Nucleon with One Dynamical Gluon

in

Basis Light-Front Quantization

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With

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Outline

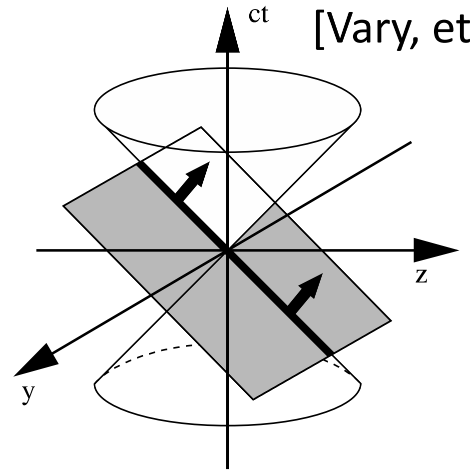
- **B**asis **L**ight-**F**ront **Q**uantization (**BLFQ**)
- Nucleon Structure in BLFQ
  - Nucleon Structure with leading Fock Sector
  - Nucleon Structure with One Dynamical Gluon
- Conclusion

Basis Light-Front Quantization

➢ Solve the time-independent Schrödinger Equation:

$$P^{-}|\beta\rangle = P_{\beta}^{-}|\beta\rangle$$

[Vary, et.al, Phys.Rev.C '10]



- $\boldsymbol{P^{-}}$: Light-Front Hamiltonian;
- $|\boldsymbol{\beta}\rangle$: Eigenstates;
- $\boldsymbol{P_{\beta}^{-}}$: Eigenvalues for eigenstates.

➢ Quantum numbers of basis states in BLFQ:

I. Longitudinal direction

— discrete longitudinal momentum (labeled by **k**): $P^{+} = \frac{2\pi}{L}k$

II. Transverse direction

— 2-dimensional harmonic oscillator (labeled by **n** , **m**) Truncation : $\begin{cases}\text{Nmax}\\ \text{Kmax}\end{cases}$

$$\Phi_{n,m}^{b}(p_{\perp}) = \frac{1}{b\sqrt{\pi}}\sqrt{\frac{n!}{(n+|m|)!}}\, e^{-\frac{p^2}{2b^2}} e^{-im\phi} \left(\frac{p}{b}\right)^{|m|} L_{n}^{|m|}\left(\frac{p^2}{b^2}\right)$$

Prof. Xingbo Zhao's Talk
At Firday noon, QCD AB II

Light-Front Hamiltonian

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$H_{trans} \sim \kappa_T^4 r^2$ -- Brodsky, Teramond arXiv: 1203.4025

$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

- Only include first Fock sector

$$H_{Interact} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$$

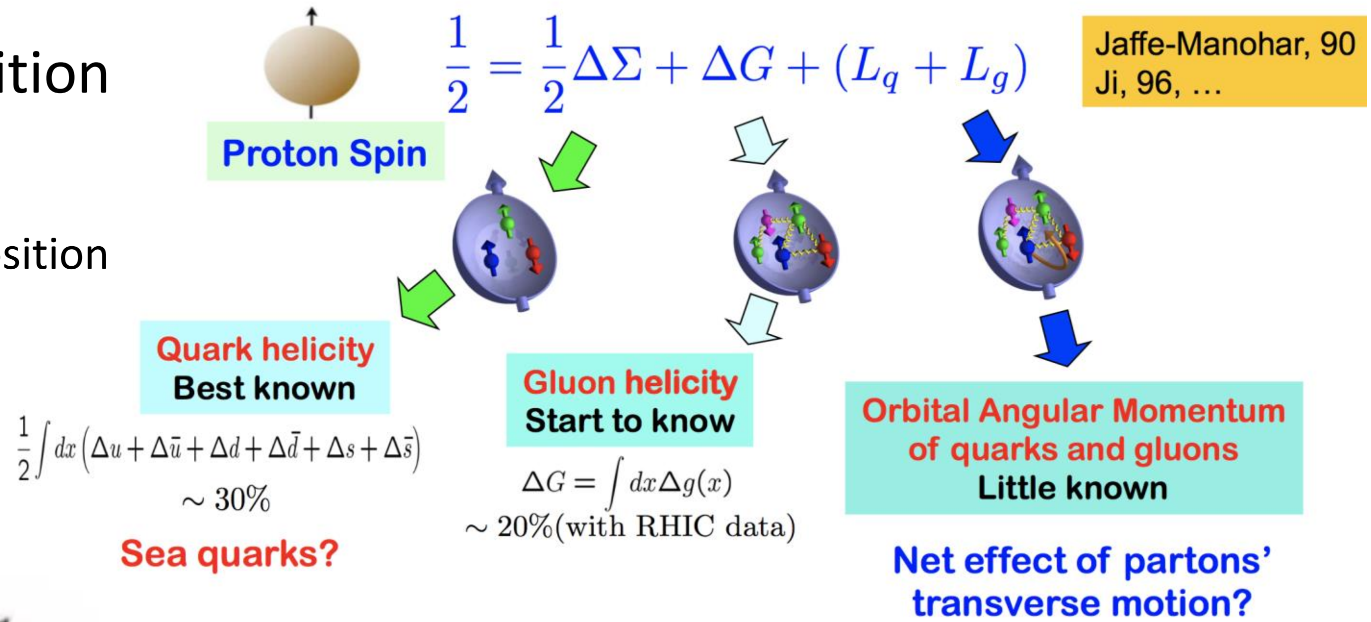
- Include the first and second Fock sector

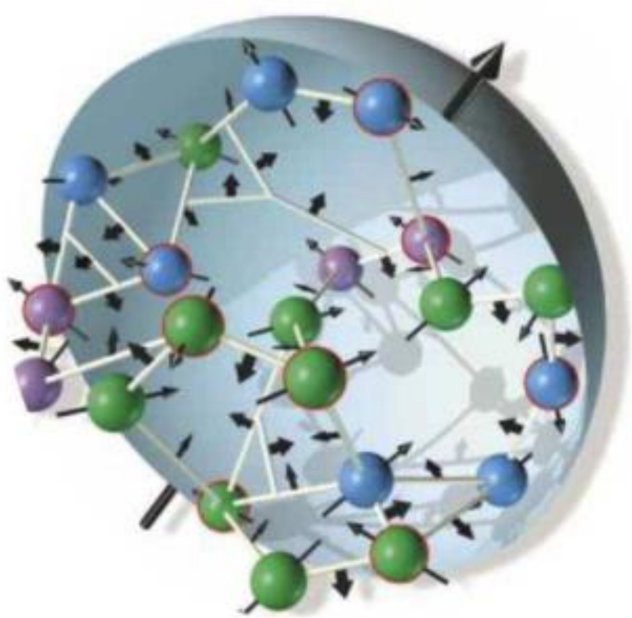
$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2} j^+ \frac{1}{(i\partial^+)^2} j^+$$

Angular Momentum Distributions

- Spin decomposition

Jaffe-Manohar Decomposition





In the quark model $\Delta\Sigma = 1$
The spin decomposition can be measured by polarized DIS

- Ji decomposition:

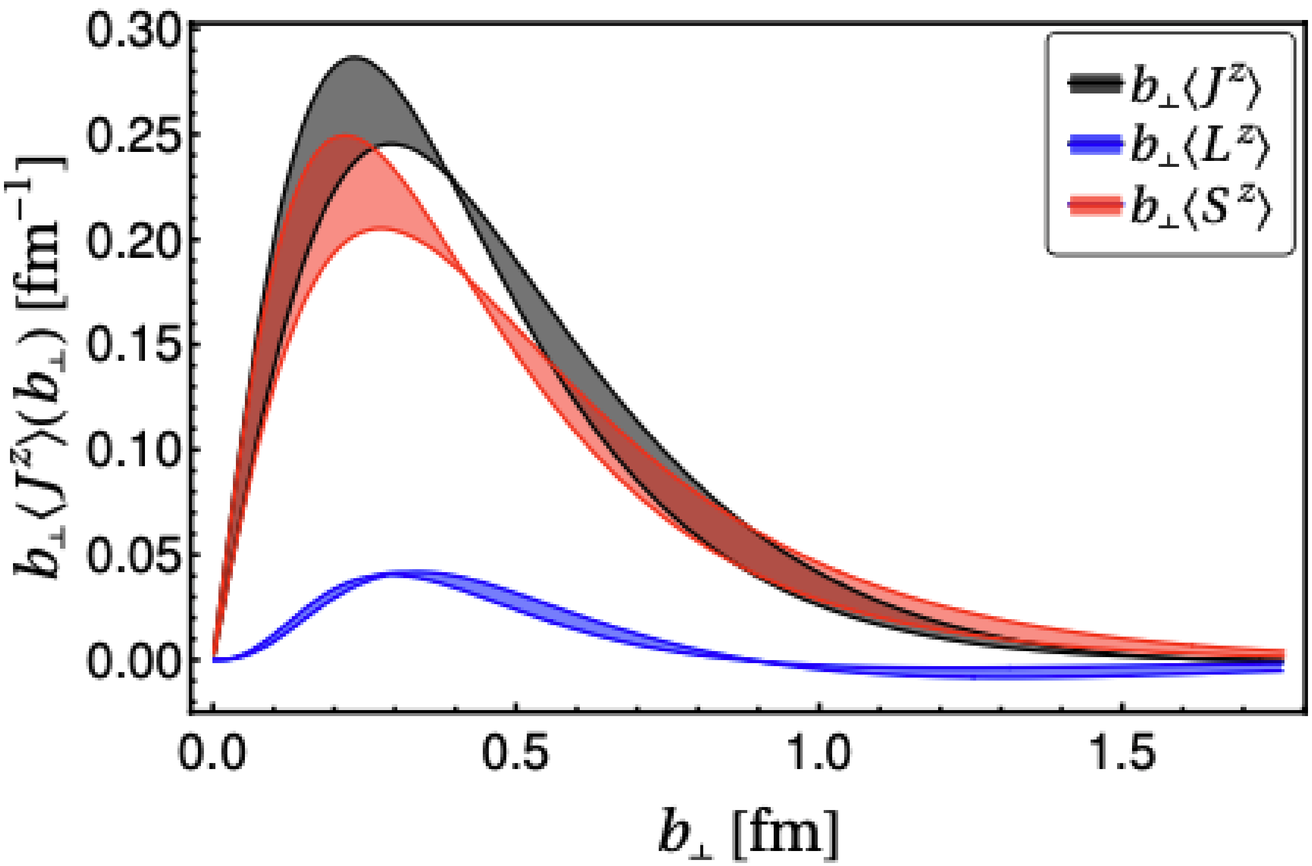
$$\frac{1}{2} = \frac{1}{2}\,\Delta\Sigma + L_{Ji}^{q} + J_g$$

Angular Momentum Distributions

- Total angular momentum density: [In preparation, Ping Yi, Siqi Xu, C. Mondal *et.al*]

$$\langle J^z\rangle(b_\perp) = \langle L^z\rangle(b_\perp) + \langle S^z\rangle(b_\perp)$$

$$\langle L^z\rangle(b_\perp) = -i\epsilon^{3jk}\int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \vec{b}_\perp} \frac{\partial\langle T^{+k}\rangle}{\partial\Delta_\perp^j} \quad , \qquad \langle S^z\rangle(b_\perp) = \frac{s^z}{2}\int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \vec{b}_\perp} G_A(-\vec{\Delta}_\perp^2)$$



$\Delta\Sigma_q \approx 0.92$

$\Delta\Sigma_\mathrm{u} \approx 1.18$

$\Delta\Sigma_\mathrm{d} \approx -0.26$

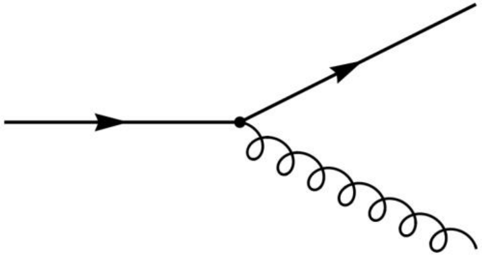
Quark Helicity > 90%

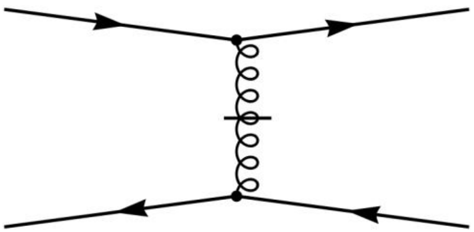
No Gluon contribution

Light-Front QCD Hamiltonian

$$|P_{baryon}\rangle = \psi_1|qqq\rangle + \psi_2|qqqg\rangle$$

$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^{\mu}T^{a}\,\psi\,A_{\mu}^{a} + \frac{g^2 C_F}{2}\,j^{+}\frac{1}{(i\partial^{+})^2}j^{+}$$





$$N_{\max} = 9, K = 16.5$$

Higher Fock Sector effect

Remove Soft Gluon effect

m_u	m_d	κ	m_g	m_{int}	b_{inst}	b	g
0.30 GeV	0.25 GeV	0.54 GeV	0.50 GeV	1.80 GeV	3.00 GeV	0.70 GeV	2.40

