zero mode contribution where \( p^+ = 0 \) is included during the \( x \) integration with \( 0 < x < 1 \).
Calculation for the LFI contribution

- The derivative technique

\[
\Sigma_{x:nF} = \frac{(-\Lambda^2)^n(m - 1)!}{(n - 1)!} \frac{\partial^{n-1}}{\partial(\Lambda^2)^{n-1}} \frac{\Sigma_{x:mF}}{(-\Lambda^2)^m}
\]  \hspace{1cm} (20)

with \(n > m\), where the \(nF\) and \(mF\) in the subscript represent the number of PV Form factor (F) multiplied to the subject \(\Sigma_x\).

- With \(m=1\) and \(n=4\) being chosen for the integrations, the final result for LFI reads

\[
\Sigma_{LFI:4F} = -\frac{g_A^2 \Lambda^8}{256 M f_{\pi}^2 \pi^2} \left( -\frac{\Lambda^4 + \frac{2m_\pi^6}{\Lambda^2} - 6\Lambda^2 m_\pi^2 + 6 m_\pi^4 \log \left[ \frac{\Lambda^2}{m_\pi^2} \right]}{2 (\Lambda^2 - m_\pi^2)^4} \right).
\]  \hspace{1cm} (21)
Calculation for the LFI contribution

- The leading non-analytic behavior for LFI is

\[
\Sigma_{LFI}^{LNA} = - \frac{3g_A^2}{32\pi f^2} \left( \frac{m^4_\pi \log[m^2_\pi]}{8M_\pi} + O(m^5_\pi) \right). \tag{22}
\]

Figure 4: Time-ordered diagram for the LF instantaneous contribution \( \Sigma_{LFI} \).
Calculation for the forward moving part in LFD

- The forward moving part of the nucleon self-energy in LFD, after some simplifications yields

\[
\Sigma_{a}^{LFD} = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4 k}{(2\pi)^4} \frac{(P_{LFD}^{\pi} \cdot k)}{D_\pi M} + \frac{(M^2 + p_N \cdot P_{LFD}^{\pi})k^2}{D_N D_\pi M}
\]

(23)

which can be decomposed as \(\Sigma_{a}^{LFD} = \Sigma - \Sigma_{LFI}\).

- The LNA behavior for OMS is

\[
\Sigma_{a}^{LFD: LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left( m_\pi^3 + \frac{3m_\pi^4 \log[m_\pi^2]}{8M_\pi} + \mathcal{O}(m_\pi^5) \right).
\]

(24)
The ID: to relate LFD with IFD

- The off-mass-shell forward and backward part

\[
\Sigma_a' = \Sigma_a + \Sigma_{RP} \\
\Sigma_b' = \Sigma_b - \Sigma_{RP}
\] (25)

\[
\Sigma_a' = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-2(p \cdot k)^2 + k^2(p \cdot k) + 2k^2M^2}{2P_N^+ M(p_{N^+} - P_{Na^+})D_\pi}
\]

\[
\Sigma_b' = -i3 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-2(p \cdot k)^2 + k^2(p \cdot k) + 2k^2M^2}{2P_N^+ M(-p_{N^+} - P_{Nb^+})D_\pi}
\] (26)

which can be further decomposed into four terms \( \Sigma_{a\pm}, \Sigma_{b\pm} \) by decomposing the pion poles in \( D_\pi \).
Rescale the variables

► Rescale: \( k_\perp = k'_\perp \sqrt{C} \), and \( k_\parallel = k'_\parallel / \sqrt{C} \) (to not alter the area spanned by the 4-momentum).

► Inference: \( k'^\parallel = k'_\parallel + \Sigma k'_\perp \), and \( dk'^\parallel = dk'_\parallel \).

