Pion to Photon Transition Form Factors with Basis Light-Front Quantization

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in collaboration with

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Overview

BLFQ-NJL Model and Applications

Pion DA & $\pi \to \gamma^* \gamma$ Transition Form Factor

$\pi \to \gamma^* \gamma^*$ Transition Form Factor

Conclusions

CM, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

- Sreeraj Nair : Tuesday, Nov. 30 at 14:30, Parallel Session 1-B
- Jiangshan Lan : Thursday, Dec. 2 at 9:05, McCartor Award Session

Basis Light-Front Quantization (BLFQ)

Vary: Nov. 29 at 11:00; Zhao: Nov. 30 at 11:50

A computational framework for solving relativistic many-body bound state problems in quantum field theories[1]

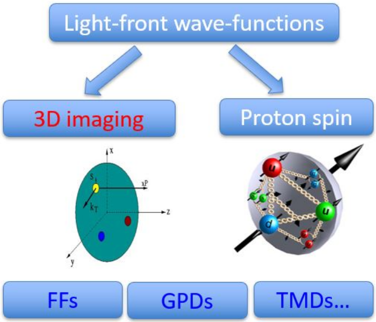
$$P^{-}P^{+}|\Psi\rangle = M^2|\Psi\rangle$$

- P^- : light-front Hamiltonian
- P^+ : longitudinal momentum
- $|\Psi\rangle$ mass eigenstate
- M^2 : mass squared eigenvalue for eigenstate $|\Psi\rangle$
- First-principles / effective Hamiltonian as input
- Evaluate observables

$$O \sim \langle\Psi|\hat{O}|\Psi\rangle$$

- Direct access to light-front wavefunction of bound states

GOAL



[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Effective Hamiltonian : BLFQ-NJL Model

$$|\,\pi\rangle_{\text{phys}} = a\,|\,q\bar{q}\rangle + b\,|\,q\bar{q}g\rangle + c\,|\,q\bar{q}q\bar{q}\rangle + ...$$

kinetic energy

transverse confining potential [2]

$$H_{\text{eff}} = \frac{\vec{k}^{\perp 2}+m_q^2}{x} + \frac{\vec{k}^{\perp 2}+m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}^{\perp 2}$$

$$+ \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big) + H^{\text{eff}}_{\text{NJL}}$$

longitudinal confining potential [3]

Nambu–Jona-Lasinio (NJL) interaction [4]

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[1] Jia and Vary, Phys. Rev. C 99, 035206 (2019)

[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).

[3] Li, Maris, Zhao and Vary, Phys. Lett. B 758, 118 (2016)

[4] Klimt, Lutz, Vogl and Weise, Nucl. Phys. A **516**, 429-468 (1990).

Meson Light-Front Wave Functions (LFWFs)

- Valence LFWFs in orthonormal bases

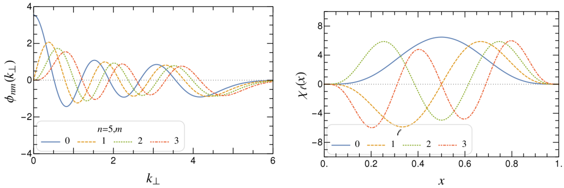
$$\psi_{rs}(x,\vec{\kappa}^{\perp}) = \sum_{n,m,l} \langle n,m,l,r,s|\psi\rangle \times \phi_{nm}\left(\vec{\kappa}^{\perp}\right)\chi_l(x)$$

  - Transverse direction (2D-HO)

$$\phi_{nm}\left(\vec{\kappa}^{\perp}\right) \sim \left(|\vec{\kappa}^{\perp}|\right)^{|m|} \times \exp\left(-\vec{\kappa}^{\perp 2}\right) L_n^{|m|}\left(\vec{\kappa}^{\perp 2}\right); \quad 0 \le n \le N_{\max}$$

  - Longitudinal direction (Jacobi polynomial basis)

$$\chi_l(x) \sim x^{\beta/2}(1-x)^{\alpha/2} P_l^{(\alpha,\beta)}(2x-1); \qquad 0 \le l \le L_{\max}$$



  - Coefficients $\langle n,m,l,r,s|\psi\rangle$: eigenvector in BLFQ basis representation.

