Axial-vector meson $a_1(1260)$ as a quasi-bound state of the $K\bar{K}^*$

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Outlines

Motivation

- In principle, the axial-vector meson resonance can be generated dynamically by pseudoscalar and vector meson interaction.

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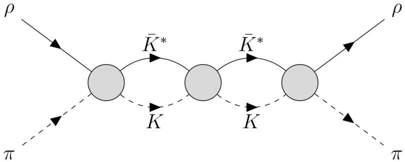

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- The full off-shell T matrix of this interaction can then be applied to other important processes.

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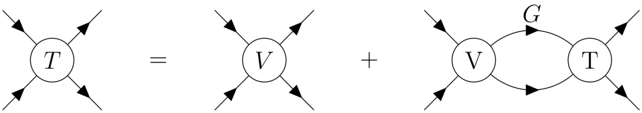
Formalism

Coupled Channel Formalism

The Bethe-Salpeter equation for two-body interaction expressed as



- The two-body propagator is not unique. We use the propagator in Blankenbecler-Sugar scheme[2][3].
  - Preserves unitarity
  - Reduces the dimensionality of the integral

[2] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966)

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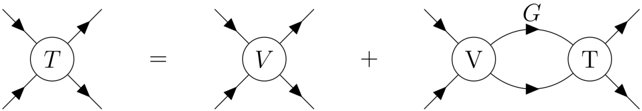

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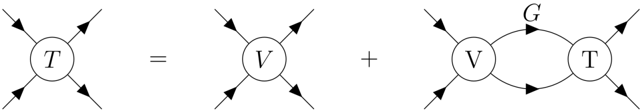

- The two-body propagator is not unique. We use the propagator in Blankenbecler-Sugar scheme[2][3].
- The three-dimensional integral equation can be reduced to one-dimensional problem by performing partial wave decomposition.
- After solving the integral equation, we transform the T-matrix to particle (LSJ) basis.

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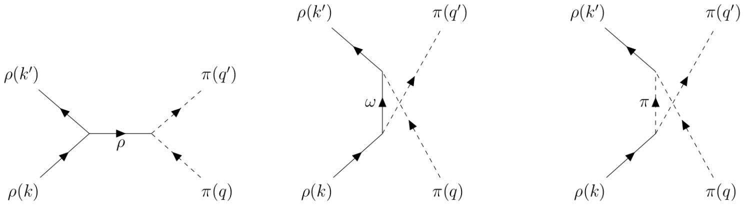
Feynman Diagrams

$\pi\rho \to \pi\rho$:



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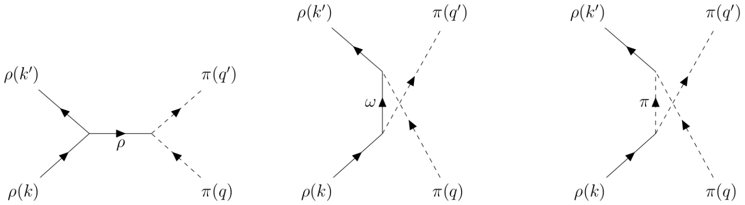
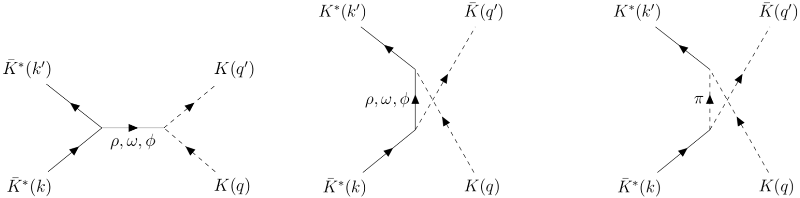


!! The a_1 pole diagram is not included explicitly.

Feynman Diagrams

$K\bar{K}^* \to K\bar{K}^*(\bar{K}K^*)$:

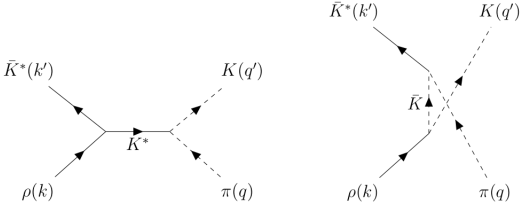


We define the $G = \pm 1$ parity state for $K\bar{K}^*$ state as

$$|K\bar{K}^*(-)\rangle = \frac{1}{\sqrt{2}}\left(|K\bar{K}^*\rangle - |\bar{K}K^*\rangle\right), \quad |K\bar{K}^*(+)\rangle = \frac{1}{\sqrt{2}}\left(|K\bar{K}^*\rangle + |\bar{K}K^*\rangle\right)$$

