RUHR-UNIVERSITÄT BOCHUM
INSTITUT FÜR THEORETISCHE PHYSIK II

Light-cone distribution amplitudes of the nucleon

June-Young Kim
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with M.V. Polyakov
H.-Ch. Kim.

Light-cone wave function (LCWF)

- Nucleon wave function in Fock basis

$$|P\rangle = \psi_{3q}|qqq\rangle + \psi_{3q+1g}|qqqg\rangle + \psi_{3q+q\bar{q}}|qqqq\bar{q}\rangle + \ldots$$

- Partial wave decomposition

$$|P,1/2\rangle^{3q} = |P,1/2\rangle^{3q}_{l_z=0} + |P,1/2\rangle^{3q}_{l_z=-1} + |P,1/2\rangle^{3q}_{l_z=1} + |P,1/2\rangle^{3q}_{l_z=-2}$$

- Leading twist DAs of the nucleon φ_N are related to the LCWF with $l_z = 0$

$$\varphi_N \sim |P,1/2\rangle^{3q}_{l_z=0}$$

- Convolution of the hard kernel of the process with the involved DAs

$$G_M \propto \varphi_N \otimes T_H \otimes \varphi_N$$

✓ /V. L. Chernyak et al (1977)/ S.Brodsky et al. (1979)/ A.V.Efremov et al. (1980)/ G.Lepage et al. (1980)/

✓ /X.d.Ji et al (2003) /X.d.Ji et al (2004)

✓ /V. Braun et al. (2000)/

✓ /V. L. Chernyak et al (1977)/ S.Brodsky et al. (1979)/ A.V.Efremov et al. (1980)/ G.Lepage et al. (1980)/

Distribution amplitudes (DAs)

- Formal definitions of the nucleon and Δ baryon DAs in position space

✓ /V. L. Chernyak et al (1977)/ G. R. Farrar et al (1989)

$$\langle 0|\epsilon^{ijk}u^i(a_1 n)u^j(a_2 n)d^k(a_3 n)|P(p_N,1/2)\rangle \sim V_N, A_N, T_N(a_i n\cdot p_N)$$

$$\langle 0|\epsilon^{ijk}u^i(a_1 n)u^j(a_2 n)u^k(a_3 n)|\Delta^{++}(p_\Delta,\lambda)\rangle \sim V_\Delta, A_\Delta, T_\Delta, \varphi_\Delta^{3/2}(a_i n\cdot p_\Delta)$$

- Values of the matrix elements at origin in position space

$$V_{N,\Delta}(0) = T_{N,\Delta}(0) = f_{N,\Delta}^{1/2}, \quad \varphi_\Delta^{3/2}(0) = f_\Delta^{3/2}$$

- Permutation symmetry of the first two quarks and isospin symmetry of the baryon

$$T_{N,\Delta} \sim \varphi_{N,\Delta}(x_i) \qquad \varphi_{N,\Delta} = [V_{N,\Delta} - A_{N,\Delta}](x_i)$$

Light-cone distribution amplitudes

- Suppressed transverse momentum dependence
- S-wave contributions of the quarks
- Baryon wave functions in terms of the DAs

✓ /V. L. Chernyak et al (1977)/
G. R. Farrar et al (1989)

Nucleon $\lambda = 1/2$

$$|P, 1/2\rangle = \frac{f_N}{4\sqrt{6}} \int \left[\frac{dx}{\sqrt{x}}\right] \varphi_N(x_i) \left[|u^{\uparrow}u^{\downarrow}d^{\uparrow}\rangle - |u^{\uparrow}d^{\downarrow}u^{\uparrow}\rangle\right],$$

Δ baryon $\lambda = 1/2$

$$|\Delta^{++}, 1/2\rangle = -\frac{f_{\Delta}^{1/2}}{8\sqrt{6}} \int \left[\frac{dx}{\sqrt{x}}\right] \varphi_{\Delta}(x_i)|u^{\uparrow}u^{\downarrow}u^{\uparrow}\rangle$$