[2] Li, Maris, and Vary, Phys. Rev. D 96 , 016022 (2017)

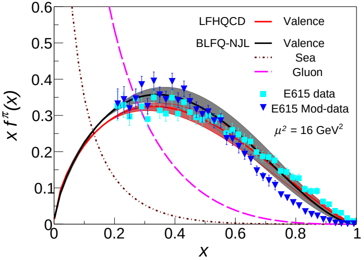
BLFQ-NJL model parameters

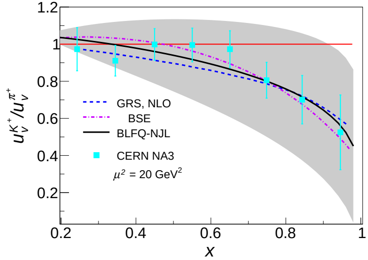
- Parameters are fixed to
  - reproduce ground state masses
  - experimental charge radii of π^+ and the K^+ [1]
- Successfully applied to
  - compute the PDAs and the EMFFs [1]
  - PDFs for the pion and the kaon and pion-nucleus induced Drell-Yan cross sections [2,3]
  - GPDs [4]
- Summary of the model parameters

Valence flavor	$N_{\max}$	$L_{\max}$	κ(MeV)	m_q(MeV)	$m_{\bar{q}}$(MeV)
$u\bar{d}$	8	8–32	227	337	337
$u\bar{s}$	8	8–32	276	308	445

[1] Jia and Vary, Phys. Rev. C 99, 035206 (2019)
[2] Lan, CM, Jia, Zhao, Vary, Phys. Rev. Lett. 122 172001 (2019)
[3] Lan, CM, Jia, Zhao, Vary, Phys. Rev. D 101, 034024 (2020)
[4] Adhikari, CM, Nair, Xu, Jia, Zhao and Vary, [arXiv:2106.04954] accepted by Phys. Rev. D

Applications: Light Meson PDFs





Light-front effective Hamiltonian, H_{eff}: ($\mu_{0\pi}^2 = 0.240 \pm 0.024$ GeV^2)

Diagonalizing H_{eff} $\Rightarrow$ LF wavefunction $\Rightarrow$ Initial PDFs $\Rightarrow$ Scale evolution [1].

$$\psi_{rs}(x, \vec{\kappa}^\perp) = \sum_{n,m,l} \langle n, m, l, r, s|\psi\rangle \times \phi_{nm}\left(\vec{\kappa}^\perp\right)\chi_l(x)$$

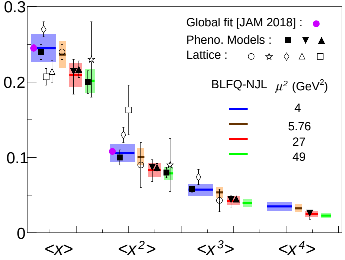
- 2D-HO $\phi_{nm}\left(\vec{\kappa}^\perp\right)$ in the transverse plane.
- Jacobi polynomial basis $\chi_l(x)$ in the longitudinal direction.

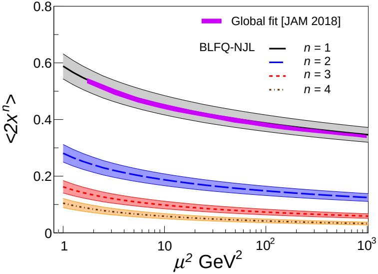
[1] Lan, CM, Jia, Zhao, Vary, Phys. Rev. Lett. 122 172001 (2019)

Moments of Pion PDF

Moments of the valence quark PDF

$$\langle x^n \rangle = \int_0^1 dx\; x^n f_v^\pi(x, \mu^2),\;\; n = 1, 2, 3, 4.$$





Consistent with global fit, lattice QCD, and phenomenological models.

