Gravitational Form Factors and Pressure Distributions For a Quark Dressed With a Gluon

Sudeep Saha

In collaboration with J. More (IIT Bombay, India), A. Mukherjee (IIT Bombay, India), S. Nair (IMP, China)

Nov 30, 2021

Contents

Gravitational Form Factors and Mechanical Properties of Nucleon

The standard parametrization used

$$\langle P', S'|\theta_i^{\mu\nu}(0)|P, S\rangle$$
$$= \bar{U}(P', S')\left[-B_i(q^2)\frac{\bar{P}^\mu \bar{P}^\nu}{M_n} + (A_i(q^2) + B_i(q^2))\frac{1}{2}(\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M_n} + \bar{C}_i(q^2) M_n g^{\mu\nu}\right]$$

$\bar{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $q^\mu = (P' - P)^\mu$, $\bar{U}(P', S')$, $U(P, S)$ are spinors and M_n is the mass of the target.

$A_i(q^2),\ B_i(q^2),\ C_i(q^2),\ \bar{C}_i(q^2)$ are Gravitational Form Factor, $i \equiv (Q, G)$

$A(0) = 1$ and it is related to the mass of the system

Ji's sum rule: $A(x) + B(x) = 2J(x) \longrightarrow B(0) = 0 \longrightarrow J(0) = \frac{1}{2}$

$$\partial^\mu \theta_{\mu\nu} = 0 \longrightarrow \bar{C} = 0$$

A. H *et al*. Phys.Lett.B 728 (2014), 63-67

M.V.P *et al*. Int.J.Mod.Phys.A 33 (2018) 26, 1830025

- The C form factor also known as the D-term is unconstrained at zero momentum transfer.
- The D-term is related to the pressure and shear distribution of the nucleon.
- These quantities can be studied in the impact parameter space by performing Fourier Transform.

$$p(\boldsymbol{b}^{\perp}) = \frac{1}{6M}\frac{1}{\boldsymbol{b}^{\perp 2}}\frac{d}{d\ \boldsymbol{b}^{\perp}}\boldsymbol{b}^{\perp 2}\frac{d}{d\ \boldsymbol{b}^{\perp}}D_Q(\boldsymbol{b}^{\perp}) - M\ \bar{C}_Q(\boldsymbol{b}^{\perp}),$$

$$s(\boldsymbol{b}^{\perp}) = -\frac{1}{4M}\boldsymbol{b}^{\perp}\frac{d}{d\boldsymbol{b}^{\perp}}\frac{1}{\boldsymbol{b}^{\perp}}\frac{d}{d\boldsymbol{b}^{\perp}}D_Q(\boldsymbol{b}^{\perp}),$$

where,

$$\mathcal{F}(\boldsymbol{b}^{\perp}) = \frac{1}{2\pi}\int_0^{\infty} d\boldsymbol{q}^{\perp 2} J_0\left(\boldsymbol{q}^{\perp}\boldsymbol{b}^{\perp}\right)\mathcal{F}(q^2),$$

where,

$$\mathcal{F} = A, B, C, \bar{C}$$

J_0 is Bessel's Function of zeroth order

$\boldsymbol{b}^{\perp}$ is impact parameter

M.V.P *et al.* Int.J.Mod.Phys.A 33 (2018) 26, 1830025

The combination of pressure and shear defines the normal and the tangential force experienced by a spherical shell of radius $\boldsymbol{b}^{\perp}$

$$F_n(\boldsymbol{b}^{\perp}) = 4\pi M \boldsymbol{b}^{\perp 2}\left(p(\boldsymbol{b}^{\perp}) + \frac{2}{3}s(\boldsymbol{b}^{\perp})\right), \qquad F_t(\boldsymbol{b}^{\perp}) = 4\pi M \boldsymbol{b}^{\perp 2}\left(p(\boldsymbol{b}^{\perp}) - \frac{1}{3}s(\boldsymbol{b}^{\perp})\right).$$

I.V.A Phys.Rev.D 99 (2019) 9, 094026
D.C *et al*. Phys.Rev.D 102 (2020), 113011

The Galilean energy density, radial pressure, tangential pressure, isotropic pressure, and pressure anisotropy (Drell-Yan Frame)

$$\mu_i(\boldsymbol{b}^{\perp}) = M\left[\frac{1}{2}A_i(\boldsymbol{b}^{\perp}) + \overline{C}_i(\boldsymbol{b}^{\perp}) + \frac{1}{4M^2}\frac{1}{\boldsymbol{b}^{\perp}}\frac{d}{d\boldsymbol{b}^{\perp}}\left(\boldsymbol{b}^{\perp}\frac{d}{d\boldsymbol{b}^{\perp}}\left[\frac{1}{2}B_i(\boldsymbol{b}^{\perp}) - 4C_i(\boldsymbol{b}^{\perp})\right]\right)\right]$$