Δ baryon $\lambda = 3/2$

$$|\Delta^{++}, 3/2\rangle = -\frac{f_{\Delta}^{3/2}}{24\sqrt{3}} \int \left[\frac{dx}{\sqrt{x}}\right] \varphi_{\Delta}^{3/2}(x_i)|u^{\uparrow}u^{\uparrow}u^{\uparrow}\rangle$$

DAs in various models

✓ Two limits of DAs: non-relativistic (NR) limit and asymptotic (AS) limit /V. L. Chernyak et al. NPB246 (1984)/

$$\varphi_N^{NR}(x_i) \propto \delta\left(x_1 - \frac{1}{3}\right)\delta\left(x_2 - \frac{1}{3}\right)\delta\left(x_3 - \frac{1}{3}\right)$$

$$\varphi_N^{\mathrm{AS}}(x_i) = 120 x_1 x_2 x_3$$

$$G_M^n > 0,\ G_M^p < 0\ \mathrm{(NR)}$$

$$G_M^p / G_M^n \to 0\ \mathrm{(AS)}$$

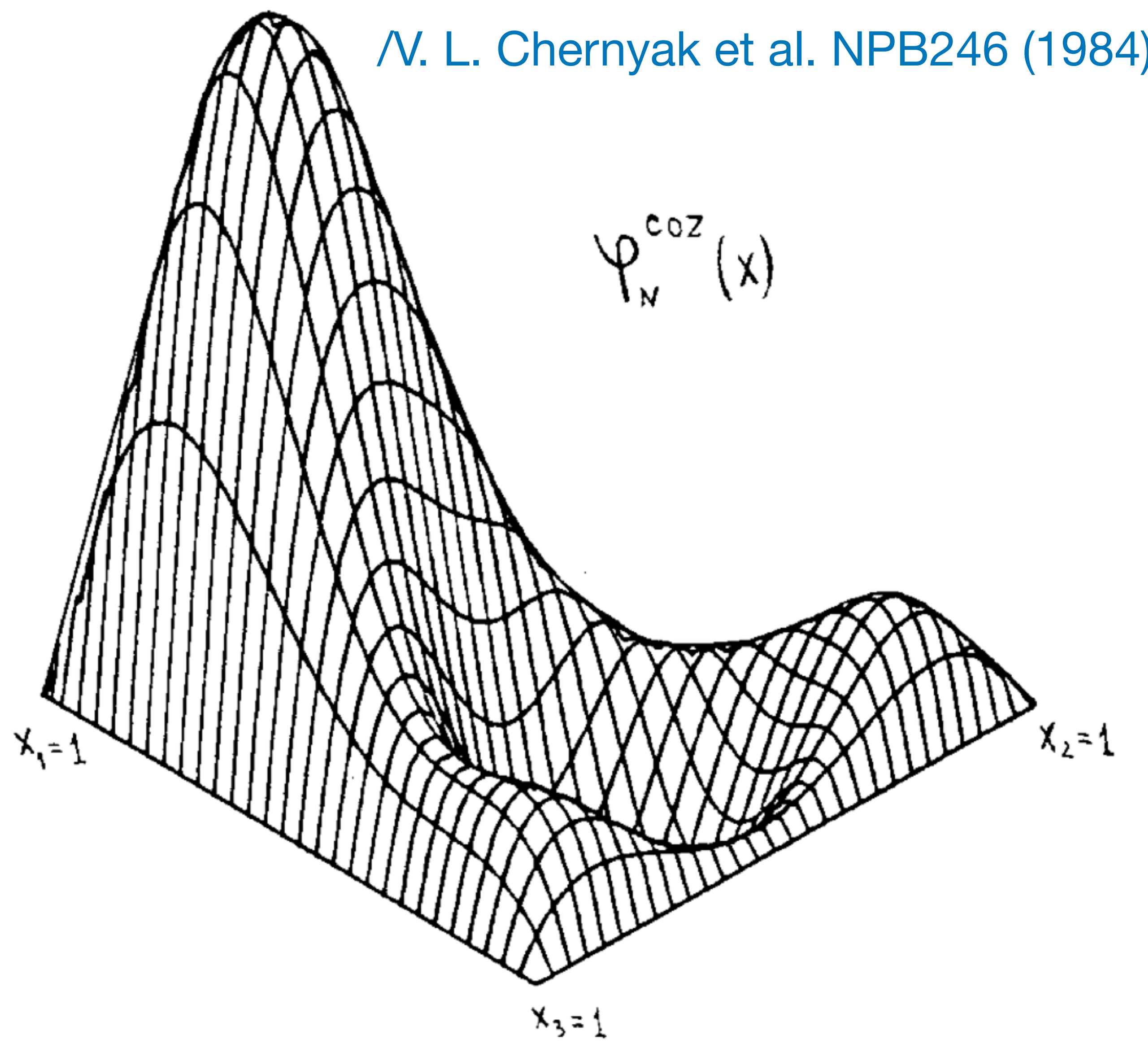
- DA is different from both limits.

