Charmonium spectrum from the instanton liquid model

Kihoon Hong

In collaboration with Hyun-Chul Kim, Ulugbek Yakhshiev (Inha University)
Mirzayousuf M. Musakhanov and Nurmukhammad Rakhimov (National University of Uzbekistan)

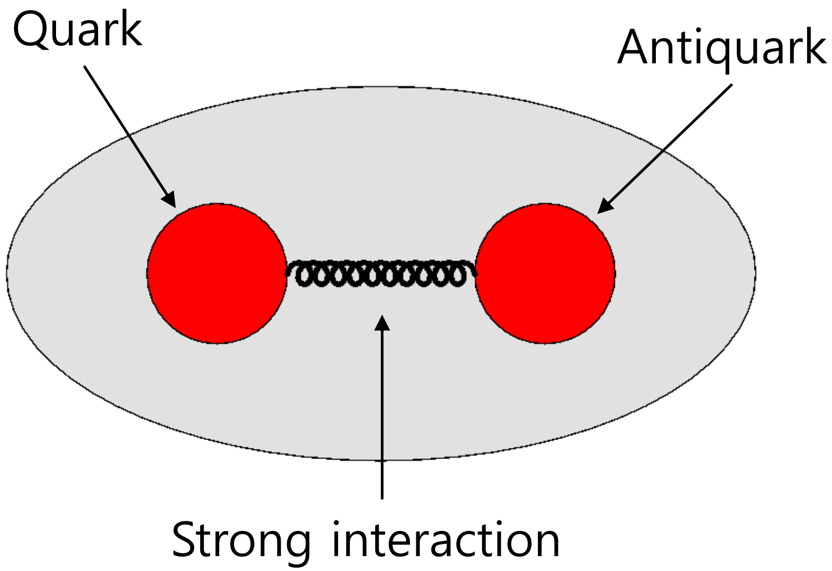
Department of Physics, Inha University, Incheon, South Korea
kihoon7065@naver.com

Contents

- Introduction
  - Heavy-Quark Potential&Instanton
  - Heavy-quark potential in the instanton vacuum
    - ❖ Heavy-quark potential with perturbative corrections (by M. Musakhanov, N. Rakhimov and U. T. Yakhshiev [1])
    - ❖Spin-dependent parts of the $Q\bar{Q}$ potential
- Results
  - ❖ Charmonium energy spectrum
  - ❖ E1 & M1 Radiative Transition
- Outlooks
  - ❖ Hadronic transition between charmonium resonances $\psi' \to \pi\pi J/\psi$
- Summary

[1] M.Musakhanov et al, PhysRevD.102.076022

Heavy-quark Potential



- Light mesons

  $E_B \gg m_q$, E_B: Bound state energy

  - Light quarks inside of a meson are relativistic (quite complicated)

- Quarkonia

  $E_B < m_Q$

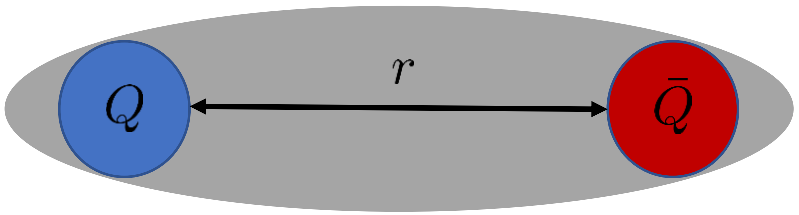
  - Relativistic effects are sufficiently small
  - $1/m_Q$ is taken to be a small parameter
  - This makes more easily study about the interaction between quark and anti-quark.

- Static heavy-quark potentials are valid.
- Cornell potential : $V(r) = -\frac{\alpha}{r} + kr$

one-gluon exchange perturbative potential ($-\frac{\alpha}{r}$)

Phenomenological confining potential (kr)

$r \to 0,\ \alpha(p^2 \to \infty) \to \epsilon$: Asymptotic freedom



: confinement

Heavy-quark Potential

- Instanton is known to contribute to the interaction between light quarks more than heavy quarks.

- Because the instanton dynamical mass is much bigger than light quark masses.

$$\Delta m_I \approx 70 \text{ MeV} : \text{ Instanton dynamical mass[1]} \qquad m_q < \Delta m_I \qquad m_Q >> \Delta m_I$$

- However, the instanton effects to the heavy quarks have been studied continuously since the instanton shows the nonperturbative effects in the heavy quark sector.

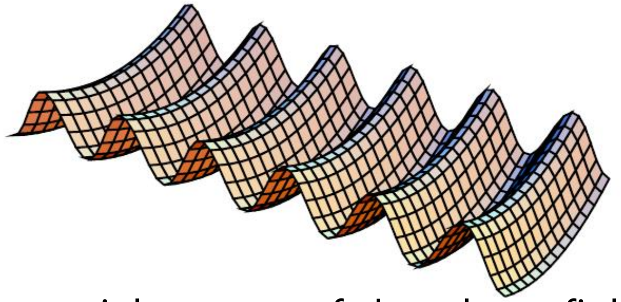
- A few years ago, the perturbative effects of instantons were also studied in Ref [2].

- Using this formalism, we derived the charmonium spectrum.