Different Mass
Asymmetry of u and d

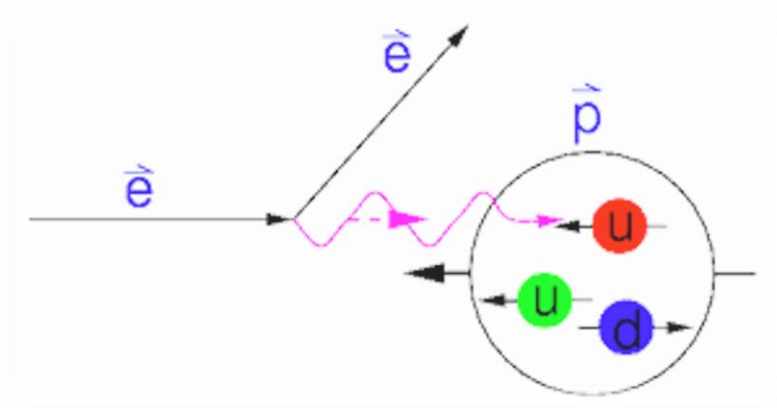
UV Cutoff
In Instantaneous term

Electromagnetic Form Factor

- Elastic scattering of proton

[*R. Hofstadter, Nobel Prize 1961*]

$$e(p) + h(P) \rightarrow e(p') + h(P')$$

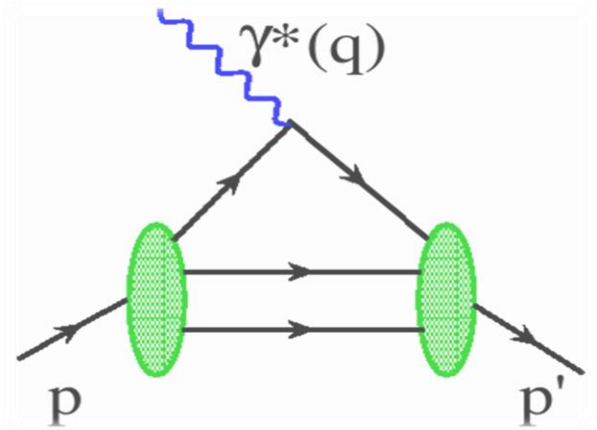


- Elastic electron scattering established the extended nature of the proton (proton radius).

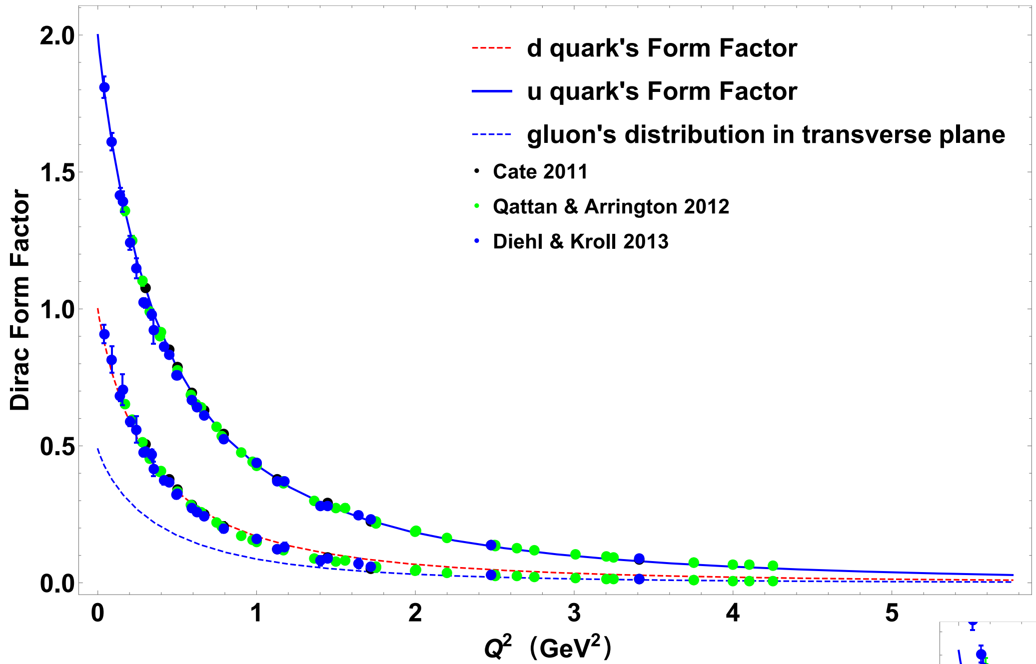
The Fourier transformation of these form factors provide spatial distributions (*charge and magnetization distributions*).

$$\langle N(p')|J^\mu(0)|N(p)\rangle =$$
$$\bar{u}(p')\left[\gamma^\mu \underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu \underbrace{F_2(q^2)}\right] u(p)$$

Dirac Form Factor — Pauli Form Factor



Form Factor with Dynamical Gluon



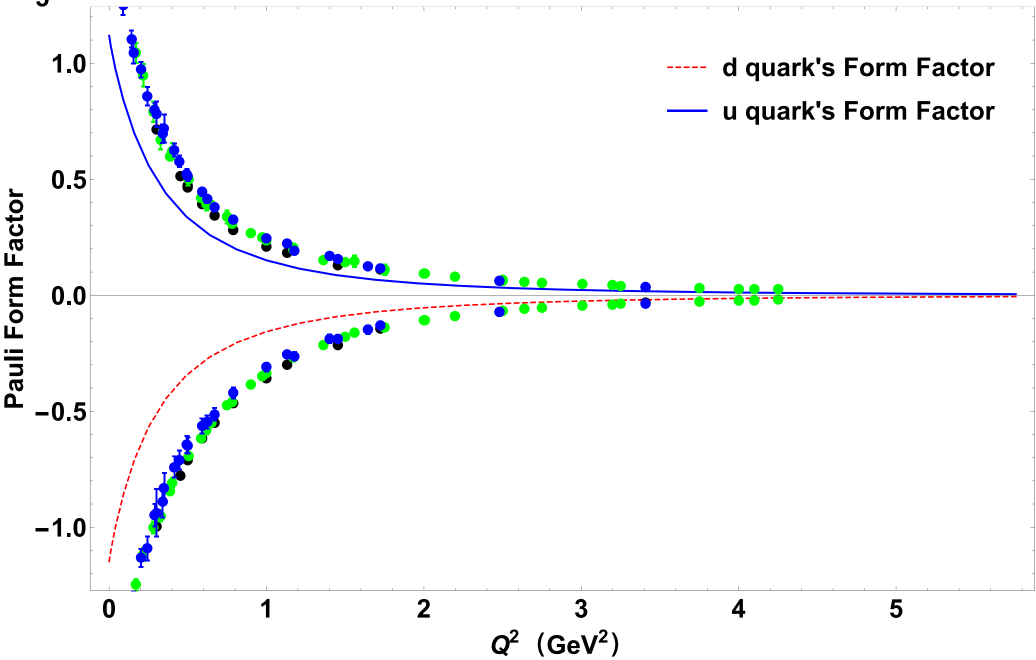
[In preparation, Siqi Xu, C. Mondal *et.al*]

Including the One Dynamical Gluon Fock Sector, the valence quark distributions are almost same with effective interaction results

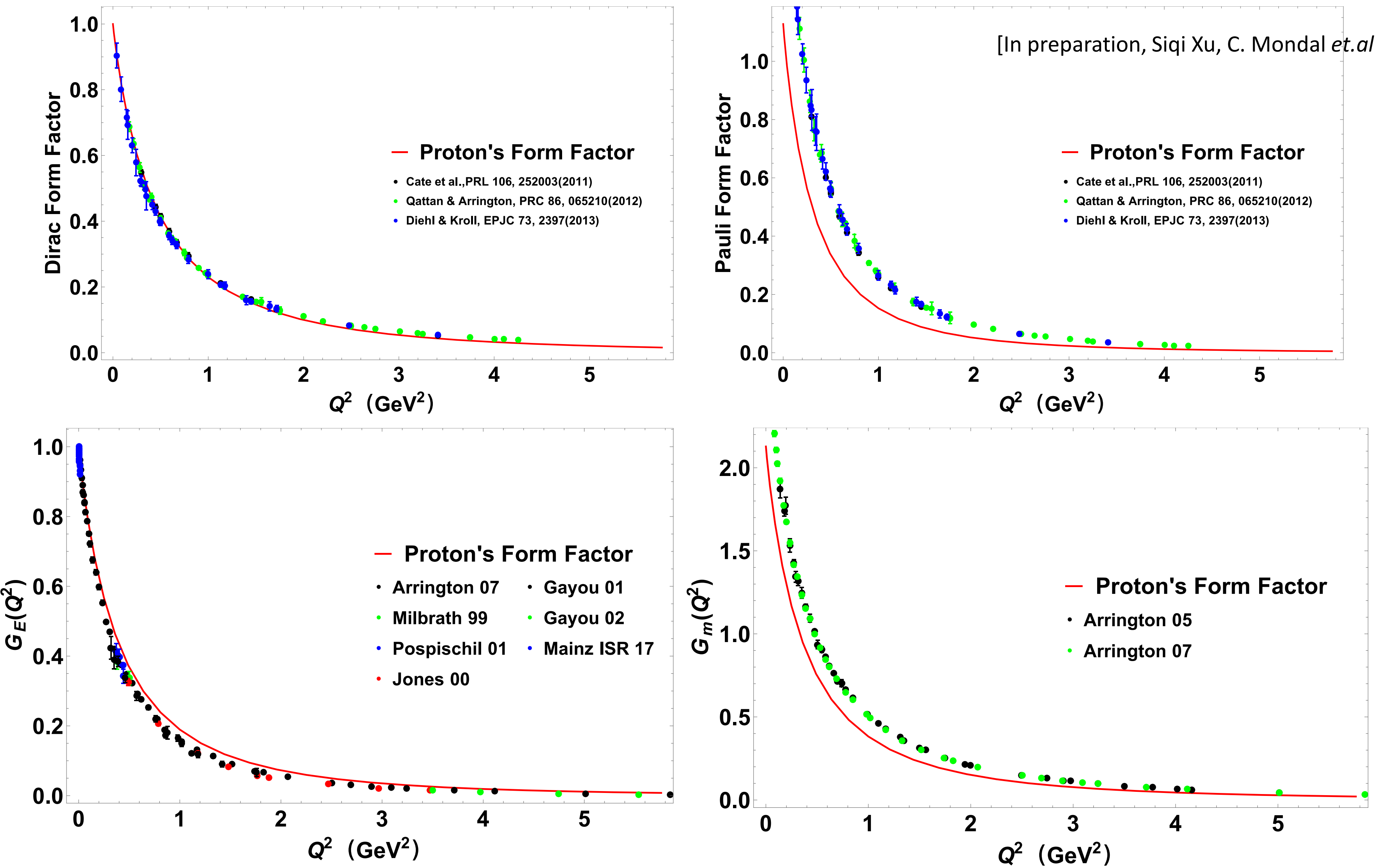
the gluon distributions are always smaller than the d quarks distributions.

$$R = -\frac{F_1(Q^2)}{Q^2}\bigg|_{Q^2=0} = \begin{cases} 0.375\ \mathrm{fm} & \text{u quark} \\ 0.341\ \mathrm{fm} & \text{d quark} \\ 0.368\ \mathrm{fm} & \text{gluon} \end{cases}$$