► The rescaled \( \Sigma'_a^\parallel \) becomes "independent" of interpolation angles

\[
\Sigma'_a^\parallel = -i 3 \left( \frac{g_A}{2 f_\pi} \right)^2 \int \frac{d^2 k_\perp dk'_\perp dk'^\parallel}{(2\pi)^4} \frac{-2(p \cdot k)^2 + k^2(p \cdot k) + 2k^2M^2}{-2P_N^\parallel M_2 \omega_k^\parallel (k'^\parallel - \kappa'^{0+})(k'^\parallel - \kappa'^{1-})}
\]

with rescaled poles

\[
\kappa'^{0\pm} = \pm \sqrt{k'^2_\perp + k^2_\perp + m^2_\pi - i\epsilon} = \pm \omega'_k
\]

\[
\kappa'^{1\pm} = \sqrt{p'^2_\perp + p^2_\perp + M^2} \pm \sqrt{p'^2_\perp + p^2_\perp + M^2 - i\epsilon} = p'^\parallel \pm P_N'^\parallel.
\]
Definition: the reference frames

- Frame X

\[ \vec{p}' = (p'_\perp, p_{\perp}) = 0, \]  

(30)

with

\[ p'_\perp = p_{\perp} / \sqrt{C} = \frac{\sqrt{M^2 + p^z_\perp \sin \delta + p^z \cos \delta}}{\sqrt{\cos 2\delta}}. \]  

(31)

In IFD (\( \delta = 0 \)): \( p_{\perp} / \sqrt{C} = p^z = 0 \) is the rest frame.

- Frame Y

\[ k'_\perp = \pm (1 - y) p'_\perp, \quad dk'_\perp = \mp p'_\perp dy, \]  

(32)

with \( p'_\perp \to \infty \) and \( p_{\perp} = 0 \). The \( \pm \) corresponds to the forward and backward moving parts.

In IFD (\( \delta = 0 \)): \( p_{\perp} / \sqrt{C} = p^z \to \infty \) is the infinite momentum frame (IMF).
In ID: the LNA for the forward and backward parts

<table>
<thead>
<tr>
<th>LNA (in the unit of $-\frac{3g^4}{32\pi f^2}$)</th>
<th>$\Sigma'_a$</th>
<th>$\Sigma'_b$</th>
<th>$\Sigma_a$</th>
<th>$\Sigma_b$</th>
<th>$\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID(Frame X)</td>
<td>$m_\pi^3 + \frac{11}{16\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$</td>
<td>$-\frac{3}{16\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$</td>
<td>$m_\pi^3 + \frac{11}{16\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$</td>
<td>$-\frac{3}{16\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$</td>
<td>$m_\pi^3 + \frac{1}{2\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$</td>
</tr>
<tr>
<td>=IFD(rest)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>=LFD($p^z \rightarrow -\infty$)</td>
<td></td>
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<tr>
<td>ID(Frame Y)</td>
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<td>$m_\pi^3 + \frac{1}{2\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$</td>
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<td>=IFD($p^z \rightarrow \infty$)</td>
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<tr>
<td>=LFD($p^z$ independent)</td>
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Table 1: Summary of the LNA terms of the forward/backward moving on/off mass shell nucleon self energy and the remnant part in different forms and frames.

- The LNA for the LF zero mode contributions $p^+ = 0$ lost in frame $X$: $\pm \frac{5}{16\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$ (in unit of $-\frac{3g^4}{32\pi f^2}$) to the $\Sigma_a$ and $\Sigma_b$; and $\pm \frac{3}{16\pi} \frac{m_\pi^4 \log[m_\pi^2]}{M}$ (in unit of $-\frac{3g^4}{32\pi f^2}$) to the $\Sigma'_a$ and $\Sigma'_b$. 
In ID: the numerical results

▷ \( \Sigma_a:4F, \Sigma_b:4F \) and \( \Sigma_{4F} \) with \( \delta, p^2 \)

Figure 5: Numerical calculations for interpolating (a) forward moving part of nucleon self-energy: \( \Sigma_a:4F \); (b) backward moving part of nucleon self-energy: \( \Sigma_b:4F \); (c) the entire nucleon self-energy: \( \Sigma_a:4F + \Sigma_b:4F \). Frame X: black lines; frame Y: blue lines.
In ID: the numerical results

- $\Sigma_{a:4F}$ and $\Sigma_{b:4F}$ with $\delta$, $p_\perp$: squeezed "I" form with $\delta$ function at the LFD end