$$\sigma_i^r(\boldsymbol{b}^{\perp}) = M\left[-\overline{C}_i(\boldsymbol{b}^{\perp}) + \frac{1}{M^2}\frac{1}{\boldsymbol{b}^{\perp}}\frac{dC_i(\boldsymbol{b}^{\perp})}{d\boldsymbol{b}^{\perp}}\right]$$

$$\sigma_i^t(\boldsymbol{b}^{\perp}) = M\left[-\overline{C}_i(\boldsymbol{b}^{\perp}) + \frac{1}{M^2}\frac{d^2C_i(\boldsymbol{b}^{\perp})}{d\boldsymbol{b}^{\perp 2}}\right]$$

$$\sigma_i(\boldsymbol{b}^{\perp}) = M\left[-\overline{C}_i(\boldsymbol{b}^{\perp}) + \frac{1}{2}\frac{1}{M^2}\frac{1}{\boldsymbol{b}^{\perp}}\frac{d}{d\boldsymbol{b}^{\perp}}\left(\boldsymbol{b}^{\perp}\frac{d\,C_i(\boldsymbol{b}^{\perp})}{d\boldsymbol{b}^{\perp}}\right)\right]$$

$$\Pi_i(\boldsymbol{b}^{\perp}) = M\left[-\frac{1}{M^2}\boldsymbol{b}^{\perp}\frac{d}{d\boldsymbol{b}^{\perp}}\left(\frac{1}{\boldsymbol{b}^{\perp}}\frac{dC_i(\boldsymbol{b}^{\perp})}{d\boldsymbol{b}^{\perp}}\right)\right]$$

$$\Pi_i = \sigma_i^r - \sigma_i^t \qquad \sigma_i = \frac{(\sigma_i^r + \sigma_i^t)}{2}$$

C.L. *et al*. Eur.Phys.J.C 79 (2019) 1, 89
D.C. *et al*. Phys.Rev.D 102 (2020), 113011

Dressed Quark Model (DQM)

- DQM: A simple relativistic spin-½ state, like a quark dressed with a gluon at one loop in QCD.
- The dressed quark state can be expanded in Fock space in terms of multi-parton light-front wave functions (LFWFs), which can be calculated analytically using light-front Hamiltonian.
- We can write the LFWFs in terms of relative momenta that are frame independent. Thus LFWFs are Boost invariant.
- This model employs a gluonic degree of freedom.

$$|P,\lambda\rangle = \psi_1(P,\lambda) b^\dagger_\lambda(P)|0\rangle + \sum_{\lambda_1,\lambda_2} \int \frac{dk_1^+ d^2\boldsymbol{k}_1^\perp}{\sqrt{2(2\pi)^3 k_1^+}} \int \frac{dk_2^+ d^2\boldsymbol{k}_2^\perp}{\sqrt{2(2\pi)^3 k_2^+}} \psi_2(P,\lambda|k_1,\lambda_1;k_2,\lambda_2)$$

$$\sqrt{2(2\pi)^3 P^+}\delta^3(P-k_1-k_2) b^\dagger_{\lambda_1}(k_1) a^\dagger_{\lambda_2}(k_2)|0\rangle$$

$\psi_1(P,\lambda)$: Normalization

$\psi_2(P,\lambda|k_1,\lambda_1;k_2,\lambda_2)$: Probability amplitude

A.H *et al*. Phys.Rev.D 59 (1999), 094013

- Two-component formulation of light-front QCD, with $A^+ = 0$
- The bad component fermion field is eliminated and can be written in terms of dynamical fields only.

$$\psi = \psi_+ + \psi_-,$$
$$\Lambda_\pm \psi = \frac{1}{2}\gamma^0\gamma^\pm\psi = \psi_\pm,$$
$$\Lambda_+ + \Lambda_- = I, \quad \Lambda_\pm^2 = \Lambda_\pm, \quad \Lambda_+\Lambda_- = 0.$$

$$\gamma^+ = \begin{bmatrix} 0 & 0 \\ 2i & 0 \end{bmatrix}, \gamma^- = \begin{bmatrix} 0 & -2i \\ 0 & 0 \end{bmatrix}$$
$$\gamma^i = \begin{bmatrix} -i\sigma^i & 0 \\ 0 & i\sigma^i \end{bmatrix}, \gamma^5 = \begin{bmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{bmatrix}.$$

$$\psi_+ = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \psi_- = \begin{bmatrix} 0 \\ \left(\frac{1}{i\partial^+}\right)[\sigma^\perp \cdot (i\partial^\perp + gA^\perp) + im]\xi \end{bmatrix}$$

$$\xi(y) = \sum_\lambda \chi_\lambda \int \frac{dk^+ d^2\boldsymbol{k}^\perp}{2(2\pi)^3\sqrt{k^+}}[b_\lambda(k)e^{-ik\cdot y} + d^\dagger_{-\lambda}(k)e^{ik\cdot y}],$$

$$A^\perp(y) = \sum_\lambda \int \frac{dk_3^+ d^2\boldsymbol{k}^\perp}{2(2\pi)^3 k^+}[\epsilon^\perp_\lambda a_\lambda(k)e^{-ik\cdot y} + \epsilon^{\perp *}_\lambda a^\dagger_\lambda(k)e^{ik\cdot y}].$$

