Basis light-front quantization approach to $\Lambda(\Sigma^0, \Sigma^+, \Sigma^-)$ and $\Lambda_c({\Sigma_c}^+, {\Sigma_c}^{++}, {\Sigma_c}^0)$

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Outline

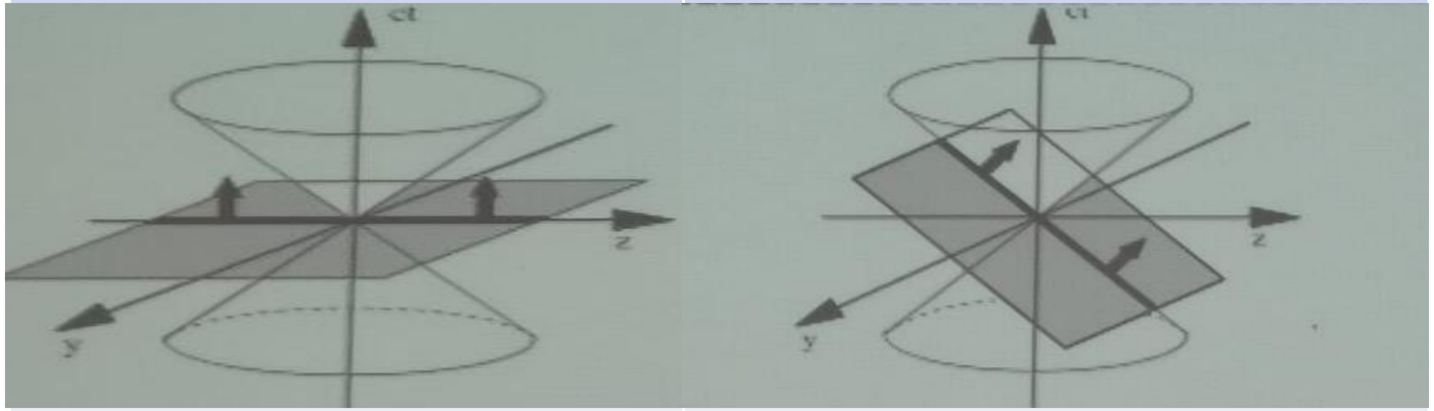
1. Basis Light-front quantization
2. Λ and Λc structure from BLFQ
3. Conclusions

Basis Light-front quantization （BLFQ）

Basia Light-front quantization

[Dirac， 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$



x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$p^- = p^0 - p^3$, $p^+ = p^0 + p^3$, $p^\perp = p^{1,2}$
$i\frac{\partial}{\partial t}\vert\varphi(t)\rangle = \mathrm{H}\vert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\vert\varphi(x^+)\rangle = \frac{1}{2}P^-\vert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$\mathrm{P}^- = \frac{m^2 + p_\perp{}^2}{P^+}$

1.Frame-independent wave function

2.No square root in Hamiltonian P^-

Basia Light-front quantization

We solve the time-independent (at fixed light-front time) Schrodinger equation:

$$\mathrm{H_{eff}}|\Phi\rangle = M^2|\Phi\rangle$$

We adopt an effective light-front Hamiltonian :

$$H_{eff} = H_{k.E} + H_{trans} + H_{longi} + H_{\boldsymbol{int}}$$

The light-front wave function is obtained by solving light-front schrodinger equation and then expanding it to the basis vector of the multi-particle Fock sector. For example,

$$|\Lambda\rangle_{phys} = a|\mathrm{uds}\rangle + b|udsg\rangle + c|udsq\bar{q}\rangle + \cdots$$

$$|\Lambda c\rangle_{phys} = a|\mathrm{udc}\rangle + b|udcg\rangle + c|udcq\bar{q}\rangle + \cdots$$

For the Fock space, we can not involve all the Fock sector directerly, so we adopt a truncation.

The current studying on Λ and Λc only considers LO, but it will be taken into account more Fock sector in the future .

Basia Light-front quantization

We represent the state of a parton by four quantum numbers (n,m,k,λ).

The longitudinal basis vector are plane wave solutions:

$$\Psi_k(x^-) = \frac{1}{2L} e^{-\frac{i}{2}kx^-}$$

The transverse basis vector are two dimensional harmonic oscillator solutions:

$$\phi_{nm} = \frac{\sqrt{2}}{b(2\pi)^{\frac{3}{2}}} \sqrt{\frac{n!}{(|m|+n)!}} \left(\frac{P_\perp}{b}\right)^{|m|} e^{\frac{-P_\perp{}^2}{2b^2}} L_n{}^{|m|} \left(\frac{P_\perp{}^2}{b^2}\right) e^{im\varphi}$$

The many-body basis states have well defined values of the total angular momentum projection $M_J = \sum_i (m_i + \lambda_i)$.

In order to calculate, except the Fock space truncation, for each Fock sector, we also need other truncation to reduce the basis to a finite dimension. On the longitudinal direction, We truncate the infinite basis by a truncation parameter K. As well as, in the transverse direction, we require the total transverse quantum number.

$$\sum_i (2n_i + |n_i| + 1) = N_\alpha \leq N_{Max}$$

Λ(uds) and Λc(udc) structure from BLFQ

PDFs(Parton distribution functions)

$$\Phi^{\Gamma(q)}(x) = \frac{1}{2}\int \frac{dz^-}{4\pi} e^{ip^+ z^-/2} \times \langle P,\Lambda|\bar{\psi}_q(0)\Gamma\psi_q(z^-)|P,\Lambda\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

With different Dirac structure $\Gamma = \gamma^+,\ \gamma^+\gamma^5,\ i\sigma^{j+}\gamma^5$,

we can get Unpolarized (f_1), Helicity(g_1) and Transversity (h_1) PDFs.

$$f^q(x) = \sum_{\{\lambda_i\}} \int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp] \times \Psi^{\uparrow *}_{\{x_i,\vec{p}_{i\perp},\lambda_i\}} \Psi^{\uparrow}_{\{x_i,\vec{p}_{i\perp},\lambda_i\}} \delta(x - x_q)$$

$$g_1^q(x) = \sum_{\{\lambda_i\}} \int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp] \times \lambda_1 \Psi^{\uparrow *}_{\{x_i,\vec{p}_{i\perp},\lambda_i\}} \Psi^{\uparrow}_{\{x_i,\vec{p}_{i\perp},\lambda_i\}} \delta(x - x_1)$$

