Pseudoscalar meson dominance and pion-nucleon coupling constant

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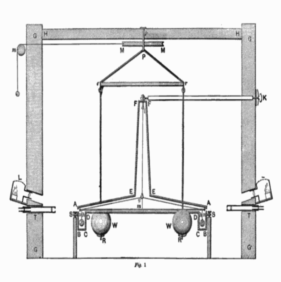
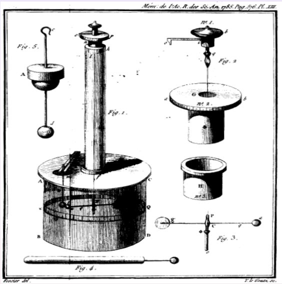
Motivation and Outline

The elementary constituents in Nuclear Physics are protons and neutrons and the interaction is mediated by pions. We want to use the concept of meson dominance to determine form factors and the strong pion-nucleon coupling constant with high precision

- Strong forces between nucleons: phenomenology
- Basic facts about meson dominance
- Sum rules and saturation
- Conclusions

Fundamental Forces





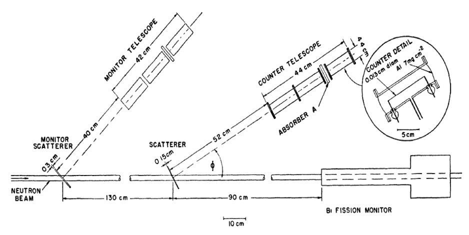
Cavendish (1798) “Experiments to determine the Density of the Earth”
Coulomb (1785) “Premier mémoire sur l’electricité et le magnétisme”

Strong Forces

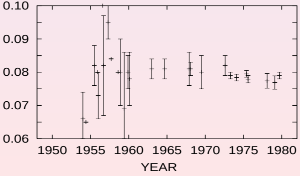
- Yukawa (1935)

$$V(r) = -f^2 \frac{e^{-mr}}{r}$$

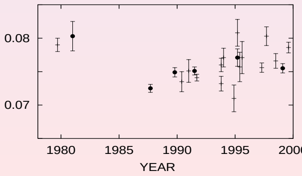
- Kemmer (1938) Isospin $\to \pi^0$
- Bethe (1940) $f^2 = 0.077 - 0.080$ (Deuteron)



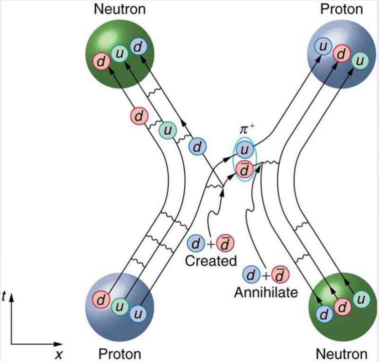
PION-NUCLEON COUPLING CONSTANT UNTIL 1980



PION-NUCLEON COUPLING CONSTANT AFTER 1980



Fundamental approach: QCD

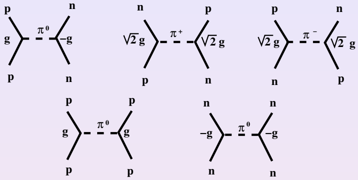


- Lattice form factor $g_{\pi NN} = 12.4(1.2)$
- Lattice NN potential $g_{\pi NN} = 12.3(1.3)$
- QCD sum rules $g_{\pi NN} \sim 13(1)$

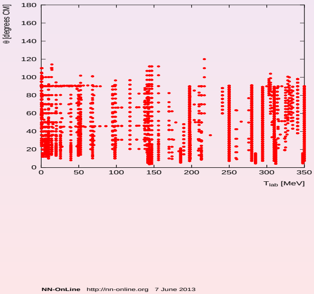
We have 10% accuracy

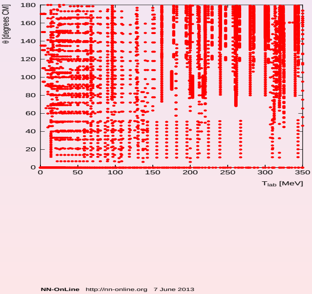
Phenomenological approach: Low energy scattering

- Nucleons exchange JUST one pion



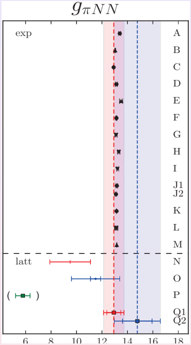
- Low energies (about pion production) 8000 pp + np scattering data (polarizations etc.)





