Designing charmonium LFWFs on a small-basis

Guangyao Chen

Jacksonville University

arXiv: 2111.07087

Co-authors: Meijian Li, Yang Li, Tuomas Lappi, and James Vary

Introduction

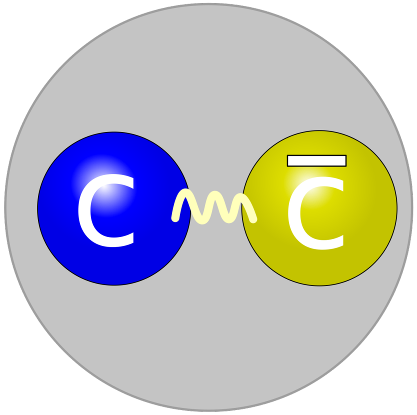
- Goals: simple-function charmonium LFWFs with few parameters!

i. Approximation to QCD.

ii. Retain more symetries.

iii. Matching the NR limit.

iv. Emphsis on decay width.



- We designed LFWFs for η_c, J/ψ, ψ' and $\psi(3770)$.

Basis functions

- LF holography/Basis LF Quantization Hamiltonian.

$$H_0 = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) r_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x)\partial_x) ,$$

i. Two parameters: m_q and κ.

ii. One-gluon interactions were treated perturbatively.

- The basis function representation.

$$\tilde{\psi}^{(m_j)}_{ss'/h}(\vec{r}_\perp, x) = \sqrt{x(1-x)} \sum_{n,m,l} \psi_h^{(m_j)}(n,m,l,s,s') \tilde{\phi}_{nm}(\sqrt{x(1-x)}\vec{r}_\perp)\chi_l(x) ,$$

Teramond and Brodsky, '09

Li et al., PLB 758, 118 (2016)

Basis functions

- Small basis for charmonium states:

$$\psi_{\text{LF}-1S} = \psi_{0,0,0} .$$

$$\psi_{\text{LF}-1P0} = \psi_{0,0,1} ,$$

$$\psi_{\text{LF}-1P\pm1} = -\psi_{0,\pm1,0} .$$

$$\psi_{\text{LF}-2S} = \sqrt{\frac{2}{3}}\psi_{1,0,0} - \sqrt{\frac{1}{3}}\psi_{0,0,2} .$$

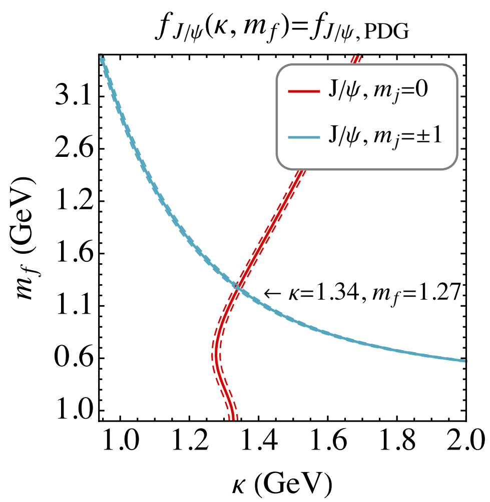
$$\psi_{\text{LF}-1D0} = \sqrt{\frac{1}{3}}\psi_{1,0,0} + \sqrt{\frac{2}{3}}\psi_{0,0,2} ,$$

$$\psi_{\text{LF}-1D\pm1} = -\psi_{0,\pm1,1} ,$$

$$\psi_{\text{LF}-1D\pm2} = \psi_{0,\pm2,0} .$$

J/ψ as a 1^{--} state

- We assume a 100% LF-1S state for J/ψ.
- Matching J/ψ decay constant to the PDG value:



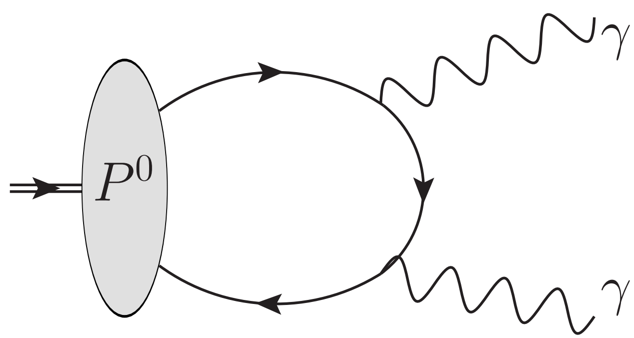
- We fix m_c and κ using the J/ψ decay constant.

η_c as a 0^{-+} state

- η_c predominantly LF-1S+LF-2S and LF-1P.

$$\psi_{\eta_c} = C_{\eta_c,1S}\psi_{\mathrm{LF-1}S,0-+} + C_{\eta_c,2S}\psi_{\mathrm{LF-2}S,0-+} + C_{\eta_c,1P}\psi_{\mathrm{LF-1}P,0-+} \,.$$

- Basis coeffecients are determined using the diphoton decay width $\Gamma(\eta_c \to \gamma\gamma)$.