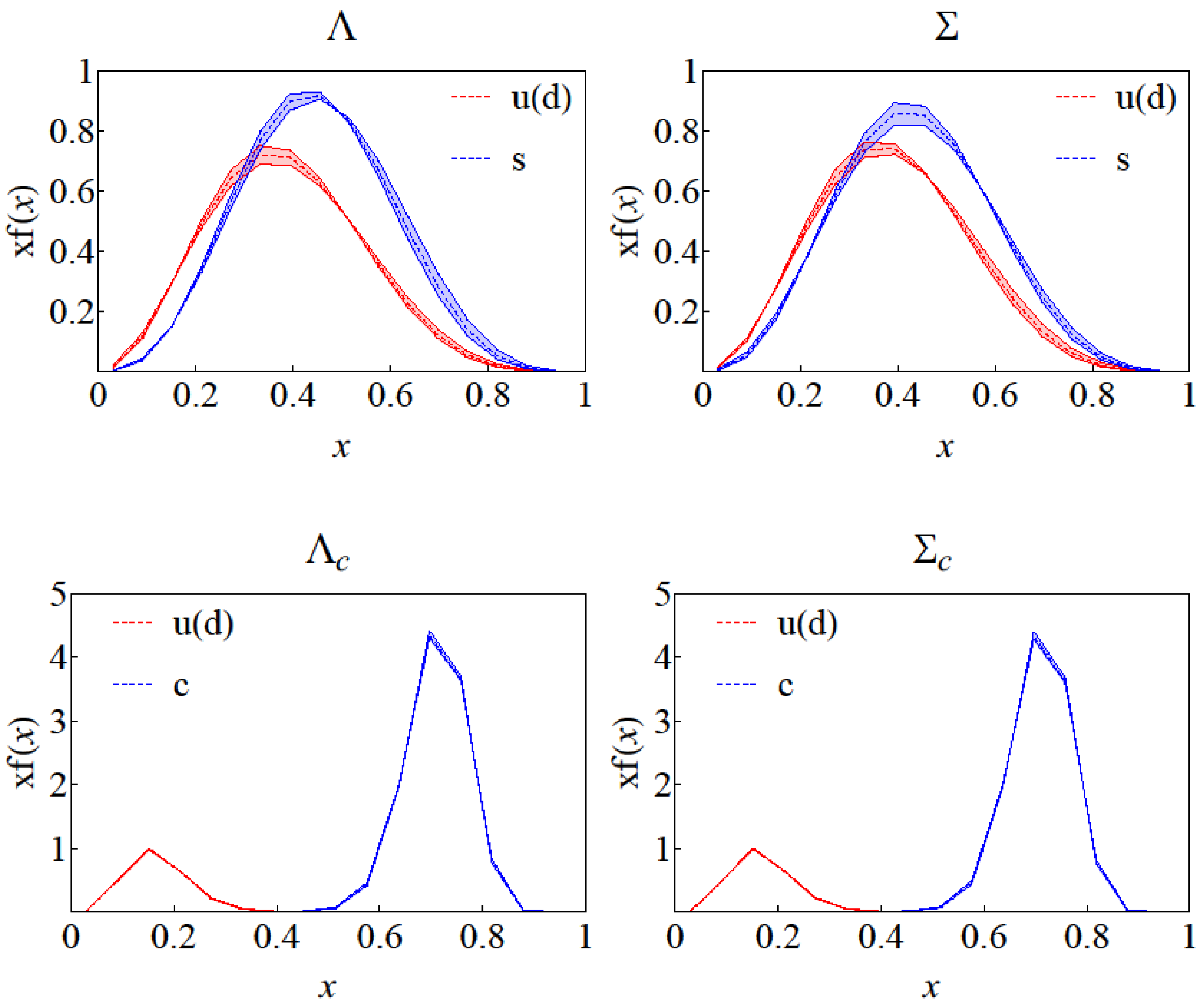
$$h_1^q(x) = \sum_{\{\lambda_i\}} \int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp] \times \left(\Psi^{\uparrow *}_{\{x_i,\vec{p}_{i\perp},\lambda'_i\}} \Psi^{\downarrow}_{\{x_i,\vec{p}_{i\perp},\lambda_i\}} + (\uparrow\leftrightarrow\downarrow)\right) \delta(x - x_1)$$

Parameter set:

	α_s	l_k/l_{oge}[GeV]	s_k/s_{oge}[GeV]	$K_L = K_T$ [GeV]	Mass[GeV]
Λ	1.06	0.300/0.200	0.390/0.290	0.377	1.115

	α_s	l_k/l_{oge}[GeV]	c_k/c_{oge}[GeV]	$K_L = K_T$ [GeV]	Mass[GeV]
Λ_c	0.57	0.300/0.200	1.580/1.480	0.377	2.286

Initial unpolarized PFDs f1 at leading Fock sector



With a 10% error band of coupling constant α_s

How to connect QCD quarks and gluons to the observed hadrons and leptons?

Here needs fundamentals of QCD factorization and evolution.

PDF f_1 compare with the Nucleon

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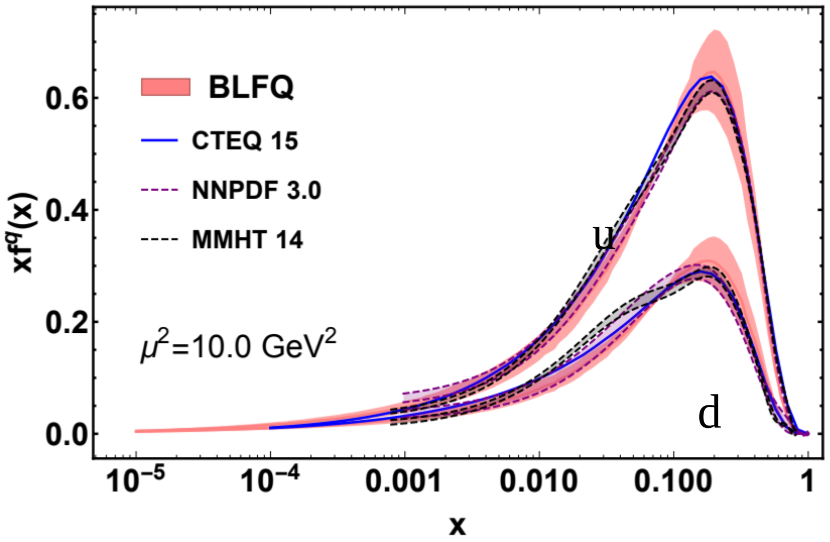
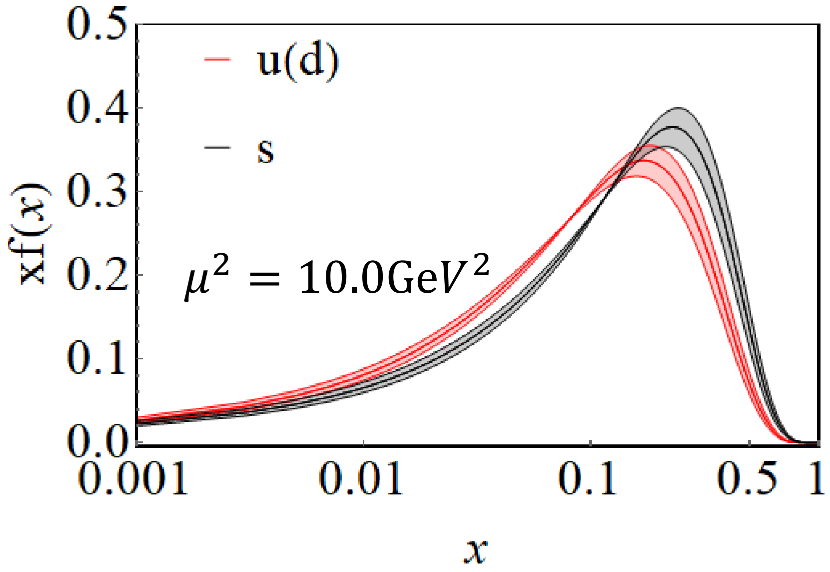
With QCD evolution equation, we can get the parton distribution functions at a higher scale.

Proton (uud)

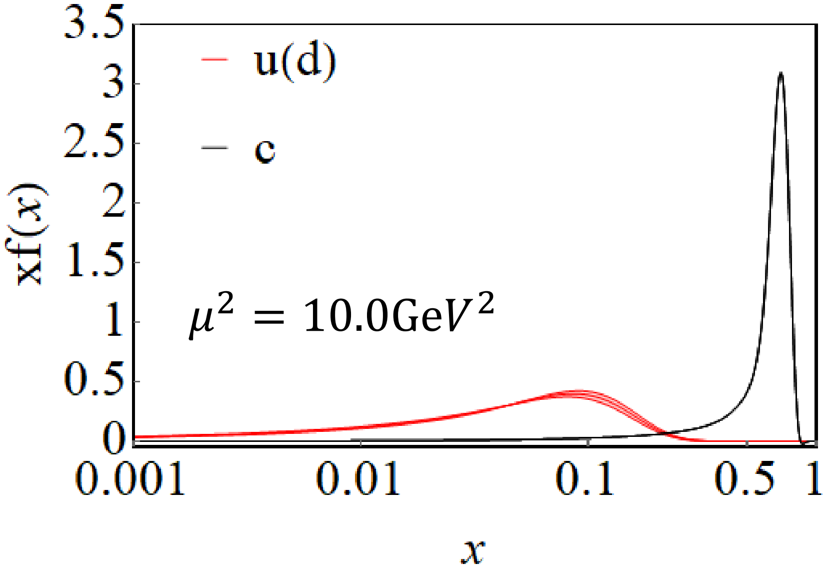


Siqi Xu et al. [BLFQ Collaboration],
Nucleon structure from basis light-front quantization