Comparison

Bali et al. JHEP (2019)



id	ref.	description
A	[134]	πN scattering; PWA
B	[135–137]	np, pp scattering; PWA
C	[138, 139]	πN scattering; PWA
D	[140]	πN scattering; PWA; GMO
E	[141]	np backward cross section
F	[142]	πN scattering; PWA; DR
G	[143]	$\pi^- p$ and $\pi^- d$ pionic atoms; GMO
H	[144]	$\pi^- p$ and $\pi^- d$ pionic atoms; GMO
I	[131]	$\pi^- p$ and $\pi^- d$ pionic atoms; GMO
J	[145]	πN scattering; DR;
J1		CERN data
J2		TRIUMF data
K	[146]	πN scattering; PWA; DR
L	[147–149]	$\pi^- p$ and $\pi^- d$ pionic atoms; GMO; including third-order ChPT corrections
M	[150]	np, pp scattering; PWA
N	[32]	$N_f = 2+1$ DWF; RBC/UKQCD; $a = 0.114$ fm; dipole ansatz
O	[52]	$N_f = 2+1$ Wilson (clover) fermions; $a = 0.114$ fm; z-exp
P	[55]	$N_f = 2+1+1$ Wilson (clover-on-HISQ) fermions; PNDME; CE; EFT ansatz
Q		**This work**; $N_f = 2+1$ Wilson (clover) fermions; RQCD; full resolution of $N\pi$ state; CE
Q1		dipole ansatz
Q2		z-exp

Granada analysis is the *most precise* (accuracy of 0.4% !!)

$$g_{\pi NN} = 12.25(5) \qquad \chi^2/\mathrm{DOF} = (2999_{pp} + 3951_{np})/(6713_{\mathrm{Dat}} - 49_{\mathrm{Par}}) = 1.043$$

Basics

- In QCD we have the relations

$$\partial_\mu(\bar{q}_f\gamma^\mu q_i) = (m_f - m_i)\bar{q}_f q_i \qquad \text{CVC} \tag{1}$$

$$\partial_\mu(\bar{q}_f\gamma^\mu\gamma_5 q_i) = (m_f + m_i)\bar{q}_f i\gamma_5 q_i \qquad \text{PCAC} \tag{2}$$

- Pion weak decay $\pi^+ \to \mu^+\bar{\nu}_\mu$

$$\langle 0|\bar{u}\gamma^\mu\gamma_5 d|\pi^+(p)\rangle = if_{\pi^+}p^\mu \underbrace{\Longrightarrow}_{\text{PCAC}} (m_u+m_d)\langle 0|\bar{u}i\gamma_5 d|\pi^+(p)\rangle = f_{\pi^+}m^2_{\pi^+}$$

- Pion dominance

$$(m_u+m_d)\bar{u}(x)i\gamma_5 d(x) = f_{\pi^+}m^2_{\pi^+}\phi_+(x) + \mathcal{O}(\phi^3)$$

- Neutron beta decay $n \to p + \bar{\nu} + e^-$

$$\langle p|\bar{u}\gamma^\mu\gamma_5 d|n\rangle = \bar{u}_p\left[\gamma^\mu G_A(q^2) + G_P(q^2)\frac{iq^\mu}{2M}\right]\gamma_5 u_n \underbrace{\Longrightarrow}_{\text{PCAC}} (m_u+m_d)\langle p|\bar{u}i\gamma_5 d|n\rangle = F_P(q^2)\bar{u}_p i\gamma_5 u_n$$

- Pseudoscalar form factor

$$F_P(q^2) = 2MG_A(q^2) + \frac{q^2}{2M}G_P(q^2) \underbrace{\Longrightarrow}_{q^2\to 0} 2Mg_A = F_P(0)$$

Dispersion relations

The pseudoscalar form factor $F_P(q^2)$ satisfies useful analytical properties in the complex $t-$plane.