[1] Lan, CM, Jia, Zhao, Vary, Phys. Rev. D 101, 034024 (2020)

GPDs → Transverse Densities

S. Nair : Nov. 30 at 14:30, Parallel Session 1-B

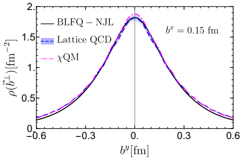
- Moments of GPDs:

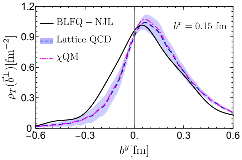
$$\int_0^1 dx\, x^{n-1} H^{\pi}(x, b_\perp^2) = A^{\pi}_{n0}(b_\perp^2)\,,$$
$$\int_0^1 dx\, x^{n-1} E^{\pi}_{T}(x, b_\perp^2) = B^{\pi}_{Tn0}(b_\perp^2)\,.$$

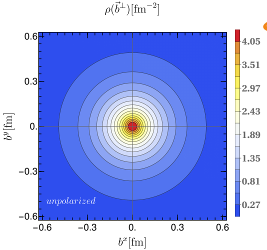
- Define density

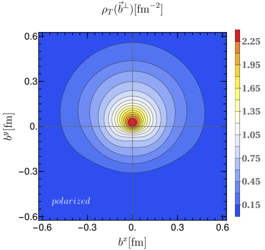
$$\rho^n(b_\perp, s_\perp) = \frac{1}{2}\left[A^{\pi}_{n0}(b_\perp^2) - \frac{s_\perp^i\, \epsilon^{ij}\, b_\perp^j}{m_\pi} B^{\pi\prime}_{Tn0}(b_\perp^2) \right],$$

- Reasonable agreement with Lattice QCD









[1] Brömmel, *et. al.*, [QCDSF and UKQCD], Phys. Rev. Lett. 101, 122001 (2008)
[2] Nam and Kim, Phys. Lett. B 700, 305-312 (2011).
[3] Adhikari, CM, Nair, Xu, Jia, Zhao and Vary, [arXiv:2106.04954] accepted by Phys. Rev. D

Distribution Amplitudes

DAs of pseudoscalar states

$$\phi(x,\mu_0) \sim \frac{1}{\sqrt{x(1-x)}} \int \frac{d^2\vec{k}_\perp}{2(2\pi)^3} \frac{(\psi_{\uparrow\downarrow} - \psi_{\downarrow\uparrow})}{\sqrt{2}}$$

- DA evolution: ERBL equations (Gegenbauer basis)

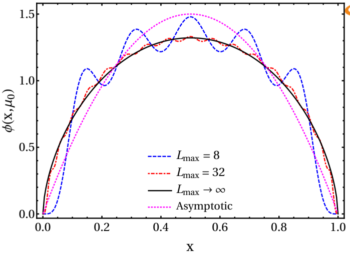
  Ruiz, *et. al.* PRD 66, (2002)

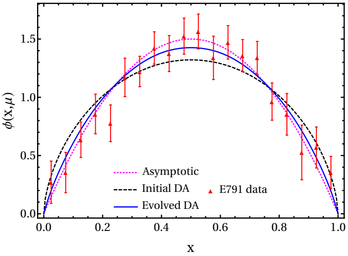
- Oscillations → Basis artifacts
- With increasing $L_{\max}$ the DA tends toward a smooth function
- Evolved DA (10 GeV^2) : Asymptotic DA

Decay constant f_π:

BLFQ (Basis [8, 32]): 145.3 MeV

Experimental data: 130.2 ± 1.7 MeV





- Consistent with the FNAL-E-791

[1] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

Moments of Pion DA

$$\langle z_p \rangle = \int_0^1 \mathrm{d}x \; z^p \; \phi(x,\mu) \,,$$

$z \equiv (2x-1)$ when $p \geq 1$ and $z \equiv x$ for $p = -1$.