A.H. *et al*. Phys.Rev.D 59 (1999), 094013

Energy-MomentumTensor (EMT)

Drell-Yan Frame:

$$P = (P^+, P^\perp, P^-) = \left(P^+, 0, \frac{M^2}{P^+}\right),$$

$$P' = (P'^+, \boldsymbol{P}'^\perp, P'^-) = \left(P^+, \boldsymbol{q}^\perp, \frac{\boldsymbol{q}^{\perp 2} + M^2}{P^+}\right),$$

$$q = (P' - P) = \left(0, \boldsymbol{q}^\perp, \frac{\boldsymbol{q}^{\perp 2}}{P^+}\right).$$

EMT:

$$\theta^{\mu\nu} = \frac{1}{2}\overline{\psi}\, i\left[\gamma^\mu D^\nu + \gamma^\nu D^\mu\right]\psi - F^{\mu\lambda a}F^\nu_{\lambda a} + \frac{1}{4}g^{\mu\nu}\left(F_{\lambda\sigma a}\right)^2 - g^{\mu\nu}\overline{\psi}\left(i\gamma^\lambda D_\lambda - m\right)\psi.$$

$$\theta^{\mu\nu}_Q = \frac{1}{2}\bar{\psi} i\left[\gamma^\mu D^\nu + \gamma^\nu D^\mu\right]\psi$$

$$iD^\mu = i\overleftrightarrow{\partial^\mu} + gA^\mu, \qquad \alpha(i\overleftrightarrow{\partial^\mu})\beta = \alpha(\frac{i}{2}\overrightarrow{\partial^\mu})\beta + \alpha(-\frac{i}{2}\overleftarrow{\partial^\mu})\beta,$$

$$\mathcal{M}^{\mu\nu}_{SS'} = \frac{1}{2}\left[\langle P', S'|\theta^{\mu\nu}_{Q}(0)|P, S\rangle\right]$$

where the Lorentz indices $(\mu, \nu) \equiv \{+, -, 1, 2\}$, $(S, S') \equiv \{\uparrow, \downarrow\}$ is the helicity of the initial and final state

$$\mathcal{M}^{++}_{\uparrow\uparrow} + \mathcal{M}^{++}_{\downarrow\downarrow} = 2\ (P^{+})^{2} A_{Q}(q^{2}),$$

$$\mathcal{M}^{++}_{\uparrow\downarrow} + \mathcal{M}^{++}_{\downarrow\uparrow} = \frac{iq^{(2)}}{M}\ (P^{+})^{2} B_{Q}(q^{2}).$$

$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} - \mathcal{M}^{22}_{\uparrow\downarrow} - \mathcal{M}^{22}_{\downarrow\uparrow} = i\left[\frac{B_{Q}(q^{2})}{4M} - \frac{C_{Q}(q^{2})}{M}\right]\left((q^{(1)})^{2}\ q^{(2)} - (q^{(2)})^{3}\right),$$

$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} + \mathcal{M}^{22}_{\uparrow\downarrow} + \mathcal{M}^{22}_{\downarrow\uparrow} = i\left[B_{Q}(q^{2})\frac{q^{2}}{4M} - C_{Q}(q^{2})\frac{3q^{2}}{M} + \overline{C}_{Q}(q^{2})2M\right]q^{(2)}.$$