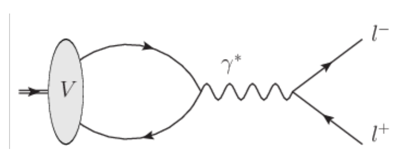


ψ' as a 1^{--} state

- A mix of LF-1S and LF-2S states for ψ'.

$$\psi_{\psi'}^{(m_j=0)} = C_{\psi',1S}^{(m_j=0)} \psi_{\mathrm{LF}-1S,1--}^{(m_j=0)} + C_{\psi',2S}^{(m_j=0)} \psi_{\mathrm{LF}-2S,1--}^{(m_j=0)} ,$$
$$\psi_{\psi'}^{(m_j=1)} = C_{\psi',1S}^{(m_j=1)} \psi_{\mathrm{LF}-1S,1--}^{(m_j=1)} + C_{\psi',2S}^{(m_j=1)} \psi_{\mathrm{LF}-2S,1--}^{(m_j=1)} ,$$
$$\psi_{\psi'}^{(m_j=-1)} = C_{\psi',1S}^{(m_j=-1)} \psi_{\mathrm{LF}-1S,1--}^{(m_j=-1)} + C_{\psi',2S}^{(m_j=-1)} \psi_{\mathrm{LF}-2S,1--}^{(m_j=-1)} .$$

- Basis coeffecients are determined using the dilepton decay constant.



$$\left| f_{\mathcal{V}}|_{m_j=0} \right| = \left| f_{\mathcal{V}}|_{m_j=\pm 1} \right| = f_{\mathcal{V},\mathrm{experiment}} .$$