Λ (uds)



Λ_c (udc)



Charm *quark has moment at x bigger than* 0.5.

unpolarized PFDs after QCD evolution

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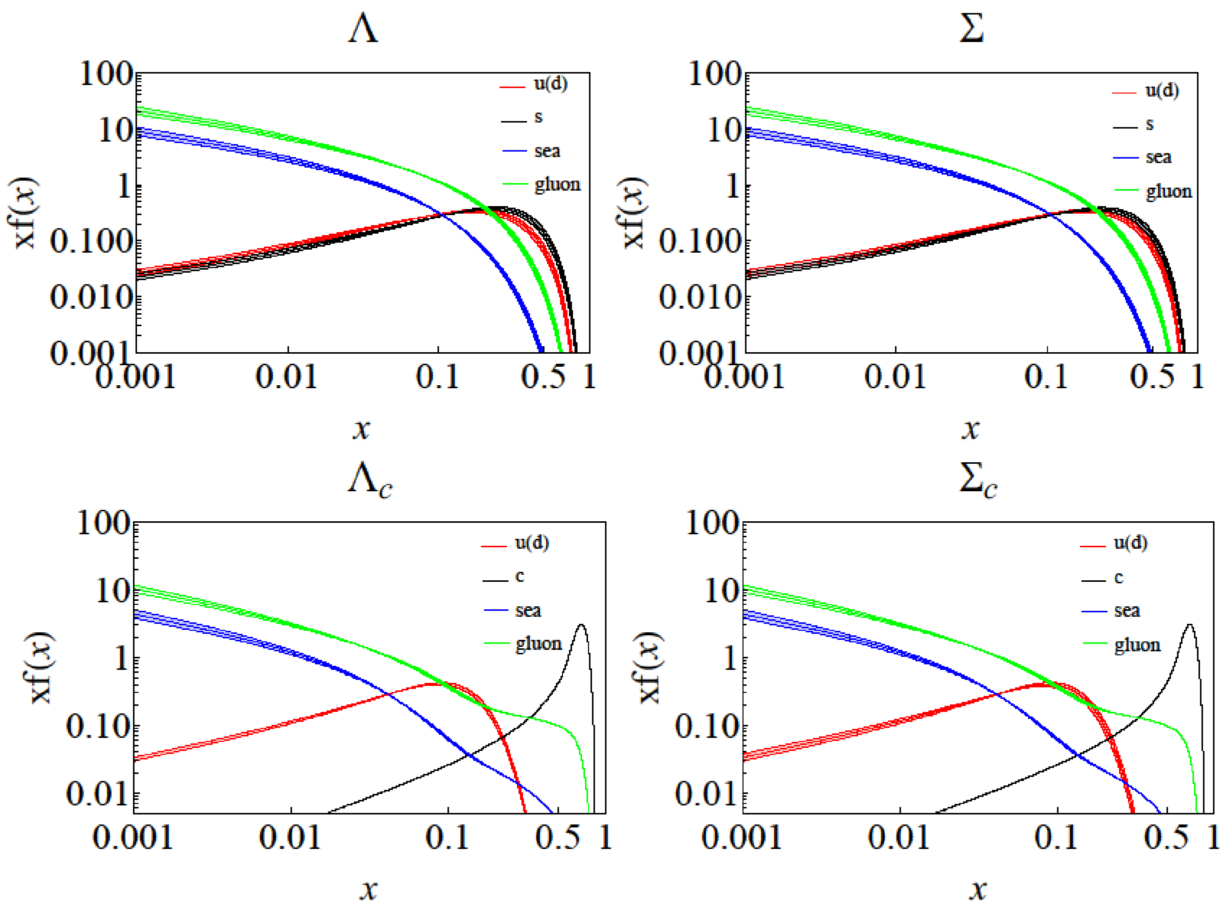
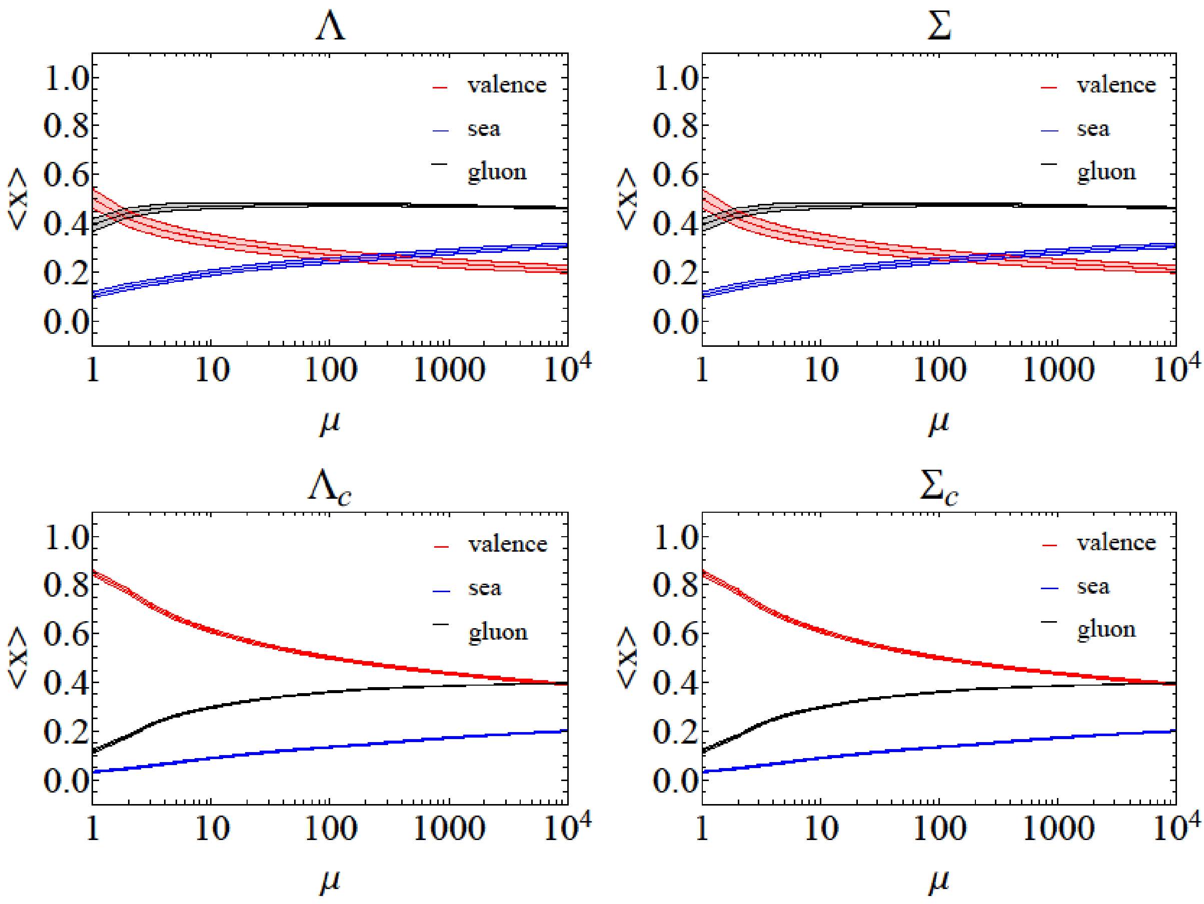
unpolarized xPDFs



The first moment



Strange quark is not much heavier than light quark

Charm quark is much heavier than light quark

FFs(Form factors)

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$$\left\langle P+q,\uparrow \left| \frac{J^+(0)}{2P^+} \right| P,\uparrow \right\rangle = F_1(Q^2),$$