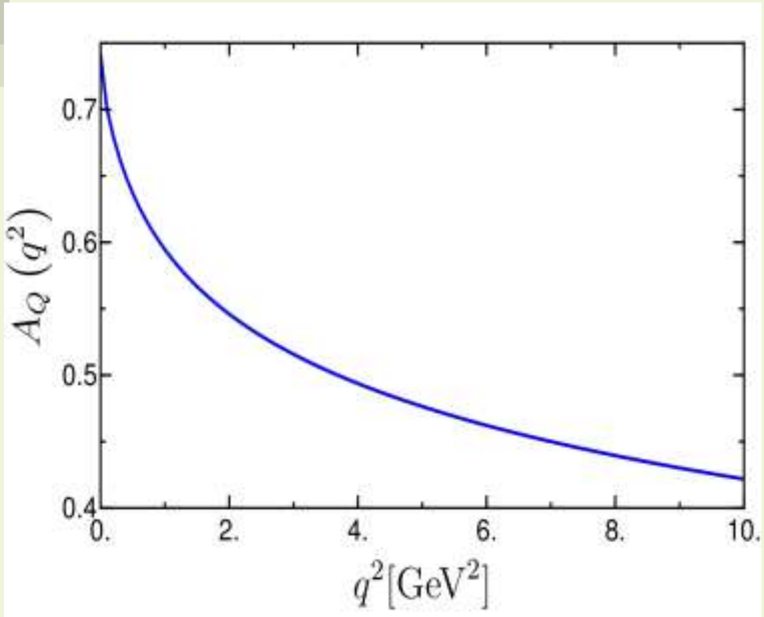
$$
\begin{aligned}
A_Q(q^2) =& 1 + \frac{g^2\ C_F}{2\pi^2}\left[\frac{11}{10} - \frac{4}{5}\left(1 + \frac{2m^2}{q^2}\right)\frac{f_2}{f_1} - \frac{1}{3}\log\left(\frac{\Lambda^2}{m^2}\right)\right], \\
B_Q(q^2) =& \frac{g^2 C_F}{12\pi^2}\ \frac{m^2}{q^2}\ \frac{f_2}{f_1}, \\
D_Q(q^2) =& \frac{5g^2 C_F}{6\pi^2}\ \frac{m^2}{q^2}\ \left(1 - f_1 f_2\right) = 4\ C_Q(q^2), \\
\overline{C}_Q(q^2) =& \frac{g^2 C_F}{72\pi^2}\ \left(29 - 30\ f_1\ f_2 + 3\ \log\left(\frac{\Lambda^2}{m^2}\right)\right),
\end{aligned}
$$

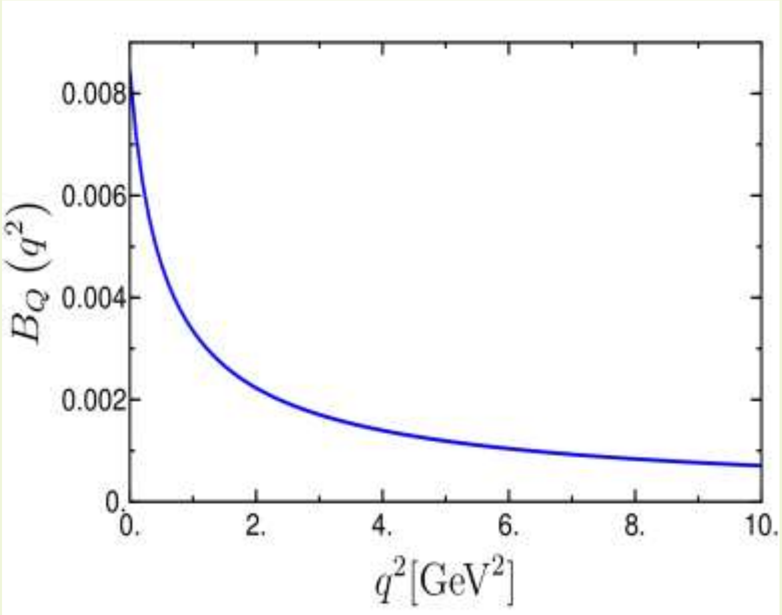
where

$$
\begin{aligned}
f_1 =& \frac{1}{2}\sqrt{1 + \frac{4m^2}{q^2}}, \\
f_2 =& \log\left(1 + \frac{q^2\ (1 + 2f_1)}{2m^2}\right)
\end{aligned}
$$

C_F is the colour factor and Λ is the ultra-violet cut-off, m is the quark mass.