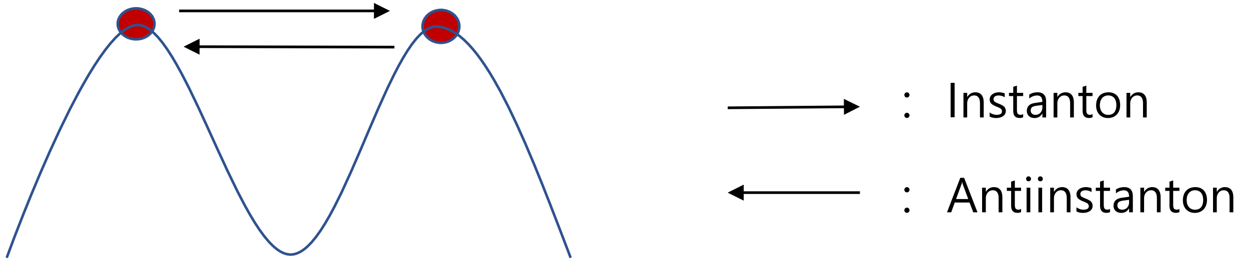
[1] Diakonov et al, Phys. Lett. B 226, 372 (1989)
[2] M.Musakhanov et al, PhysRevD.102.076022

Instanton

- In the Chern-Simon coordinate, the instanton is a large fluctuation of the gluon field corresponding to quantum tunneling from one vacuum(minimum of the potential energy) to the neighboring one:



Potential energy of the gluon field



Classical trajectory in Euclidean space

- To find the best tunneling trajectory having the largest amplitude one has thus to minimize the YM action, which becomes

$$S = \frac{1}{4g^2}\int d^4x\; F^a_{\mu\nu}F^a_{\mu\nu} = \frac{8\pi^2}{g^2}.$$

$F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}$: self-duality condition

- Gauge field satisfying the self-duality equation can be written as:

$$A_{I,\mu}(x, z_I) = \frac{\eta^{-a}_{\mu\nu}(x - z_I)_\mu \lambda^a \rho^2}{(x - z_I)^2((x - z_I)^2 + \rho^2)},$$

ρ : Average instanton size =1/3 fm

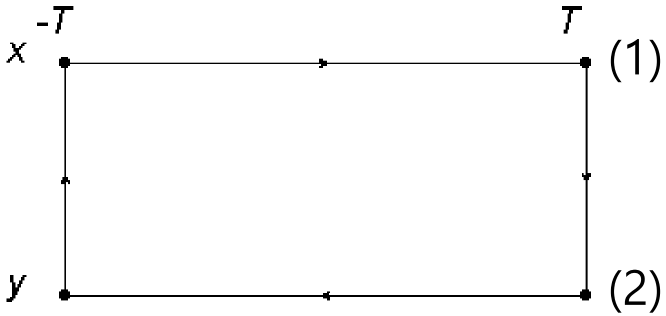
z_I : A position of instanton

Heavy-quark potential in the instanton vacuum (Including perturbative corrections) [2]

$$\theta^{-1} = \frac{d}{dt}$$

$$D = \theta^{-1} - g\sum_I A_I$$

- In Ref. [2], they considered the one-gluon exchange(OGE) perturbation part.
- They used the instanton packing parameter $\lambda = \frac{\rho^4}{R^4} \sim 0.01$ as the running coupling constant $\alpha_s \sim \lambda^{\frac{1}{2}}$.
- Averaged Wilson loop($Q\bar{Q}$ correlator) can be written as



$$W = \int D\xi \exp\left[\frac{1}{2}\sum_{i\neq j=1}^{2}\left(\frac{\delta}{\delta a_a^{(i)}} S_{ab}^{(ij)} \frac{\delta}{\delta a_b^{(j)}}\right)\right] \frac{1}{D^{(1)} - ga^{(1)}} \frac{1}{D^{(2)} - g\bar{a}^{(2)}},$$

order of $\alpha_s (\propto g^2)$ ⇒ $$W^{-1} = \int D\xi \left(D^{(1)}D^{(2)} - g^2 \frac{\lambda_a}{2}\frac{\bar{\lambda}_b}{2} S_{ab}\right)$$

- Using the Fourier transform of W^{-1}: $W^{-1}(\omega) = i\omega + f(\omega) + g(\omega)$

$$\langle t_1 | W | t_2 \rangle = \int \frac{d\omega}{2\pi} e^{i\omega(t_1 - t_2)} \frac{1}{W^{-1}(\omega)}$$

- Correlation function from the Fourier transformation :

$$\exp\left(-V_I^1 T\right) = \exp\left[-(V_I^{1,(\mathrm{NP})} + V_I^{1,(\mathrm{P})})T\right] = \exp[-(f(0) + g(0))T]$$

[1] Diakonov et al, Phys. Lett. B 226, 372 (1989)
[2] M.Musakhanov et al, PhysRevD.102.076022

Heavy-quark potential in the instanton vacuum (Including perturbative corrections) [2]

$$V_I^{1,(\mathrm{NP})} = \frac{N}{2VN_c}\sum_{\pm}\int d_3z_{\pm}\,\mathrm{Tr}_c\left[1 - P\exp\left(i\int_{-\infty}^{\infty} dx_4 A_{\pm,4}\right) P\exp\left(-i\int_{-\infty}^{\infty} dy_4 A_{\pm,4}\right)\right]$$

$$V_I^{1,(\mathrm{P})} = g^2 \frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2}\int \frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2 + M_g^2(q)}$$

$M_g(q) = \frac{2\pi}{\rho}\left(\frac{6\lambda}{N_c^2-1}\right)^{1/2} q\rho K_1(q\rho)$: Momentum dependent gluon mass

K_1 : Modified Bessel function of the second type

$\rho = 1/3$ fm

$R = 1$ fm

In the color-singlet state

$$= -\frac{4\alpha_s}{3r}\left(1 - \frac{2r}{\pi}\int_0^{\infty} dq j_0(qr)\frac{3\pi^2\lambda K_1^2(q\rho)}{1 + 3\pi^2\lambda K_1^2(q\rho)}\right)$$
$$= V_C^{1,(\mathrm{P})}(r) + V_I^{1,(\mathrm{SCR})}(r),$$

$$\left(\frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2}\right)_S = -\frac{N_c^2 - 1}{2N_c} I,$$
$$\left(\frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2}\right)_A = \frac{1}{2N_c} I.$$

$$V_C^{1,(\mathrm{P})}(r) \equiv -\frac{4\alpha_s}{3r},$$
$$V_I^{1,(\mathrm{SCR})}(r) \equiv \frac{8\alpha_s}{3\pi}\int_0^{\infty} dq j_0(qr)\frac{3\pi^2\lambda K_1^2(q\rho)}{1 + 3\pi^2\lambda K_1^2(q\rho)}$$

