Generalized parton distributions and spin structures of light mesons from a light-front Hamiltonian approach

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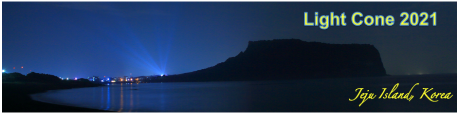
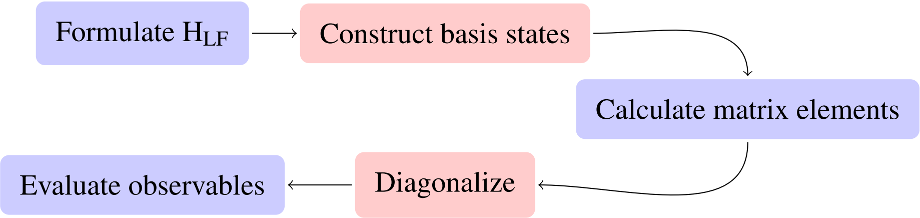
Based on : arXiv-2110.05048 [accepted by PRD]



Nov 30, 2021

Basis Light-Front Quantization (BLFQ)

- Nonperturbative approach
- Solve the light-front eigenvalue equation : $\mathrm{H_{LF}} \mid \psi\rangle = M^2 \mid \psi\rangle$
- General BLFQ algorithm :



—— J. P. Vary, H. Honkanen, J. Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, and C. Yang, Phys. Rev. C 81, 035205 (2010).

—— P. Wiecki, Y. Li, X. Zhao, P. Maris, and J. P. Vary, Phys. Rev. D 91, 105009 (2015).

Effective Hamiltonian : BFLQ-NJL Model

$$|\,\pi\rangle_{\text{phys}} = a\,|\,q\bar{q}\rangle + b\,|\,q\bar{q}g\rangle + c\,|\,q\bar{q}q\bar{q}\rangle + \dots$$

kinetic energy

transverse confining potential

$$H_{\text{eff}} = \frac{\vec{k}^{\perp 2} + m_q^2}{x} + \frac{\vec{k}^{\perp 2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}^{\perp 2}$$

$$+ \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big) + H_{\text{NJL}}^{\text{eff}}$$

longitudinal confining potential

Nambu–Jona-Lasinio (NJL) interaction [1,2]

[1] S. Jia and J. P. Vary, Phys. Rev. C **99**, 035206 (2019).

[2] S. Klimt, M. F. M. Lutz, U. Vogl and W. Weise, Nucl. Phys. A **516**, 429-468 (1990).

Meson Light Front Wave Functions (LFWFs)

- The valence LFWFs in orthonormal bases [1]

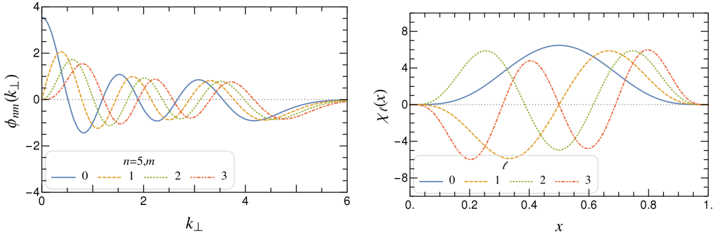
$$\psi_{rs}(x, \vec{\kappa}^{\perp}) = \sum_{n,m,l} \langle n, m, l, r, s | \psi \rangle \times \phi_{nm}\left(\vec{\kappa}^{\perp}\right) \chi_l(x)$$

  - transverse direction

$$\phi_{nm}\left(\vec{\kappa}^{\perp}\right) \sim \left(|\vec{\kappa}^{\perp}|\right)^{|m|} \times \exp\left(-\vec{\kappa}^{\perp 2}\right) L_n^{|m|}\left(\vec{\kappa}^{\perp 2}\right)$$

  - longitudinal direction

$$\chi_l(x) \sim x^{\beta/2}(1-x)^{\alpha/2} P_l^{(\alpha,\beta)}(2x-1)$$



[1] Y. Li, P. Maris and J. P. Vary, Phys. Rev. D **96**, 016022 (2017).