- $F_P(t)$ is real in the space-like region, $t < (3m_\pi)^2$,
- It has a pion pole at $t = m_\pi^2$,
- It has a branch cuts along the odd number of pions production thresholds, $t = (3m_\pi)^2, (5m_\pi)^2, \ldots,$
- Its value at the origin is $F_P(0) = 2Mg_A$.
- it falls off as $m_q(\Lambda_N^4 \alpha(Q^2)/Q^2)^2$ (Alvegard:1979ui,Lepage:1979za,Brodsky:1980sx) in the deep Euclidean region, $t = -Q^2 \to -\infty$.

All this implies the dispersion relation

$$F_P(t) = \frac{2f_\pi m_\pi^2 g_{\pi NN}}{m_\pi^2 - t} + \frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} F_P(s)}{s-t} \underbrace{\to}_{t\to-\infty} \frac{m_q \Lambda_N^4 \alpha_s(t)^2}{t^2} , \quad \Lambda_N \sim 300\mathrm{MeV}$$

In particular , for $t = 0$

$$2Mg_A = 2f_\pi g_{\pi NN} + \underbrace{\frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} F_P(s)}{s}}_{3\pi, 5\pi, \ldots}$$

For

$$M_p = 938.27231\mathrm{MeV} \,,\, M_n = 939.56563\mathrm{MeV} \,,\, g_A = 1.2723(23) \,,\, f_{\pi^+} = 92.28(9)\mathrm{MeV} \,,\, g_{\pi NN} = 12.25(5)$$

the Goldberger-Treiman discrepancy is *small*

$$\Delta_{\mathrm{GT}} = 1 - \frac{g_A M_N}{g_{\pi NN} f_\pi} = 2.3(4)\% \implies \frac{1}{2f_\pi g_{\pi NN}}\int_{(3m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} F_P(s)}{s} \ll 1$$

The Goldberger-Treiman relation

- Axial current is conserved if $f_\pi m_\pi^2 = 0$ (Goldstone alternative)
- $f_\pi = 0$ (Wigner mode) $\implies$ Normal multiplets
- $m_\pi = 0$ (Goldstone mode) $\implies$ Chiral symmetry + Pions are Goldstone bosons
- Quark level $m_\pi \to 0 \implies m_u, m_d \to 0$
- Gell-Mann–Oakes-Renner relation

$$f_\pi^2 m_\pi^2 = m_u \langle \bar{u} u \rangle + m_d \langle \bar{d} d \rangle + \mathcal{O}(m_u^2, m_d^2) \implies m_u, m_d = \mathcal{O}(m_\pi^2)$$

- For $m_\pi \to 0$ (the chiral limit) PCAC implies

$$2MG_A(q^2) + \frac{q^2}{2M} G_P(q^2) = 0 \underbrace{\implies}_{q^2 \to 0} G_P(q^2) \to -g_A \frac{4M^2}{q^2}$$

- Pion dominance implies the Goldberger-Treiman relation *unphysical* quantities, i.e.

$$\mathring{M}_N \mathring{g}_A = \mathring{f}_\pi \mathring{g}_{\pi NN} \qquad \mathring{A} = A|_{m_\pi \to 0}$$

The Goldberger-Treiman discrepancy

- Since 1958 till 1985 discrepancies were large $\Delta_{\mathrm{GT}} = 5 - 10\%$ (Wikipedia says 10%)

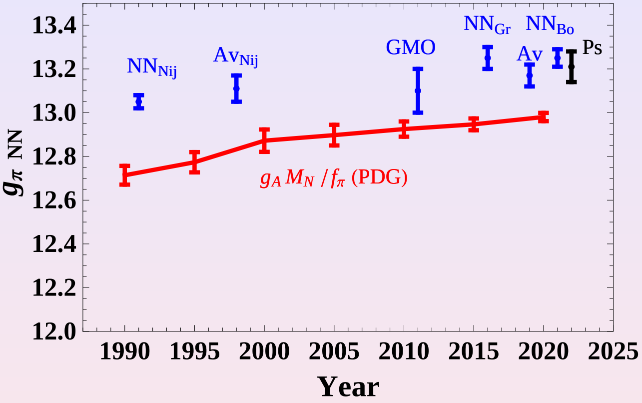


Figure: History of determinations of the pion-nucleon coupling constant compared to the historical PDG averages of the ratio $g_A M_N / f_\pi$. Here NN_{Gr} stands for the NN Granada determination. NN_{Bo} the chiral Bochum. The difference is percentage of both determinations corresponds to the GT discrepancy and is a direct measure of the chiral symmetry breaking in QCD.