Heavy-quark potential in the instanton vacuum (Including perturbative corrections) [2]

$$V_I^{1,(\mathrm{NP})}(r) = \frac{4\pi\lambda}{N_c\rho}\mathcal{I}_{\mathrm{NP}}\left(\frac{r}{\rho}\right),$$

$$V_I^{1,(\mathrm{P})}(r) = V_C^{1,(\mathrm{P})}(r) + V_I^{1,(\mathrm{SCR})}$$

$$V_C^{1,(\mathrm{P})}(r) = -\frac{4\alpha_s}{3r}, \qquad V_I^{1,(\mathrm{SCR})} = \frac{8\alpha_s}{3\pi\rho}\mathcal{I}_{\mathrm{SCR}}\left(\frac{r}{\rho}\right)$$

In the color-singlet state

$$\mathcal{I}_0^d = 4.41625, \qquad \mathcal{I}_0^s = 0.578695$$

$$a^d = \begin{pmatrix} -1 \\ 0.128702 \\ -1.1047 \end{pmatrix}, \qquad a^s = \begin{pmatrix} 1 \\ 0.121348 \\ 2.71619 \end{pmatrix}$$

$$b^d = \begin{pmatrix} 0.404875 \\ 0.453923 \\ 0.420733 \end{pmatrix}, \qquad b^s = \begin{pmatrix} 0.144123 \\ 0.189758 \\ 0.144123 \end{pmatrix}$$

$$\mathcal{I}_{\mathrm{NP}}(x) = \mathcal{I}_0^{\mathrm{d}}\left\{1 + \sum_{i=1}^{2}\left[a_i^d x^{2(i-1)} + a_3^d(-b_3^d x)^i\right]e^{-b_i^d x^2} + \frac{a_3^d}{x}\left(1 - e^{-b_3^d x^2}\right)\right\},$$

$$\mathcal{I}_{\mathrm{SCR}}(x) = \mathcal{I}_0^s\left\{\sum_{i=1}^{2}\left[a_i^s x^{2(i-1)} + a_3^s(-b_3^s x)^i\right]e^{-b_i^s x^2} + \frac{a_3^s}{x}\left(1 - e^{-b_3^s x^2}\right)\right\}$$

Spin-dependent parts of the $Q\bar{Q}$ potential

- The spin-dependent parts of the Cornell potential and the one gluon exchange correction of the instanton effects are represented by the Breit Fermi equation [3, 4]

$$V_{SD} = V_{SS}\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} + V_{LS}\mathbf{L}\cdot\mathbf{S} + V_T\left[3(\mathbf{S}_Q\cdot\hat{\mathbf{n}})(\mathbf{S}_{\bar{Q}}\cdot\hat{\mathbf{n}}) - \mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}}\right]$$

$$\mathbf{S} = \mathbf{S}_Q + \mathbf{S}_{\bar{Q}}$$

$$V_V = -\frac{4\alpha_s}{3r}$$

$$V_S = kr$$

$$V_{SS}(r) = \frac{2}{3m_Q^2}\nabla^2 V_V = \frac{32\pi\alpha_s}{9m_Q^2}\delta(r)$$

$$= \frac{32\alpha_s\sigma^3}{9m_Q^2\sqrt{\pi}}e^{-\sigma^2 r^2}$$

$$V_{LS}(r) = \frac{1}{2m_Q^2 r}\left(3\frac{dV_V}{dr} - \frac{dV_S}{dr}\right)$$

$$V_T(r) = \frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV_V}{dr} - \frac{d^2V_V}{dr^2}\right)$$

- The spin-dependent parts of the instanton case is defined as [3]

$$V_{SD}^I = V_{SS}^I\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} + V_{LS}^I\mathbf{L}\cdot\mathbf{S} + V_T^I\left[3(\mathbf{S}_Q\cdot\hat{\mathbf{n}})(\mathbf{S}_{\bar{Q}}\cdot\hat{\mathbf{n}}) - \mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}}\right]$$

$$V_{SS}^I(r) = \frac{1}{3m_Q^2}\nabla^2 V_I^{(\mathrm{NP})}$$

$$V_{LS}^I(r) = \frac{1}{2m_Q^2 r}\frac{dV_I^{(\mathrm{NP})}}{dr}$$

$$V_T^I(r) = \frac{1}{3m_Q^2}\left(\frac{d^2V_I^{(\mathrm{NP})}}{dr^2} - \frac{1}{r}\frac{dV_I^{(\mathrm{NP})}}{dr}\right)$$

[3] E. Eichten and F. Feinberg, Phys. Rev. D173090 (1981)
[4] M. B. Voloshin, Progress in Particle and Nucl Phys. 61 (2008) 455-511

Eigenvalues of the Hamiltonian

$$H\chi(r) = \left[-\frac{\hbar^2}{m_Q}\frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{m_Q r^2} + V_C + V_I + V_{SD} + V_{SD}^I\right]\chi(r)$$

$$m_{Q\bar{Q}} = 2m_Q + E_B$$

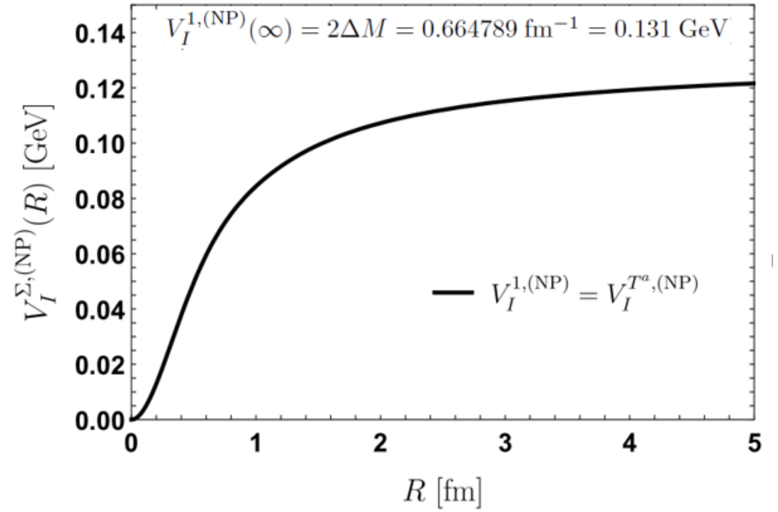
$$\chi(r) = r\psi(r)$$

$$V_C = \underbrace{-\frac{4\alpha_s}{3r}}_{V_V} + \underbrace{kr + V_0}_{V_s}$$

$m_{Q\bar{Q}}$: mass of the quarkonia
E_B: Bound state energy (eigenvalue of the Hamiltonian)