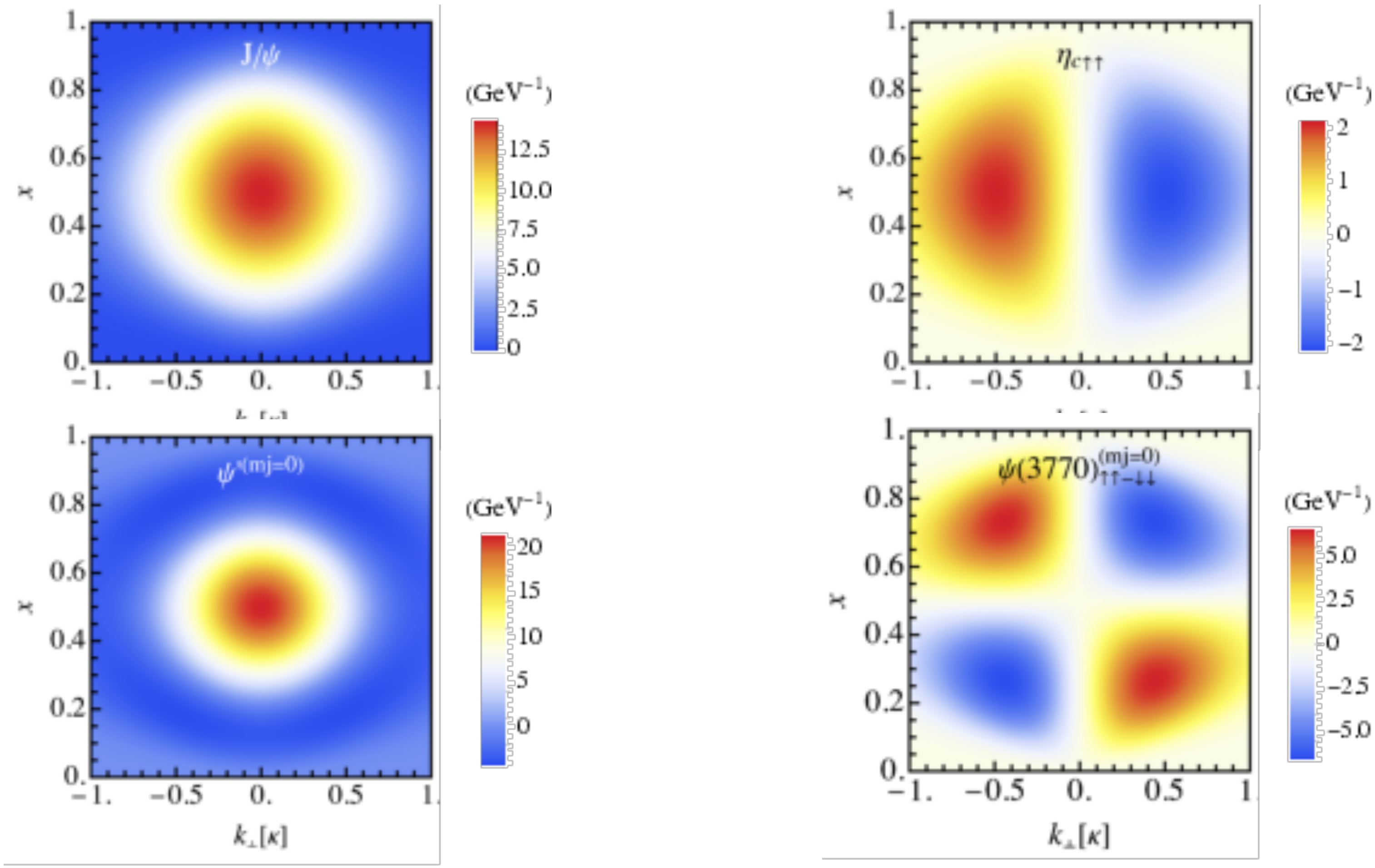
ψ(3770) as a 1⁻⁻ state

- A mix of LF-1S, LF-2S, LF-1D states for ψ(3770), LF-1D is dominating.

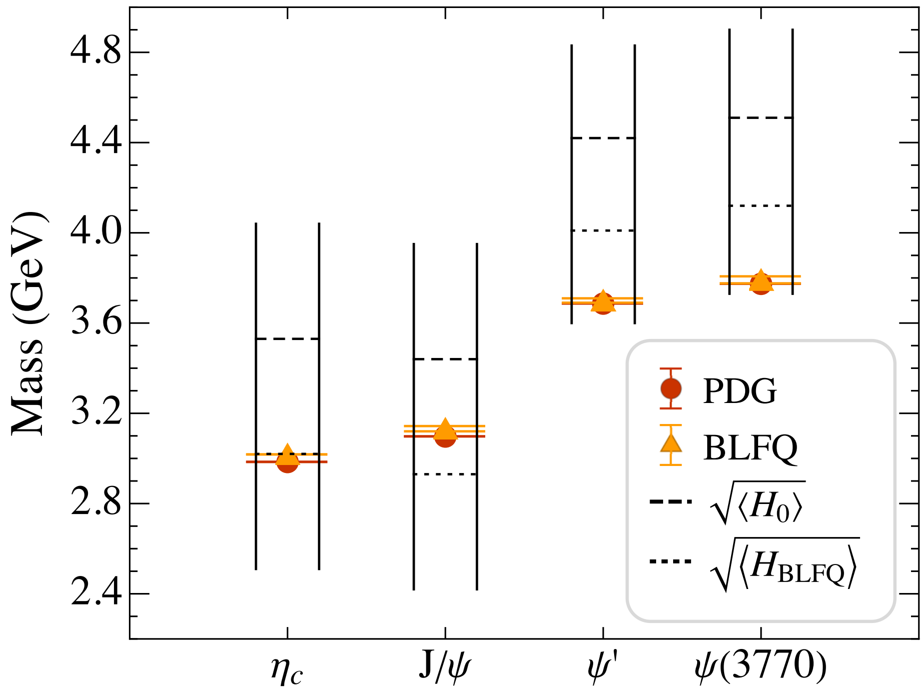
$$\begin{aligned}\psi^{(m_j=0)}_{\psi(3770)} =& C^{(m_j=0)}_{\psi(3770),1S}\psi^{(m_j=0)}_{\mathrm{LF}-1S,1--} \\ &+ C^{(m_j=0)}_{\psi(3770),2S}\psi^{(m_j=0)}_{\mathrm{LF}-2S,1--} \\ &+ C^{(m_j=0)}_{\psi(3770),1D}\psi^{(m_j=0)}_{\mathrm{LF}-1D,1--},\end{aligned}$$

- Basis coeffecients are determined by requiring orthogonality between ψ' and ψ(3770).

LFWFs by design



The mass spectrum



$$\left(\tilde{M}_h^{(m_j)}\right)^2 = \sum_{n,m,l,s,\bar{s}} \sum_{n',m',l',s',\bar{s}'} \psi_h^{(m_j)}(n,m,l,s,\bar{s}) \times \psi_h^{(m_j)*}(n',m',l',s',\bar{s}') \times \Big[M^2_{n,m,l}\delta_{n,n'}\delta_{m,m'}\delta_{l,l'}\delta_{s,s'}\delta_{\bar{s},\bar{s}'} + \langle \beta_{n',m',l',s',\bar{s}'} | \Delta H | \beta_{n,m,l,s,\bar{s}} \rangle \Big] .$$

$$V_{\rm OGE} = -\frac{C_F 4\pi\alpha_s(q^2)}{q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}') ,$$

Masses calculated from small-basis LFWFs should be regarded as Estimated!