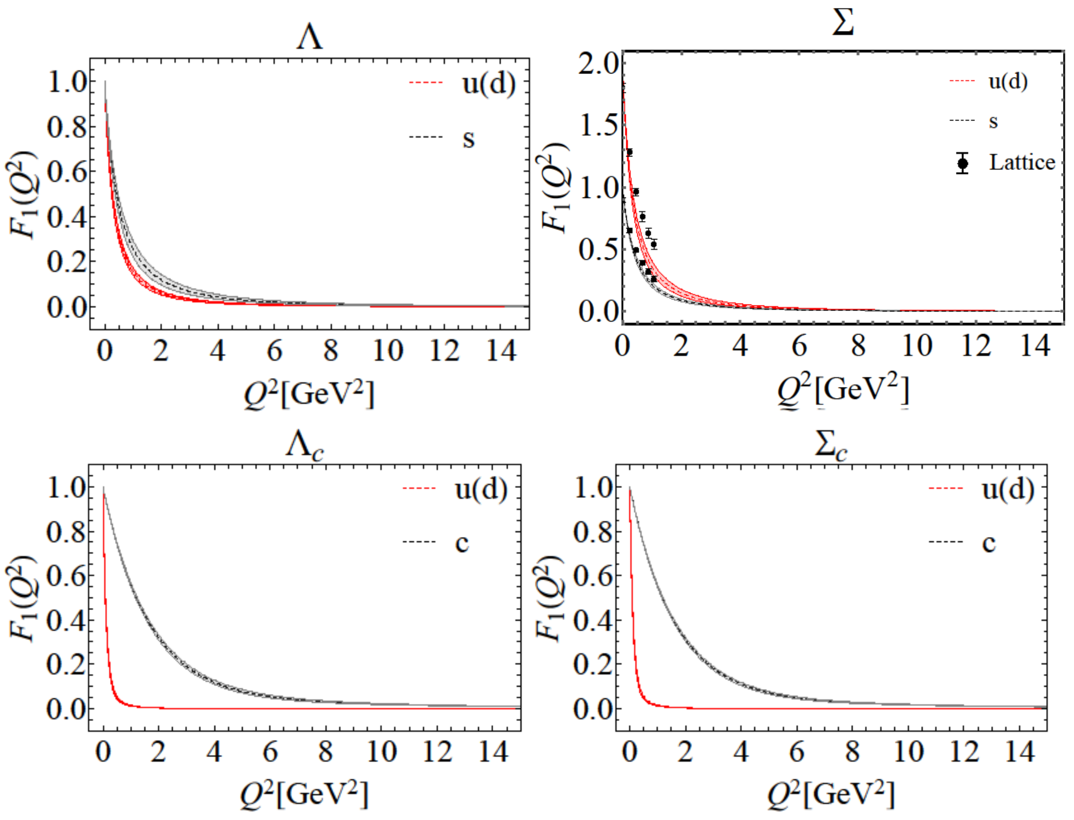
$$\left\langle P+q,\uparrow \left| \frac{J^+(0)}{2P^+} \right| P,\downarrow \right\rangle = -\frac{(q^1-iq^2)}{(2M)} F_2(Q^2)$$

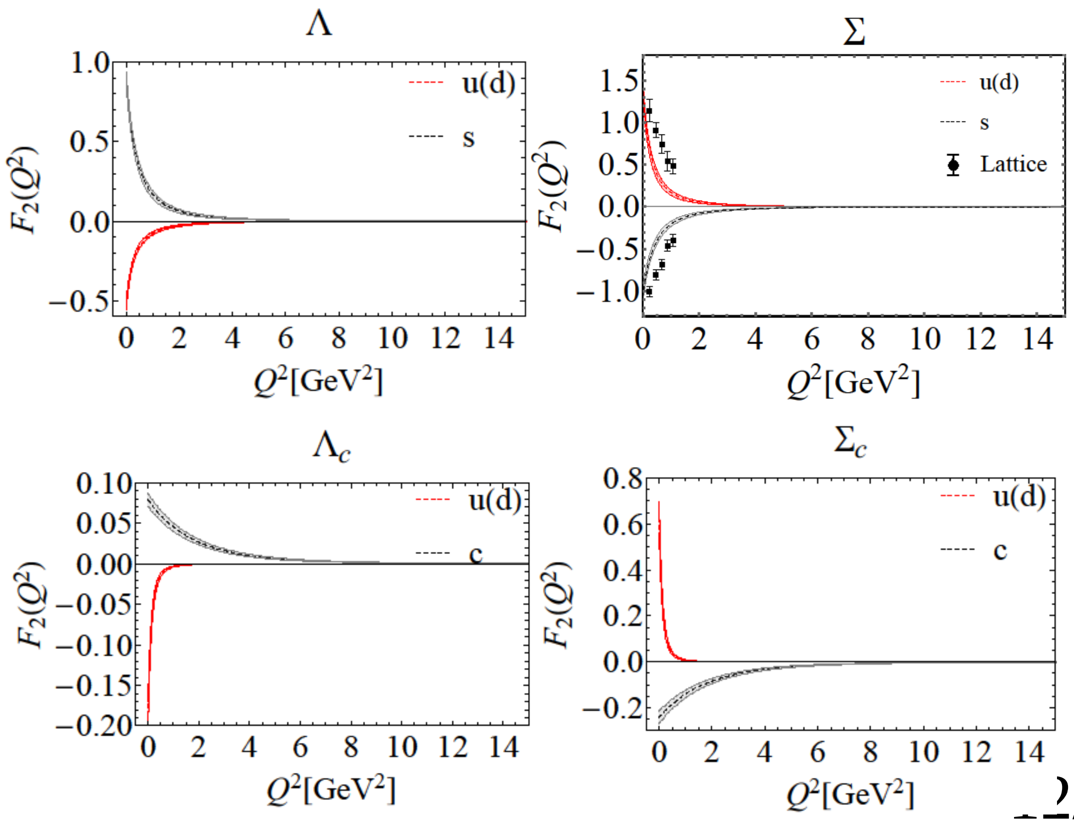
$$F_1^q(Q^2) = \int_D \Psi^{\uparrow *}_{\{x_i',\vec{p}'_{\perp i},\lambda_i\}} \Psi^{\uparrow}_{\{x_i,\vec{p}_{\perp i},\lambda_i\}},$$

$$F_2^q(Q^2) = -\frac{2M}{(q^1-iq^2)} \int_D \Psi^{\uparrow *}_{\{x_i',\vec{p}'_{\perp i},\lambda_i\}} \Psi^{\downarrow}_{\{x_i,\vec{p}_{\perp i},\lambda_i\}}$$