- For the charmonium case, we use the mass values as the input data:

State	Exp(MeV)	Input
$J/\psi\ (1^3S_1)$	3096.900 ± 0.006	3097
$\eta_c\ (1^1S_0)$	2983.9 ± 0.5	2984
$\psi\ (2^3S_1)$	3686.097 ± 0.5	3686
$\eta_c\ (2^1S_0)$	3637.6 ± 1.2	3638
$J/\psi\ (4^3S_1)$	4421 ± 4	4421



NP potential of instanton cannot explain the quark confinement.

Results: Charmonium spectrum

SetA	ρ_I (fm)	R (fm)	α_s(-)	k(GeV^2)	σ(GeV)	m_Q(GeV)	V_0(GeV)
SetIa	1/3	1	0.5141	0.1432	1.136	1.3634	0
SetIIa	0.36	0.89	0.4783	0.1375	1.174	1.3251	0
SetB	ρ_I (fm)	R (fm)	α_s(-)	k(Ge[illegible]	[illegible](GeV)	m_Q(GeV)	V_0(GeV)
SetIb	1/3	1	0.5098	0.144[illegible]	[illegible]66	1.3932	-0.0563
SetIIb	0.36	0.89	0.4773	0.1375	[illegible]	1.3211	0.0062

Parameter	Linear potential model	Screened potential model
m_c (GeV)	1.4830	1.4110
α_s	0.5461	0.5070
b (GeV^2)	[illegible]425	0.2100
σ (GeV)	[illegible]	1.1600
r_c (fm)	[illegible]	0.180
μ (GeV)	…	0.0979

Fitting parameters

State	Exp	SetIa	Se[illegible]	[illegible]etIb	SetIIb
$J/\psi(1^3S_1)$	3096[illegible]				
$\eta_c\ (1^1S_0)$	[illegible]				
$\psi\ (2^3S_1)$	36[illegible]				
$\eta_c\ (2^1S_0)$					
$\psi\ (3^3S_1)$					
$\eta_c\ (3^1S_0)$					
$\psi\ (4^3S_1)$					
$\eta_c\ (4^1S_0)$					
$\chi_{c2}\ (1^3P_2)$	3[illegible]				
$\chi_{c1}\ (1^3P_1)$	3[illegible]				
$\chi_{c0}\ (1^3P_0)$	3[illegible]				
$h_c\ (1^1P_1)$	3[illegible]				
$\chi_{c2}\ (2^3P_2)$					
$\chi_{c1}\ (2^3P_1)$					
$\chi_{c0}\ (2^3P_0)$					
$h_c\ (2^1P_1)$					

[4]

n[illegible]	name	J^{PC}	Exp. [6]	LP	SP
				[illegible]097	3097
				[illegible]3	2984
					3679
					3637
					4030
					4004
					4281
					4264
				[illegible]2	3553
				[illegible]5	3521
				[illegible]5	3415
				[illegible]2	3526
				[illegible]7	3937
				[illegible]7	3914
				[illegible]69	3848
2^1P_1	$h_c(2P)$	1^{+-}		3940	3916

$$V_s = \begin{cases} kr & : \text{LP} \\ \frac{k}{\mu}(1-e^{-\mu r}) & : \text{SP} \end{cases}$$

α_s	without instanton	with instanton	
		ρ=1/3 fm	ρ=0.36 fm
SetI	0.5461	0.5141	0.4783
SetII	0.5461	0.5098	0.4773

$\alpha_s(\mu)$ in pQCD:

μ [GeV]	1.275
$\alpha_s(\mu)$	0.4258

With instanton case gives closer value to pQCD α_s than without instanton case.

[4] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

Results: E1 & M1 Radiative Transitions

E1 radiative partial width [10]

$$\Gamma_{E1}(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} C_{fi} \delta_{SS'} e_c^2 \alpha |\langle \psi_f | r | \psi_i \rangle|^2 E_\gamma^3 \frac{E_f^{(c\bar{c})}}{M_i^{(c\bar{c})}}$$

M1 radiative partial width [10]

$$\Gamma_{M1}(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} \frac{2J'+1}{2L+1} \delta_{LL'} \delta_{S,S'\pm 1} e_c^2 \frac{\alpha}{m_c^2} |\langle \psi_f | \psi_i \rangle|^2 E_\gamma^3 \frac{E_f^{(c\bar{c})}}{M_i^{(c\bar{c})}}$$

$$C_{fi} = \max(L, L')(2J'+1) \left\{ \begin{array}{ccc} L' & J' & S \\ J & L & 1 \end{array} \right\}^2$$

E_γ: final photon energy, $E_f^{c\bar{c}}$: total energy of the final $c\bar{c}$ state

$M_i^{c\bar{c}}$: mass of the initial $c\bar{c}$ state, C_{fi}: angular matrix element

E1 transition								
		Γ_{theory} (keV)						
Initial state	Final state	This work				[10]		
		SetIa	SetIIa	SetIb	SetIIb	NR	GI	Exp.
$\psi'(2S)$	$\chi_{c2}(1P)$	49.7	58.3	48.8	58.8	38	24	27 ± 4
	$\chi_{c1}(1P)$	47.8	51.0	46.3	51.3	54	29	27 ± 3
	$\chi_{c0}(1P)$	32.0	33.0	31.2	33.0	63	26	27 ± 3
$\chi_{c2}(1P)$	$J/\psi(1S)$	437.7	446.0	428.4	446.6	424	313	426 ± 51
$\chi_{c1}(1P)$		360.0	376.3	355.2	376.9	314	239	291 ± 48
$\chi_{c0}(1P)$		173.9	184.1	171.4	184.0	152	114	119 ± 19
$\psi(1D)$	$\chi_{c2}(1P)$	7.64	8.8	7.62	8.8	4.9	3.3	≤ 330 (90% c.l.)
	$\chi_{c1}(1P)$	151.5	164.7	149.3	165.4	125	77	280 ± 100
	$\chi_{c0}(1P)$	337.1	355.1	331.4	356.0	403	213	320 ± 120
M1 transition								
$J/\psi(1S)$	$\eta_c(1S)$	2.9	2.6	2.9	2.6	2.9	2.4	1.1 ± 0.3
$\psi'(2S)$	$\eta_c(1S)$	5.6	5.4	5.6	5.4	4.6	9.6	0.8 ± 0.2

[10] T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev D. 72, 054026 (2005)

Summary & Outlooks

- We obtained the charmonium spectrum from the instanton vacuum model.