✓ QCD sum rules /V. L. Chernyak et al. NPB246 (1984)/

- strong asymmetry ($x_1 \leftrightarrow x_2$)
- normalization constant f_N

$$f_N(\mu = 1\ \mathrm{GeV}) = (5.0 \pm 0.3) \times 10^{-3}\ \mathrm{GeV}^2$$

/V. L. Chernyak et al. NPB246 (1984)/



DAs in various models

✓ Lattice QCD /G. S. Bali et al. EPJA (2019)/

- weak asymmetry($x_1 \leftrightarrow x_2$) and almost symmetric
- normalization constant f_N

$$f_N^{N_f=2+1}(\mu = 2\ \mathrm{GeV}) = 3.54^{+6}_{-4} \times 10^{-3}\mathrm{GeV}^2$$

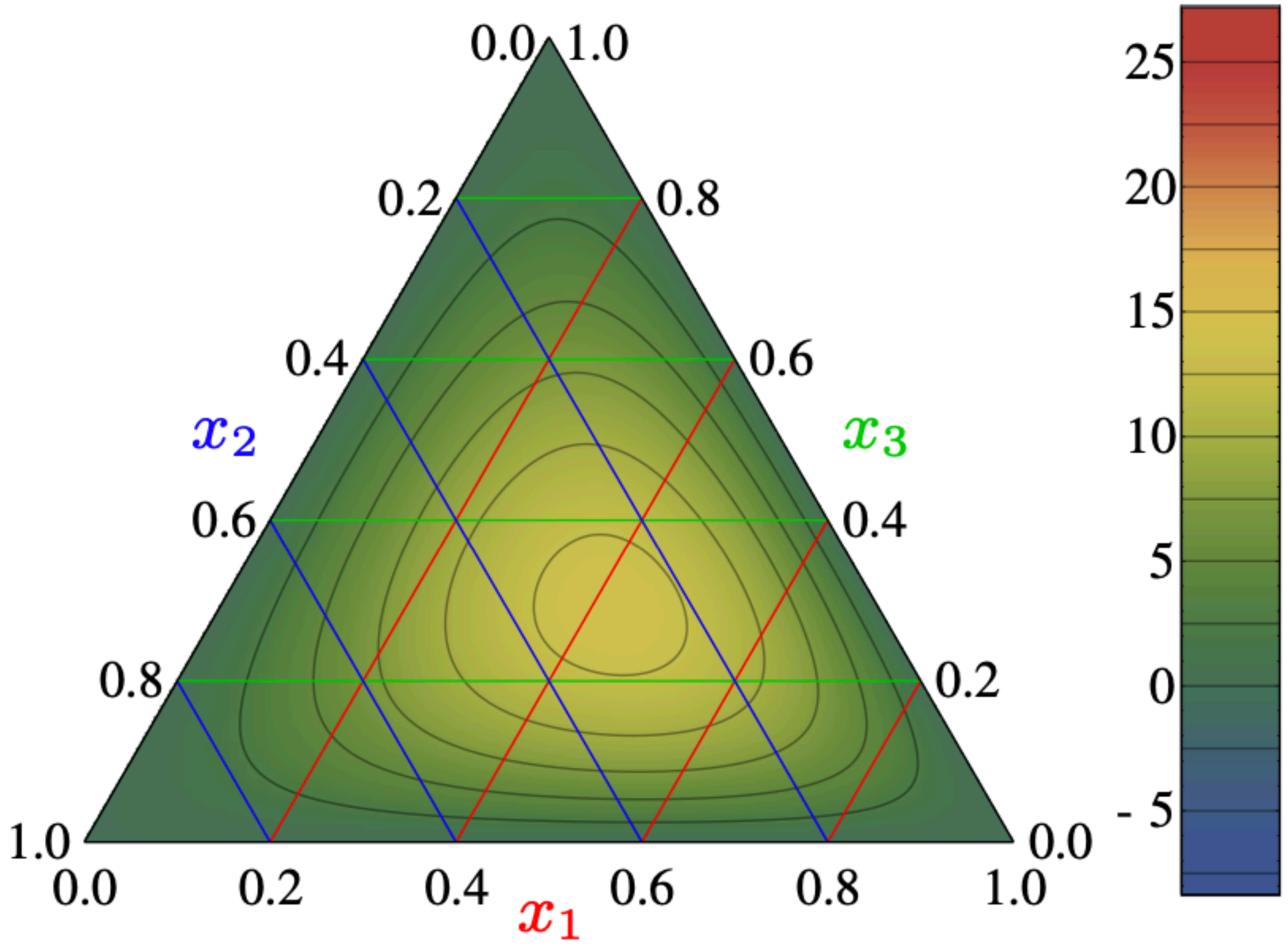
/G. S. Bali et al. EPJA (2019)/

$$f_N^{N_f=2}(\mu = 2\ \mathrm{GeV}) = 2.84^{+33}_{-33} \times 10^{-3}\mathrm{GeV}^2$$

/V.M. Braun. Et al. PRD89 (2014)/

- discrepancy between predictions from lattice QCD and QCD sum rules.

/V.M. Braun. Et al. PRD89(2014)/



✓ Models based on a dynamical Ansatz

✓ Z.Dziembowski PRD37 (1988),

✓ Z.Dziembowski, et al. PRD42 (1990),

✓ J.Bolz, et al. Z.Phys.A356 (1996),

✓ B.Pasquini, et al. PRD80 (2009).

Chiral quark-soliton model

✓ Effective chiral Lagrangian