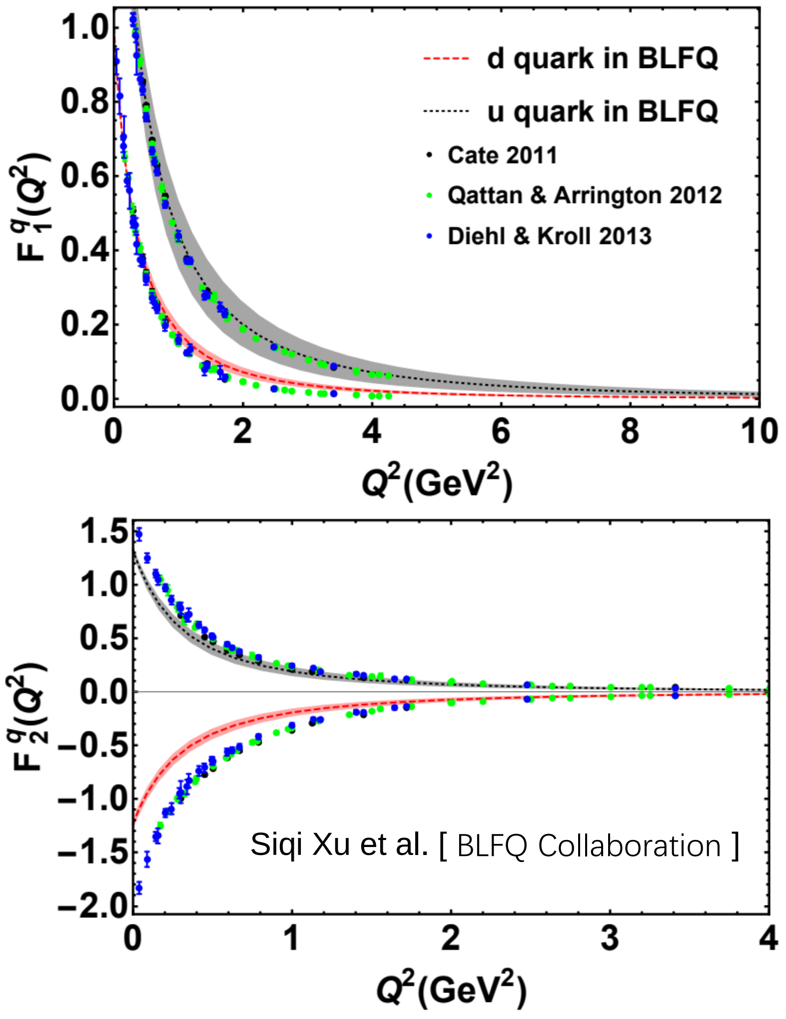
Dirac FFs

Pauli FFs



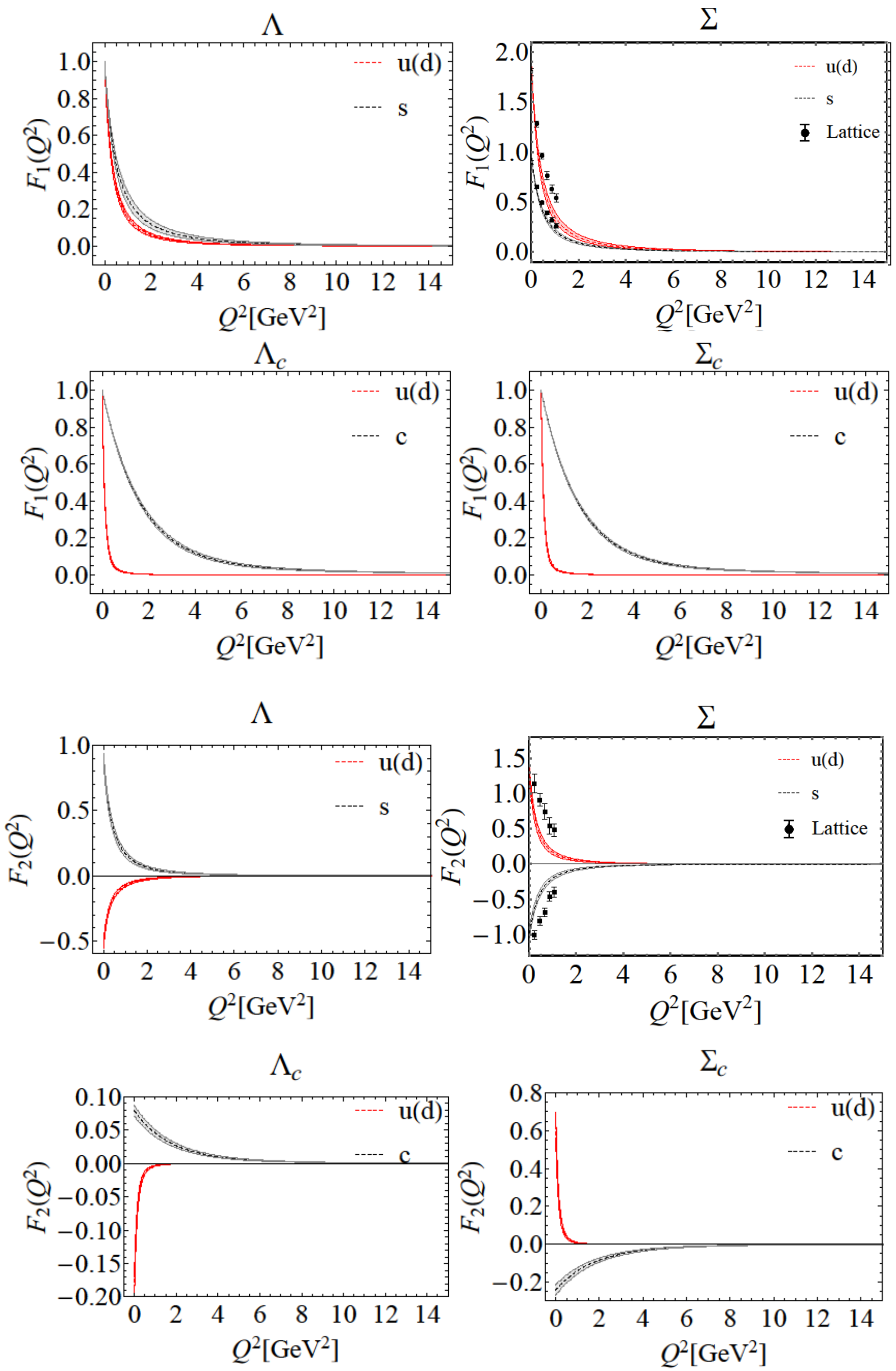


FFs (Form factors)



Dirac FFs

Pauli FFs



Light quark can give large contribution to Pauli FFs

Heave quark gives more contributions to Dirac FFs

Baryons Electromagnetic FFs

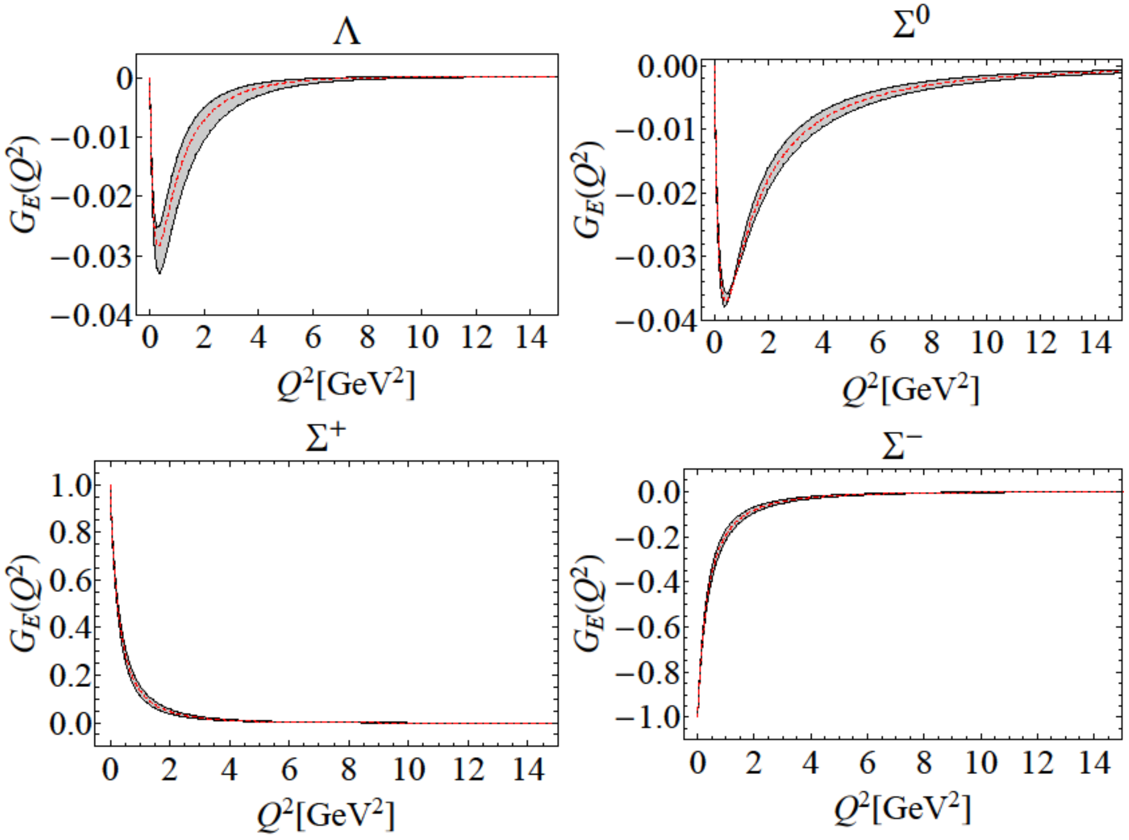
$$F_i^{\Lambda} = e_u F_i^u + e_d F_i^d + e_s F_i^s$$
$$F_i^{\Lambda_c} = e_u F_i^u + e_d F_i^d + e_c F_i^c$$