Feynman Diagrams

$\pi\rho \to K\bar{K}^*$



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Lagrangian

We use the SU(3) symmetric Lagrangian given by

$$\mathcal{L}_{PPV} = g\,\mathrm{Tr}\left([P, \partial_\mu P]_-\, V^\mu\right)$$
$$\mathcal{L}_{VVV} = -\frac{1}{2} g\,\mathrm{Tr}\left[(\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu\right]$$
$$\mathcal{L}_{PVV} = \frac{g}{m_V}\,\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(\partial_\mu V_\nu \partial_\alpha V_\beta P\right)$$

with

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

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with

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0_\mu + \frac{1}{\sqrt{2}}\omega_\mu & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & -\frac{1}{\sqrt{2}}\rho^0_\mu + \frac{1}{\sqrt{2}}\omega_\mu & K^{*0}_\mu \\ K^{*-}_\mu & \bar{K}^{*0}_\mu & \phi_\mu \end{pmatrix}$$

Note that the trace operation is only for SU(3) matrices. By using this Lagrangian, we can calculate the potentials and SU(3) symmetric and isospin factors for each diagrams.

Form factor

Since hadron has a finite size, we need to introduce form factor in each vertex in the diagrams.

$$F(n,k,k') = \left(\frac{n\Lambda^2 - m^2}{n\Lambda^2 + k^2 + k'^2}\right)^n$$

The cut-off mass Λ is determined by adding $500 - 700$ MeV to the exchange mass. However, for the reaction involving strangeness we apply higher cut-off mass value[4] especially for ϕ exchange the cut-off mass is 1700 MeV higher than its mass.

[4] D. Lohse, J. W. Durso, K. Holinde and J. Speth, Nucl. Phys. A **516** (1990)

Results and Discussion

General parameter

Graph	I	SIF	Λ[MeV]
π ρ ; π ; ρ π	0	-8	800
	1	4	
π ρ ; ω ; ρ π	0	4	1300
	1	-4	
ρ π ; ρ ; ρ π	0	-8	1300
	1	-4	
K $\bar{K}^*$; $\bar{K}$; ρ π	0	$\sqrt{6}\,(-\sqrt{6})$	1450
	1	$-2(2)$	
$\bar{K}^*$ K ; K^* ; ρ π	0	$\sqrt{6}\,(-\sqrt{6})$	1950
	1	$2(-2)$	

Graph	I	SIF	Λ[MeV]
$\bar{K}^*(\bar{K})$ $K(K^*)$; ρ ; $\bar{K}^*$ K	0	$-3(-3)$	1800
	1	$1(-1)$	
$\bar{K}^*(\bar{K})$ $K(K^*)$; ω ; $\bar{K}^*$ K	0	$-1(-1)$	1800
	1	$-1(1)$	
$\bar{K}^*(\bar{K})$ $K(K^*)$; ϕ ; $\bar{K}^*$ K	0	$-2(-2)$	2700
	1	$-2(2)$	
$\bar{K}$ K^* ; η ; $\bar{K}^*$ K	0	3	1600
	1	-3	

* Here we use the general coupling[5]

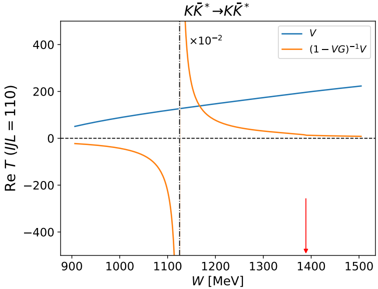
$$g^2/4\pi = 0.71$$

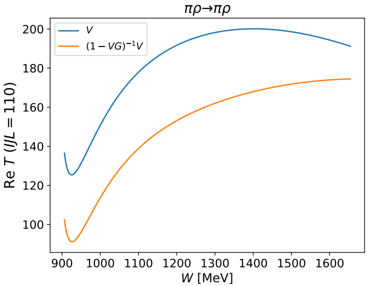
* For the ϕ-exchange, it differ by 16%.

[5] G. Janssen, K. Holinde and J. Speth, Phys. Rev. C **49**, 2763 (1994).

a_1 resonance

The singularity arises as a result of integral equation in the region below $K\bar{K}^*$ threshold.