Li et al., PLB 758, 118 (2016)

The charge radii

- Defined in terms of the slope of the charge form factor at zero momentum transfer.

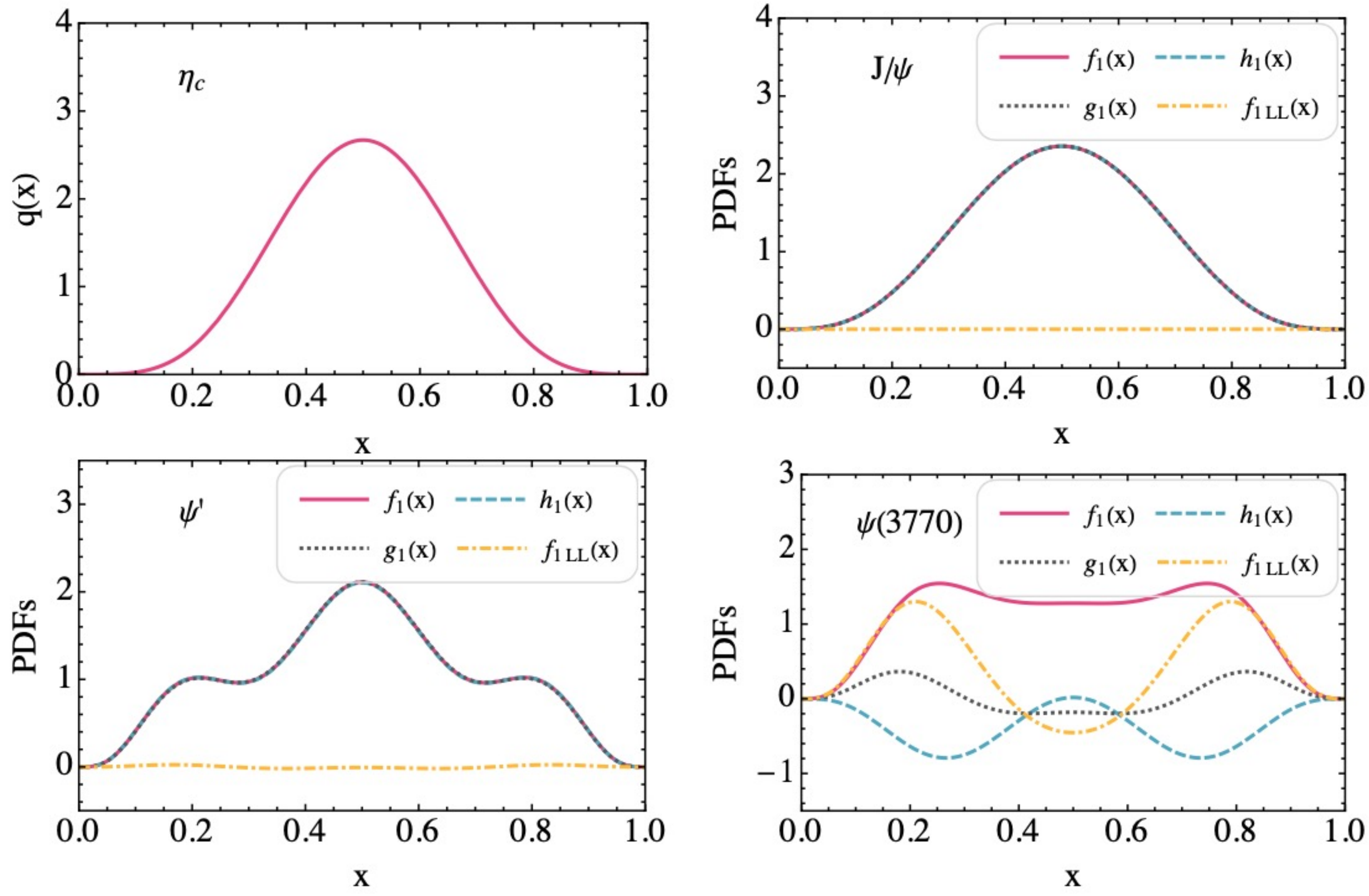
$$\langle r_h^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_0(Q^2) \Big|_{Q\to 0} .$$

(fm^2)	$\langle r^2_{\eta_c} \rangle$	$\langle r^2_{J/\psi} \rangle$	$\langle r^2_{\psi'} \rangle$	$\langle r^2_{\psi(3770)} \rangle$
this work	0.098	0.046	0.154	0.138
BLFQ [27]	0.029(1)	0.0402(2)	0.13(0)	0.13(0)

- J/ψ, ψ' and $\psi(3770)$ radii consistent with BLFQ calculations.
- A large size η_c!