BLFQ-NJL model parameters

- Parameters are fixed to
  - reproduce the ground state masses
  - experimental charge radii of π^+ and the K^+ [1]
- Successfully applied to
  - compute the PDAs and the EMFFs [1]
  - PDFs for the pion and the kaon and pion-nucleus induced Drell-Yan cross sections [2,3]
- Summary of the model parameters [1]

Valence flavor	$N_{\max}$	$L_{\max}$	κ(MeV)	m_q(MeV)	$m_{\bar{q}}$(MeV)
$u\bar{d}$	8	8−32	227	337	337
$u\bar{s}$	8	8−32	276	308	445

[1] S. Jia and J. P. Vary, Phys. Rev. C **99**, 035206 (2019).
[2] J. Lan, C. Mondal, S. Jia, X. Zhao and J. P. Vary, Phys. Rev. D 101, 034024 (2020).
[3] J. Lan, C. Mondal, S. Jia, X. Zhao and J. P. Vary, Phys. Rev. Lett. 122, 172001 (2019).

Generalized Parton Distributions (GPDs) : Spin-0 Meson

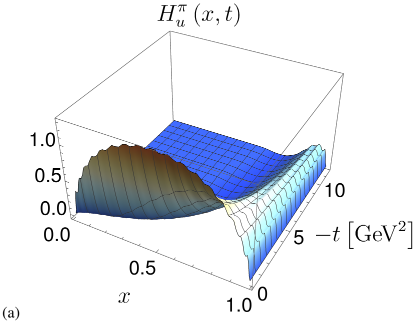
two independent GPDs at leading twist

$$H^{\mathcal{P}}(x,\zeta,t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle \mathcal{P}(P')|\bar{\Psi}_q(0)\gamma^+\Psi_q(z)|\mathcal{P}(P)\rangle|_{z^+=\mathbf{z}^\perp=0}$$

$$\frac{i\epsilon^\perp_{ij} q^\perp_i}{2M_{\mathcal{P}}} E^{\mathcal{P}}_T(x,\zeta,t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle \mathcal{P}(P')|\bar{\Psi}_q(0) i\sigma^{j+}\gamma_5\Psi(z)_q|\mathcal{P}(P)\rangle|_{z^+=\mathbf{z}^\perp=0}$$

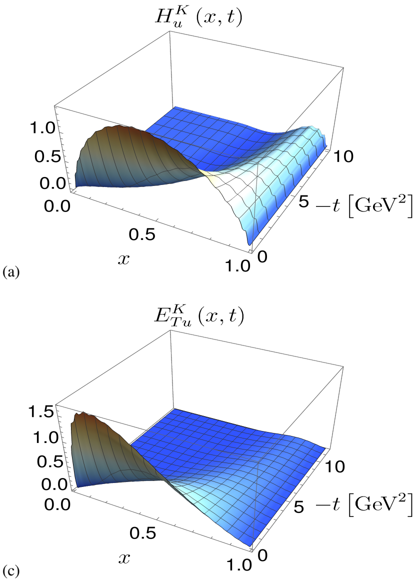
- $H \Rightarrow$ chirally even unpolarized quark GPD
- $E_T \Rightarrow$ chirally odd transversely polarized quark GPD
- $P(P')$ denotes the meson momentum of initial (final) state of the meson $(\mathcal{P})$.
- We choose $A^+ = 0$ and the kinematical region: $0 < x < 1$ at zero skewnes ($\zeta = 0$).