With the quarks charge, $e_u(e_c) = +\frac{2}{3}$, $e_d(e_s) = -\frac{1}{3}$

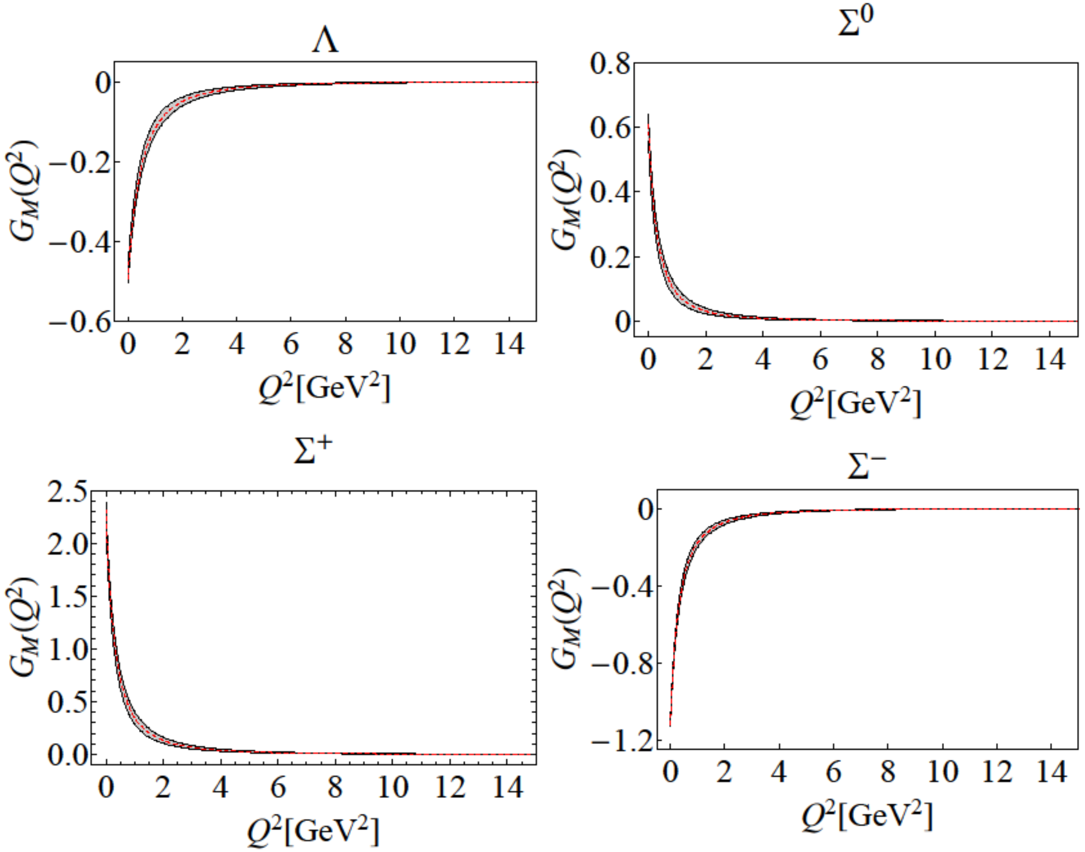
we can get the baryons FFs through the flavor FFs

$$G_E^{\Lambda(\Lambda_c)}(Q^2) = F_1^{\Lambda(\Lambda_c)}(Q^2) - \frac{Q^2}{4M^2} F_2^{\Lambda(\Lambda_c)}(Q^2)$$
$$G_M^{\Lambda(\Lambda_c)}(Q^2) = F_1^{\Lambda(\Lambda_c)}(Q^2) + F_2^{\Lambda(\Lambda_c)}(Q^2),$$

Electric FFs G_E



Magnetic FFs G_M



Baryon Electromagnetic FFs

$$F_i^{\Lambda} = e_u F_i^u + e_d F_i^d + e_s F_i^s$$
$$F_i^{\Lambda_c} = e_u F_i^u + e_d F_i^d + e_c F_i^c$$

With the quark charge, $e_u(e_c) = +\frac{2}{3}, e_d(e_s) = -\frac{1}{3}$

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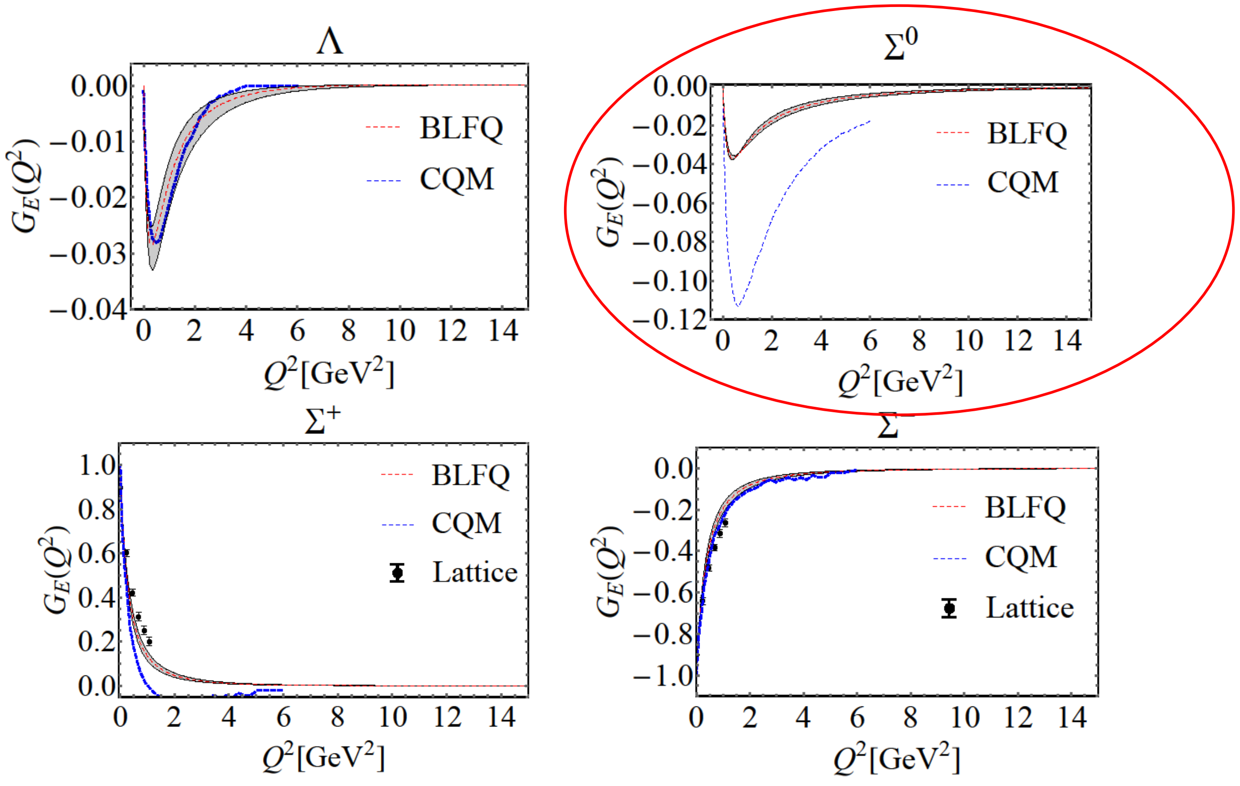
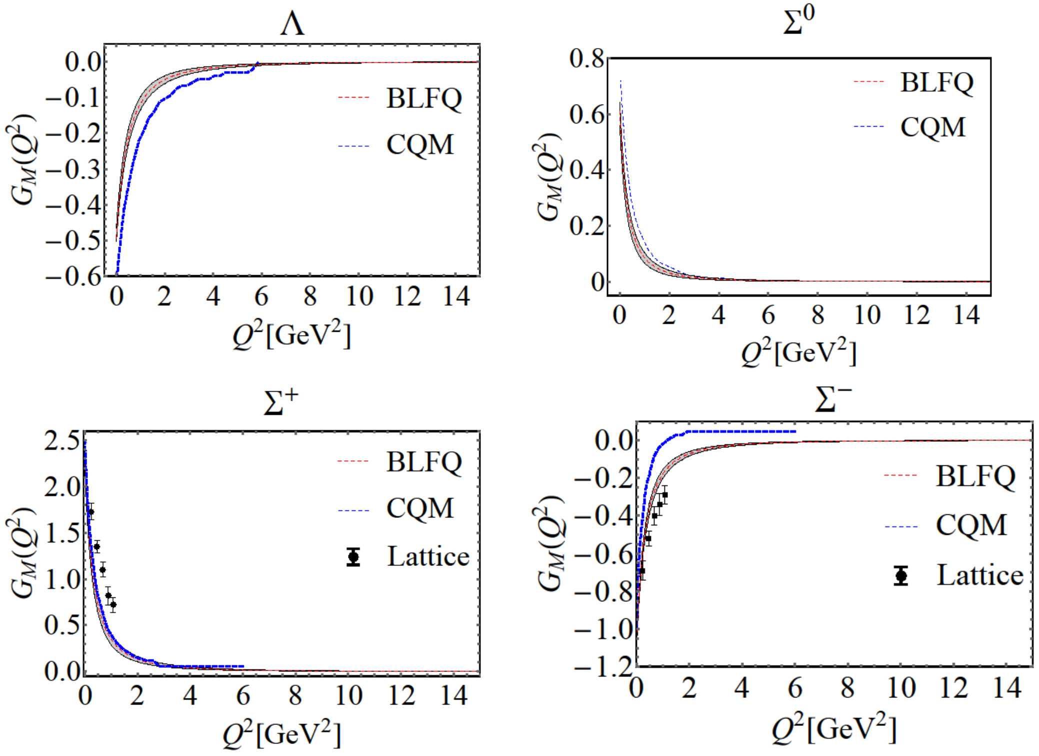
Electric FFs G_E