Li et al., PLB 758, 118 (2016)

Parton Distribution Functions (PDFs)



Dipole model of diffractive VM production

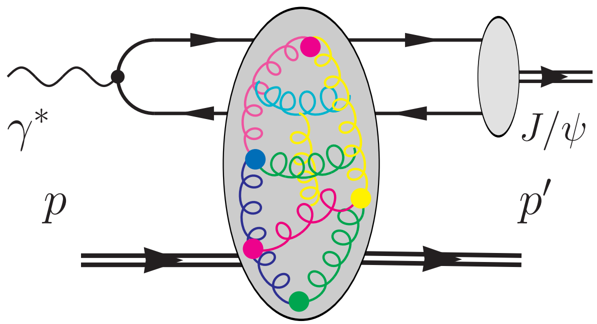
- The exclusive VM production amplitude:

$$\mathcal{A}_{T,L}^{\gamma^* p \to E p}(x, Q, \Delta) = \mathrm{i} \int d^2\vec{r} \int_0^1 \frac{dz}{4\pi} \int d^2\vec{b}\, (\Psi_E{}^* \Psi)_{T,L} \times \mathrm{e}^{-\mathrm{i}[\vec{b}-(1-z)\vec{r}]\cdot\vec{\Delta}} \frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}$$

i. Ψ_E : LFWF of vector meson

ii. Ψ : Photon LFWF

iii. $\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}$: dipole cross section

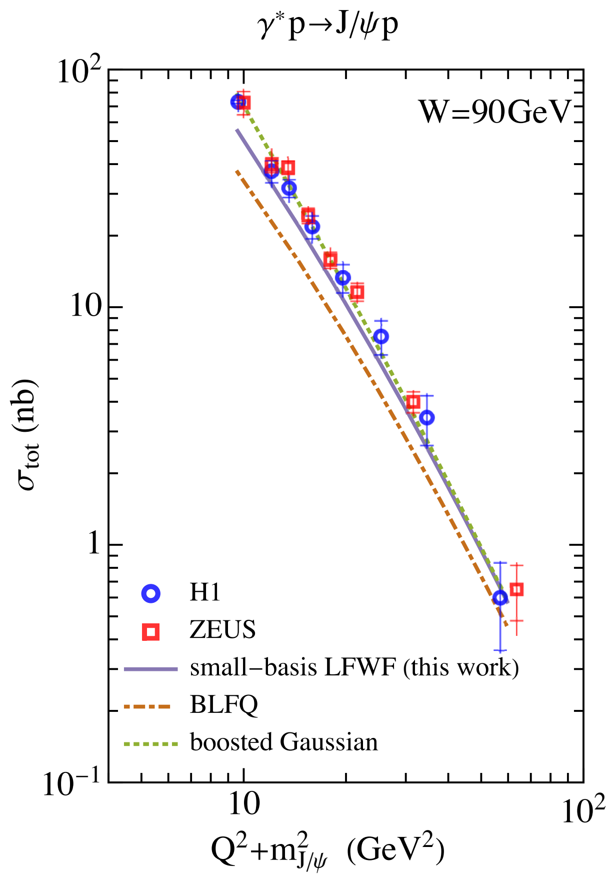


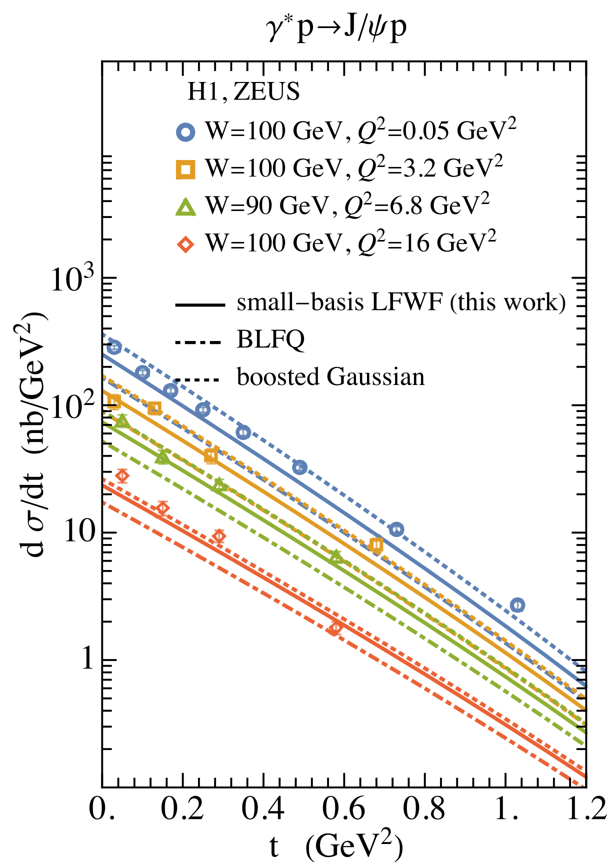
- LFWFs of vector meson is the KEY!