Figure 6: Numerical calculations ($\delta$ versus $p_\perp$) for interpolating (a) forward moving part of nucleon self-energy: $\Sigma_{a:4F}$; (b) backward moving part of nucleon self-energy: $\Sigma_{b:4F}$; Frame X: black lines; frame Y: blue lines.
In ID: the numerical results

- $\Sigma_{a:4F}$ and $\Sigma_{b:4F}$ with $\delta$, $p'_\perp$: "I" form showing dynamical form invariance

Figure 7: Numerical calculations ($\delta$ versus $p'_\perp$) for interpolating (a) forward moving part of nucleon self-energy: $\Sigma_{a:4F}$; (b) backward moving part of nucleon self-energy: $\Sigma_{b:4F}$; Frame X: black lines; and frame Y is in the infinite $p'_\perp$ which is out of the range.
In ID: the numerical results

<table>
<thead>
<tr>
<th>Numerical results (GeV)</th>
<th>$\Sigma'_a:4F$</th>
<th>$\Sigma'_b:4F$</th>
<th>$\Sigma_a:4F$</th>
<th>$\Sigma_b:4F$</th>
<th>$\Sigma_4F$</th>
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<tr>
<td>ID(Frame X)</td>
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</tr>
<tr>
<td>=IFD(rest)</td>
<td>$-0.202$</td>
<td>$-0.030$</td>
<td>$-0.228$</td>
<td>$-0.004$</td>
<td>$-0.232$</td>
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<tr>
<td>=LFD($p^z \to -\infty$)</td>
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<tr>
<td>ID(Frame Y)</td>
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<td>=IFD($p^z \to \infty$)</td>
<td>$-0.232$</td>
<td>$0$</td>
<td>$-0.275$</td>
<td>$0.043$</td>
<td>$-0.232$</td>
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<td>=LFD($p^z$ independent)</td>
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Table 2: Summary of numerical results of $\Sigma'_a:4F$, $\Sigma'_b:4F$, $\Sigma_a:4F$, $\Sigma_b:4F$, and $\Sigma_4F$ in frame X and Y.

- The LF zero mode contributions at $p^z = -\infty$ to the $\Sigma'_a$ and $\Sigma'_b$ are $\mp 0.03$ GeV, and are $\mp 0.047$ GeV to the $\Sigma_a$ and $\Sigma_b$.
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<th>Discussions and Conclusions</th>
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Introduction

Anatomy of the Nucleon self-energy

Anatomy of the reduced nucleon self-energy

Discussions and Conclusions
Reducing the nucleon self-energy

- The nucleon self-energy in Eq.(9), after applying PV regularization, reads

\[
\Sigma_{2F} = -i3\Lambda^4 \left( \frac{g_A}{2f_\pi} \right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-\left[2(p \cdot k)(k \cdot p_N) - (p \cdot p_N)k^2\right] + M^2k^2}{D_ND_\pi D_\Lambda^2}.
\]  

(33)

- Substitution

\[
k^2 = D_\pi + m_\pi^2, \quad p \cdot k = \frac{1}{2}(D_\pi - D_N + m_\pi^2)
\]  

(34)

- Reduced nucleon self-energy

\[
\Sigma_{2F} = -\frac{3ig_A^2M}{32f_\pi^2\Lambda^4}(I_{2F} + m_\pi^2l_{2F}).
\]  

(35)

with nucleon-pion and nucleon propagating parts

\[
l_{2F} = \int d^4k \frac{\Lambda^4}{D_ND_\pi D_\Lambda^2}, \quad l_{2F}' = \int d^4k \frac{\Lambda^4}{D_ND_\pi D_\Lambda^2}
\]  

(36)
Distinguish forward and backward parts

- Take $I'_1F$ as an example, which can be split into four poles (rescaled)

$$I'_1F = \int d^4k \frac{-\Lambda^2}{D_\Lambda D_N} = \int d^2k_\perp \int dk'_- \int \frac{-\Lambda^2 dk'_+}{(k'_+ - \kappa'_1+)(k'_+ - \kappa'_1-)(k'_+ - \kappa'_2+)(k'_+ - \kappa'_2-)},$$

with the form factor poles

$$\kappa'_2\pm = \pm \sqrt{k'_2 - k^2_\perp + \Lambda^2 - i\epsilon} = \pm \omega'_k \frac{1}{k\Lambda}. \quad (38)$$

The poles with $+/-$ are in the lower/upper half plane (L/UHP).