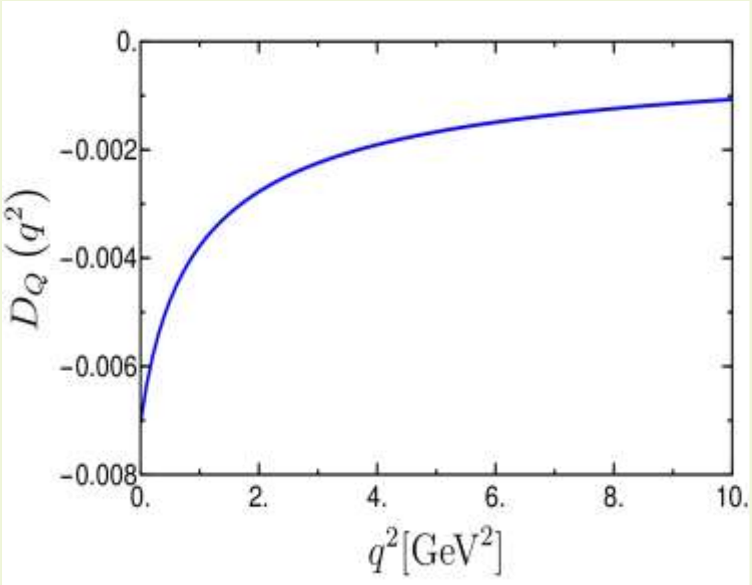
Numerical Analysis

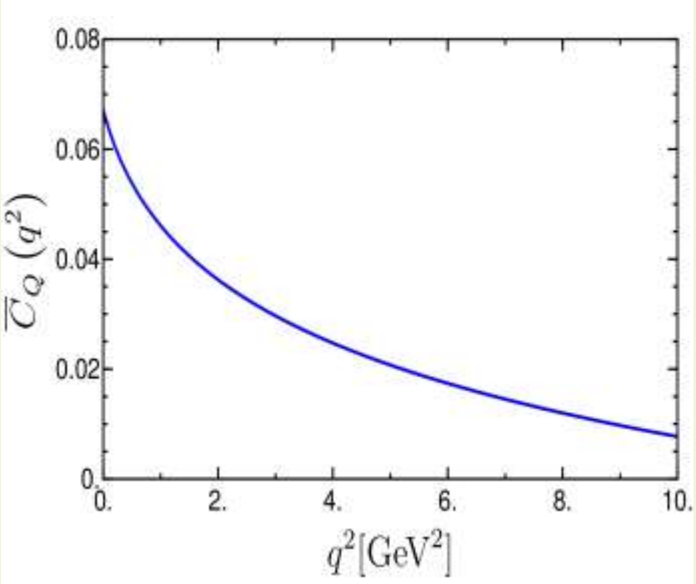




$A_Q(0) = 0.7412$ $B_Q(0) = 0.0084$

$$A(0) + B(0) = 1 = 2J(0) \longrightarrow J(0) = \frac{1}{2}$$





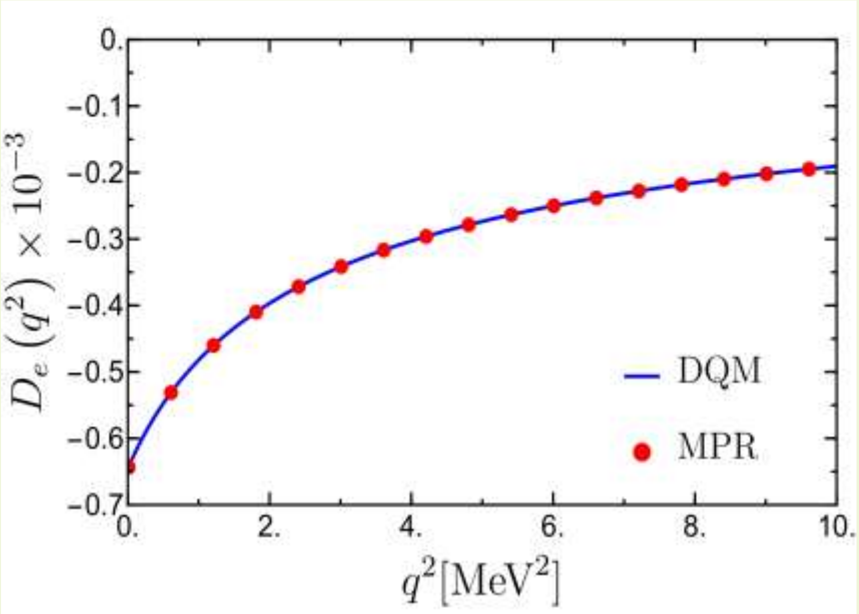
$$D_Q(0) = -0.007$$

$$\bar{C}_Q(0) = 0.067$$

$$\bar{C}_Q + \bar{C}_G = 0$$

Comparison with MPR

$D_e(q^2)$: Electron D-term



DQM: Dressed Quark Model

MPR: Metz *et al.*

(Phys.Lett.B 820 (2021), 136501)

$$m = 0.511\mathrm{MeV},$$

$$g^2 = 4\pi\alpha, \quad \alpha \text{ is fine structure constant.}$$

D-term and Pressure Distribution

In order to study the spatial distribution, we have taken the wave packet states in position space centred at origin instead of plane waves. The most prevalent forms are *Gaussian wave packets*.