$$\mathcal{L} = \overline{\psi}(x)(i\partial\!\!\!/ - MU^{\gamma^5})\psi(x)$$

✓ Saddle point equation of motion

$$\frac{\delta M_N[U]}{\delta U} = 0$$

✓ Relativistic invariance

- stationary mean field - nucleon at rest
- time-dependent mean field - moving nucleon

$$U(t, \boldsymbol{x}) = \bar{U}\left(\frac{\boldsymbol{x} - \boldsymbol{v}t}{\sqrt{1 - v^2}}\right)$$

✓ E.Witten NPB160 (1979)

✓ E.Witten NPB223 (1984)

✓ D.I.Diakonov, V.Y.Petrov, P.V.Pobylitsa. NPB306 (1988)

✓ C.V. Christov et al, PPNP37 (1996)

✓ D.I. Diakonov *et al. Nucl.Phys.B* 480 (1996)

✓ D.I. Diakonov et al. PRD56 (1997)

Light-cone Chiral quark-soliton model

✓ Baryon wave function

$$|B\rangle = \prod_{\text{color}}^{N_c} \int \frac{d^3k}{(2\pi)^3} F(\boldsymbol{k}) a^\dagger(\boldsymbol{k}) |\Omega\rangle$$

✓ Vacuum wave function

- quark-antiquark pair wave function W(p, p’)
- Generate higher Fock states 5Q, 7Q, 9Q, …

$$|\Omega\rangle = \exp\left(\int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} a^\dagger(\boldsymbol{p}) W(\boldsymbol{p}, \boldsymbol{p'}) b^\dagger(\boldsymbol{p'}) \right) |0\rangle$$