- We obtained the running coupling constant that is closer to the pQCD running coupling constant than another model.

- Using this charmonium spectrum, we evaluated the radiative transition width of E1 and M1 radiative transitions.

- SetIb model ($\rho_I = 1/3$ fm, Constant confining potential on) is the best model to describe the E1 and M1 radiative transition.

- We can conclude that instanton makes meaningful contributions in the heavy quark sector also.

Thank you for your attention

Back Up

Outlooks

- For the case of the hadronic transition amplitude [5] of the two-pion transition between n^3S_1 states:

$$A(\psi' \to \pi^+\pi^- J/\psi) = \frac{1}{2}\langle \pi^+\pi^- | E_i^a E_j^a | 0 \rangle \alpha_{ij}^{(12)}$$

- The $\psi' \to J/\psi$ transition in the chromo-electric field is described by the effective Hamiltonian

$$H_{\text{eff}} = -\frac{1}{2}\alpha_{ij}^{(12)} E_i^a E_j^a,$$

with the chromo-polarizability given by

$$\alpha^{(12)} = \frac{1}{48}\langle 1S|\xi^a r_i G r_i \xi^a|2S\rangle = \frac{1}{9}\langle 1S|r_i \frac{1}{H_o - E_{2S}} r_i|2S\rangle, \qquad \xi^a = \frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}$$

where G is the Green's function of the heavy quark pair in the color octet state.

[5] M. B. Voloshin, Progress in Particle and Nucl Phys. 61 (2008) 455-511

Results: Charmonium spectrum

SetA	ρ_I (fm)	R (fm)	α_s(-)	k(GeV^2)	σ(GeV)	m_Q(GeV)	V_0(GeV)
SetIa	1/3	1	0.5141	0.1432	1.136	1.3634	0
SetIIa	0.36	0.89	0.4783	0.1375	1.174	1.3251	0

SetB	ρ_I (fm)	R (fm)	α_s(-)	k(GeV^2)	σ(GeV)	m_Q(GeV)	V_0(GeV)
SetIb	1/3	1	0.5098	0.1444	1.166	1.3932	-0.0563
SetIIb	0.36	0.89	0.4773	0.1375	1.174	1.3211	0.0062

State	Exp	SetIa	SetIIa	SetIb	SetIIb
χ_{c2} (3^3P_2)		4318	4313	4318	4313
χ_{c1} (3^3P_1)		4286	4287	4287	4287
χ_{c0} (3^3P_0)		4236	4235	4235	4235
h_c (3^1P_1)		4290	4287	4290	4287
ψ_3 (1^3D_3)		3810	3806	3810	3806
ψ_2 (1^3D_2)	3822.2 ± 1.2	3808	3806	3809	3806
ψ (1^3D_1)	3778.1 ± 1.2	3787	3791	3789	3791
η_{c2} (1^1D_2)		3806	3804	3807	3804
ψ_3 (2^3D_3)		4172	4169	4172	4168
ψ_2 (2^3D_2)		4168	4166	4168	4165
ψ (2^3D_1)	4191 ± 5	4140	4148	4143	4148
η_{c2} (2^1D_2)		4167	4164	4167	4164

Parameter	Linear potential model	Screened potential model
m_c (GeV)	1.4830	1.4110
α_s	0.5461	0.5070
b (GeV^2)	0.1425	0.2100
σ (GeV)	1.1384	1.1600
r_c (fm)	0.202	0.180
μ (GeV)	...	0.0979

Fitting parameters

$n^{2S+1}L_J$	name	J^{PC}	Exp. [6]	LP	SP
3^3P_2	$\chi_{c2}(3P)$	2^{++}		4310	4211
3^3P_1	$\chi_{c1}(3P)$	1^{++}		4284	4192
3^3P_0	$\chi_{c0}(3P)$	0^{++}		4230	4146
3^1P_1	$h_c(3P)$	1^{+-}		4286	4193
1^3D_3	$\psi_3(1D)$	3^{--}		3811	3808
1^3D_2	$\psi_2(1D)$	2^{--}	3823	3807	3807
1^3D_1	$\psi_1(1D)$	1^{--}	3778	3787	3792
1^1D_2	$\eta_{c2}(1D)$	2^{-+}		3806	3805
2^3D_3	$\psi_3(2D)$	3^{--}		4172	4112
2^3D_2	$\psi_2(2D)$	2^{--}		4165	4109
2^3D_1	$\psi_1(2D)$	1^{--}	4191?	4144	4095
2^1D_2	$\eta_{c2}(2D)$	2^{-+}		4164	4108

[4]

$$V_s = \begin{cases} kr & \text{: LP} \\ \frac{k}{\mu}(1-e^{-\mu r}) & \text{: SP} \end{cases}$$

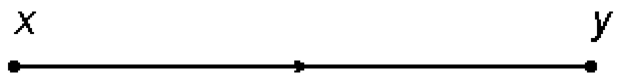
[4] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