Results: Pion GPDs



- $H_u^\pi(x,0) \Rightarrow$ symmetric with peak at $x = 0.5$
- $E_{Tu}^\pi(x,0) \Rightarrow$ asymmetric with peak below $x = 0.5$
- peak in the GPDs shift towards higher values of x
- oscillations are numerical artifacts due to longitudinal cutoff L_{max}

Results: Kaon GPDs



Generalized Form Factors

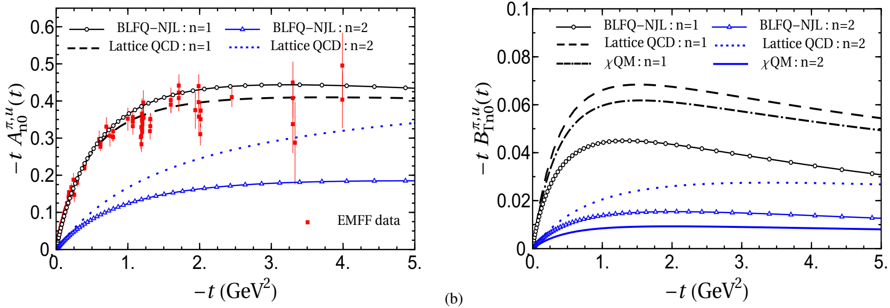
The Mellin moments of the valence GPDs give the generalized FFs

$$A^q_{n0}(t) = \int_0^1 dx\, x^{n-1}\, H^q(x,0,t)$$

$$B^q_{Tn0}(t) = \int_0^1 dx\, x^{n-1}\, E^q_T(x,0,t)$$

- $H^q(x,0,t) \xrightarrow{\text{first moment}} A^q_{10}(t)$ electromagnetic FF of an unpolarized quark
- $E^q_T(x,0,t) \xrightarrow{\text{first moment}} B^q_{T10}(t)$ tensor FF when the quark is transversely polarized
- The second moments $\rightarrow$ gravitational FFs of the quarks

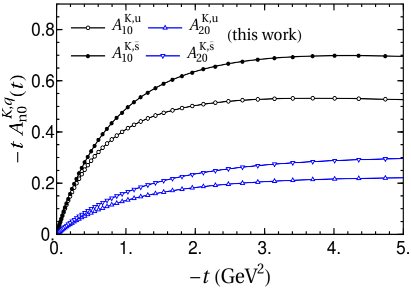
Results : Pion Generalized Form Factors



(a) (b)

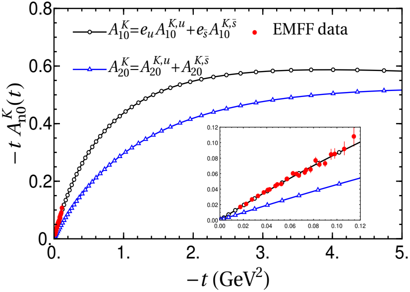
- The EMFF $A_{10}(t)$ of the pion is compared with the experimental data and the lattice QCD result [1]
- The gravitational FF $A_{20}(t)$ is compared with the parameterization of lattice QCD simulations at $\mu^2 = 4$ GeV^2 [2]
- $B_{T10}(t)$ and $B_{T20}(t)$ are compared with lattice QCD and the χQM [3] results at the same scale $\mu^2 = 4$ GeV^2

[1] D.Brömmel et al. [QCDSF/UKQCD], Eur. Phys. J. C 51, 335-345 (2007).
[2] D.Brömmel et al. [QCDSF/UKQCD], Phys. Rev. Lett. 101, 122001 (2008).
[3] S. i. Nam and H. C. Kim, Phys. Lett. B 700, 305-312 (2011).



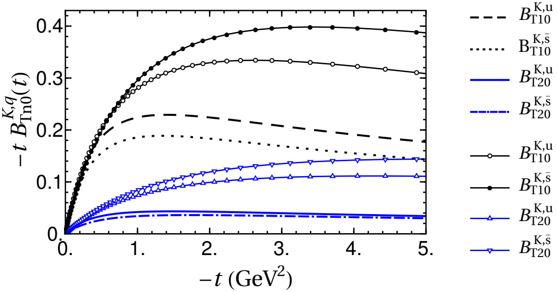
$A_{10}^{K,u}$
$A_{20}^{K,u}$
(this work)
$A_{10}^{K,\bar{s}}$
$A_{20}^{K,\bar{s}}$
$-t\,A_{n0}^{K,q}(t)$
$-t$ (GeV^2)