- The discrepancy has been diminishing and one recent analysis (GOM) provides *almost vanishing* $\Delta_{\mathrm{GT}} = 1.4(1.2)\%$
- Granada NN yields $\Delta_{\mathrm{GT}} = 2.3(4)\%$ small but non-vanishing.

Non-positivity of the spectral function

- Define $\rho(s) \equiv \text{Im}\, \hat{m} F_P(s)$. The three QCD sum rules provide

$$
\begin{aligned}
2g_A M &= 2f_\pi g_{\pi NN} + \frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} ds \rho(s)/s, && (3)\\
0 &= 2f_\pi m_\pi^2 g_{\pi NN} + \frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} ds \rho(s), && (4)\\
0 &= 2f_\pi m_\pi^4 g_{\pi NN} + \frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} ds \rho(s) s\,. && (5)
\end{aligned}
$$

- *Assume* $\rho(s)$ with well-defined sign

$$\rho(s) = \text{sign}(\rho(s))\sqrt{|\rho(s)/s|}\sqrt{|\rho(s)s|}$$

- Owing to Schwartz's inequality we get ,

$$(2f_\pi m_\pi^2 g_{\pi NN})^2 = \left|\int ds \rho(s)\right|^2 \leq \int ds |\rho(s)| s \int ds |\rho(s)|/s \underbrace{=}_{\rho(s)>0} \int ds \rho(s) s \int ds \rho(s)/s$$

$$= 2f_\pi m_\pi^4 g_{\pi NN}(2f_\pi g_{\pi NN} - g_A M_N) \qquad \Longrightarrow 1 \leq \Delta_{\rm GT} \underbrace{=}_{\rm exp} 0.02(1)!!$$

- If $\rho(s) > 0$, we can intert the three sum rules and we get $1 \leq \Delta_{\rm GT}$, a *contradiction* with phenomenological estime of $\Delta_{\rm GT} = 0.02$
- There must exist at least one point where $\rho(s_0) = 0$ and $\rho'(s_0) \neq 0$

Asymptotic spectral function

- Asymptotic freedom from space-like region $q^2 = -Q^2 \to -\infty$ to the time-like region $q^2 = s \to +\infty$ by analytical continuation $s = -Q^2 \equiv \lim_{\theta\to\pi} e^{-i\theta} Q^2$

$$\alpha_S(-Q^2) = \frac{4\pi}{\beta_0}\frac{1}{\ln Q^2/\Lambda^2} \Longrightarrow \alpha_S(s) = \frac{4\pi}{\beta_0}\frac{1}{\ln s/\Lambda^2 - i\pi}, \qquad \begin{cases} \beta_0 = (11N_c - 2N_f)/3 = 9 \\ \Lambda \equiv \Lambda_{\rm QCD} \sim 0.240 \mathrm{GeV} \end{cases}$$

- Pseudoscalar form factor

$$F_P(s) \to \frac{m_q \Lambda_N^4}{s^2}\left[\frac{4\pi}{\beta_0}\frac{1}{\ln s/\Lambda^2 - i\pi}\right]^2 \Longrightarrow \mathrm{Im} F_P(s) \to \frac{m_q\Lambda_N^4}{s^2}\left(\frac{4\pi}{\beta_0}\right)^2 \frac{2\pi \ln s/\Lambda^2}{(\ln^2 s/\Lambda^2 + \pi^2)^2}$$

- Convergent small contributions to the sum rules for $\sqrt{s_H} = m_{\pi''} \sim 2.2 \mathrm{GeV} \sim 10\Lambda_{QCD}$

$$\begin{aligned}
\int_{s_H}^\infty \mathrm{Im} F_P(s)/s ds &\sim \frac{m_q\Lambda_N^4}{\Lambda_{QCD}^4} 2.4\times 10^{-6} \sim 6\times 10^{-8}\mathrm{GeV} \ll 2(g_A M_N - g_{\pi NN} f_\pi) \sim 0.1\mathrm{GeV} \\
\int_{s_H}^\infty \mathrm{Im} F_P(s) ds &\sim \frac{m_q\Lambda_N^4}{\Lambda_{QCD}^2} 4.2\times 10^{-4} \sim 6\times 10^{-7}\mathrm{GeV}^3 \ll 2g_{\pi NN} f_\pi m_\pi^2 \sim 0.05\mathrm{GeV}^3 \\
\int_{s_H}^\infty \mathrm{Im} F_P(s) s ds &\sim m_q\Lambda_N^4 2.0\times 10^{-1} \sim 1.6\times 10^{-5}\mathrm{GeV}^5 \ll 2g_{\pi NN} f_\pi m_\pi^4 \sim 10^{-3}\mathrm{GeV}^5
\end{aligned}$$