R is a variable related to the radii of particle distribution



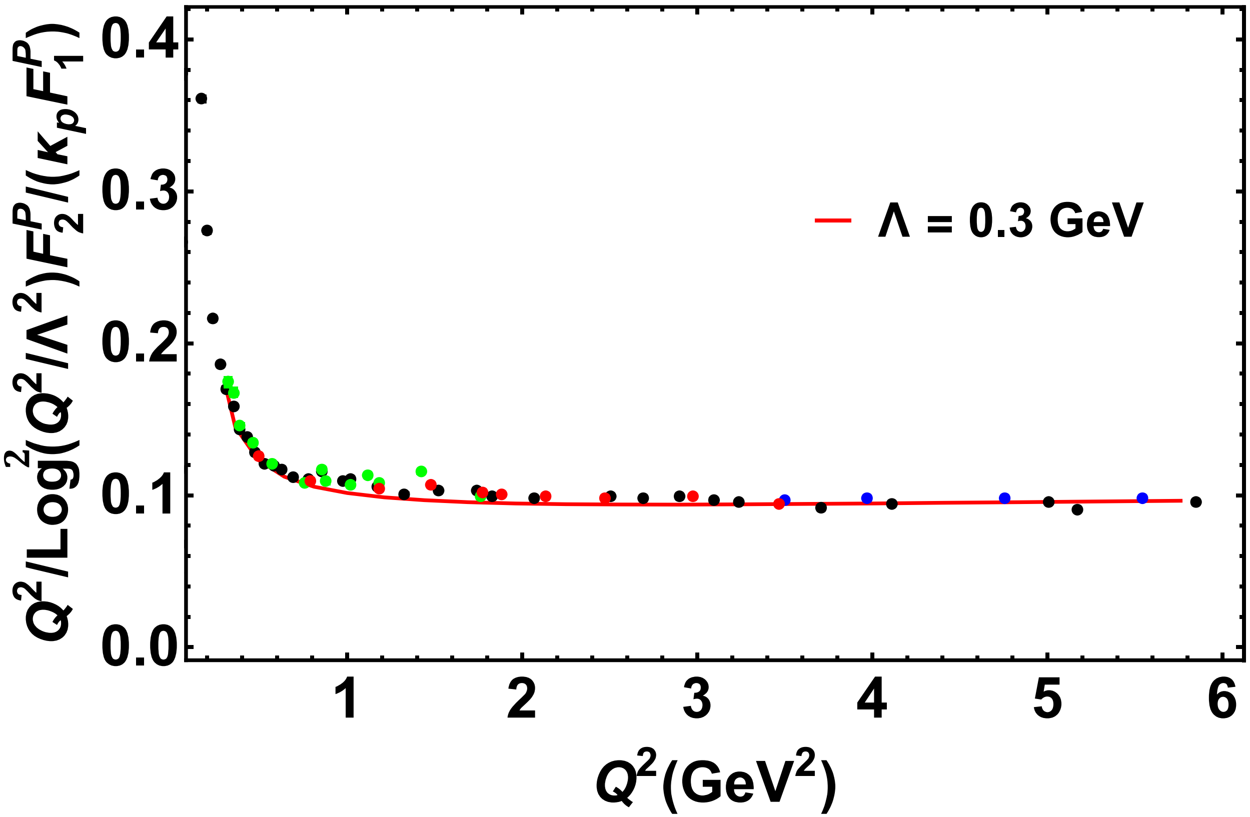
Form Factor with Dynamical Gluon



Including the One Dynamical Gluon Fock Sector, the valence quark distributions are almost same with effective interaction results

Form Factor with Dynamical Gluon

[In preparation, Siqi Xu, C. Mondal *et.al*]



At $Q^2 \gg m^2 = 0.09\ \mathrm{GeV}^2$, we find our Form Factor ratio is proportional to $\log^2(Q^2/\Lambda)/Q^2$.

In our calculation, we use the quark mass around $0.3\ \mathrm{GeV}$, and fix the proton mass around 0.94 GeV

Nucleon Observable

- Nucleon radii and magnetic moment [In preparation, Siqi Xu, C. Mondal *et.al*]

Quantity	BLFQ (no gluon)[c]	BLFQ (gluon)	Measurement[a]	Lattice[b]
μ_p	2.443 ± 0.027	2.228	2.79	2.43(9)
r_E^P [fm]	0.802 ± 0.04	0.847	0.833 ± 0.01	0.742(13)
r_M^P [fm]	0.834 ± 0.029	0.88	0.851 ± 0.026	0.710(26)

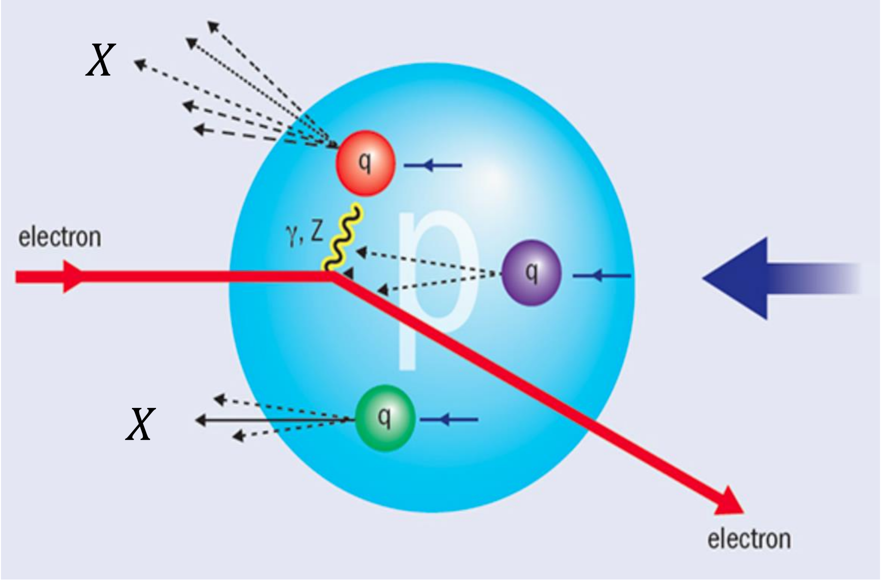
a. C. Alexandrou et al. Phys. Rev. D 96, no. 11, 114509

b. M.Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no.3, 030001

c. 2108.03909 [hep-ph], Siqi Xu, C. Mondal, *et al.* , accepted by PRD

Parton Distribution Functions (PDF)

- **Deep Inelastic Scattering (SLAC 1968)**



$$e(p) + h(P) = e'(p') + X(P')$$

✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1 \text{ fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1 \text{ fm}$$

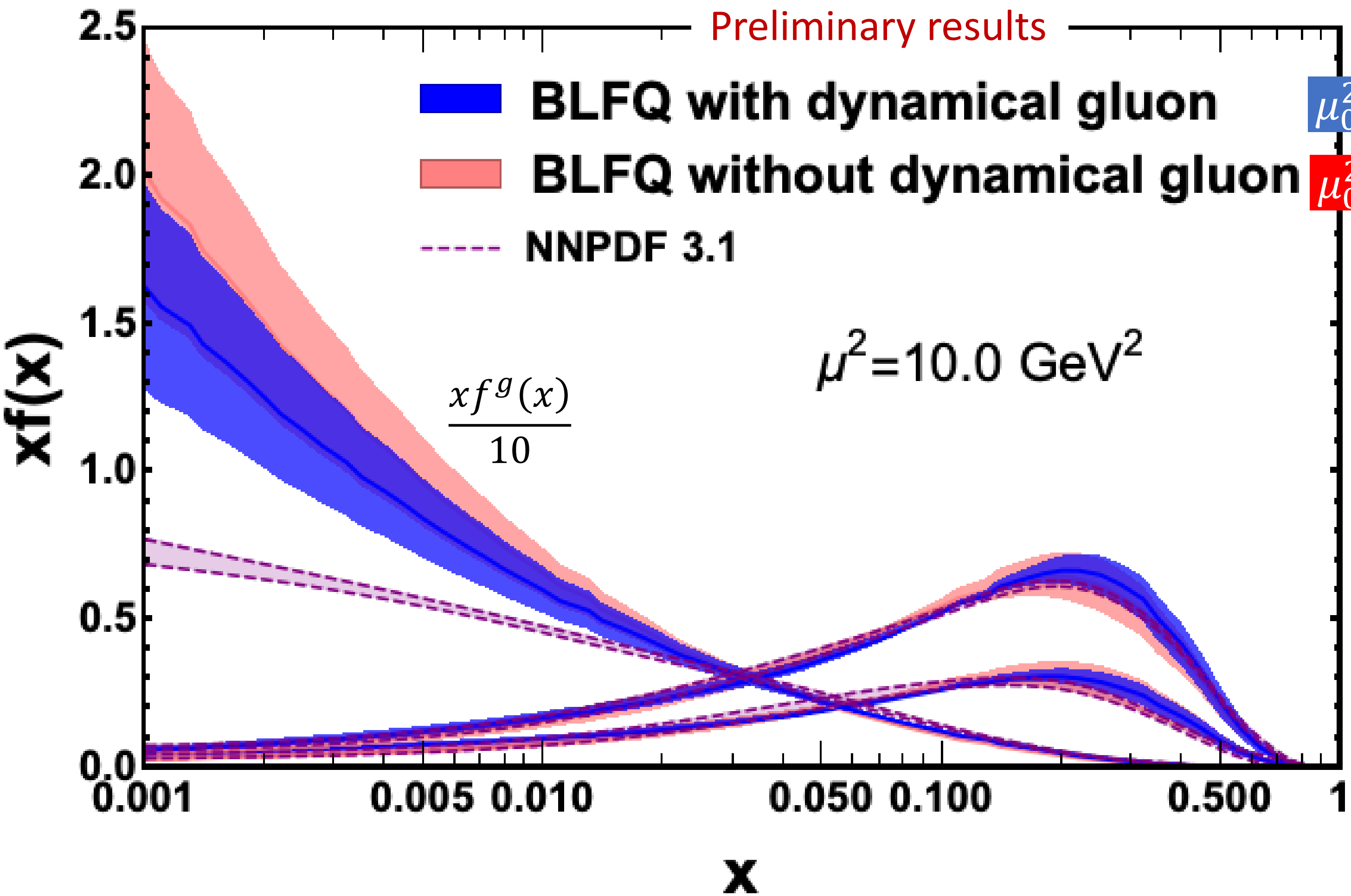
Discovery of spin ½ quarks and partonic structure

- **Parton distribution functions** (PDFs) are extracted from DIS processes.

$$\Phi^{[\gamma^+]}(x, Q^2) = \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2} \langle P, \Lambda | \bar{\psi}(z)\gamma^+\psi(0) | P, \Lambda \rangle$$

PDFs encode the distribution of longitudinal momentum and polarization carried by the constituents