✓ 3Q wave function in IMF

- translational and rotational zero modes
- normalization constant c_0

$$|B_\lambda\rangle^{3q} = c_0 \int [dk_\perp] \int \left[\frac{dx}{\sqrt{x}} \right] \int dR B^*_\lambda(R) \epsilon^{\alpha_1\alpha_2\alpha_3} \prod_{n=1}^{N} R^{f_n}_{j_n} F^{j_n\sigma_n}(x_n, \boldsymbol{k}^n_\perp) a^\dagger_{\alpha_n f_n \sigma_n}(x_n, \boldsymbol{k}^n_\perp) |0\rangle.$$

✓ V.Y.Petrov, M.V. Polyakov [hep-ph/0307077] (2002),

✓ D.I.Diakonov, V.Y.Petrov Annalen Phys 13 (2004),

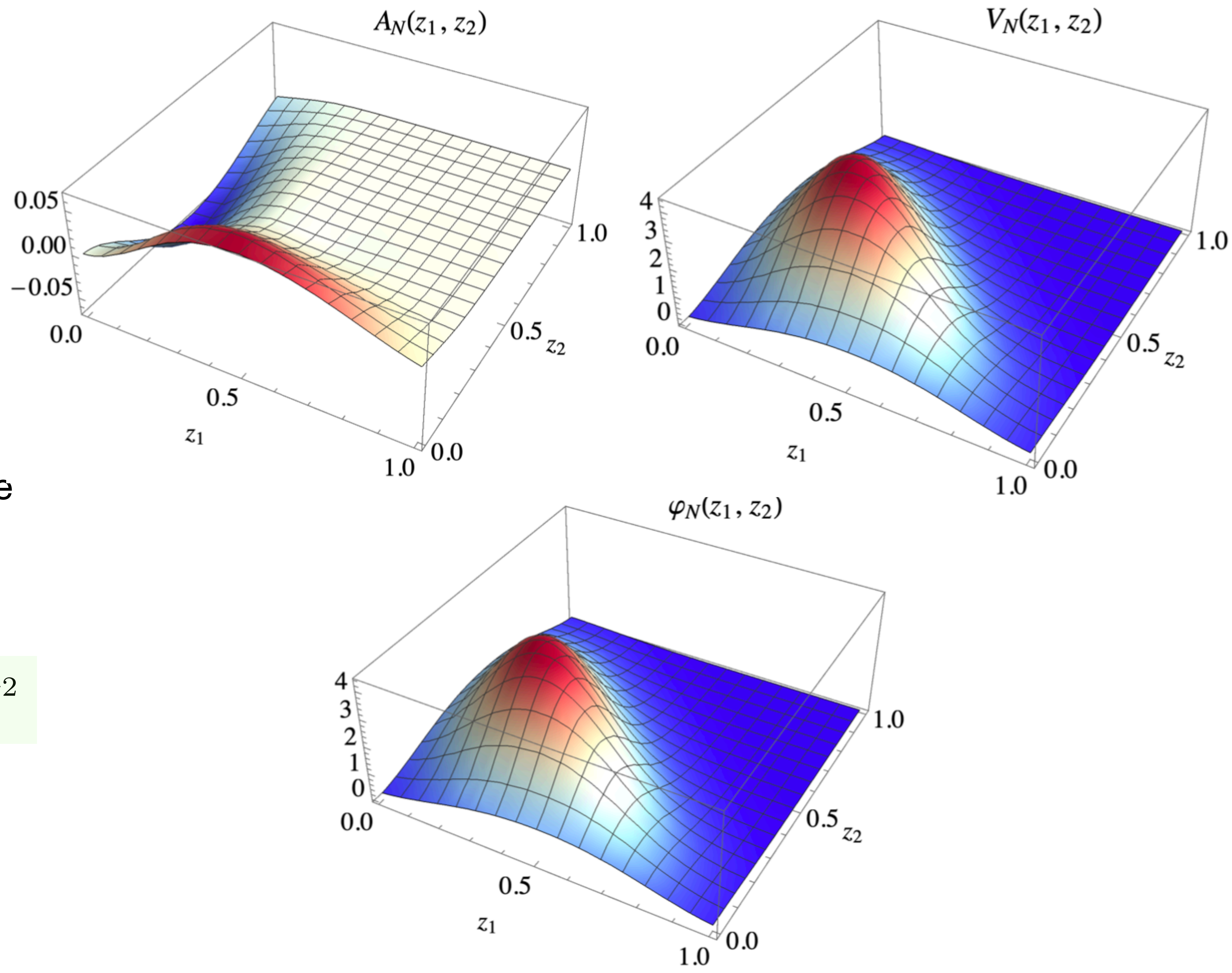
✓ D.I.Diakonov, V.Y.Petrov PRD72 (2005)