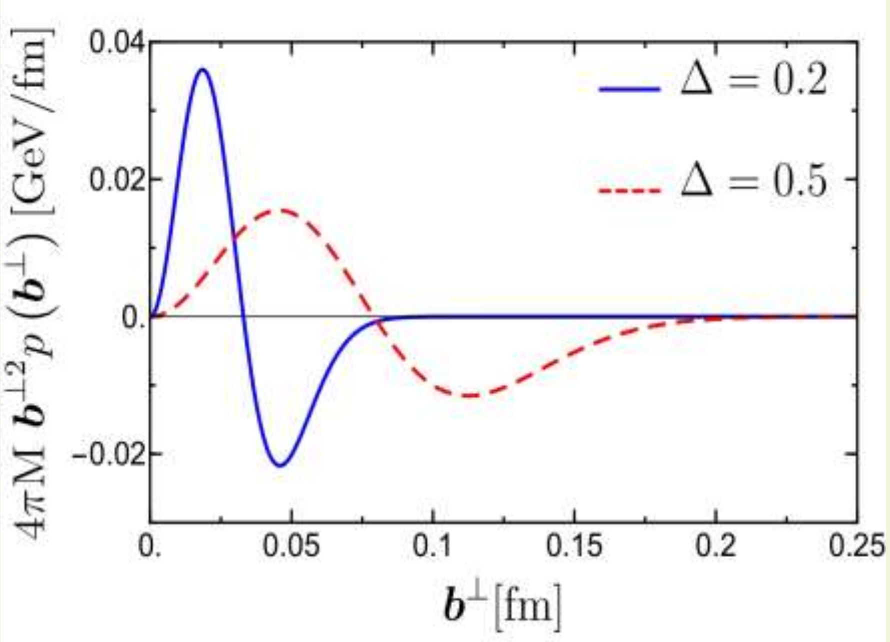
$$\frac{1}{16\pi^3}\int \frac{d^2\boldsymbol{p}^{\perp} dp^+}{p^+}\phi(p)|p^+,\boldsymbol{p}^{\perp},\lambda\rangle$$

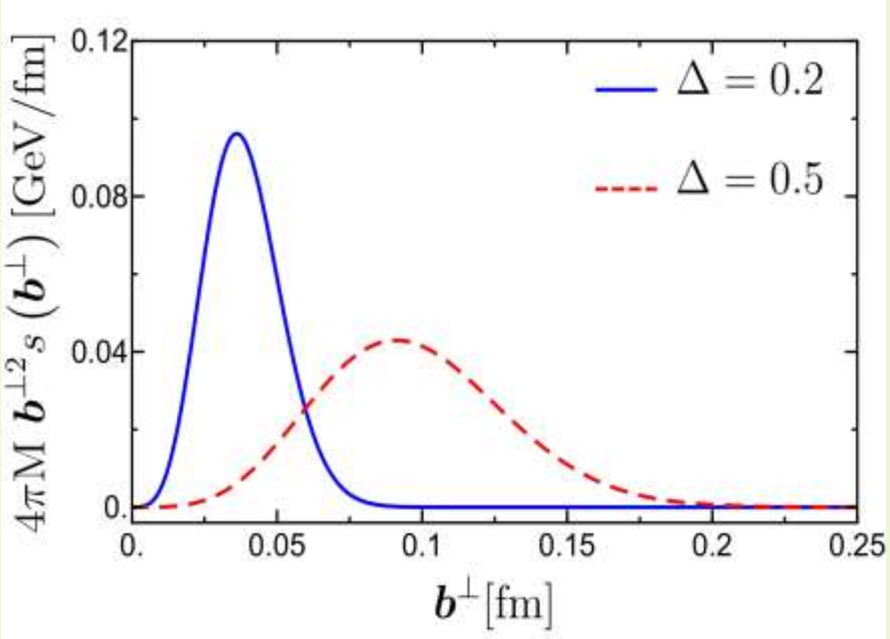
$$\phi(p) = p^+\delta\left(p^+ - p_0^+\right)\phi\left(\boldsymbol{p}^{\perp}\right),$$

$$\phi\left(\boldsymbol{p}^{\perp}\right) = e^{-\frac{\boldsymbol{p}^{\perp 2}}{2\Delta^2}}$$

Δ is Gaussian width.