Unpolarized Parton Distribution Functions



$\mu_0^2 = 0.25 \pm 0.02\ \mathrm{GeV}^2$

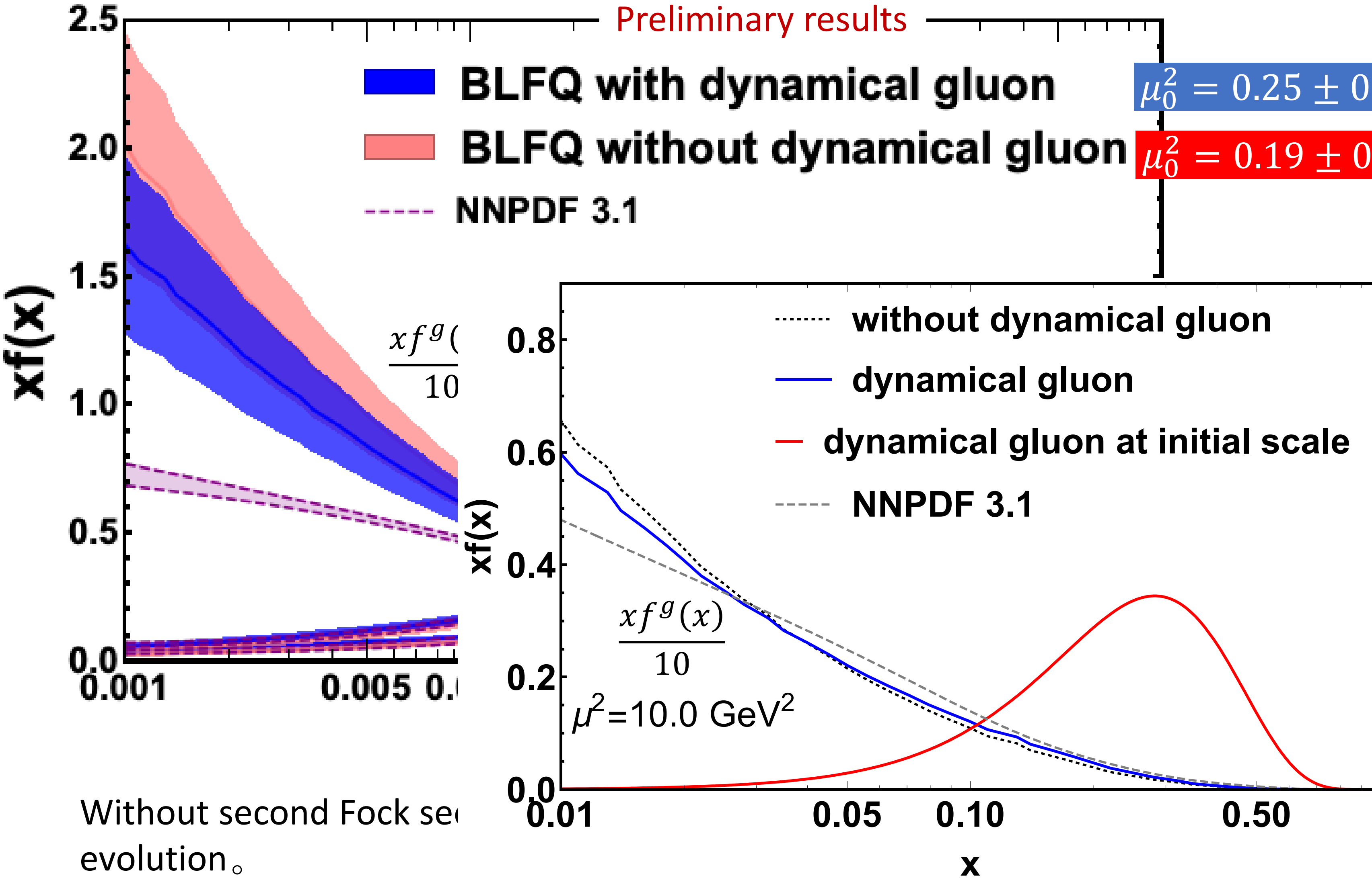
$\mu_0^2 = 0.19 \pm 0.02\ \mathrm{GeV}^2$

Without second Fock sector $|qqqg\rangle$, the gluon is generated dynamically from the DGLAP evolution。

Including the One Dynamical Gluon Fock Sector, the gluon distribution is closer to the global fit.

[Work in progress, C. Mondal, Siqi Xu, *et.al*]   [EPJC 77 (2017) 663]

Unpolarized Parton Distribution Functions



$\mu_0^2 = 0.25 \pm 0.02\ \mathrm{GeV}^2$

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Without second Fock se ... GLAP evolution。

Including the One Dynamical Gluon Fock Sector, the gluon distribution is closer to the global fit.

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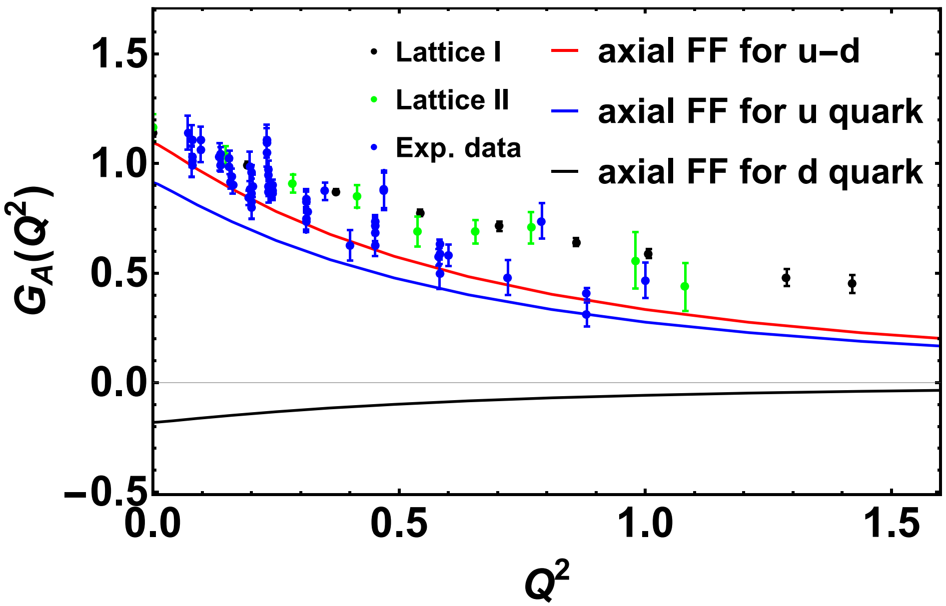
Axial Form Factor of The Proton

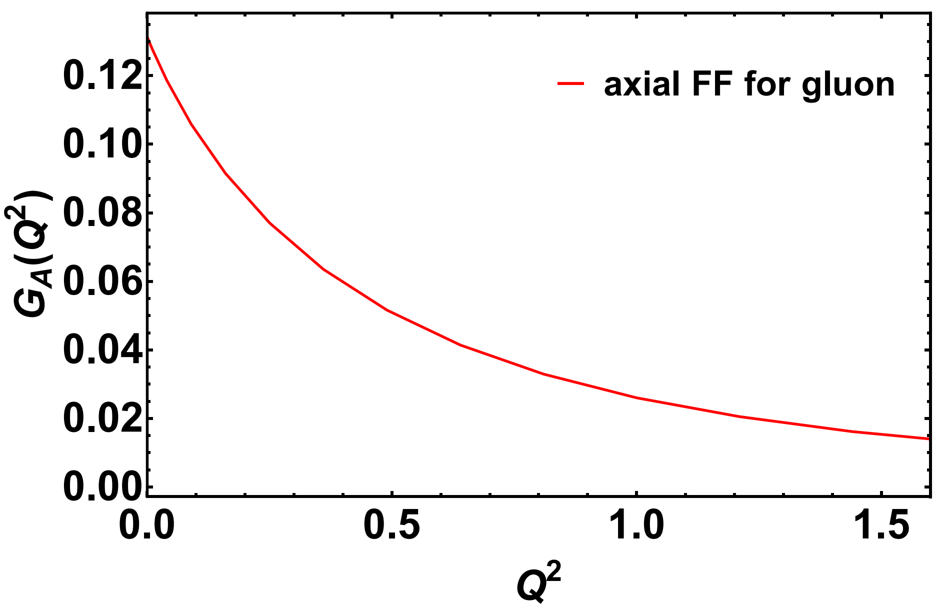
- Provide information on spin-isospin distributions

$$\langle N(p')|A^a_\mu|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu G_A(t) + \frac{(p'-p)_\mu}{2m} G_P(t)\right]\gamma_5 \frac{\tau^a}{2} u(p) \qquad A^a_\mu = \bar{q}\gamma_\mu\gamma_5 T^a q$$

Including the dynamic gluon, the u quark's contribution is suppressed and closer to the experimental data results.

$\Delta\Sigma_q \approx 0.7$ $\Delta\Sigma_u \approx 0.86$ $\Delta\Sigma_d \approx 0.16$ $\Delta G \approx 0.13 < 0.2$ (COMPASS)





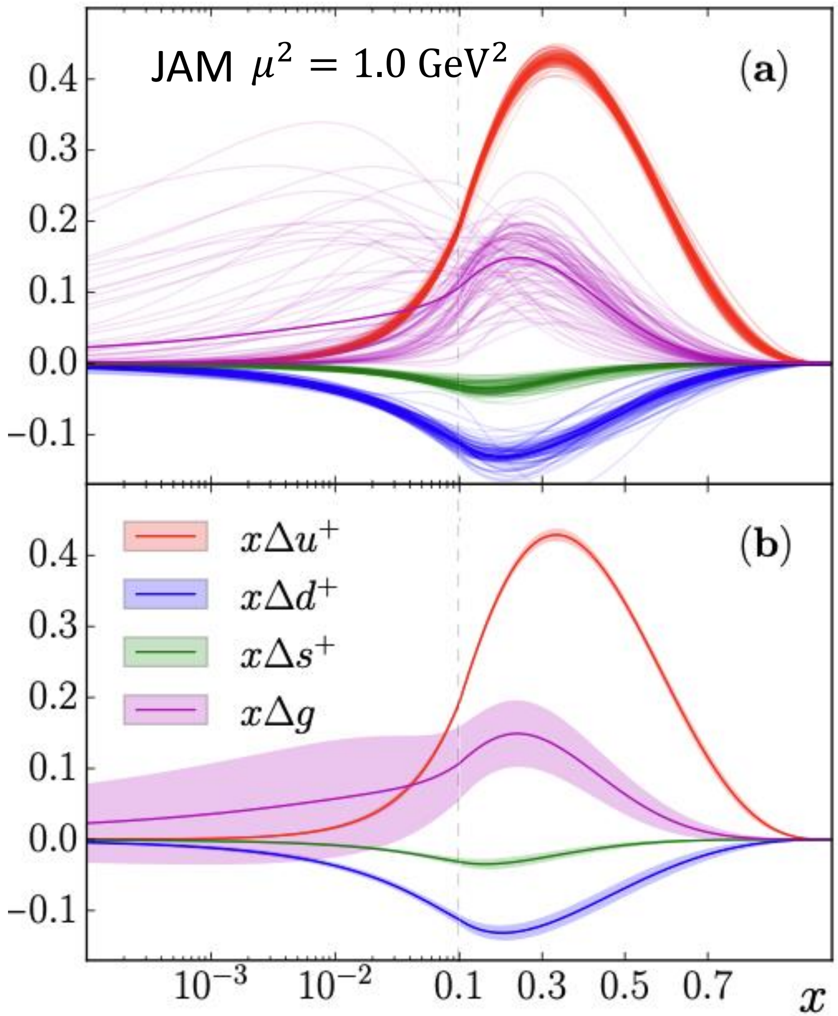
[Chandan Mondal, EPJC 2017] [In preparation, Siqi Xu, C. Mondal *et.al*] [COMPASS, EPJC 77 (2017) 209]

Prediction of Other Approach

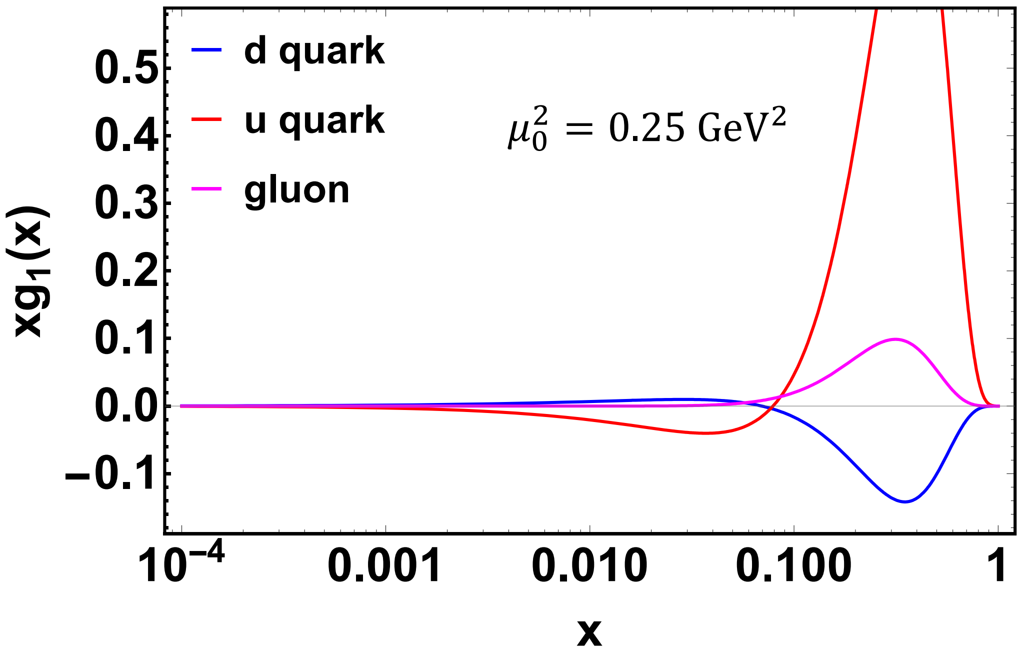
[Alexandre Deur *et al* 2019 *Rep. Prog. Phys.* **82** 076201]