Potential from Wilson Loop

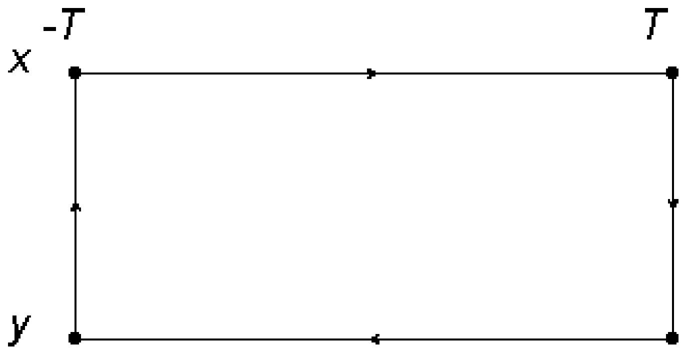
- The $Q\bar{Q}$ state evaluates in time T and can be represented as

$$|\Phi(\vec{x},T;\vec{y},T)\rangle = \bar{Q}(\vec{x})U(x,y)Q(\vec{y})|0\rangle$$

- $U(x,y) = P\exp\left(ig\int_x^y \frac{\lambda_a}{2}A_\mu^a(z)dz_\mu\right)$: Wilson line



- We assume that the heavy quark and antiquark masses $m_{Q,\bar{Q}} \to \infty$ and they are in static state during $T \to \infty$.



$T \gg r\ (= x - y)$

- Heavy quark potential from the correlation function:

$$W(C) = U(\vec{x},-T;\vec{x},T)U(\vec{y},T;\vec{x},-T)$$

$$\langle\Phi(\vec{y},-T;\vec{x},-T)|\Phi(\vec{x},T;\vec{y},T)\rangle = \langle e^{-HT}\rangle \sim e^{-VT} = \langle P\exp\left(\oint A_4(z)dz_4\right)\rangle \longrightarrow V = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W(C)\rangle$$

Results

$$V_I^{1,(\mathrm{NP})}(\infty) = 2\Delta M = 0.664789\ \mathrm{fm}^{-1} = 0.131\ \mathrm{GeV} = C$$

$$V^1 = V_C^{1,(\mathrm{P})} + \sigma r + C, \qquad V^{T^a} = V_C^{T^a,(\mathrm{P})} + \sigma r + C$$

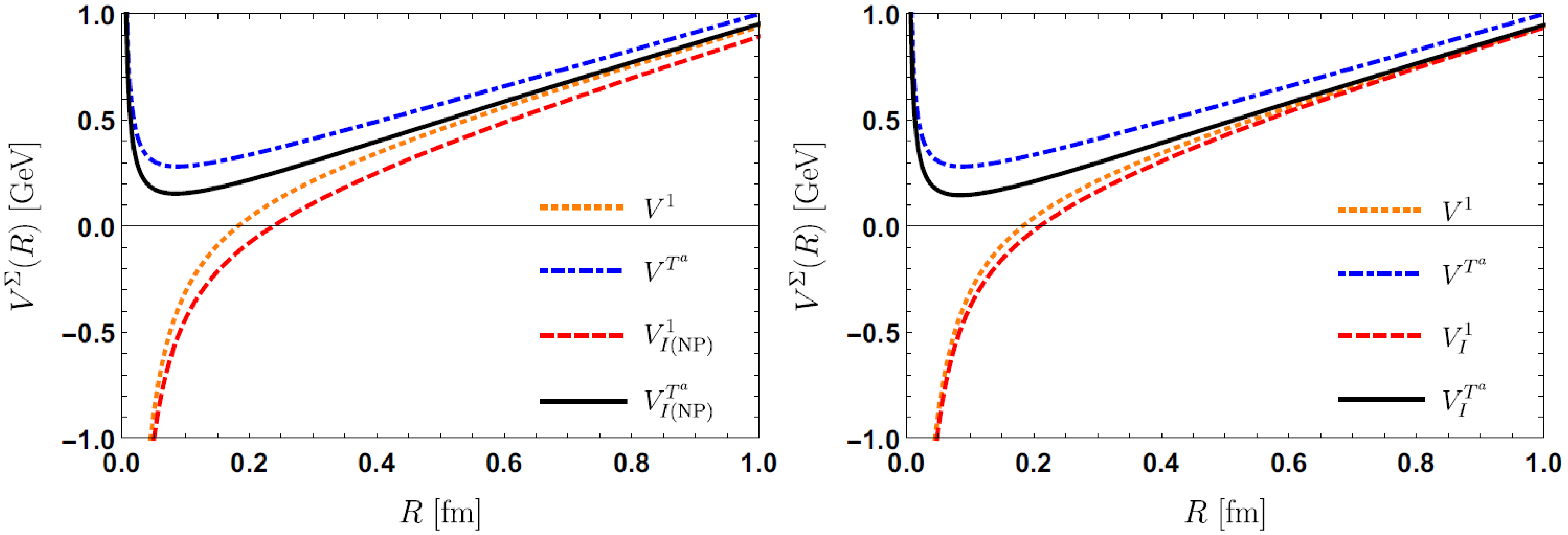
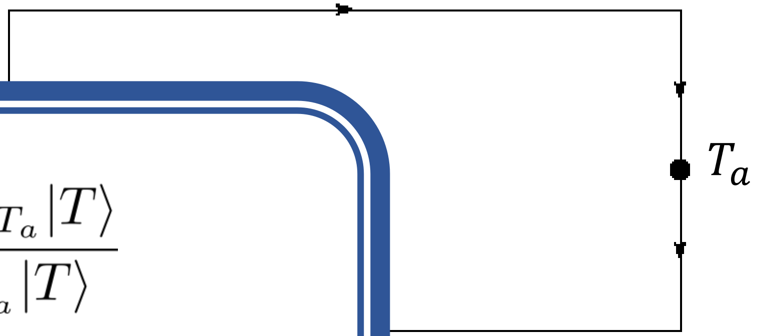


FIG. 2. The left panel is the total color siglet and octet potential without considering the instanton effect of the one gluon exchange. Otherwise, in the right panel, we considered all of instanton effects. Orange dotted lines are color singlet potential and the blue dot-dashed lines are color-octet potential without the instanton effects. The red lines are $V_{1,\mathrm{Ins}}$ and $V_{1,\mathrm{Ins,ge}}$, respectively. The black solid lines represent $V_{T^a,\mathrm{Ins}}$ and $V_{T^a,\mathrm{Ins,ge}}$. Here we set the parameters $\alpha_s = 0.2$, $\sigma = 0.17\ \mathrm{GeV}^2$, $C = 0.131183$ GeV, $\rho = 0.33$ fm and $R = 1$ fm.