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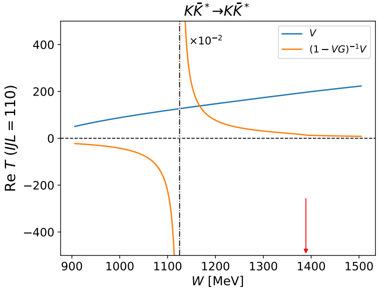
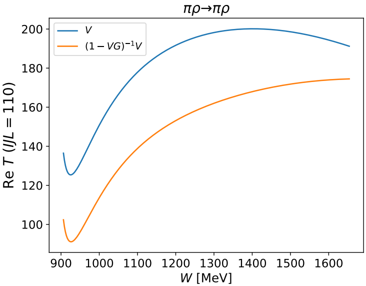
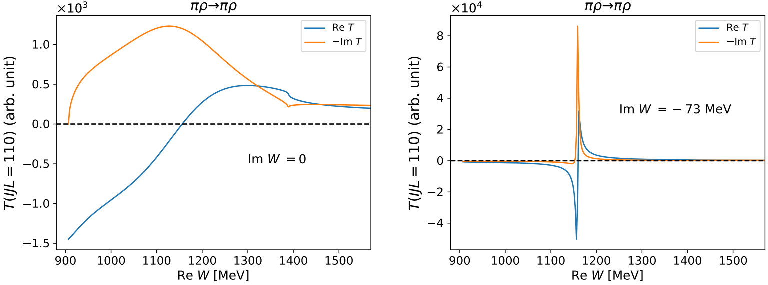




We can interpret a_1 (1260) resonance as molecular state of $K\bar{K}^*$.

a_1 resonance

After both channel is coupled, we can obtain the resonance structure in the $\pi\rho$ channel.



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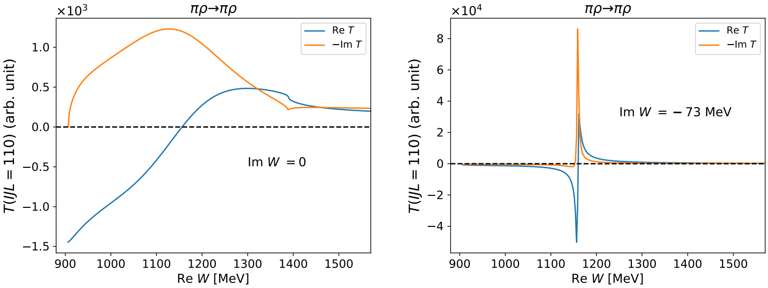
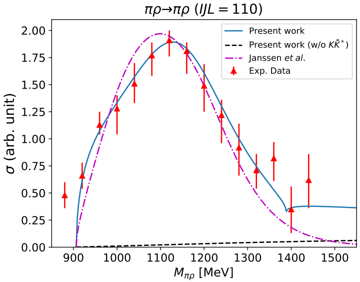


We obtained the pole position of a_1 resonance is $W_p = 1159 - i73$ MeV.

a_1 resonance

We compare the model to experimental data from charge exchange reaction ($\pi p \to 3\pi n$)[6]. Here we assume that the t dependence is small thus it will not affect the shape of the mass spectrum.



where we define[7]

$$\sigma \equiv \sigma_{\pi\rho}\left(t = m_\rho^2, M_{\pi\rho}\right) = -2\,\mathrm{Im}\left[T_{\pi\rho}(M_{\pi\rho})\right]$$

[6] J. A. Dankowych, *et al.* Phys. Rev. Lett. **46**, 580 (1981).

[7] G. Janssen, K. Holinde and J. Speth, Phys. Rev. C **49**, 2763 (1994).

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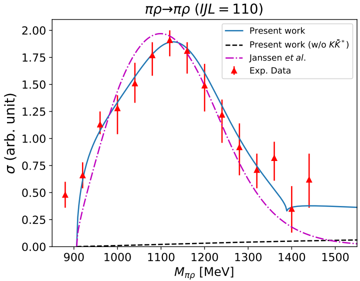
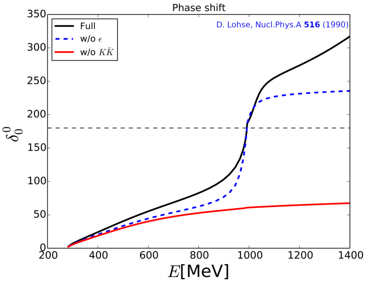