Chiral corrections

For $m_\pi \to 0$ we can use chiral perturbation theory (Kaiser+Passemar) *below* the resonance region

$$\mathrm{Im}\,\hat{m}F_P(t)|_{\mathrm{HBChPT}} = \frac{m_\pi^2 M g_A}{36864\pi^3 F_\pi^4}\left(\left(5+\frac{68\pi^2}{35}\right)g_A+1\right) \times \frac{\left(\sqrt{t}-3m_\pi\right)^2\left(8m_\pi+\sqrt{t}\right)^2}{(m_\pi^2-t)} \tag{6}$$

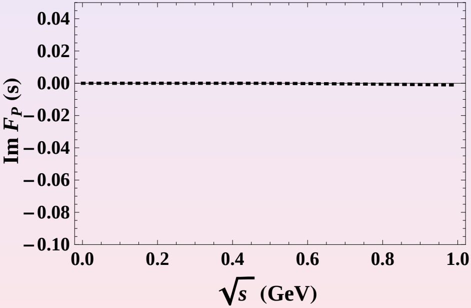
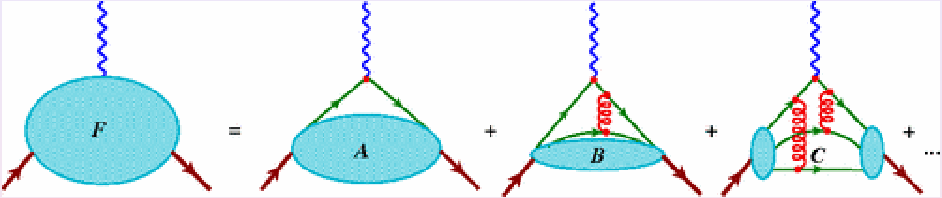


Figure: Chiral correction to the Goldberger-Treiman discrepancy according to chiral perturbation theory

pQCD corrections

Anatomy of the form factors (Braun et al.)



$$F_1 \sim A(Q^2) + \left(\frac{\alpha_S(Q^2)}{\pi}\right)\frac{B(Q^2)}{Q^2} + + \left(\frac{\alpha_S(Q^2)}{\pi}\right)^2 \frac{C}{Q^4}$$

where

$$A(Q^2) \leq 1/Q^6 \qquad B(Q^2) \leq 1/Q^4$$

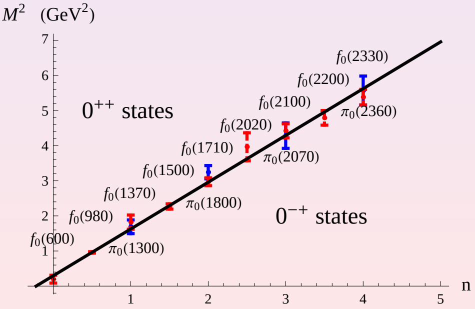
Pseudoscalar spectrum

- There are 5 pseudoscalar 0^{-+} isovector states appearing in the PDG.
- Only $\pi(140)$, $\pi(1300)$ and $\pi(1800)$ have been seen experimentally in many ocassions.
- The other two, $\pi(2070)$ and $\pi(2360)$, have only been extracted in $\bar{p}p$ anihilation and appear in "further states".
- These states are identified as radial n^1S_0 excitations

$$\pi(1300) \to \rho\pi \to 3\pi \qquad \pi(1800) \to \sigma\pi \to 3\pi$$

- Half-width rule allows for radial linear Regge trajectories

$$M_n^2 \pm \Gamma_n M_n = 1.27(27)n + M_1^2 \quad (\mathrm{GeV}^2) \qquad n \neq 0$$