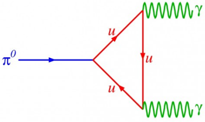
	μ(GeV)	$\langle z_2 \rangle$	$\langle z_4 \rangle$	$\langle z_6 \rangle$	$\langle x^{-1} \rangle$
BLFQ-NJL (this work)	1, 2	0.221, 0.217	0.099, 0.097	0.057, 0.055	3.21, 3.17
Playkurtic [90]	2	$0.220^{+0.009}_{-0.006}$	$0.098^{+0.008}_{-0.005}$	...	$3.13^{+0.14}_{-0.10}$
NLC sum rules [93]	2	$0.248^{+0.016}_{-0.015}$	$0.108^{+0.05}_{-0.03}$	...	3.16(9)
LF quark model [44]	∼1	0.24(22)	0.11(9)	0.07(5)	...
Sum rules [91]	1	0.24	0.11	...	...
AdS/QCD [55]	∼1	0.25	0.125	0.078	3.98
LF holographic ($B=0$) [46]	1, 2	0.180, 0.185	0.067, 0.071	...	2.81, 2.85
LF holographic ($B \gg 1$) [46]	1, 2	0.200, 0.200	0.085, 0.085	...	2.93, 2.95
Renormalon model [98]	1	0.28	0.13	...	...
Instanton vacuum (MIA 1) [92]	1, 2	0.237, 0.218	0.112, 0.094	0.066, 0.052	...
Instanton vacuum (MIA 2) [92]	1, 2	0.239, 0.220	0.113, 0.096	0.067, 0.053	...
Sum rules [2]	2	0.343	0.181	...	4.25
Dyson-Schwinger [RL, DB] [99]	2	0.280, 0.251	0.151, 0.128	...	5.5, 4.6
QCD background field theory sum rule [47]	1	0.271(13)	0.138(10)	0.087(6)	3.95
QCD background field theory sum rule [47]	2	0.254(10)	0.125(7)	0.077(6)	3.33
Lattice QCD [100]	2	0.28(1)(2)	...	...	...
Lattice QCD [94]	2	0.2361(41)(39)	...	...	...
Lattice QCD [101]	2	0.27(4)	...	...	...
Lattice QCD [95]	2	0.2077(43)	...	...	...
Lattice QCD [96]	2	0.234(6)(6)	...	...	...
Lattice QCD [97]	2	0.244(30)	...	...	...
Asymptotic QCD	∞	0.200	0.086	0.048	3.00

[1] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

Radiative Decay Width

- Alder-Bell-Jackiw (ABJ) anomaly[1,2] relations

$$F_{\pi\gamma}^{\mathrm{ABJ}}(0) = \frac{1}{2\sqrt{2}\pi^2 f_{\pi^0}}$$



- The radiative decay width

$$\Gamma_{\pi\to\gamma\gamma} = \frac{\pi}{4}\alpha_{\mathrm{EM}}^2 M_\pi^3 |F_{\pi\gamma}(0)|^2$$

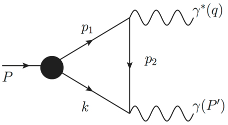
$[N_{\max}, L_{\max}] \rightarrow$	[8, 8]	[8, 16]	[8, 32]	Experimental data [102]
f_π (MeV)	142.8	144.8	145.3	130.2
$\Gamma_{\pi\to 2\gamma}$ (keV)	7.22×10^{-3}	7.03×10^{-3}	6.98×10^{-3}	$(7.82 \pm 0.22) \times 10^{-3}$

- Demonstrate a good convergence trend.

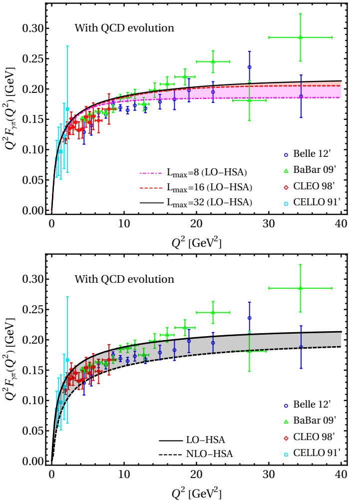
[1] Adler, Phys. Rev. 177, 2426 (1969)
[2] Bell and Jackiw, Nuovo Cimento A 60, 47 (1969)
[3] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

$\pi \to \gamma^* \gamma$ Transition Form Factor

$$\langle \gamma(P-q) | J^\mu | P(P) \rangle = -ie^2 F_{P\gamma}(Q^2) \epsilon^{\mu\nu\rho\sigma} P_\nu \epsilon_\rho q_\sigma \, ,$$



- Evaluated from convolution of a hard scattering amplitude (HSA) with DA.
- Results for $\{N_{\max}, L_{\max}\} \equiv \{8, 8\}$, $\{8, 16\}$, and $\{8, 32\}$ (upper panel)
- Good convergence trend.
- Consistent with Brodsky-Lepage limit [1]: $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = \text{const.}$
- ERBL evolution effects & α_s order correction considered (lower panel).