Golec-Biernat and Wusthoff , 1999
Kowalski and Teaney , 2001

J/ψ production at HERA

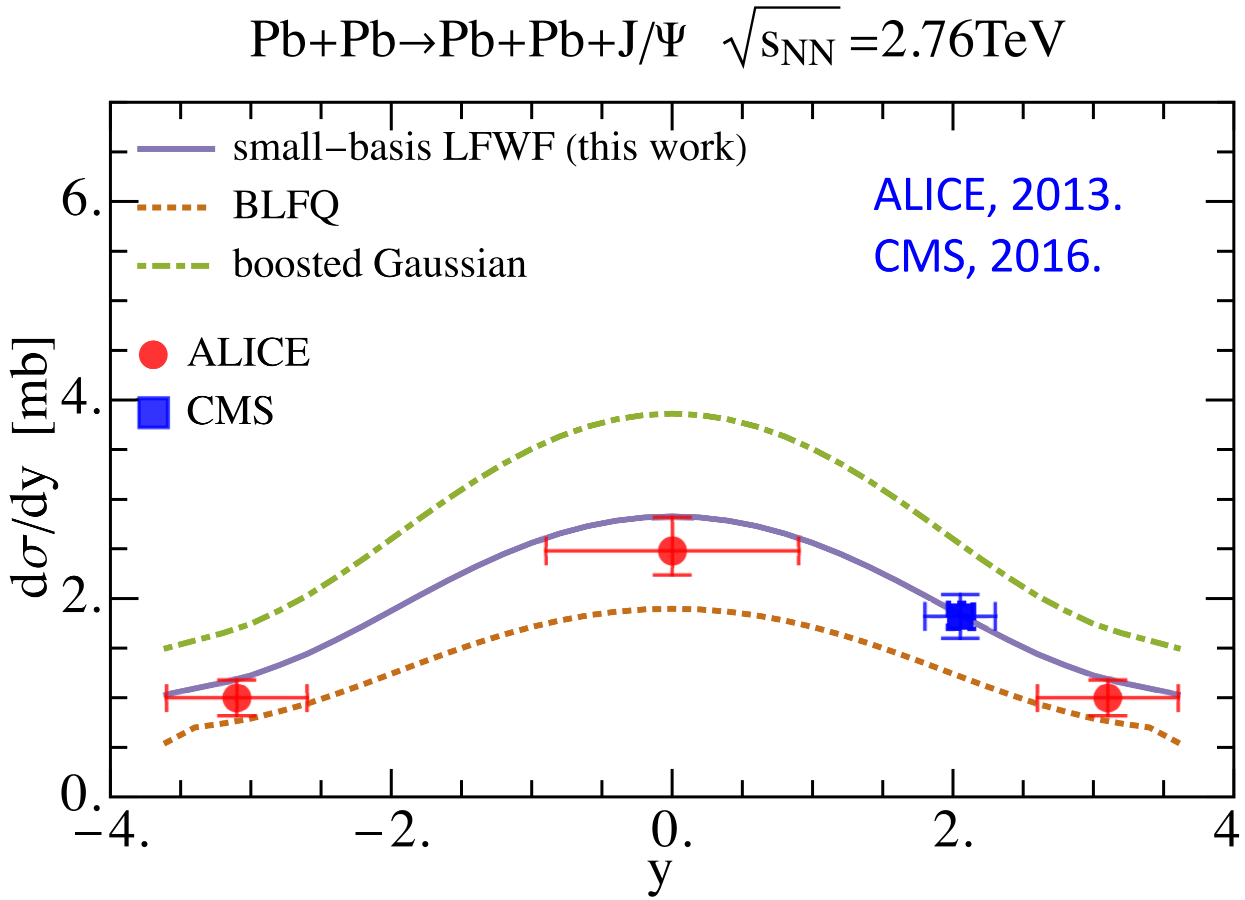
ZEUS, 2004.
H1, 2006.