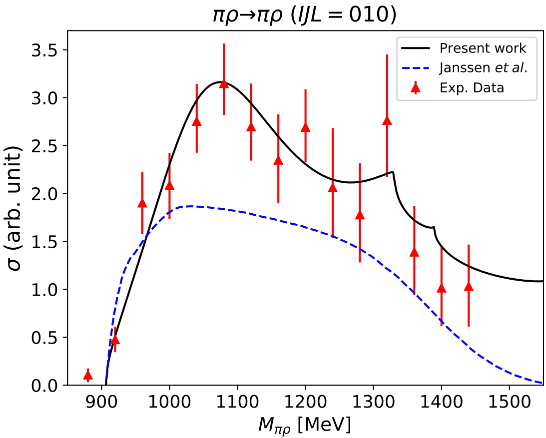
We found the similar phenomena as in the pseudoscalar meson interaction[7].

[6] J. A. Dankowych, *et al.* Phys. Rev. Lett. **46**, 580 (1981).
[7] D. Lohse, *et al.* Nucl. Phys. A **516** (1990).

Summary and Conclusion

- We investigated the axial-vector meson a_1 (1260) resonance from $\pi\rho$ scattering based on the fully off-mass-shell coupled channel formalism.
- The $a_1(1260)$ resonance can be theoretically interpreted as the $K\bar{K}^*$ molecular state.
- We also present the comparison of the model calculation to the experimental data from charge exchange reaction and it reproduce the data very well. From that, we extracted the pole position of $a_1(1260)$, $W_p = 1159 - i73$.
- The present result may be applied to the description of other process.
- For our next project, we will extend this work to study all the low-lying axial-vector meson resonances.

$h_1(1170)$ resonance



Experimental data from charge exchange reaction ($\pi p \to 3\pi n$)[8].
The other theoretical model includes $h_1(1170)$ resonance explicitly[9].

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[8] J. A. Dankowych, *et al.* Phys. Rev. Lett. **46**, 580 (1981).
[9] G. Janssen, K. Holinde and J. Speth, Phys. Rev. C **54**, 2218 (1996).

Thank You

Backup Slide

Blanckenbecler-Sugar scheme

The Bethe-Salpeter equation for two-body interaction express as

$$T(p,p';s) = V(p,p';s) + \frac{1}{(2\pi)^4}\int d^4q V(p,q;s)G(q;s)T(q,p';s)$$

The two-meson propagator is given by[10,11].

$$G(q;s) = \delta\left(q_0 - \frac{1}{2}E_1 + \frac{1}{2}E_2\right)\frac{\pi}{E_1E_2}\frac{E_1+E_2}{s-(E_1+E_2)^2}$$

with $E_i = (\mathrm{q}^2 + m_i)^{1/2}$ and we define $E = E_1 + E_2$. The BS equation is then

$$T(p,p';s) = V(p,p';s) + \frac{1}{(2\pi)^3}\int \frac{d^3q}{2E_1(q)E_2(q)}V(p,q;s)\frac{E(q)}{s-E^2(q)}T(q,p';s)$$

[10] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966)

[11] R. Aaron, R. D. Amado and J. E. Young, Phys. Rev. **174**, 2022 (1968).

Partial wave decomposition

Through the partial wave decomposition of potential V and T matrix, the BS equation becomes

$$T^{fi}_{\lambda'\lambda}(p,p') = V^{fi}_{\lambda'\lambda}(p,p') + \frac{1}{(2\pi)^3}\sum_{g}\sum_{\lambda_g}\int \frac{q^2 dq}{2E_1(q)E_2(q)} \times V^{fg}_{\lambda'\lambda_g}(p,q)\frac{E(q)}{s-E^2(q)}T^{gi}_{\lambda_g\lambda}(q,p'),$$

where

$$\mathcal{V}^{fi}_{\lambda'\lambda}(p',p) = 2\pi\int \mathrm{d}(\cos\theta) d^{J}_{\lambda'\lambda}(\theta)\mathcal{V}^{fi}_{\lambda'\lambda}(p',p,\theta),$$

λ', λ and λ_g denote the helicity of final (f), initial (i) and intermediate (g) state, respectively. Note that the total energy argument and the total angular momentum J are not written explicitly in the potential V and T matrix.