✓ Leading twist nucleon DAs

- almost symmetric
- a rather small asymmetry
- non-zero values at the end-points
- Nucleon LCWF is normalized to the 3Q state

$$f_N(\mu \approx 0.6\ \mathrm{GeV}) = 4.7 \times 10^{-3}\ \mathrm{GeV}^2$$

✓ /J.-Y. Kim, H.-Ch.Kim, M.V. Polyakov, JHEP (2021)/



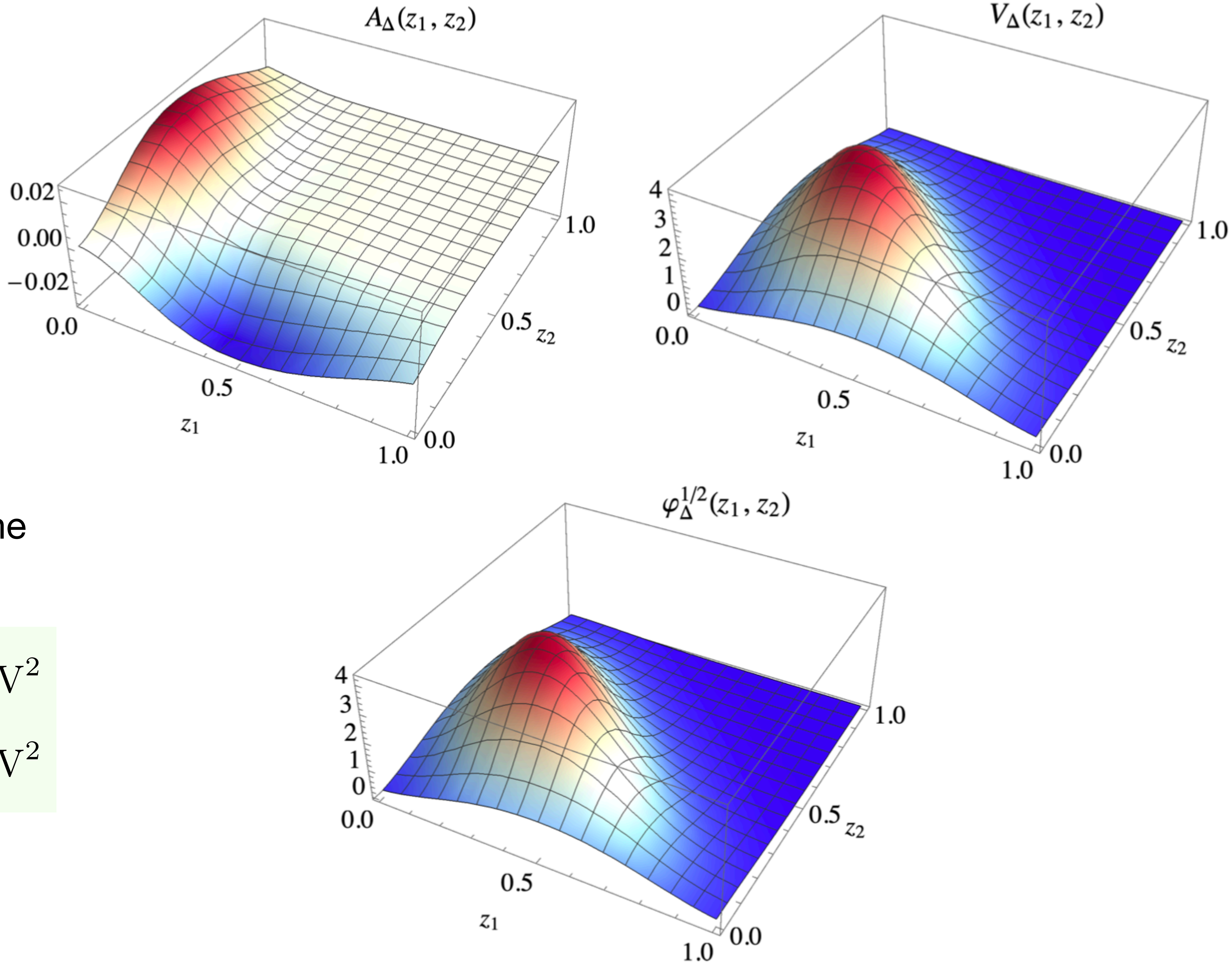
✓ Leading twist Δ baryon DAs

- Similar to the nucleon DAs
- QCD sum rules also predict almost symmetric DAs of the Δ baryon
- totally symmetric $\varphi_\Delta^{3/2}$
- Δ baryon LCWF is normalized to the 3Q state

$$f_\Delta^{1/2}(\mu \approx 0.6\text{GeV}) = 14.6 \times 10^{-3}\ \text{GeV}^2$$

$$f_\Delta^{3/2}(\mu \approx 0.6\text{GeV}) = 15.9 \times 10^{-3}\ \text{GeV}^2$$

✓ /J.-Y. Kim, H.-Ch.Kim, M.V. Polyakov, JHEP (2021)/



Beyond 3Q states

✓ Normalization c_0 of the LCWF

- Valence-quark dominance?

$$\langle P|P'\rangle \overset{?}{\approx}{}^{3q} \langle P|P'\rangle^{3q}$$

- 5Q component is substantial

$$\langle P|P'\rangle ={}^{3q} \langle P|P'\rangle^{3q} +{}^{5q} \langle P|P'\rangle^{5q}\ldots$$

$$g_A^{3q} = 1.67, \quad g_A^{5q} = 1.36, \quad g_A^{\mathrm{ex}} = 1.27$$

- Its impact on $f_{N,\Delta}$ is also important.

✓ D.I.Diakonov, V.Y.Petrov Annalen Phys 13 (2004)

✓ D.I.Diakonov, V.Y.Petrov PRD72 (2005)

✓ C.Lorce Phys.Rev.D 79 (2009)