GC et al., PLB 769, 477, 2017
GC et al., PRC 100, 025208, 2019

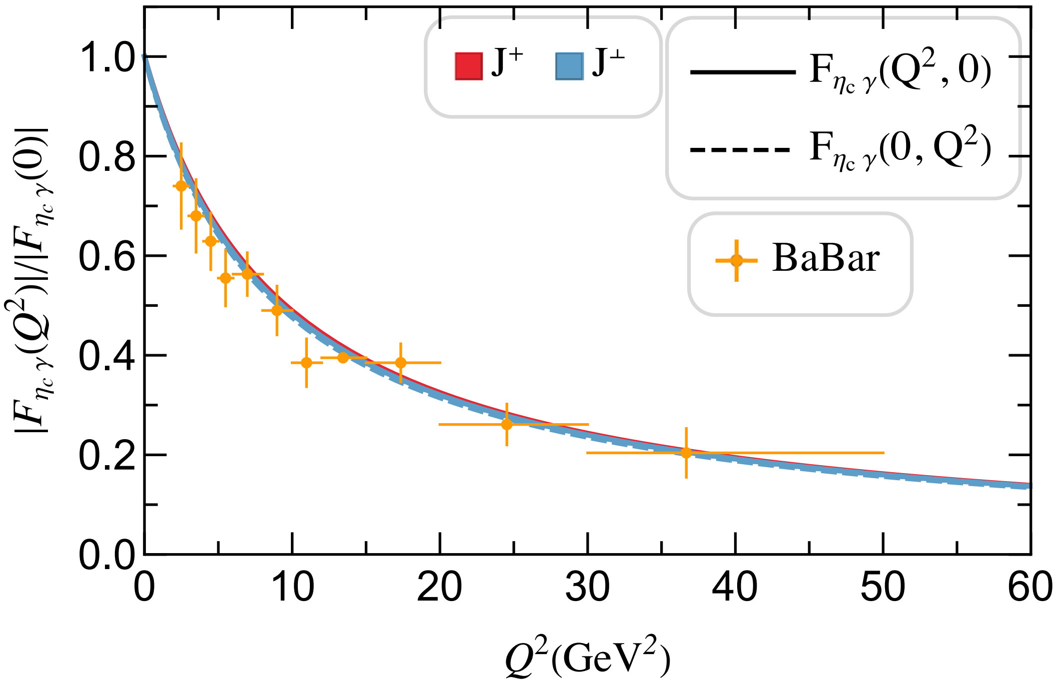
J/ψ production at LHC



GC et al., PLB 769, 477, 2017

GC et al., PRC 100, 025208, 2019

$\gamma^*\gamma$ -> η_c Transition Form Factor



$$I^{\mu}_{\lambda_1}(P, q_1) \equiv \langle \gamma^*(q_1, \lambda_1)|\, J^{\mu}(0)\, |\mathcal{P}(P)\rangle$$
$$= -\, ie^2 F_{\mathcal{P}\gamma}(Q_1^2, Q_2^2)\epsilon^{\mu\alpha\beta\sigma} P_{\alpha} q_{1\beta} \epsilon^*_{\sigma,\lambda_1}(q_1)\ ,$$

BaBar, 2010.

Summary

- η_c , J/ψ, ψ' and $\psi(3770)$ LFWFs in a simple functional form.
- Physical observables calculated:

i. Masses and charge radii.

ii. PDFs.

iii. J/ψ production at HERA and LHC.

iv. η_c diphoton transition form factor.

- Outlook: simultaneous global analysis in determining model parameters.

Thank you!

- Collaborators: Meijian Li, Yang Li, Tuomas Lappi, James Vary
- This work is supported by Department of Energy, USA, and European Research Council
- Acknowledgement: ILCAC

Backup Slides

Basis Function

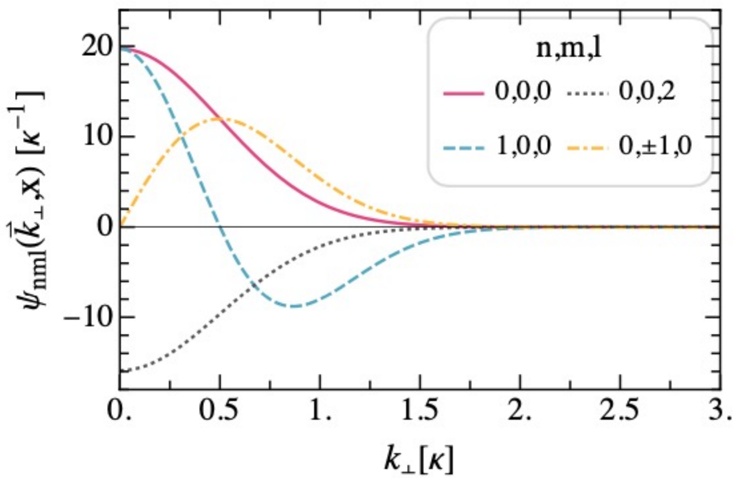
- Transverse:

$$\phi_{nm}(\vec{k}_\perp) = \kappa^{-1} \sqrt{\frac{4\pi n!}{(n+|m|)!}} \left(\frac{k_\perp}{\kappa}\right)^{|m|} \exp(-k_\perp^2/(2\kappa^2))$$
$$L_n^{|m|}({k_\perp}^2/\kappa^2) \exp(im\theta_k) \ ,$$