(a)



$A_{10}^{K}=e_u A_{10}^{K,u}+e_{\bar{s}} A_{10}^{K,\bar{s}}$
EMFF data
$A_{20}^{K}=A_{20}^{K,u}+A_{20}^{K,\bar{s}}$
$-t\,A_{n0}^{K}(t)$
$-t$ (GeV^2)

(b)



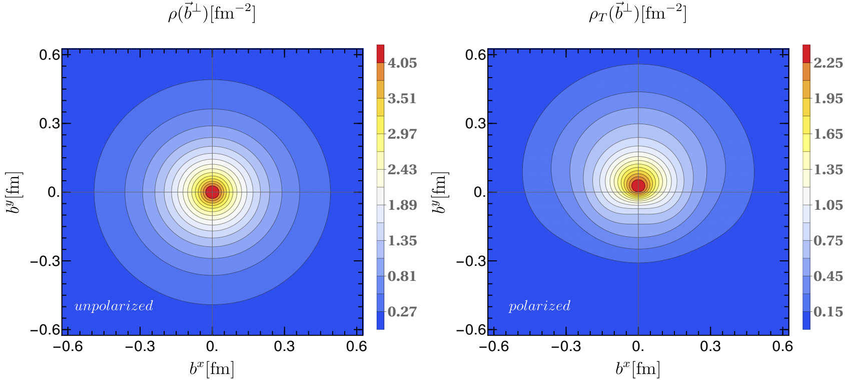
$B_{T10}^{K,u}$
$B_{T10}^{K,\bar{s}}$
$B_{T20}^{K,u}$
$B_{T20}^{K,\bar{s}}$
$B_{T10}^{K,u}$
$B_{T10}^{K,\bar{s}}$
$B_{T20}^{K,u}$
$B_{T20}^{K,\bar{s}}$
$-t\,B_{Tn0}^{K,q}(t)$
$-t$ (GeV^2)

(c)

Spin densities in Impact Parameter Space

The density of quarks with transverse spin $\vec{s}^{\perp}$

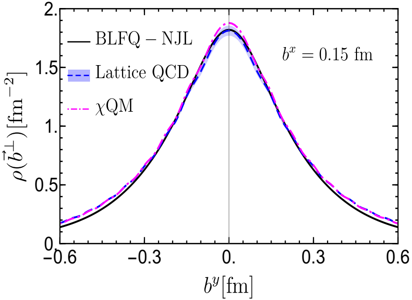
$$\rho^n(\vec{b}^{\perp}, \vec{s}^{\perp}) = \frac{1}{2}\left[\mathcal{A}^q_{n0}(\vec{b}^{\perp}) - \frac{\vec{s}^{\perp}_i \epsilon^{\perp}_{ij} \vec{b}^{\perp}_j}{M_{\mathcal{P}}} \mathcal{B}^{q\prime}_{Tn0}(\vec{b}^{\perp})\right]$$