M.D. Eur.Phys.J.C 25 (2002)
D.C. *et al.* Phys.Rev.D 72 (2005), 034013



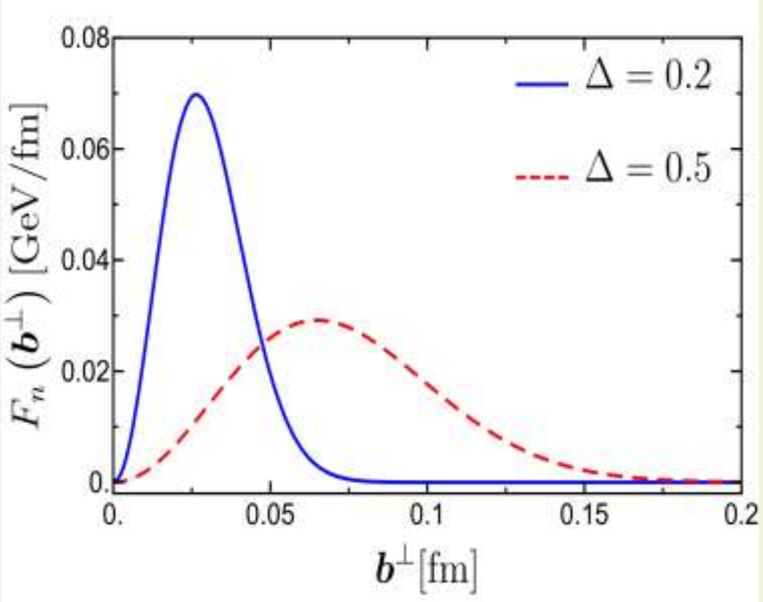


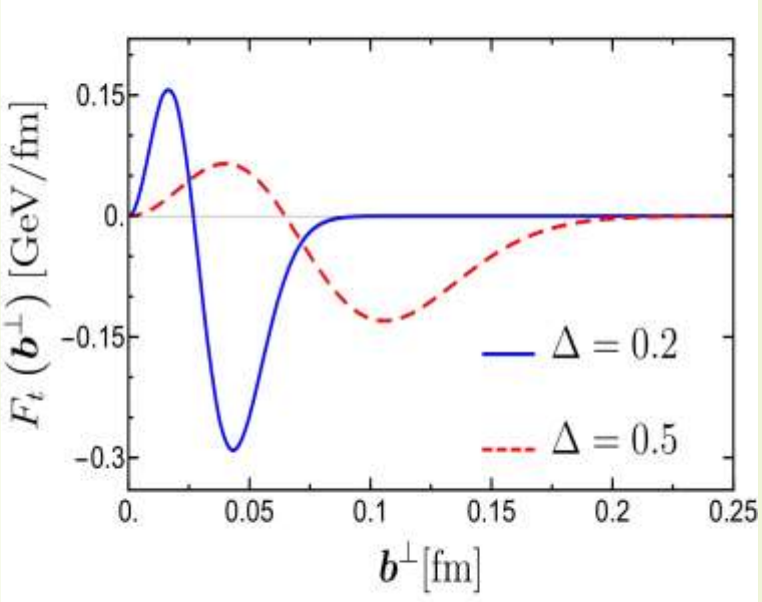
von Laue Condition:

$$\int_0^{\infty} d\boldsymbol{b}^{\perp 2} p(\boldsymbol{b}^{\perp}) = 0$$

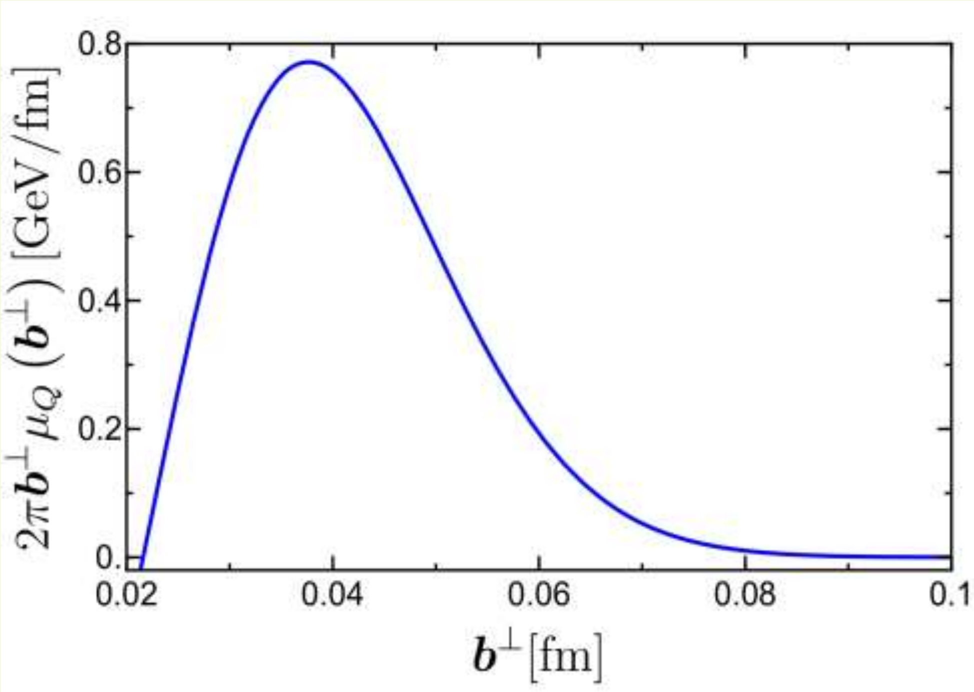
M.V.P *et al.* Int.J.Mod.Phys.A 33 (2018) 26, 1830025
V.D.B *et al.* Nature 557 (2018) 7705, 396-399