Minimal hadronic model

- The minimal ansatz fulfilling two constraints (two resonances)

$$\mathrm{Im}F_P(s) = Z_1\mathrm{Im}D_1(s) + Z_2\mathrm{Im}D_2(s)$$

- Where generally

$$D_i(s) = \frac{1}{s - M_i^2 + iM_i\Gamma_i(s)} \implies \mathrm{Im}D_i(s) = -\frac{M_i\Gamma_i(s)}{(s - M_i^2)^2 + M_i^2\Gamma_i(s)^2}$$

- The sum rules imply

$$0 = f_\pi m_\pi^2 + Z_1 \int_{(3m_\pi)^2} \mathrm{Im}D_1(s)ds + Z_2 \int_{(3m_\pi)^2} \mathrm{Im}D_2(s)ds \quad (7)$$

$$0 = f_\pi m_\pi^4 + Z_1 \int_{(3m_\pi)^2} s\mathrm{Im}D_1(s)ds + Z_2 \int_{(3m_\pi)^2} s\mathrm{Im}D_2(s)ds \quad (8)$$

$$\implies g_{\pi NN}^{-1} = \frac{f_\pi}{M_N} + Z_1 \int_{(3m_\pi)^2} \frac{\mathrm{Im}D_1(s)}{s}ds + Z_2 \int_{(3m_\pi)^2} \frac{\mathrm{Im}D_2(s)}{s}ds \quad (9)$$

- Defining $\rho_i(s) \equiv \mathrm{Im}D_i(s)/s$

$$\Delta = m^2 \int dsdu\rho_1(s)\rho_2(u)(s-u)\left(m^2 - s - u\right) / \int dsdu\rho_1(s)\rho_2(u)su(u-s) \quad (10)$$

- Note that for $m_\pi \to 0$ we have $Z_1 = Z_2 = 0$, so that GT is satisfied.
- Chiral corrections (are tiny)

$$\int_{(3m_\pi)^2} \mathrm{Im}D(s)ds \to \int_{(3m_\pi)^2}^{s_R} \mathrm{Im}D_\chi(s)ds + \int_{s_R} \mathrm{Im}D(s)ds$$

Spectral profiles

- Resonances are poles in the second Riemann sheet.
- The shape of the profile depends on the particular process, typically

$$\rho_R(s) = \frac{1}{\pi}\mathrm{Im}\left[\frac{1}{M_R^2 - s + iM_R\Gamma_R(s)}\right]$$

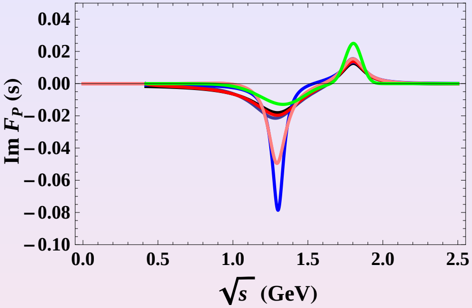
- The relevant resonances are *below* the $\bar{N}N$ threshold.
- Resonance overlapp. Shape falloff.
- Blatt-Weisskopf–von Hippel-Quigg centrifugal barrier kinematical terms with relative orbital angular momentum L and interaction radius $R = 1/q_R = 1\mathrm{fm}$ suppreses Froissart bound violations

$$\begin{aligned}
\Gamma(s) &= \Gamma_0 \frac{q}{\sqrt{s}}\frac{M_0}{q_0}\left[\frac{F_L(q)}{F_L(q_0)}\right]^2 \to \Gamma_0 \frac{M_0}{\sqrt{s}}\left(\frac{q}{q_0}\right)^{2L+1}, \quad \sqrt{q^2+m_A^2}+\sqrt{q^2+m_B^2}=\sqrt{s} \\
F_0^2(q) &= 1 \\
F_1^2(q) &= 2q/(q+q_R)
\end{aligned}$$

- All of this is VERY model dependent

6 Spectral functions

- We take 6 different profiles incorporating mass and width but different functional form



- Result

$$g_{\pi NN} = 13.2376, 13.2769, 13.2978, 13.2658, 13.2038, 13.2287 \implies g_{\pi NN} = 13.25(3)$$

- The effect of the *finite width* is small but dominates errors.
- Taking *zero width limit* $\Gamma_R \to 0$ (large N_c) and the half-width rule

$$g_{\pi NN} = \frac{M_N g_A}{f_\pi} \frac{1}{1 - m_\pi^2 / m_\pi'^2} \frac{1}{1 - m_\pi^2 / m_\pi''^2} \to 12.25(5)$$