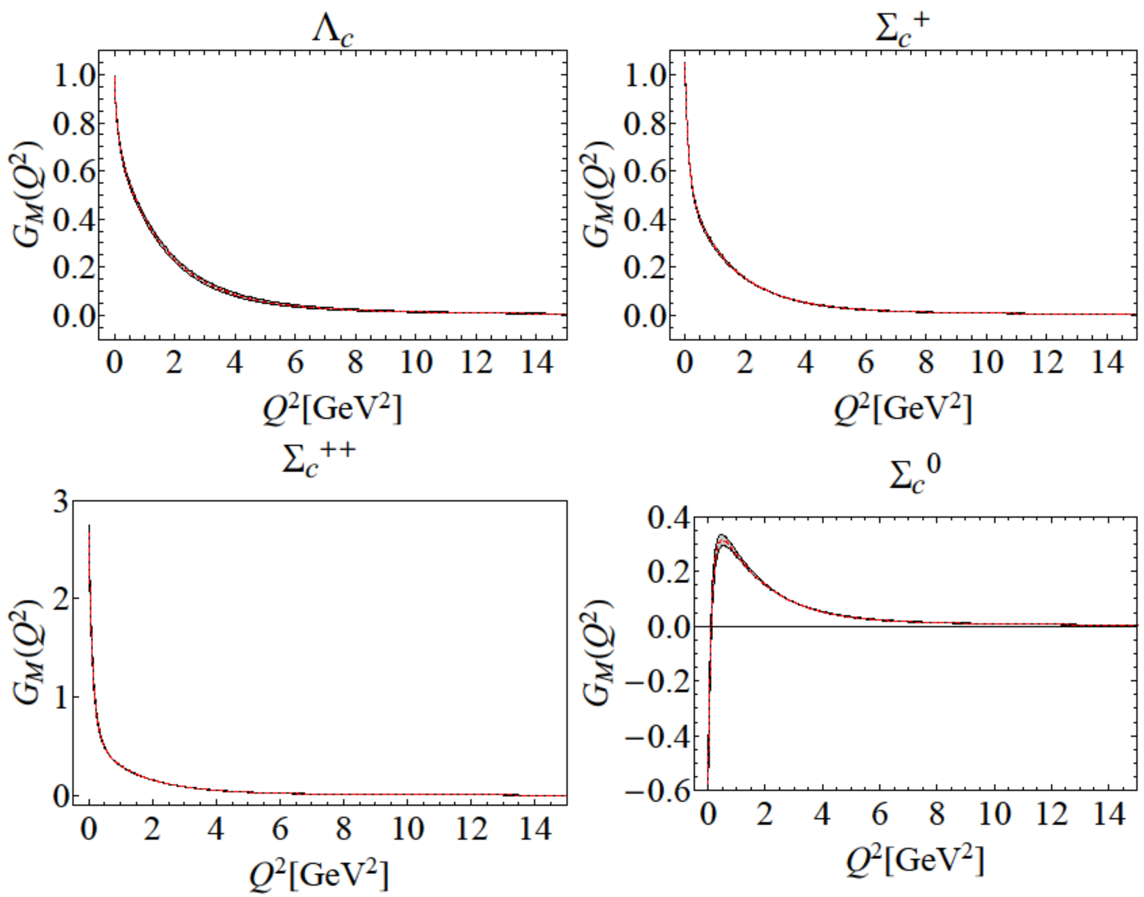


Magnetic FFs G_M

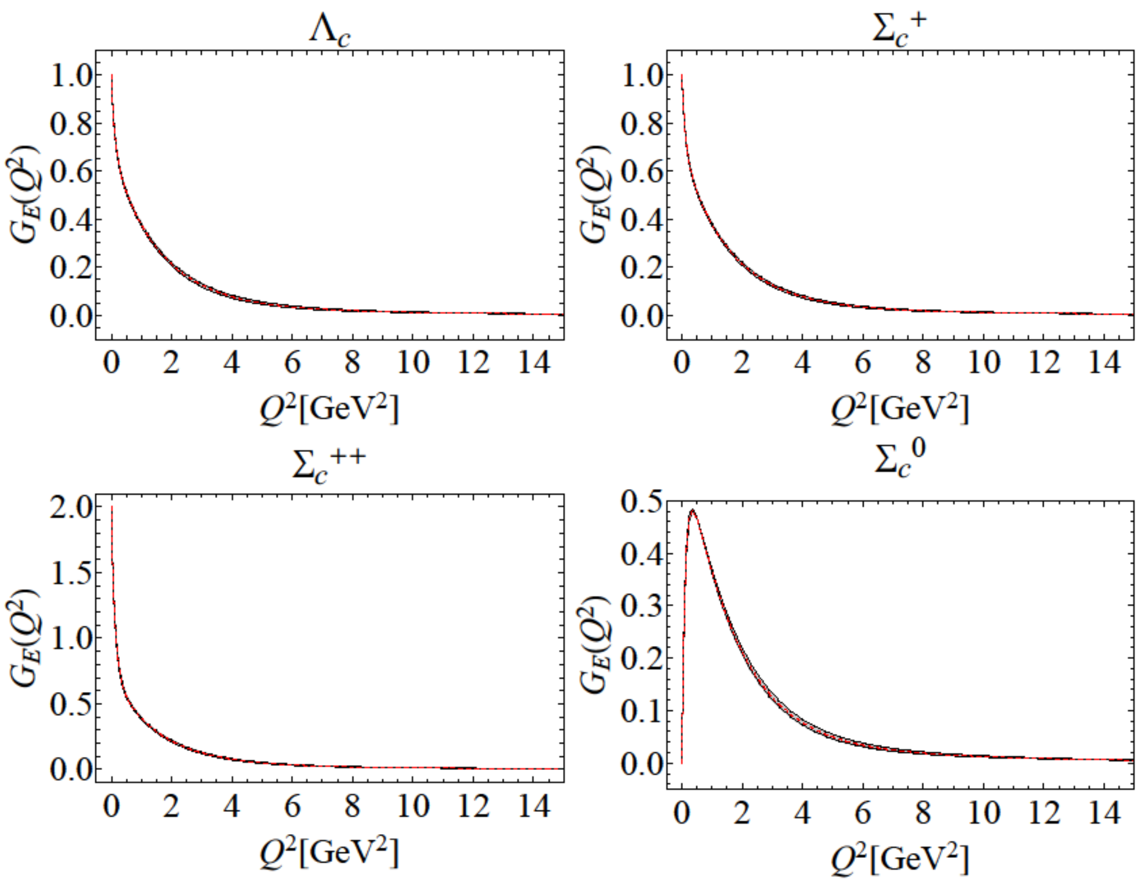


Constituent quark model and lattice QCD.

Electric FFs G_E



Magnetic FFs G_M



Magnetic moment and Electromagnetic radius

From the zero point value of the G_M, we can get the magnetic moment μ = G_M (0)

$$\langle r_E^2 \rangle = -\frac{6}{G_E(0)} \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

From the ratio of the G_E and G_M at zero point, we can get the electromagnetic radius like this:

$$\langle r_M^2 \rangle = -\frac{6}{G_M(0)} \frac{dG_M(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

	μ_{BLFQ}/μ_N	μ_{exp}/μ_N	$< r_E^2 >_{BLFQ}/[fm^2]$	$< r_E^2 >_{exp}/[fm^2]$	$< r_M^2 >_{BLFQ}/[fm^2]$	$< r_M^2 >_{exp}/[fm^2]$
Λ	$-0.494^{+0.028}_{-0.010}$	-0.613 ± 0.004	$0.07^{+0.01}_{-0..01}$	-	$0.52^{+0.01}_{-0.01}$	-
Σ^0	$0.610^{+0.032}_{-0.051}$	-	$0.07^{+0.00}_{-0..01}$	-	$0.82^{+0.00}_{-0.01}$	-
Σ^+	$2.323^{+0.067}_{-0.111}$	2.458 ± 0.010	$0.79^{+0.05}_{-0..05}$	-	$0.79^{+0.00}_{-0.00}$	-
Σ^-	$-1.124^{+0.011}_{-0.007}$	-1.160 ± 0.025	$0.65^{+0.02}_{-0..02}$	$0.60 \pm 0.08 \pm 0.08$	$0.70^{+0.02}_{-0.02}$	-

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	μ_{BLFQ}/μ_N	$< r_E^2 >_{BLFQ}/[fm^2]$	$< r_M^2 >_{BLFQ}/[fm^2]$
Λ_c	$0.99^{+0.00}_{-0.00}$	$0.73^{+0.02}_{-0.02}$	$0.64^{+0.02}_{-0.02}$
Σ_c^+	$1.05^{+0.0.01}_{-0.001}$	$0,74^{+0.02}_{-0.02}$	$0.78^{+0.01}_{-0.01}$
Σ_c^{++}	$2.67^{+0.49}_{-0.08}$	$1.33^{+0.03}_{-0.03}$	$1.54^{+0.01}_{-0.01}$
Σ_c^0	$-0.58^{+0.07}_{-0.07}$	$-1.19^{+0.04}_{-0.03}$	$3.37^{+0.33}_{-0..27}$

Magnetic moment

	μ_{BLFQ}	μ_{exp}[20]
Λ	$-0.494^{+0.028}_{-0.010}$	-0.613±0.004
Σ^0	$0.610^{+0.032}_{-0.051}$	-
Σ^+	$2.323^{+0.067}_{-0.112}$	2.458±0.010
Σ^-	$-1.124^{+0.011}_{-0.007}$	-1.160±0.025

Agree with available experimental data

	μ_{BLFQ}	[36]	[37]	[38]	[39]	[40]	[41]	[42]	[43]	[44]	S-I[45]	S-II[45]
Λ_c	$0.99^{+0.00}_{-0.00}$	0.41	0.42	0.392	0.341	0.411	-	0.37	0.385	-	0.24	0.24