[1] Lepage and Brodsky, Phys. Rev. D 22, 2157 (1980)
[2] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

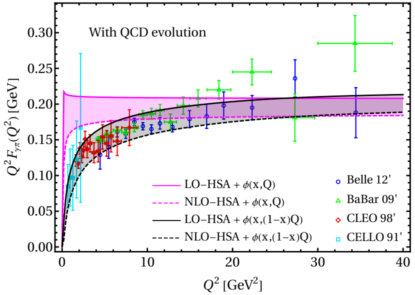
$\pi \to \gamma^* \gamma$ Transition Form Factor

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \mathrm{d}x\, T_\mathrm{H}(x, Q^2)\, \phi(x, \bar{x}Q),$$

- The hard scattering amplitude (HSA)[1 2]

$$T_\mathrm{H}(x, Q^2) = \frac{1}{1-x} + \mathcal{O}(\alpha_s) + \dots$$

- Assumption $\phi(x, (1-x)Q) \simeq \phi(x, Q)$: reasonable at $Q^2 \to \infty$[2].
- NOT well justified below the asymptotic region.
- Need to take into account the ERBL evolution effects.



[1] Braaten, Phys. Rev. D 28, 524 (1983)
[2] Brodsky, Cao, and de Teramond, Phys. Rev. D 84, 033001 (2011)
[3] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

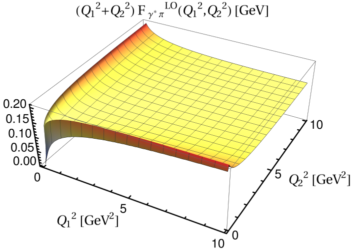
$\pi \to \gamma^* \gamma^*$ Transition Form Factor

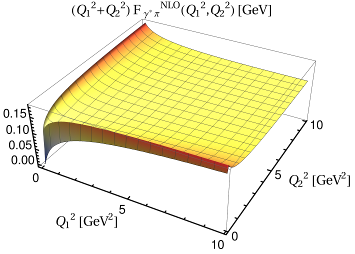
$$F_{\pi\gamma^*}(Q_1^2, Q_2^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \mathrm{d}x \, T_{\mathrm{H}}^{\gamma^*\gamma^* \to \pi^0}(x, Q_1^2, Q_2^2) \, \phi(x, \bar{Q})$$

$$T_{\mathrm{H}}^{\gamma^*\gamma^* \to \pi^0} = \frac{1}{(1-x)Q_1^2 + xQ_2^2} + \mathcal{O}(\alpha_s) + \dots$$

- $F_{\pi\gamma^*}(Q_1^2, Q_2^2) \sim 1/(Q_1^2 + Q_2^2)$ when $(Q_1^2, Q_2^2) \to \infty$.
- Consistent with PQCD prediction[1].
- Vector meson dominance (VMD) model: $F_{\pi\gamma^*}^{\mathrm{VMD}}(Q_1^2, Q_2^2) \sim 1/(Q_1^2 Q_2^2)$.
- Qualitative behavior → consistent with the LFQM results.

  Choi, Ryu and Ji, PRD 99, 076012 (2019)
- Singly virtual TFF → by setting one of the momentum transfers to zero.





[1] Lepage and Brodsky, Phys. Rev. D 22, 2157 (1980)

[2] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

$\pi\to\gamma^*\gamma^*$ Transition Form Factor

$$F_{\pi\gamma^*}(Q_1^2, Q_2^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \mathrm{d}x\, T_{\mathrm{H}}^{\gamma^*\gamma^*\to\pi^0}(x, Q_1^2, Q_2^2)\, \phi(x, \bar{Q})$$

(Q_1^2, Q_2^2)	$F_{\pi\gamma^*}^{\mathrm{LO}}$ (this work)	$F_{\pi\gamma^*}^{\mathrm{NLO}}$ (this work)	PQCD LO	PQCD NLO	LFQM [54]	VMD [54]
(6.48, 6.48)	10.39–10.56	9.59–9.75	9.52	8.78	9.08	1.957 ± 0.022
(16.85, 16.85)	3.99–4.06	3.73–3.79	3.66	2.69	3.58	0.322 ± 0.004
(14.83, 4.27)	7.55–7.72	7.00–7.14	6.91	6.39	6.76	1.301 ± 0.014
(38.11, 14.95)	2.65–2.69	2.48–2.52	2.42	2.27	2.40	0.163 ± 0.002
(45.63, 45.63)	1.47–1.50	1.39–1.41	1.35	1.33	1.33	0.046 ± 0.001