unpolarized quark $\rightarrow \vec{s}^{\perp} = \vec{0}$

transversely polarized quark $\rightarrow \vec{s}^{\perp} = (+1, 0)$

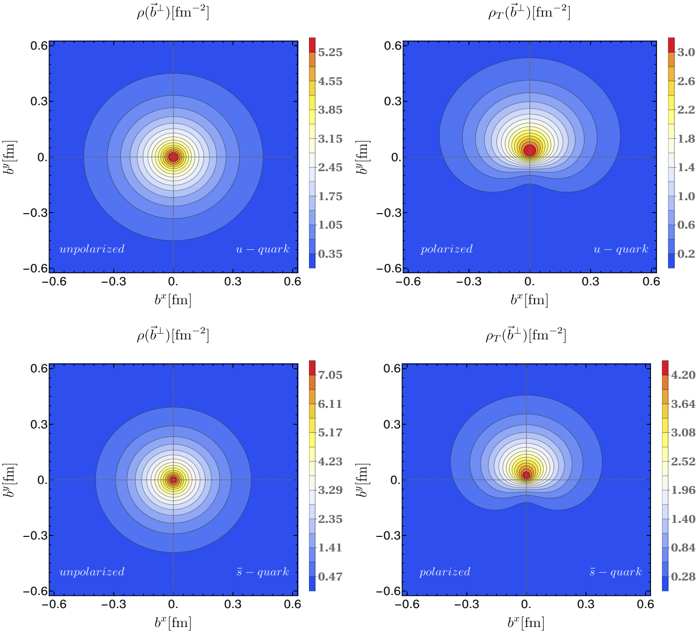
Spin Densities in Impact Parameter Space



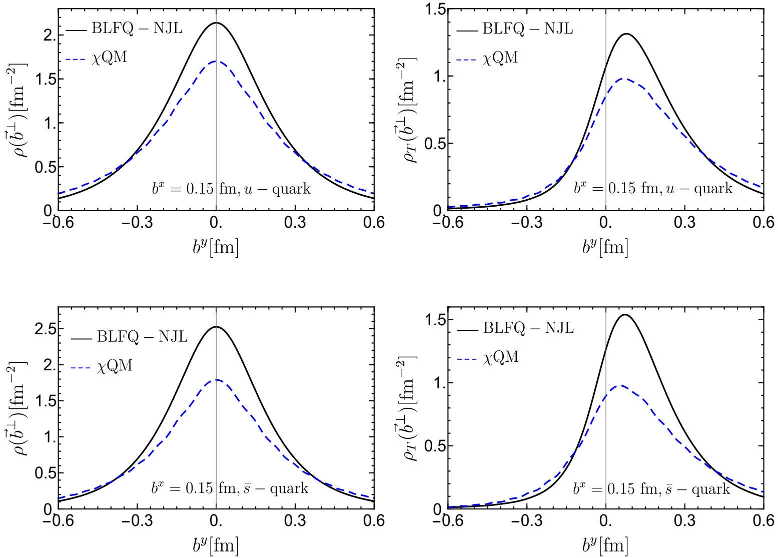
- probability densities as a function of b_y at fixed $b_x = 0.15$ fm
- results are found to be consistent with the results of lattice QCD [1] and the χQM [2].

[1] D.Brömmel et al. [QCDSF/UKQCD], Phys. Rev. Lett. 101, 122001 (2008).
[2] S. i. Nam and H. C. Kim, Phys. Lett. B 700, 305-312 (2011).

Spin densities in Impact Parameter Space : Kaon



Spin densities in Impact Parameter Space : Kaon



Average transverse shift $\langle b_y^\perp \rangle_n$

average transverse shift of the peak position along b_y direction [1]

$$\langle b_y^\perp \rangle_n = \frac{\int d^2\vec{b}^\perp \, b_y^\perp \, \rho^n(\vec{b}^\perp, \vec{s}^\perp)}{\int d^2\vec{b}^\perp \, \rho^n(\vec{b}^\perp, \vec{s}^\perp)} = \frac{1}{2M_{\mathcal{P}}} \frac{B^q_{Tn0}(0)}{A^q_{n0}(0)}$$