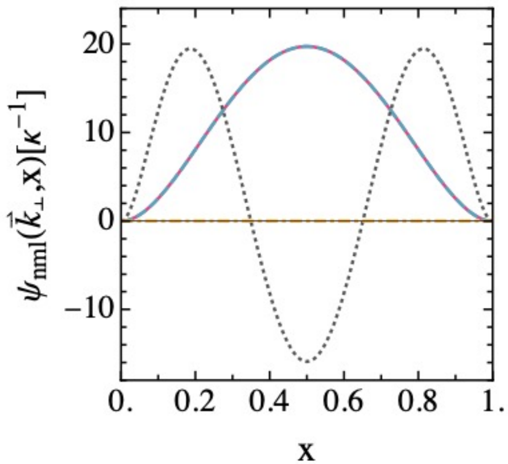
- Longitudinal:

$$\chi_l(x) = \sqrt{4\pi(2l+\alpha+\beta+1)} \sqrt{\frac{\Gamma(l+1)\Gamma(l+\alpha+\beta+1)}{\Gamma(l+\alpha+1)\Gamma(l+\beta+1)}}$$
$$x^{\beta/2}(1-x)^{\alpha/2} P_l^{(\alpha,\beta)}(2x-1) \ ,$$

Sample Basis Function



(a) $\psi_{nml}(k_\perp, \theta_k = 0, x = 0.5)$



(b) $\psi_{nml}(\vec{k}_\perp = \vec{0}_\perp, x)$

Mirror Parity and Chargeconjugation

TABLE I. Basis states $\psi_{nml}\sigma_{s\bar{s}}$ categorized by the mirror parity m_{P} according to Eq. (17).

m_j	m	$m_{\mathsf{P}} = 1$	$m_{\mathsf{P}} = -1$
0	0	$\psi_{n,0,l}\sigma_{+}$	$\psi_{n,0,l}\sigma_{-}$
	± 1	$\frac{1}{\sqrt{2}}(\psi_{n,-1,l}\sigma_{\uparrow\uparrow} - \psi_{n,1,l}\sigma_{\downarrow\downarrow})$	$\frac{1}{\sqrt{2}}(\psi_{n,-1,l}\sigma_{\uparrow\uparrow} + \psi_{n,1,l}\sigma_{\downarrow\downarrow})$
$1, -1$	0	$\psi_{n,0,l}\sigma_{\uparrow\uparrow}, \psi_{n,0,l}\sigma_{\downarrow\downarrow}$	$\psi_{n,0,l}\sigma_{\uparrow\uparrow}, -\psi_{n,0,l}\sigma_{\downarrow\downarrow}$
	± 1	$\psi_{n,1,l}\sigma_{\pm}, \mp\psi_{n,-1,l}\sigma_{\pm}$	$\psi_{n,1,l}\sigma_{\pm}, \pm\psi_{n,-1,l}\sigma_{\pm}$
	± 2	$\psi_{n,2,l}\sigma_{\downarrow\downarrow}, \psi_{n,-2,l}\sigma_{\uparrow\uparrow}$	$\psi_{n,2,l}\sigma_{\downarrow\downarrow}, -\psi_{n,-2,l}\sigma_{\uparrow\uparrow}$

TABLE II. Basis states $\psi_{nml}\sigma_{s\bar{s}}$ categorized by the charge conjugation C according to Eq. (18).

$m + l$	$\mathsf{C} = 1$	$\mathsf{C} = -1$
even	$\psi_{n,m,l}\sigma_{-}$	$\psi_{n,m,l}\sigma_{+}, \psi_{n,m,l}\sigma_{\uparrow\uparrow}, \psi_{n,m,l}\sigma_{\downarrow\downarrow}$
odd	$\psi_{n,m,l}\sigma_{+}, \psi_{n,m,l}\sigma_{\uparrow\uparrow}, \psi_{n,m,l}\sigma_{\downarrow\downarrow}$	$\psi_{n,m,l}\sigma_{-}$

J/ψ Decay Constant

$$f_{\mathcal{V}}|_{m_j=0} = \sqrt{2N_c}\int_0^1 \frac{\mathrm{d}x}{\sqrt{x(1-x)}}\int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \psi_{+/\mathcal{V}}^{(m_j=0)}(\vec{k}_\perp, x)\ ,$$

$$f_{\mathcal{V}}|_{m_j=1} = \frac{\sqrt{N_c}}{2m_{\mathcal{V}}}\int_0^1 \frac{\mathrm{d}x}{[x(1-x)]^{3/2}}\int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \Big\{k^L[(1-2x)\psi_{+/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x) - \psi_{-/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x)] - \sqrt{2}m_f\psi_{\uparrow\uparrow/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x)\Big\}\ ,$$