The one-dimensional integral equation can be expressed as

$$T_{\alpha\beta}(q_i^\alpha, q_l^\beta) = V_{\alpha\beta}(q_i^\alpha, q_l^\beta) + \sum_\gamma \sum_j^{n+1} V_{\alpha\gamma}(q_i^\alpha, q_j^\gamma)\, \tilde{G}_j^\gamma\; T_{\gamma\beta}(q_j^\gamma, q_l^\beta).$$

where α, β and γ denote the final, initial and transition states (represent two meson state and helicity state). The weight $\tilde{G}_j^\gamma$ is given by

$$\tilde{G}_j^\gamma = \frac{1}{(2\pi)^3} \frac{E(q_j^\gamma)}{2E_1(q_j^\gamma)E_2(q_j^\gamma)} \frac{(q_j^\gamma)^2}{s - E^2(q_j^\gamma)} \omega_j, \qquad \text{for } j = 1, 2, \cdots, n$$

$$\tilde{G}_{n+1}^\gamma = -\frac{1}{(2\pi)^3} \sum_{r=1}^{n} \frac{q_r^\gamma E(q_r^\gamma)}{2E_1(q_r^\gamma)E_2(q_r^\gamma)} \frac{q_{n+1}^\gamma}{s - E^2(q_r^\gamma)} \omega_r + \frac{1}{(2\pi)^3} \frac{q_{n+1}^\gamma}{4\sqrt{s}} \left\{ \ln \left| \frac{\sqrt{s} - m_1^\gamma - m_2^\gamma}{\sqrt{s} + m_1^\gamma + m_2^\gamma} \right| - i\pi \right\},$$

where $E^2(q_{n+1}^\gamma) = s$ and m^γ denotes the mass of particle in the two meson channel.

Matrix inversion method

We build matrix V with dimension enough to contain two meson channel, helicity and momentum points. Therefore, the T matrix can be calculated by

$$T = \left(1 - V\tilde{G}\right)^{-1} V.$$

The T matrix for this study in two meson channel basis can be expressed as

$$T_{I=0} = \begin{pmatrix} T^{\pi\rho,\pi\rho} & T^{\pi\rho,K\bar{K}^*} & T^{\pi\rho,\eta\omega} & T^{\pi\rho,\eta\phi} \\ T^{K\bar{K}^*,\pi\rho} & T^{K\bar{K}^*,K\bar{K}^*} & T^{K\bar{K}^*,\eta\omega} & T^{K\bar{K}^*,\eta\phi} \\ T^{\eta\omega,\pi\rho} & T^{\eta\omega,K\bar{K}^*} & T^{\eta\omega,\eta\omega} & T^{\eta\omega,\eta\phi} \\ T^{\eta\phi,\pi\rho} & T^{\eta\phi,K\bar{K}^*} & T^{\eta\phi,\eta\omega} & V^{\eta\phi,\eta\phi} \end{pmatrix},$$

$$T_{I=1} = \begin{pmatrix} T^{\pi\rho,\pi\rho} & T^{\pi\rho,K\bar{K}^*} \\ T^{K\bar{K}^*,\pi\rho} & T^{K\bar{K}^*,K\bar{K}^*} \end{pmatrix}.$$

LSJ basis

The T matrix can be expressed in LSJ basis as

$$T^J_{L',L} = \langle JML'S|T|JMLS\rangle = \sum_{\lambda_1',\lambda_2',\lambda_1,\lambda_2} \langle JML'S|JM\lambda_1'\lambda_2'\rangle T^J_{\lambda',\lambda} \langle JM\lambda_1\lambda_2|JMLS\rangle.$$

In the pseudoscalar and vector meson interaction, it becomes

$$T^J_{J,J} = T^J_1 - T^J_2$$

$$T^J_{J-1,J-1} = \frac{1}{\sqrt{2J+1}}\left[JT^J_0 + (J+1)(T^J_1 + T^J_2) + \sqrt{2J(J+1)}(T^J_3 + T^J_4)\right]$$

$$T^J_{J+1,J-1} = \frac{1}{\sqrt{2J+1}}\left[-\sqrt{J(J+1)}\,T^J_0 + \sqrt{J(J+1)}(T^J_1 + T^J_2) + \sqrt{2}JT^J_3 - \sqrt{2}(J+1)T^J_4)\right]$$

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$$T^J_{J+1,J+1} = \frac{1}{\sqrt{2J+1}}\left[(J+1)T^J_0 + J(T^J_1 + T^J_2) - \sqrt{2J(J+1)}(T^J_3 + T^J_4)\right].$$

where we define $T_0 = T_{0,0}$; $T_1 = T_{1,1}$; $T_2 = T_{1,-1} = T_{-1,1}$;
$T_3 = T_{1,0} = T_{-1,0}$; $T_4 = T_{0,1} = T_{0,-1}$