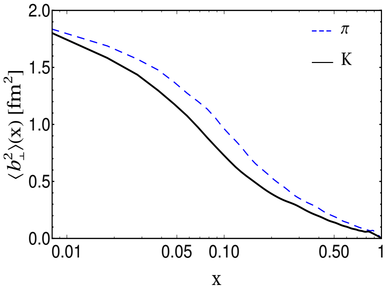
Approach	$\langle b_y^\perp \rangle_1^{q,\pi}$ fm	$\langle b_y^\perp \rangle_2^{q,\pi}$ fm	$\langle b_y^\perp \rangle_1^{u,K}$ fm	$\langle b_y^\perp \rangle_1^{\bar{s},K}$ fm	$\langle b_y^\perp \rangle_2^{u,K}$ fm	$\langle b_y^\perp \rangle_2^{\bar{s},K}$ fm
BLFQ-NJL (this work)	0.162 ± 0.003	0.131 ± 0.003	0.164 ± 0.003	0.141 ± 0.002	0.114 ± 0.002	0.114 ± 0.002
Lattice QCD [1]	0.151 ± 0.024	0.106 ± 0.028	...	...	...	...
χQM [2]	0.152	...	...	...	...	...
χQM (model I) [3]	...	...	0.168	0.166	...	...
χQM (model II) [3]	...	...	0.139	0.100	...	...
CCQM [4]	0.090 ± 0.001	0.080 ± 0.001	...	...	...	...
NJL model [5]	...	...	0.116	...	0.083	...

[1] D.Brömmel et al. [QCDSF/UKQCD], Phys. Rev. Lett. 101, 122001 (2008).
[2] S. i. Nam and H. C. Kim, Phys. Lett. B 700, 305-312 (2011).
[3] S. i. Nam and H. C. Kim, Phys. Lett. B 707, 546-552 (2012).
[4] C. Fanelli, E. Pace, G. Romanelli, G. Salme and M. Salmistraro, Eur. Phys. J. C 76, 253 (2016).
[5] J. L. Zhang and J. L. Ping, Eur. Phys. J. C 81, 814 (2021).

Transverse squared radius $\langle b_\perp^2 \rangle^q(x)$

squared radius of the quark density [1]

$$\langle b_\perp^2 \rangle^q(x) = \frac{\int d^2\vec{b}^\perp \, (\vec{b}^\perp)^2 q(x, \vec{b}^\perp)}{\int d^2\vec{b}^\perp \, q(x, \vec{b}^\perp)}; \qquad \langle b_\perp^2 \rangle = \sum_q e_q \frac{1}{N_q} \int_0^1 dx H^q(x,0,0) \langle b_\perp^2 \rangle^q(x)$$



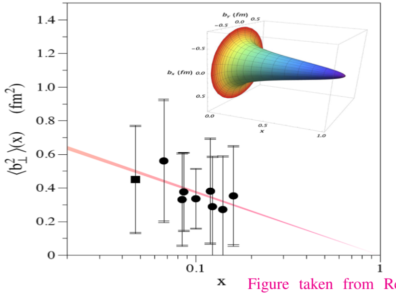


Figure taken from Ref. [1]

- $\langle b_\perp^2 \rangle^\pi = 0.285 \text{ fm}^2$ $\qquad \langle b_\perp^2 \rangle^\pi_{\text{exp}} = 0.301 \pm 0.014 \text{ fm}^2$ [2]
- $\langle b_\perp^2 \rangle^K = 0.223 \text{ fm}^2$ $\qquad \langle b_\perp^2 \rangle^K_{\text{exp}} = 0.209 \pm 0.047 \text{ fm}^2$ [2]

[1] R. Dupre, M. Guidal and M. Vanderhaeghen, Phys. Rev. D 95 , 011501 (2017).
[2] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, 030001 (2018).

Conclusion

- We investigated the valence quark GPDs of the light pseudoscalar mesons in the BLFQ-NJL model.
- We also calculated the spin densities of the unpolarized and transversely polarized quark inside the pion and the kaon.
- Our results were found to be consistent with lattice QCD and χQM results.

Thank You

Pion to photon transition form factors with basis light-front quantization

Dec 1, 2021, 3:10 PM
20m
Emerald Hall B (Jeju Booyoung Hotel)

Contributed talk | Parallel Session

Speaker

Chandan Mondal (Institute of Modern ...

Light meson structure on BLFQ

Dec 2, 2021, 9:05 AM
25m
Emerald Hall 1+2 (Jeju Booyoung Hotel)

Invited talk | McCartor Session

Speaker

Jiangshan Lan (Institute of Modern ...