Normal Force and Tangential Force





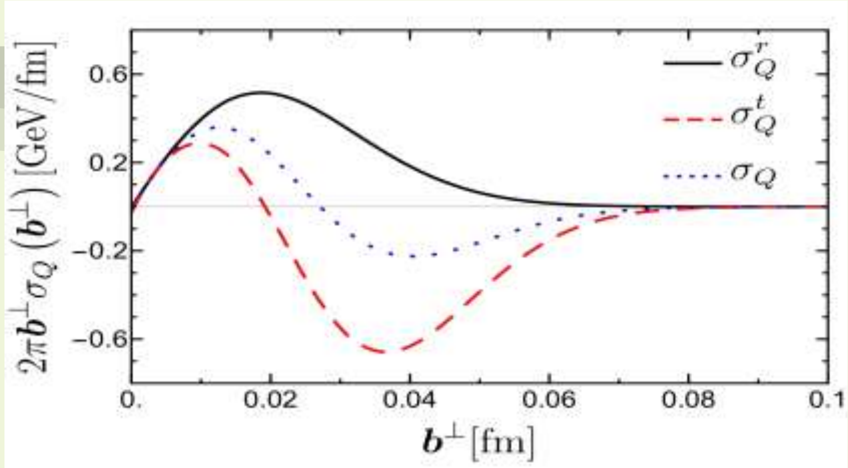
2-Dimensional Energy Density



$\boldsymbol{b}^{\perp} = 0.038\,\mathrm{fm}$ $\Delta = 0.2$

D. C. *et al*. Phys.Rev.D 102 (2020), 113011
C. L. *et al*. Eur.Phys.J.C 79 (2019) 1, 89

2-Dimensional Pressure Distributions

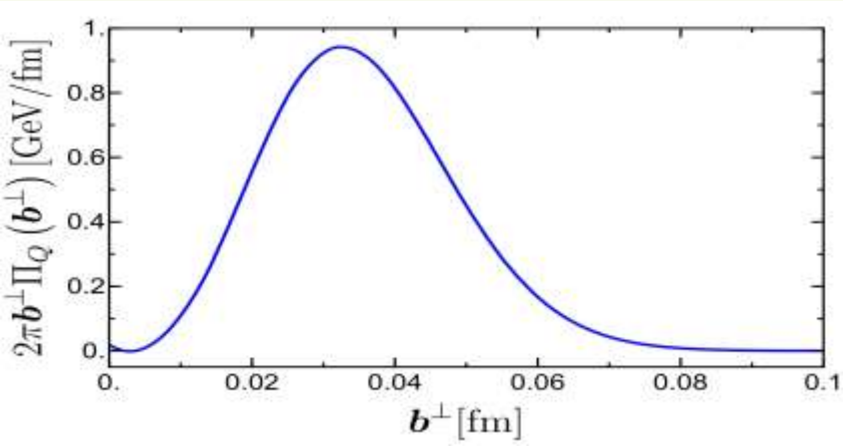


$$\sigma_i^r(\boldsymbol{b}^\perp) = M\left[-\overline{C}_i(\boldsymbol{b}^\perp) + \frac{1}{M^2}\frac{1}{\boldsymbol{b}^\perp}\frac{dC_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^\perp}\right]$$

$$\sigma_i^t(\boldsymbol{b}^\perp) = M\left[-\overline{C}_i(\boldsymbol{b}^\perp) + \frac{1}{M^2}\frac{d^2C_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^{\perp 2}}\right]$$

$$\sigma_i(\boldsymbol{b}^\perp) = M\left[-\overline{C}_i(\boldsymbol{b}^\perp) + \frac{1}{2}\frac{1}{M^2}\frac{1}{\boldsymbol{b}^\perp}\frac{d}{d\boldsymbol{b}^\perp}\left(\boldsymbol{b}^\perp\frac{d\,C_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^\perp}\right)\right]$$

$$\Pi_i(\boldsymbol{b}^\perp) = M\left[-\frac{1}{M^2}\boldsymbol{b}^\perp\frac{d}{d\boldsymbol{b}^\perp}\left(\frac{1}{\boldsymbol{b}^\perp}\frac{dC_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^\perp}\right)\right]$$



$$\Delta = 0.2$$

$$\boldsymbol{b}^\perp = 0.03\text{fm}$$

C. L. *et al.* Eur.Phys.J.C 79 (2019) 1, 89

Conclusion

- We have studied the four Gravitational Form Factors in a composite spin-1/2 system, a quark dressed with a gluon at one loop level in QCD.
- We have also analysed the pressure and shear distributions in this model.
- Future work will focus on gluon contribution of the GFFs.

Thank You