- Transform the integrand

$$I'_1F = -\Lambda^2 \int d^2k_\perp \int dk'_- \int dk'_+ \left( \frac{1}{k'_+ - \kappa'_1+} - \frac{1}{k'_+ - \kappa'_1-} \right) \frac{1}{\kappa'_1+ - \kappa'_1-} \times \left( \frac{1}{k'_+ - \kappa'_2+} - \frac{1}{k'_+ - \kappa'_2-} \right) \frac{1}{\kappa'_2+ - \kappa'_2-}. \quad (40)$$
Distinguish the forward and backward parts

Only the combination of the different sides poles contribute:

\[- \frac{3i g_A^2 M}{32 f_\pi^2 \pi^4} l_{1F}' = - \frac{3i g_A^2 M(-\Lambda^2)}{32 f_\pi^2 \pi^4} \int d^2 k_\perp \int dk'_- \int dk'_+ \frac{-1}{\kappa'^1+ - \kappa'^1-} \frac{1}{\kappa'^2+ - \kappa'^2+} \]

\times \left[ \frac{1}{(k'^+ - \kappa'^1-)(k'^+ - \kappa'^2+)} + \frac{1}{(k'^+ - \kappa'^1+)(k'^+ - \kappa'^2-)} \right]

= \Sigma^{NF:-+}_{1F} + \Sigma^{NF:+-}_{1F} \tag{41} \]

The superscript "NF : −+" means that the nucleon pole (N) and the form factor pole (F) are chosen as \( \kappa'^1- (-) \) and \( \kappa'^2+ (+) \).

The \( l_{1F}' \) can be similarly decomposed into

\[- \frac{3i g_A^2 M}{32 f_\pi^2 \pi^4} m_\pi^2 l_{1F} = \frac{m_\pi^2}{D_\Lambda - D_\pi} \left( \Sigma^{NP:-+}_{1F} + \Sigma^{NP:+-}_{1F} - \Sigma^{NF:-+}_{1F} - \Sigma^{NF:+-}_{1F} \right) \]

\[= \Sigma^{N(P-F):-+}_{1F} + \Sigma^{N(P-F):+-}_{1F} \tag{42} \]

−+: the normal diagram (Fig. 3a); +−: the "Z" graph (Fig. 3b).
In ID: LNA results

- Using Eq.(20), we can update $\Sigma_{1F}$ with

$$\Sigma_{2F} = \Sigma_{2F}^{NF:+} + \Sigma_{2F}^{NF:-} + \Sigma_{2F}^{NPF:+} + \Sigma_{2F}^{NPF:-}. \quad (43)$$

- LNA results

<table>
<thead>
<tr>
<th>LNA (in the unit of $-\frac{3g_A^2}{32\pi f_x^2}$)</th>
<th>$\Sigma_{2F}^{NF:+}$</th>
<th>$\Sigma_{2F}^{NF:-}$</th>
<th>$\Sigma_{2F}^{N(P-F):-}$</th>
<th>$\Sigma_{2F}^{N(P-F):+}$</th>
<th>$\Sigma_{2F}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID(Frame X)</td>
<td>$0$</td>
<td>$0$</td>
<td>$m^3_\pi + \frac{3}{4\pi} m^4_\pi \log[m^2_\pi] - \frac{1}{4\pi} m^4_\pi \log[m^2_\pi] \frac{1}{M}$</td>
<td>$m^3_\pi + \frac{1}{2\pi} m^4_\pi \log[m^2_\pi] \frac{1}{M}$</td>
<td></td>
</tr>
<tr>
<td>=IFD(rest)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>=LFD($p^z \rightarrow -\infty$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ID(Frame Y)</td>
<td>$0$</td>
<td>$0$</td>
<td>$m^3_\pi + \frac{1}{2\pi} m^4_\pi \log[m^2_\pi] \frac{1}{M}$</td>
<td>$0$</td>
<td>$m^3_\pi + \frac{1}{2\pi} m^4_\pi \log[m^2_\pi] \frac{1}{M}$</td>
</tr>
<tr>
<td>=IFD($p^z \rightarrow \infty$)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>=LFD($p^z$ independent)</td>
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</tr>
</tbody>
</table>