Color-octet heavy-quark potential (NP correction)

- Color-octet Wilson loop W_{T^a} can be represented by inserting the color exchange operator T_a

$$W_{T_a} = \int D\xi \exp\left[\frac{1}{2} \cdots\right.$$



$$V_I^{T_a} = -\lim_{T\to\infty}\frac{1}{T}\ln\langle T|W_{T_a}|T\rangle = -\lim_{T\to\infty}\frac{\frac{d}{dt}\langle T|W_{T_a}|T\rangle}{\langle T|W_{T_a}|T\rangle}$$

$$= \lim_{T\to\infty}\frac{2N_c^2\Delta M e^{-(2\Delta M - V_I^1)T} - V_I^1}{N_c^2 e^{-(2\Delta M - V_I^1)T} - 1} = V_I^1$$

$$W_{T_a} = \frac{1}{2(N_c^2-1)}\langle \mathrm{Tr} \cdots$$

$$= \frac{N_c^2}{2(N_c^2-1)} w^{(1)} w^{(2)} - \frac{1}{2(N_c^2-1)} W_1,$$

$\langle\langle \mathcal{W}(\vec{x},\Delta t)\rangle\rangle$

$\langle\langle \mathcal{W}^\dagger(\vec{y},\Delta t)\rangle\rangle$

$\langle\langle \mathcal{W}\rangle\langle \mathcal{W}^\dagger\rangle\rangle = \langle\langle \mathcal{W}\rangle\rangle\langle\langle \mathcal{W}^\dagger\rangle\rangle$:cluster decomposition for one-instanton calculation

$$\langle T|W_{T_a}|T\rangle = \frac{N_c^2}{2(N_c^2-1)}\langle T|w^{(1)}|-T\rangle\langle -T|w^{(2)}|T\rangle - \frac{1}{2(N_c^2-1)}\langle T|W_1|T\rangle$$

$$= \frac{N_c^2}{2(N_c^2-1)} e^{-2\Delta M T} - \frac{1}{2(N_c^2-1)} e^{-V_I^{1,(\mathrm{NP})}T}.$$

Eigenvalues of the Color-octet Hamiltonian

State	Exp	SetIa	SetIIa	SetIb	SetIIb
1^3S_1		3328	3330	3380	3328
1^1S_0		3332	3334	3384	3332
2^3S_1		3796	3797	3826	3796
2^1S_0		3800	3800	3829	3800
3^3S_1		4165	4165	4186	4165
3^1S_0		4168	4168	4189	4168
4^3S_1		4485	4485	4503	4485
4^1S_0		4488	4488	4506	4488
1^3P_2		3589	3590	3623	3589
1^3P_1		3617	3618	3651	3617
1^3P_0		3636	3637	3668	3636
1^1P_1		3604	3605	3638	3604
2^3P_2		3988	3989	4011	3988
2^3P_1		4012	4012	4034	4012
2^3P_0		4027	4028	4049	4027
2^1P_1		4001	4002	4024	4001
3^3P_2		4326	4327	4345	4326
3^3P_1		4347	4348	4365	4347
3^3P_0		4361	4362	4379	4361
3^1P_1		4339	4339	4357	4339

Bound state energy (E_B)

Unobservable quantities.
Physical implications are yet unknown.

Why we use the instanton?

- In the pQCD, the running coupling constant α_s at the one loop level is given by the expression [3-6]:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0} \frac{1}{\ln(\mu^2/\Lambda^2_{QCD})}$$

$$\beta_0 = (11N_c - 2N_f)/3, \qquad \Lambda_{QCD} = 0.217 \text{ GeV}$$

Δm_I (GeV)	μ (GeV)	α_s (GeV)
0	1.275	0.4258
0.067	1.343	0.4137
0.1357	1.411	0.4029

$$\mu = m_c + \Delta m_I$$

Δm_I: Dynamical mass (Instanton mass) [1]
m_c: charm-quark mass=1275 MeV

The model	ρ (fm)	R (fm)	Δm_I (GeV)	α_s (GeV)
MWOI	Not applicable	Not applicable	Not applicable	0.2068
M-I	0.33	1.00	0.0676	0.3447
M-IIb	0.36	0.89	0.1357	0.4588

Table 1 [8]. The result using Cornell potential gives a 51% difference from one of pQCD. On the other hand, the instanton effects as in M-I and M-Iib give the difference 17% and 14%, respectively.

[3] M. Peter, Phys. Rev. Lett. 78, 602 (1997).
[4]M. Peter, Nucl. Phys. B501, 471 (1997).
[5] Y. Schroder, Phys. Lett. B 447, 321 (1999).
[6] A. V. Smirnov, V. A. Smirnov, and M. Steinhauser, Phys. Rev. Lett. 104, 112002 (2010).
[7] C. Anzai, Y. Kiyo, and Y. Sumino, Phys. Rev. Lett. 104, 112003 (2010).
[8] Yakhshiev et al, PhysRevD.98.114036