Reference	Q^2 (GeV^2)	$\Delta\Sigma$	Remarks
—	—	1	Naive quark model
[109]	—	0.75 ± 0.05	Relativistic quark model
[107]	—	0.58 ± 0.12	Ellis-Jaffe SR
[106]	—	0.60	Quark parton model
[113]	**10.7**	$\mathbf{0.14 \pm 0.23}$	**EMC**
[109]	**10.7**	$\mathbf{0.01 \pm 0.29}$	**EMC (Jaffe-Manohar analysis)**
[414]	—	0.30	Skyrme model
[415]	—	0.09	Instanton model
[271]	**10**	$\mathbf{0.28 \pm 0.16}$	**SMC**
[255]	**—**	$\mathbf{0.41 \pm 0.05}$	**Global analysis**
[268]	**3**	$\mathbf{0.33 \pm 0.06}$	**E143**
[32]	**10**	$\mathbf{0.31 \pm 0.07}$	**BBS**
[416]	—	0.37	χ quark model
[299]	**1**	$\mathbf{0.5 \pm 0.1}$	**Global fit**
[123]	**4**	**0.168**	**GRSV 1995**
[267]	**2**	$\mathbf{0.39 \pm 0.11}$	**E142**
[256]	**5**	$\mathbf{0.20 \pm 0.08}$	**E154**
[302]	**4**	**0.342**	**LSS 1997**
[417]	—	0.4	Relativistic quark model
[300]	**1**	$\mathbf{0.45 \pm 0.10}$	**ABFR 1998**
[309]	**5**	$\mathbf{0.26 \pm 0.02}$	**AAC 2000**
[257]	**5**	$\mathbf{0.23 \pm 0.07}$	**E155**
[316]	**5**	**0.197**	**StandardGRSV2000**
		0.273	**SU(3)$_f$ breaking**
[336]	4	0.282	Stat. model
[304]	**1**	$\mathbf{0.21 \pm 0.10}$	**LSS 2001**
[301]	**4**	**0.198**	**ABFR 2001**
[418]	**5**	$\mathbf{0.16 \pm 0.08}$	**Global analysis**
[375]	**4**	**0.298**	**BB 2002**
[310]	**5**	$\mathbf{0.213 \pm 0.138}$	**AAC 2003**
[367]	**5**	$\mathbf{0.35 \pm 0.08}$	**Neutron (^{3}He) data (section 6.9.1)**
[282]	**5**	$\mathbf{0.169 \pm 0.084}$	**Proton data (section 6.9.1)**
[419]	—	0.366	χ Quark soliton model
[124, 420]	∞	0.33	Chiral quark soliton model. $n_f = 6$
[311]	**5**	$\mathbf{0.26 \pm 0.09}$	**AAC 2006**
[274]	**5**	$\mathbf{0.330 \pm 0.039}$	**HERMES Glob. fit**
[272]	**10**	$\mathbf{0.35 \pm 0.06}$	**COMPASS**
[312]	**5**	$\mathbf{0.245 \pm 0.06}$	**AAC 2008**

Helicity Parton Distribution Functions



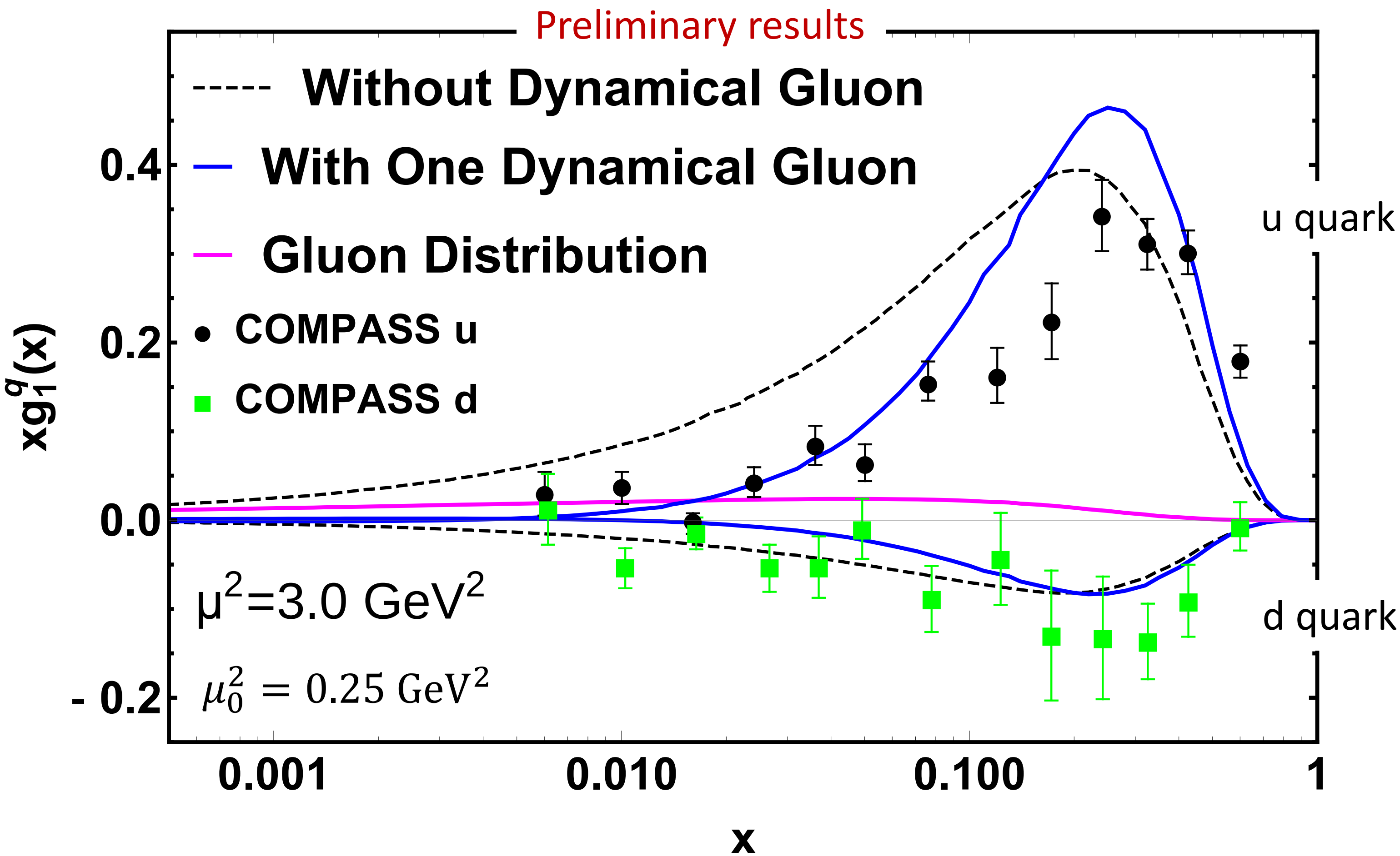
Preliminary results



Including the One Dynamical Gluon Fock Sector, the valence quark distributions at small x region are improved.

[In preparation, Siqi Xu, C. Mondal *et.al*]

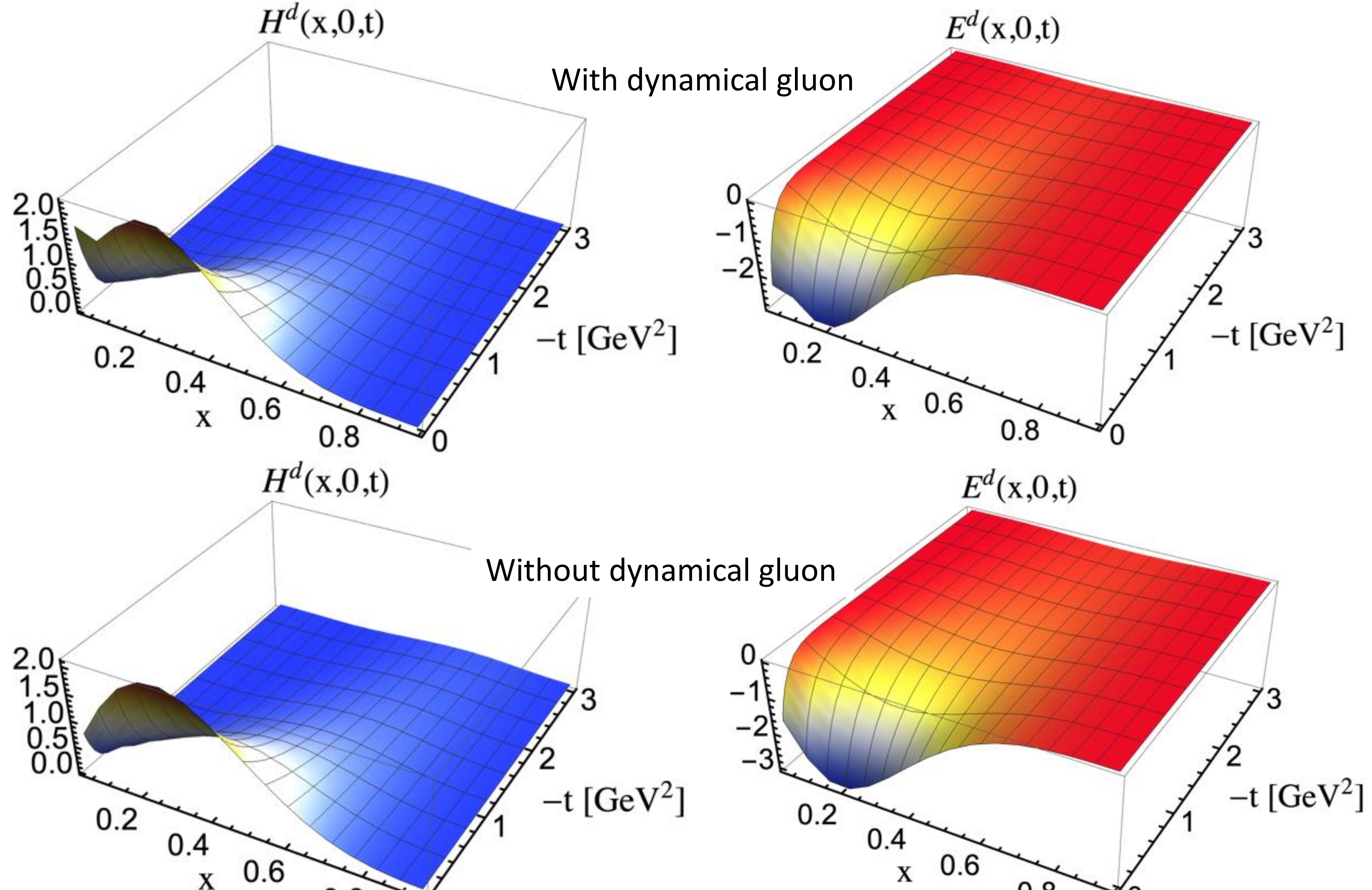
Helicity Parton Distribution Functions



- Without second Fock sector $|qqqg\rangle$, the gluon is generated dynamically from the DGLAP evolution。
- Including the One Dynamical Gluon Fock Sector, the valence quark distributions at small x region and x larger than 0.5 region are improved.