Table 3: Summary of the LNA terms of the $\Sigma_{2F}^{NF:+\mp}$, $\Sigma_{2F}^{N(P-F):+\mp}$ and their summation in frame $X$ and $Y$.

- The zero mode contributes $\mp \frac{1}{4\pi} m^4_\pi \log[m^2_\pi] \frac{1}{M}$ (in unit of $\frac{3g_A^2}{32\pi f_x^2}$) to the $\Sigma_{2F}^{N(P-F):+\mp}$ in the frame $X$ with $p^z \rightarrow -\infty$. 
In ID: numerical results

<table>
<thead>
<tr>
<th>Numerical results (GeV)</th>
<th>$\Sigma_{2F}^{NF:--}$</th>
<th>$\Sigma_{2F}^{NF:--}$</th>
<th>$\Sigma_{2F}^{N(P-F):++}$</th>
<th>$\Sigma_{2F}^{N(P-F):--}$</th>
<th>$\Sigma_{2F}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID (Frame X)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>=IFD (rest)</td>
<td>-0.763</td>
<td>-0.342</td>
<td>0.026</td>
<td>0.005</td>
<td>-1.074</td>
</tr>
<tr>
<td>=LFD ($p^z \to -\infty$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ID (Frame Y)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>=IFD ($p^z \to \infty$)</td>
<td>-1.105</td>
<td>0</td>
<td>0.031</td>
<td>0</td>
<td>-1.074</td>
</tr>
<tr>
<td>=LFD ($p^z$ independent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Summary of numerical results of $\Sigma_{2F}^{NF:\mp\pm}$, $\Sigma_{2F}^{N(P-F):\mp\pm}$ and their summation in frame X and Y.

- The LF zero mode contributions at $p^z = -\infty$ to the $\Sigma_{2F}^{NF:\mp\pm}$ and $\Sigma_{2F}^{N(P-F):\mp\pm}$ are $\mp 0.342$ GeV and $\pm 0.005$ GeV respectively.
Introduction

Anatomy of the Nucleon self-energy

Anatomy of the reduced nucleon self-energy

Discussions and Conclusions
Discussions: Contrast between the LFD and IFD

- LFD has frame invariance \((p^\prime_- = p_-/C = p^+/0)\)
  - In frame X: i.e., \(p^\prime_- = 0 \Rightarrow\) light-front zero mode \(p^+ = 0 \Rightarrow\)
    \(p^0 = -p^z \Rightarrow \sqrt{(p^z)^2 + M^2} = -p^z \Rightarrow p^z = -\infty\).
  - In frame Y: i.e., \(p^\prime_- \to \infty \Rightarrow p^+ > 0 \Rightarrow \sqrt{(p^z)^2 + M^2 + p^z} > 0 \Rightarrow\)
    \(p^z \neq -\infty\) or \(p^z > -\infty\).
  - LFD frame invariance in \(-\infty \leq p^z \leq +\infty \Rightarrow \Sigma_{Frame \ Y} = \Sigma_{Frame \ X}\).

- IFD has no frame invariance: \(\Sigma_{Frame \ Y} \neq \Sigma_{Frame \ X}\) for each anatomy part.

- The IMF in IFD is literally taking a particular frame, i.e. \(p^z \to +\infty\), and is not equivalent with LFD.
Conclusions

- Figure out how much the LFI contribute and how it is related with backward moving part of nucleon self-energy in IFD, numerically and analytically, as well as the corresponding results for forward moving part.

- Identify the LF zero mode contribution missed at $p^2 \to -\infty$ in the LFD for each anatomic part of the (reduced) nucleon self-energy numerically and analytically.
Thanks for your attention.