Color-singlet & octet $Q\bar{Q}$ potential

A. $\rho = 0.33$ **fm,** $R = 1$ **fm**

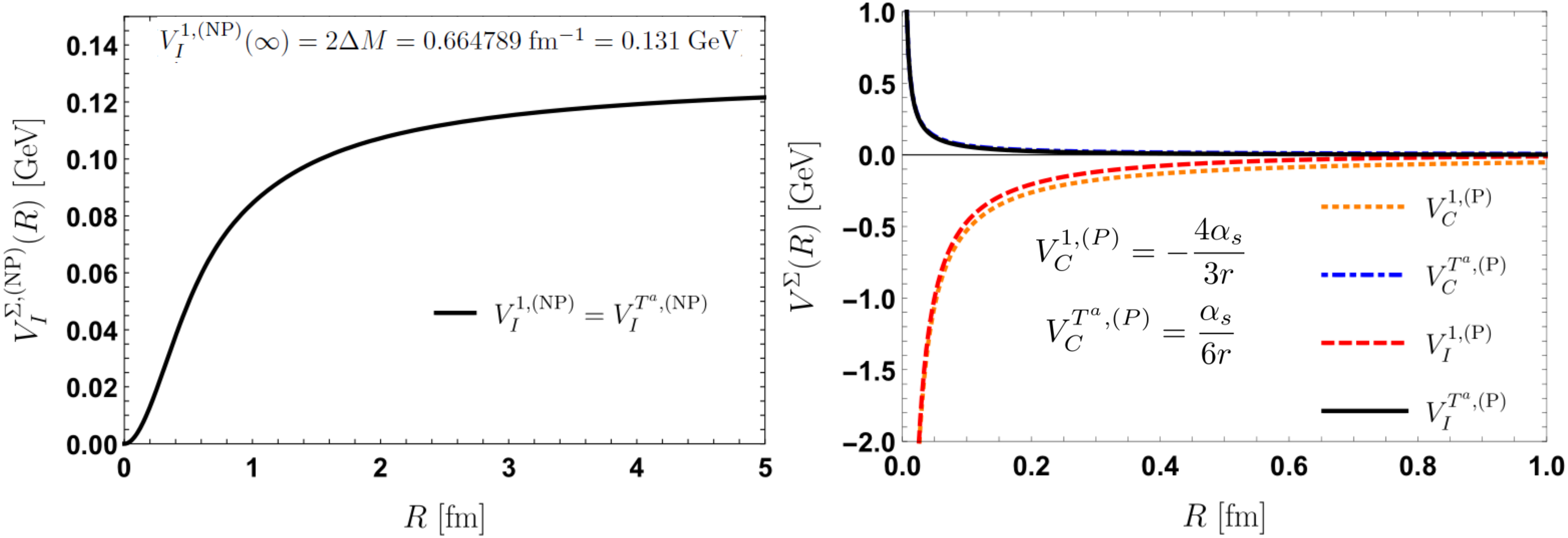


FIG. 1. Left panel(Non-perturbative potential) : The Non-perturbative instanton potential is not affected by color-state, which is black solid line. Right panel(Perturbative potential) : $V_C^{1,(P)} = -4\alpha_s/3r$ and $V_C^{T^a,(P)} = \alpha_s/6r$ are the perturbative color singlet and octet potential without instanton effect, respectively. $V_I^{1,(P)}$ and $V_I^{T^a,(P)}$ are including the instanton effects. We set $\alpha_s = 0.2$, $\rho = 0.33$ fm and $R = 1$ fm.

Results: Charmonium spectrum

State	Exp	Method 1: Instanton effects Off	Method 1: Instanton effects On	Method 2
$J/\psi(1^3S_1)$	3096.900 ± 0.006	3099	3098	3121
$\eta_c\ (1^1S_0)$	2983.9 ± 0.5	2984	2984	2996
$\psi\ (2^3S_1)$	3686.097 ± 0.025	3682	3684	3682
$\eta_c\ (2^1S_0)$	3637.6 ± 1.2	3637	3639	3619
$\psi\ (3^3S_1)$	4039 ± 1	4085	4085	4084
$\eta_c\ (3^1S_0)$		4054	4053	4034
$\psi\ (4^3S_1)$	4421 ± 4	4422	4421	4424
$\eta_c\ (4^1S_0)$		4397	4396	4382
$\chi_{c2}\ (1^3P_2)$	3556.17 ± 0.07	3557	3552	3526
$\chi_{c1}\ (1^3P_1)$	3510.67 ± 0.05	3510	3510	3500
$\chi_{c0}\ (1^3P_0)$	3414.71 ± 0.30	3415	3415	3415
$h_c\ (1^1P_1)$	3525.38 ± 0.17	3520	3520	3508
$\chi_{c2}\ (2^3P_2)$	3927.2 ± 2.6	3976	3971	3950
$\chi_{c1}\ (2^3P_1)$		3933	3934	3925
$\chi_{c0}\ (2^3P_0)$	3862^{+26+40}_{-32-13}	3874	3874	3859
$h_c\ (2^1P_1)$		3942	3942	3931
$\chi_{c2}\ (3^3P_2)$		4323	4318	4303
$\chi_{c1}\ (3^3P_1)$		4281	4283	4278
$\chi_{c0}\ (3^3P_0)$		4236	4236	4222
$h_c\ (3^1P_1)$		4290	4290	4284

- For the case of the instanton effects off: $V_I = 0,\ V_{SD}^I = 0$
- We used the instanton parameters $\rho = 1/3$ fm and $R = 1$ fm

Old version : Method 1

Instanton	m_Q	α_s	k	σ
Off	1.4796	0.5426	0.1444	1.1510
On	1.36530	0.51770	0.14280	1.12900

New version : Method 2

Instanton	m_Q	α_s	k	σ
On	1.3353	0.41495	0.14679	1.79984

fitting parameters

The running coupling constant at the one-loop level:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0} \frac{1}{\log(\mu^2/\Lambda_{\rm QCD}^2)}$$

$$\beta_0 = \frac{11N_c - 2N_f}{3}, \quad \Lambda_{\rm QCD} = 0.217\ \text{GeV}$$

$$\mu = m_Q = m_Q^0 + \Delta m_I^{\rm pert} = 1.275 + 0.1454\alpha_s\ [\text{GeV}]$$

Summary & Outlook

- We showed the non-perturbative color-octet heavy quark potential in the instanton vacuum:

$$V_I^{T^a,(\mathrm{NP})}(r) = V_I^{1,(\mathrm{NP})}(r)$$

- The perturbative one gluon exchange instanton effects make the color-singlet(octet) heavy quark potential a little weaker → Instanton makes the screening effects.

- We obtained the charmonium spectrum and the bound energies in the color-octet states.

- Using the color-octet potential derived in the present work, we are going to calculate chromo-polarizabilities of quarkonia.

- From this chromo-polarizability, we will show hadronic transition between charmonium resonances.