Lattice QCD

$$f_N^{N_f=2+1}(\mu = 2\ \mathrm{GeV}) = 3.54^{+6}_{-4} \times 10^{-3}\mathrm{GeV}^2$$

/G. S. Bali et al. EPJA

$$f_N^{N_f=2}(\mu = 2\ \mathrm{GeV}) = 2.84^{+33}_{-33} \times 10^{-3}\mathrm{GeV}^2$$

/V.M. Braun. Et al. PRD89

✓ Normalizations of the DAs to the 5Q states ($\mu = 2$ GeV)

$$|f_N| = 3.9 \times 10^{-3}\ \mathrm{GeV}^2, \quad |f_\Delta^{1/2}| = 11.7 \times 10^{-3}\ \mathrm{GeV}^2, \quad |f_\Delta^{3/2}| = 12.3 \times 10^{-3}\ \mathrm{GeV}^2.$$

✓ /J.-Y. Kim, H.-Ch.Kim, M.V. Polyakov, JHEP (2021)/

Conclusions

- We obtained the nucleon and Δ baryon distribution amplitudes in the chiral quark-soliton model motivated by the QCD instanton vacuum.
- The light cone wave function of a baryon is expressed in terms of the wave function of the discrete level in the presence of the pion mean field and the quark-antiquark pair wave functions.
- We found that the results for the nucleon and Δ baryon DAs are almost symmetric, and they have rather small asymmetries.
- The normalization constant f_N is comparable to that from the lattice QCD.

Outlook

- It would be interesting to compare the normalization constants $f_\Delta^{1/2}$, $f_\Delta^{3/2}$ for the Δ baryon with those from lattice QCD.
- Higher twist-4,5,6 distribution amplitudes are under investigation.

Thank you very much!