[In preparation, Siqi Xu, C. Mondal *et.al*]

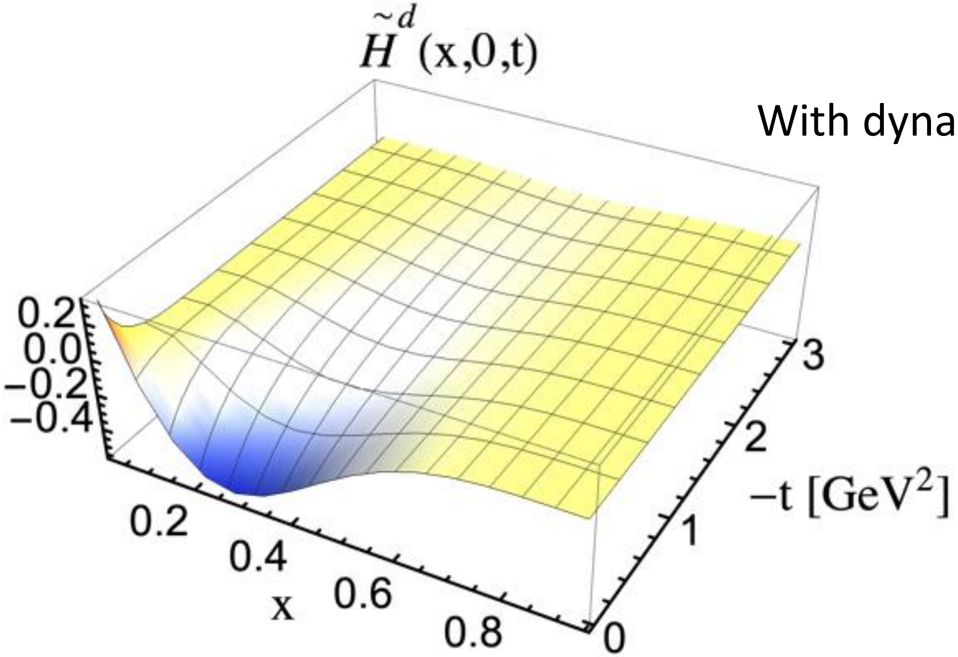
Generalized Parton Distribution Functions (GPD)

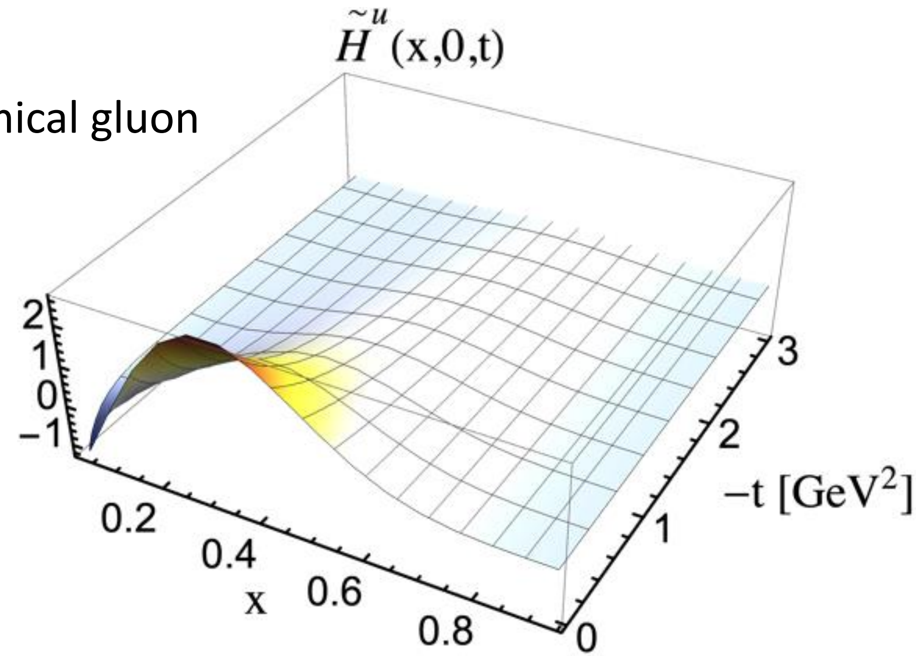


[In preparation, Siqi Xu, C. Mondal *et.al*]

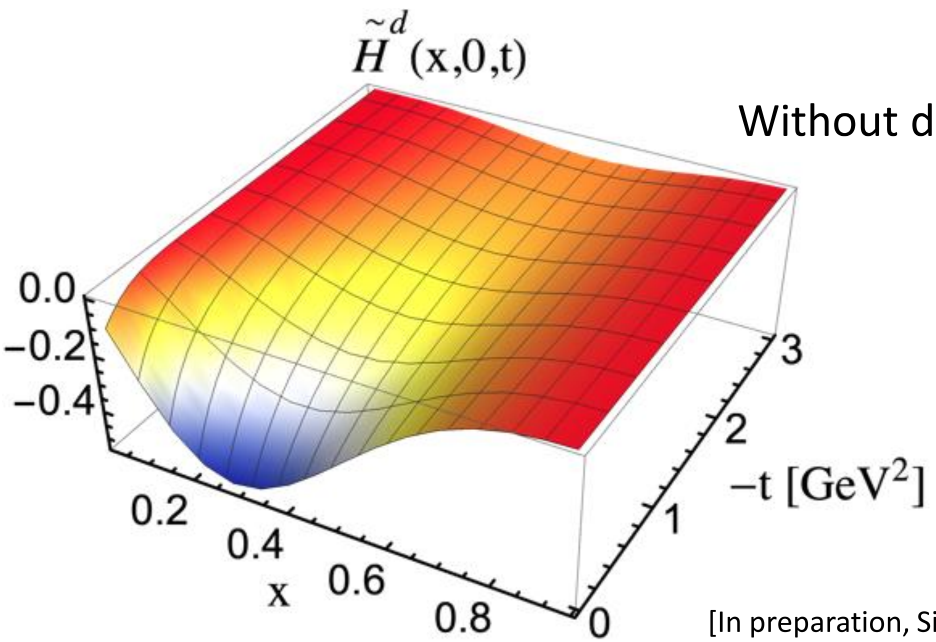
Generalized Parton Distribution Functions (GPD)

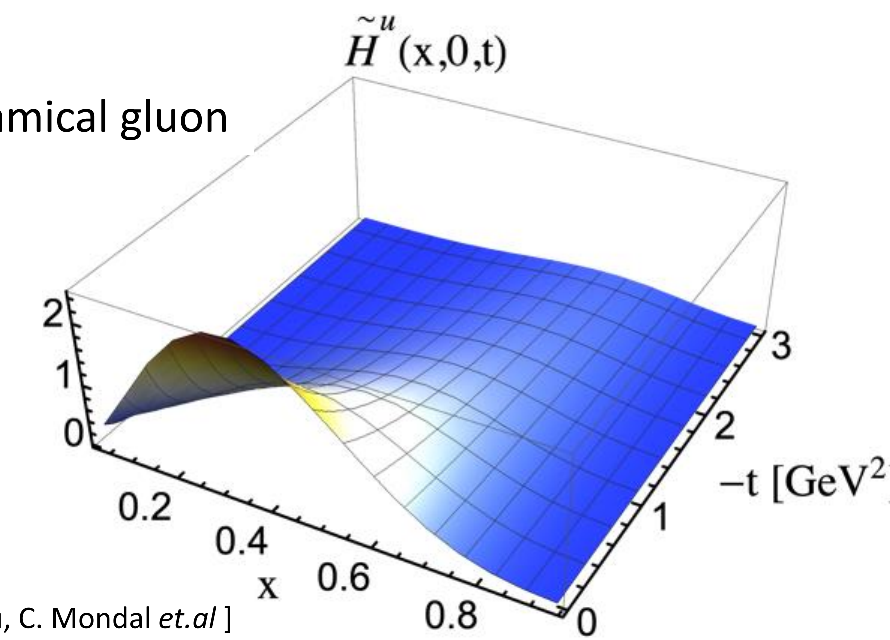
With dynamical gluon





Without dynamical gluon

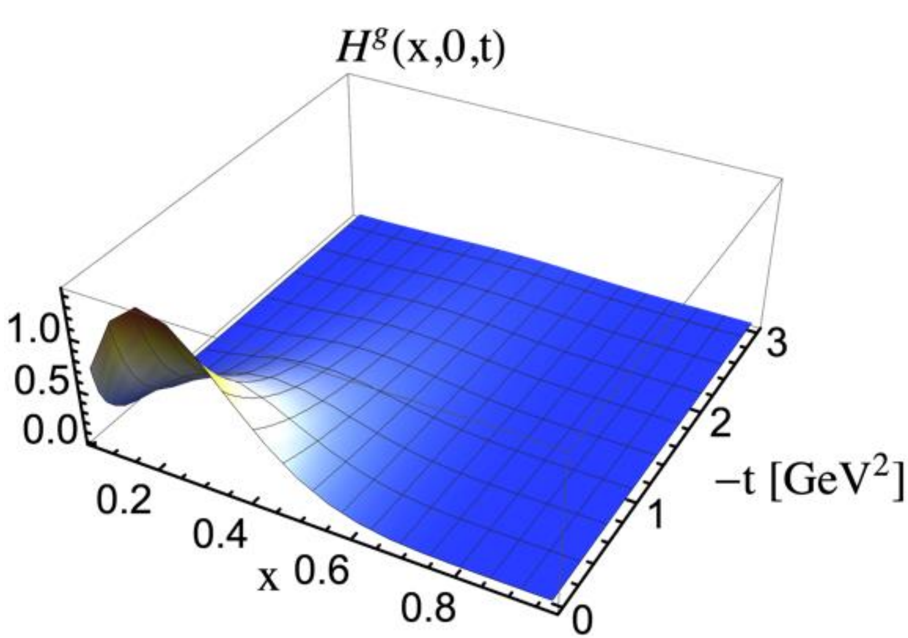


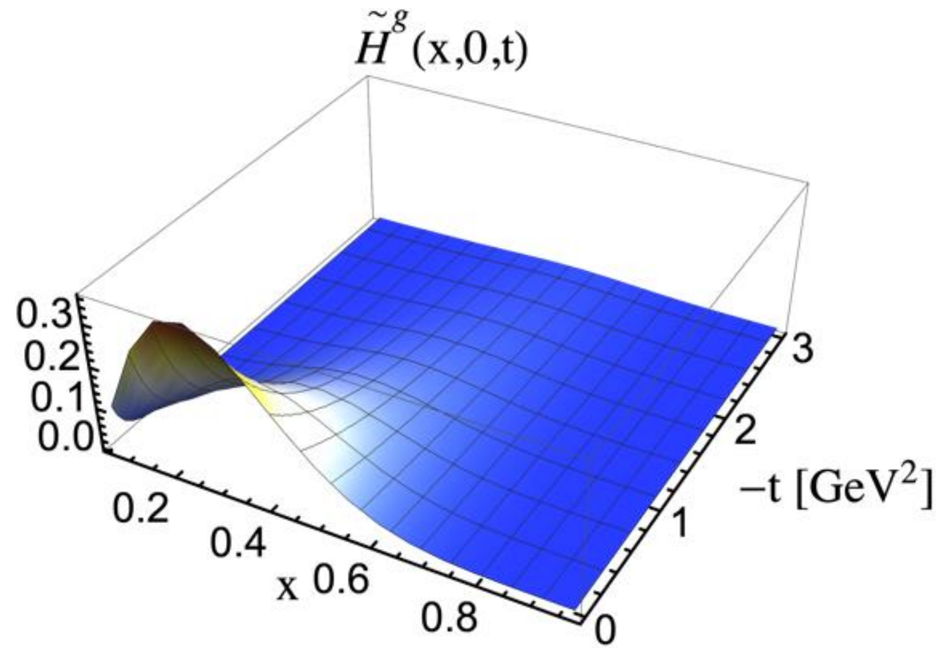


[In preparation, Siqi Xu, C. Mondal *et.al*]

Generalized Parton Distribution Functions (GPD)

➤ Generalized Parton Distribution Functions For Gluon





Including the One Dynamical Gluon Fock Sector, we can calculate the gluon distribution at initial scale and Increase the distribution of gluon at large x region.