Σ_c^+	$1.05^{+0.01}_{-0.01}$	0.65	0.36	0.30	0.525	0.318	-	0.63	0.501	0.46(3)	0.26	0.30
Σ_c^{++}	$2.67^{+0.49}_{-0.08}$	3.07	1.76	2.20	2.44	1.679	2.1(3)	2.18	2.279	2.15(10)	1.50	1.50
Σ_c^0	$-0.58^{+0.06}_{-0.07}$	-1.78	-1.04	-1.60	-1.391	-1.043	-1.6(2)	-1.17	-1.015	-1.24(5)	-0.97	-0.91

Electric radius ${r_E}^2$

Neutral particle

	BLFQ	HB[30]		IR[31]		HBχPT[32]		RQM[36]		exp.[33]
		$O(q^3)$	$O(q^4)$	$O(q^3)$	$O(q^4)$	$O(1/\Lambda^2_{chi})$	$O(1/\Lambda^2_\chi M_N)$	I	II	
Λ	0.07±0.01	0.14	0.00	0.05	0.11 ± 0.02	-0.150	-0.050	-0.01	0.02	-
Σ^0	$0.07^{+0.00}_{-0.01}$	-0.14	-0.08	-0.05	-0.03 ± 0.01	-	-	0.02	0.02	-
Σ^+	0.79±0.05	0.59	0.72	0.63	0.60 ± 0.02	1.522	1.366	0.47	0.66	-
Σ^-	0.65±0.02	0.87	0.88	0.72	0.67 ± 0.03	0.977	0.798	0.41	0.64	0.60±0.08±0.08

	BLFQ	[36]		
		Instant	Point	Front
Λ_c	$0.73^{+0.02}_{-0.02}$	0.5	0.2	0.4
Σ_c^+	$0.74^{+0.02}_{-0.02}$	0.5	0.2	0.4
Σ_c^{++}	$1.33^{+0.03}_{-0.03}$	1.7	0.4	1.4
Σ_c^0	$-1.19^{0.039}_{-0.03}$	0.7	-0.0	-0.6

Magnetic radius ${r_M}^2$

Chiral perturbative theory

	BLFQ	$O(q^4)HB[30]$	$O(q^4)IR[31]$
Λ	0.52 ± 0.01	0.30 ± 0.11	0.48 ± 0.09
Σ^0	$0.82^{+0.00}_{-0.01}$	0.20 ± 0.10	0.45 ± 0.08
Σ^+	0.79 ± 0.00	0.74 ± 0.06	0.80 ± 0.05
Σ^-	0.70 ± 0.02	1.33 ± 0.16	1.20 ± 0.13

	$< r_M^2 >$
Λ_c	$0.64^{+0.03}_{-0.03}$
Σ_c^+	$0.78^{+0.01}_{-0.01}$
Σ_c^{++}	$1.54^{+0.01}_{-0.01}$
Σ_c^0	$3.37^{+0.33}_{-0.27}$

Conclusions

1. Λ, Λ_c and their isospin triplet baryons structure from BLFQ.

2. Compare Λ(uds) and Λ_c(udc) with p(uud).

For PDFs, we find with addition of a strange quark or charm quark, they will be different .

For FFs, the light part will give a big contribution to Pauli FFs and heave quark will give more contribution to Dirac FFs in heave-light system.

3. The electromagnetic radii and the magnetic moment are found to be consistent with the available experimental data. We also show a comparison with other theoretical calculation on the electromagnetic properties of these baryons.

Thank you for your attention!