[1] Choi, Ryu and Ji, Phys. Rev. D 99, 076012 (2019)

[1] Mondal, Nair, Jia, Zhao and Vary, Phys. Rev. D 104, 094034 (2021)

Effective Hamiltonian with One Dynamical Gluon

$$|\,\pi\rangle_{\text{phys}} = a\,|\,q\bar{q}\rangle + b\,|\,q\bar{q}g\rangle + c\,|\,q\bar{q}q\bar{q}\rangle + \ldots$$

kinetic energy

transverse confining potential [2]

$$H_{\text{eff}} = \frac{\vec{k}^{\perp 2}+m_q^2}{x} + \frac{\vec{k}^{\perp 2}+m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}^{\perp 2}$$

$$+ \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right) + H_{\text{vertex}} + H_{\text{inst}}$$

longitudinal confining potential [3]

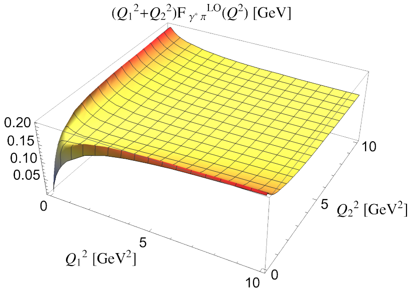
QCD interactions [4]

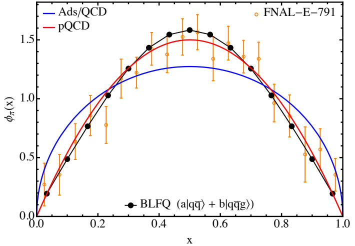
[1] Lan, Fu, CM, Zhao and Vary, arXiv:2106.04954 [hep-ph].
[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016).
[4] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).

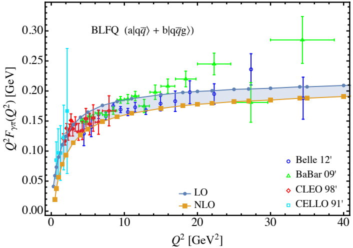
DA & Transition Form Factors

$$|\,\pi\rangle_{\text{phys}} = a\,|\,q\bar{q}\rangle + b\,|\,q\bar{q}g\rangle + c\,|\,q\bar{q}q\bar{q}\rangle + \ldots$$

$$\{N_{\max}, L_{\max}\} = \{14, 15\}$$







Decay constant f_π:

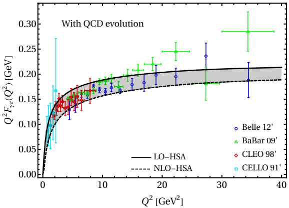
BLFQ : 138 MeV

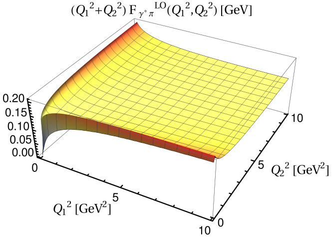
Experimental data: 130.2 ± 1.7 MeV

[1] Work in progress; Jiangshan Lan: Thursday, Dec. 2 at 9:05, McCartor Award Session

Conclusions

- Pion structure from the eigenstates of light-front effective Hamiltonians
- $|q\bar{q}\rangle$ (BLFQ-NJL model) and $|q\bar{q}\rangle + |q\bar{q}g\rangle$ (with QCD interactions).
- LF Hamiltonian $\Rightarrow$ Wavefunctions $\Rightarrow$ Observables.
- $\pi \to \gamma^* \gamma$ TFF : Consistent with Belle data; deviates from BaBaR data.
- $F_{\pi\gamma^*}(Q_1^2, Q_2^2) \sim 1/(Q_1^2 + Q_2^2)$ when $(Q_1^2, Q_2^2) \to$ large.





Thank You