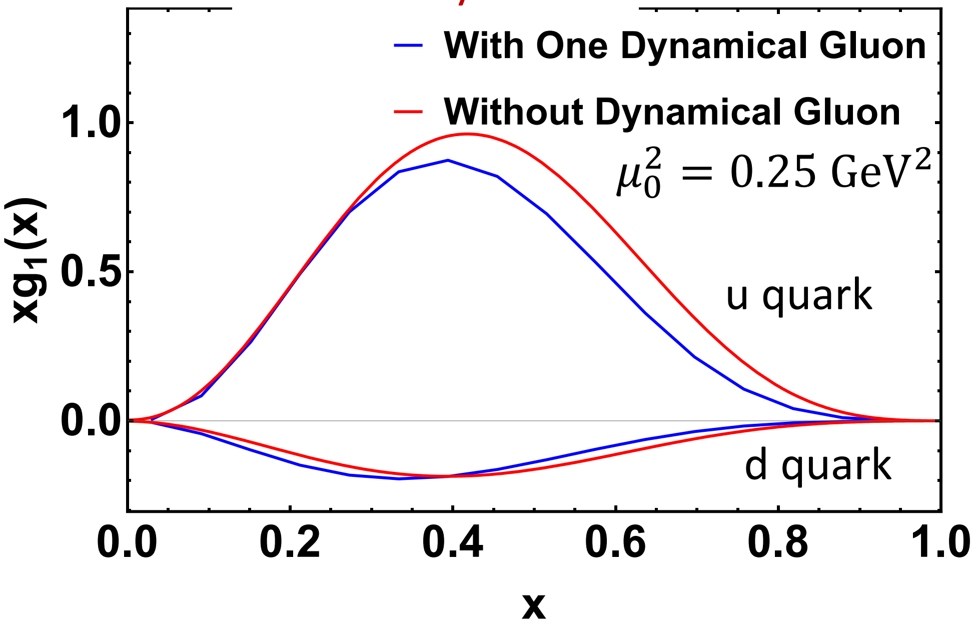
[In preparation, Siqi Xu, C. Mondal *et.al*]

Transversity Parton Distribution Functions

[In preparation, Siqi Xu, C. Mondal *et.al*]

[Phys. Rev. D87, 094019(2013)]

Preliminary results



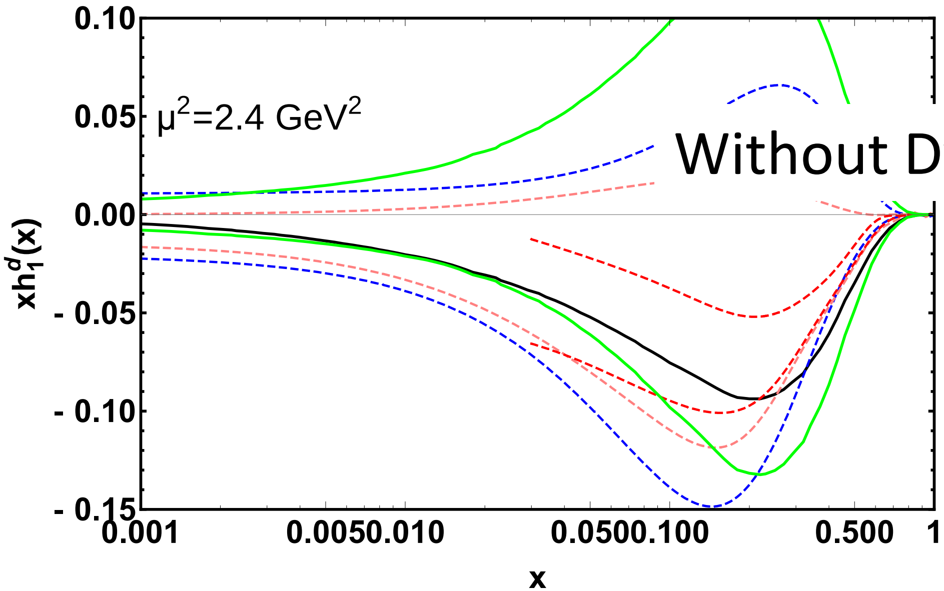
We are still working on the calculation of the gluon transversity PDF.

Due to the gluon distribution, we can't evolve the PDF from initial scale to experimental scale.

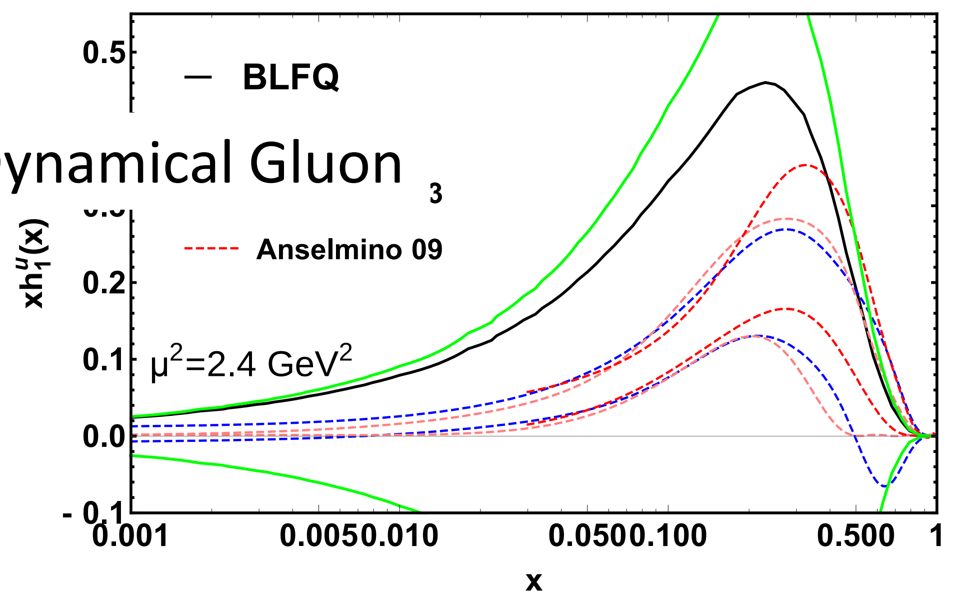
$g_T^u = 0.55, g_T^d = -0.29$ dynamical Gluon

$g_T^u = 0.94, g_T^d = -0.20$ no dynamical Gluon

$g_T^u = 0.39^{+0.18}_{-0.12}, g_T^d = -0.25^{+0.30}_{-0.10}$ Extracted



Without Dynamical Gluon



Conclusion

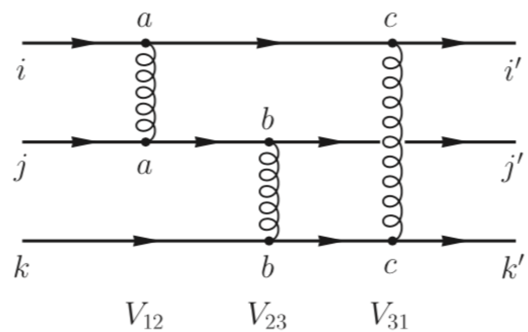
- Light-front Hamiltonian approach: mass spectrum and structure.
- Investigate the structure of the nucleon from the eigenstates of effective Hamiltonians and one dynamical gluon effective Hamiltonians.
- Wavefunctions lead to a good description of various observables such as electromagnetic form factors, PDFs, GPDs, etc.
- While including higher Fock Sectors, the effective interaction is replaced by the QCD vertex function and the gluon distribution can be explored.
- Including the one dynamical gluon Fock Sector, the Axial Form Factor of u quark is suppressed, and the d quark's Axial FF is almost same with only the leading Fock sector case.
- As we include the gluon contribution at initial scale, the gluon distributions are closer to the NNPDF results.

Thank you

Effective Hamiltonian

$$|P_{baryon}\rangle = \psi_1 |qqq\rangle$$

$$\boldsymbol{H_{Interact} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^{\mu} u_{s_i}(k_i) \bar{u}_{s_j'}(k_j')\gamma_{\mu} u_{s_j}(k_j)}$$



$$\boxed{N_{\max} = 10, K = 16.5}$$

$m_{q/k}$	$m_{q/g}$	κ	α_s
0.3 GeV	0.2 GeV	0.34 GeV	1.1 ± 0.1

Note: In our calculation, we fix the basis scale b equal to 0.6 GeV

Nucleon Radii and Axial Charges

- The magnetic moment of the proton and neutron

Quantity	BLFQ	Measurement[a]	Lattice
μ_{p}	2.443 ± 0.027	2.79	$2.43(9)$
μ_{n}	-1.405 ± 0.026	-1.91	$-1.54(6)$

- The radii of the proton and neutron

Quantity	BLFQ	Measurement	Lattice
$r_{\mathrm{E}}^{\mathrm{p}}$ [fm]	$0.802^{+0.042}_{-0.040}$	0.833 ± 0.010	$0.742(13)$
$r_{\mathrm{M}}^{\mathrm{p}}$ [fm]	$0.834^{+0.029}_{-0.029}$	0.851 ± 0.026	$0.710(26)$
$\langle r_{\mathrm{E}}^2 \rangle^{\mathrm{n}}$ [fm^2]	-0.033 ± 0.198	-0.1161 ± 0.0022	$-0.074(16)$
$r_{\mathrm{M}}^{\mathrm{n}}$ [fm]	$0.861^{+0.021}_{-0.019}$	$0.864^{+0.009}_{-0.008}$	$0.716(29)$

- The axial charge and axial radius

Quantity	BLFQ	Extracted data	Lattice
g_{A}^{u}	1.16 ± 0.04	0.82 ± 0.07	$0.830(26)$
g_{A}^{d}	-0.248 ± 0.027	-0.45 ± 0.07	$-0.386(16)$
g_{A}^{u-d}	1.41 ± 0.06	1.2723 ± 0.0023	$1.237(74)$
$\sqrt{\langle r_{\mathrm{A}}^2 \rangle}$ fm	$0.680^{+0.070}_{-0.073}$	0.667 ± 0.12	$0.512(34)$

[2108.03909 [hep-ph], Siqi Xu, C. Mondal, *et.al*], accepted by PRD

The Quark Tensor Charge in The Proton

Quantity	BLFQ	Extracted data	Lattice
g_T^u	$0.94^{+0.06}_{-0.15}$	$0.39^{+0.18}_{-0.12}$	$0.784(28)$
g_T^d	$-0.20^{+0.02}_{-0.04}$	$-0.25^{+0.30}_{-0.10}$	$-0.204(11)$
$\langle x \rangle_T^{u-d}$	$0.229^{+0.019}_{-0.048}$	$-$	$0.203(24)$

- The first moment of the transversitv PDF

$$g_T^q = \int dx\, h_1^q(x, \mu^2). \qquad \mu^2 = 2.4\ \text{GeV}^2$$

The BLFQ predicts the tensor charges for the down quark in good agreement with the global QCD qnqlysis

- The second moment of the transversity PDF

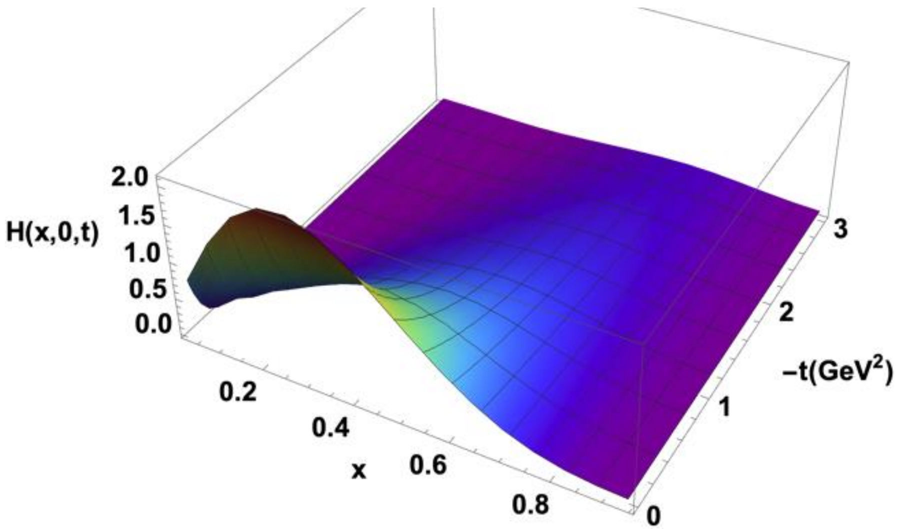
$$\langle x \rangle_T^{u-d} = \int dx\, x \left(h_1^u(x, \mu^2) - h_1^d(x, \mu^2) \right), \qquad \mu^2 = 2.4\ \text{GeV}^2$$

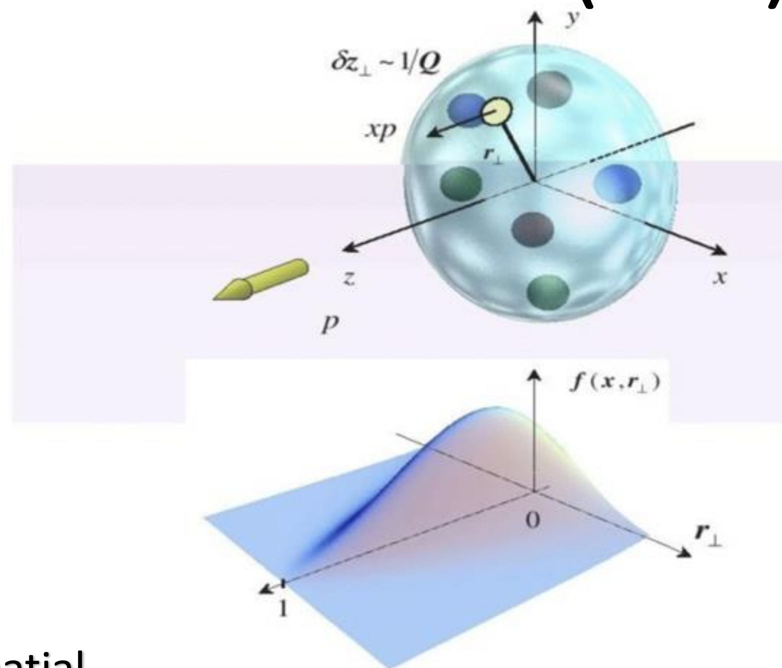
The BLFQ prediction for $\langle x \rangle_T^{u-d}$agrees reasonably well with the lattice data.