- Extended PCAC (C. Dominguez, 70's)

$$\partial^\mu (\bar{q}\gamma^\mu \gamma_5 \vec{\tau} q) = 2 \sum_n f_{\pi n} m_{\pi n}^2 \vec{\phi}_n(x)$$

Form factors in extended PCAC

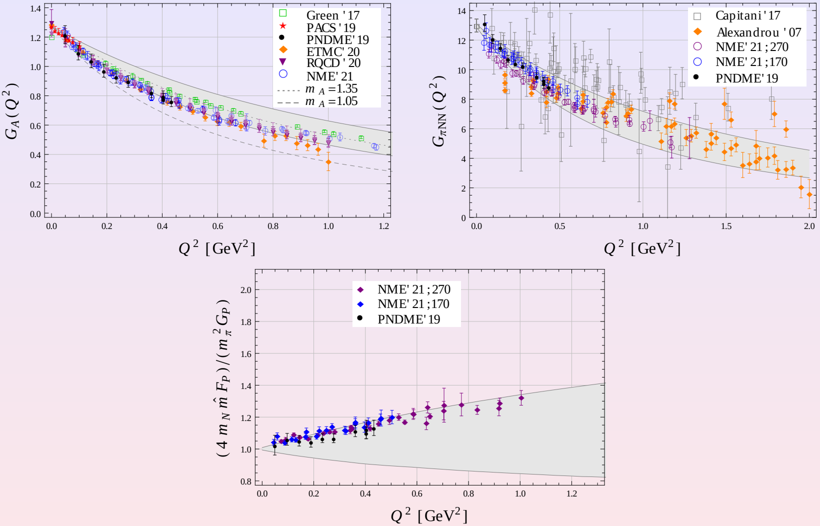
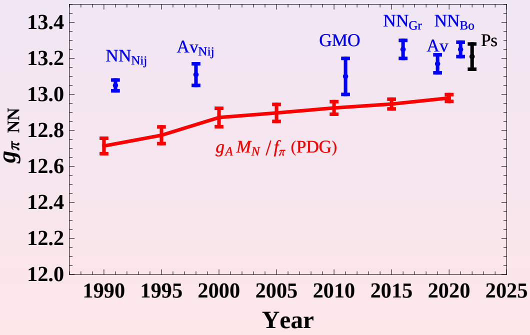


Figure: Our band; different lattice data; dipole pars from MiniBooNE/Electroproduction.

Conclusions

- Meson dominance of form factors is a large N_c inspired scheme where errors may be estimated using the half-width rule: take the mass as a distribution around its width values.
- The pion-nucleon coupling constants based on pseudoscalar meson dominance leading to extended PCAC agree with latest and most accurate determinations from NN scattering based on the Granada database

$$g_{\pi NN} = 13.21(^{+0.12}_{-0.06})(0.12)_{F_\pi}, \quad \Delta_{GT} = 1.75(^{+0.86}_{-0.41})\% \quad \mathrm{E-PCAC}$$
$$g_{\pi NN} = 13.25^{+0.05}_{-0.05}, \quad \Delta_{GT} = 2.3 \pm 0.4\% \quad \mathrm{Granada-NN}$$



- Form factors agree well with data and/or lattice determinations

Appendix: Gravitational form factor of the Nucleon

(Masjuan, Broniowski, RA, 2012)

$$\langle p'|\Theta^q_{\mu\nu}|p\rangle = \bar{u}(p')\left[A^q_{20}(t)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^q_{20}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M_N} + C^q_{20}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{M_N}\right]u(p), \quad ($$

The spin-2 (normalized) component becomes

$$F^q_T(t) = \frac{A^q_{20}(t)}{A^q_{20}(0)} \tag{12}$$

Use minimal number of spin-2 resonances f_2, f_2' $m_{f_2} = 1.320$ GeV, $\Gamma_{f_2} = 0.185$ GeV, $m_{f_2'} = 1.525$ GeV, $\Gamma_{f_2'} = 0.073$

$$F_T(Q^2)/F_T(0) = \frac{m^2_{f_2}}{m^2_{f_2} + Q^2}\,\frac{m^2_{f_2'}}{m^2_{f_2'} + Q^2}$$