[2108.03909 [hep-ph], Siqi Xu, C. Mondal, *et.al*], accepted by PRD

Generalized Parton Distribution Functions (GPD)

[2108.03909 [hep-ph], Siqi Xu, C. Mondal, *et.al*]





Encode the information about three dimensional spatial structure the spin and orbital angular momentum

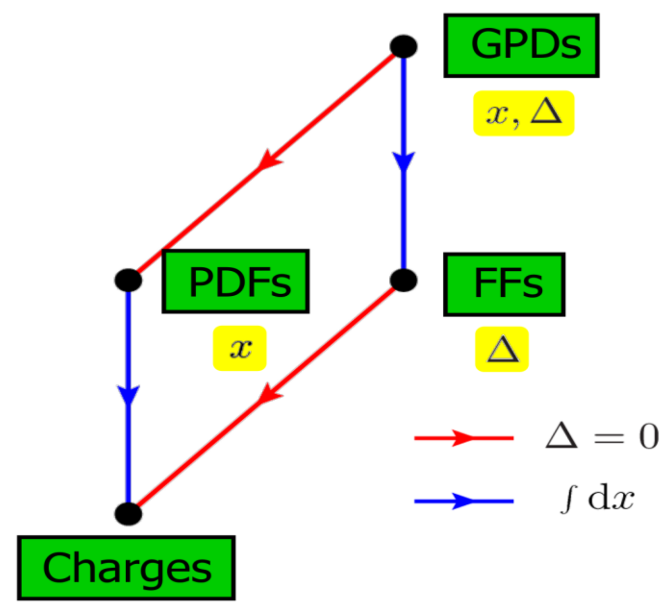
With increasing momentum transfer (t), the peaks of distributions shift to larger x;

$$t = \Delta^2, x = \frac{k^+}{P^+}, \boldsymbol{\zeta = \frac{\Delta^+}{P^+} = 0} \qquad b_\perp \xrightarrow{FT} \Delta_\perp$$

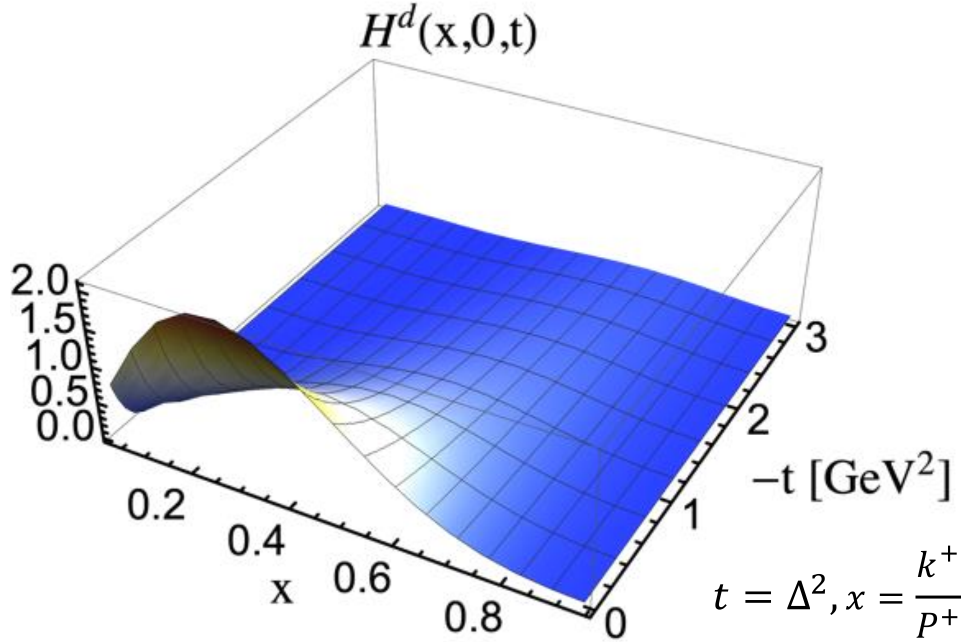
Impact parameter distribution ($b_\perp$):

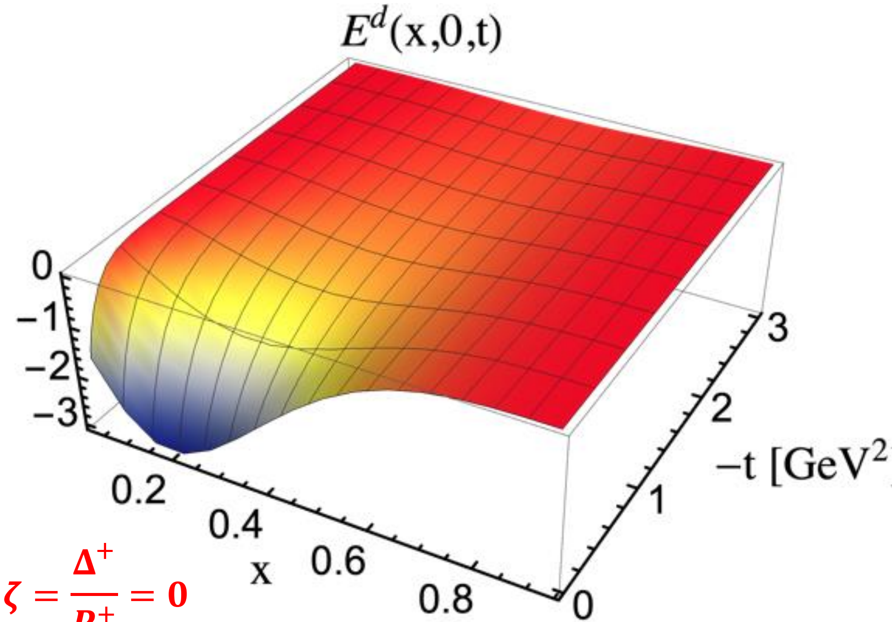
$$\langle b_\perp^2 \rangle^q(x) = -4 \frac{\partial}{\partial \vec{q}_\perp^{\,2}} \ln H^q(x, 0, -\vec{q}_\perp^{\,2}) \Big|_{\vec{q}_\perp = 0}$$

$$\langle b_\perp^2 \rangle(x) = 2e_u \langle b_\perp^2 \rangle^u(x) + e_d \langle b_\perp^2 \rangle^d(x)$$



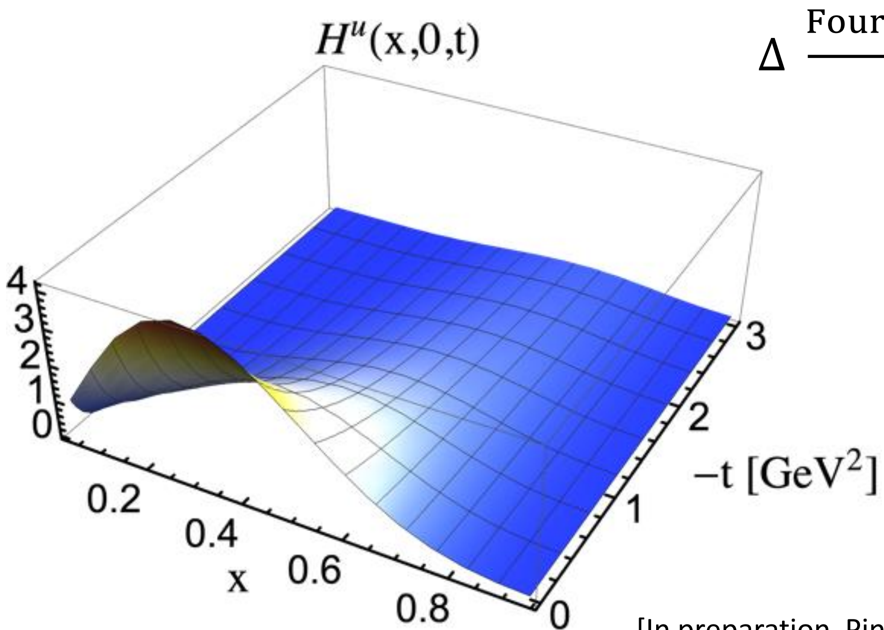
Generalized Parton Distribution Functions (GPD)

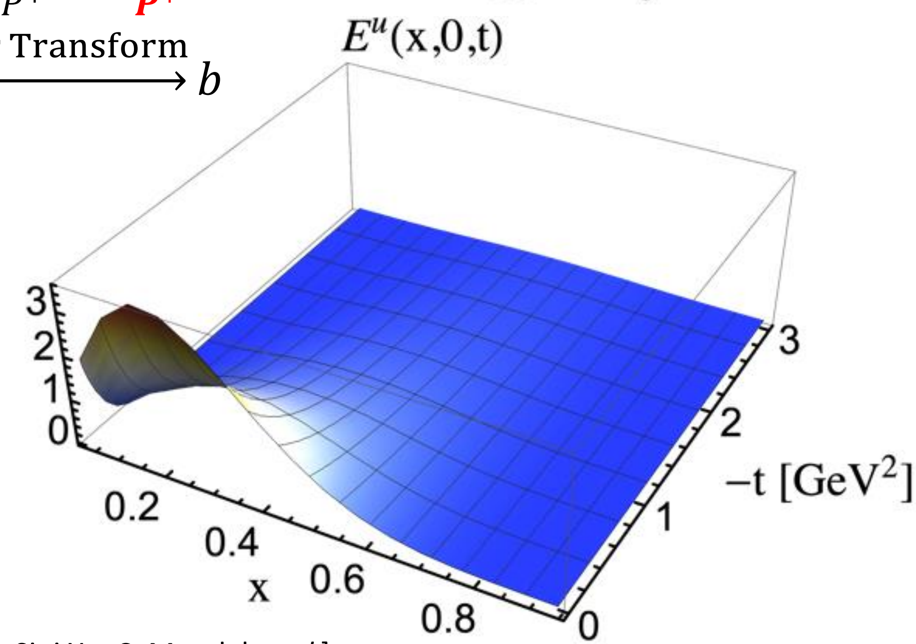




$$t = \Delta^2, x = \frac{k^+}{P^+}, \boldsymbol{\zeta = \frac{\Delta^+}{P^+} = 0}$$

$$\Delta \xrightarrow{\text{Fourier Transform}} b$$





[In preparation, Ping Yi, Siqi Xu, C. Mondal *et.al*]

Angular Momentum Distributions

- Total angular momentum density: [In preparation, Ping Yi, Siqi Xu, C. Mondal *et.al*]

$$\langle J^z\rangle(b_\perp) = \langle L^z\rangle(b_\perp) + \langle S^z\rangle(b_\perp)$$

$$\langle L^z\rangle(b_\perp) = -i\epsilon^{3jk}\int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \vec{b}_\perp} \frac{\partial\langle T^{+k}\rangle}{\partial \Delta_\perp^j} \quad , \qquad \langle S^z\rangle(b_\perp) = \frac{s^z}{2}\int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \vec{b}_\perp} G_A(-\vec{\Delta}_\perp^2)$$

- Belinfante-improved tensors

$$\langle J^z\rangle(b_\perp) = \langle J^z_{Bel}\rangle(b_\perp) + \langle M^z\rangle(b_\perp)$$

Quark Helicity > 90%

$$T^{\mu\nu}_{\rm Bel}(x) = T^{\mu\nu}(x) + \partial_\lambda G^{\lambda\mu\nu}(x),$$

$$J^{\mu\alpha\beta}_{\rm Bel}(x) = J^{\mu\alpha\beta}(x) + \partial_\lambda\left[x^\alpha G^{\lambda\mu\beta}(x) - x^\beta G^{\lambda\mu\alpha}(x)\right],$$