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All-charm tetraquark using BLFQ

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Problem

- Are there any $cc\bar{c}\bar{c}$ bound states?

Motivations

- Confinement
- Open channels, scattering
- Identical particles, color
- $X(6900)$

In principle:

1. Write H_{QCD}

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2. Spectrum of H_{QCD}

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Problem:

None of the above steps can be done exactly

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In practice:

1. Write H_{QCD}
2. Spectrum of H_{QCD}
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Problem:

None of the above steps can be done exactly

~~In principle:~~
In practice:

1. ~~Write H_{QCD}~~
   Write H_{model}
2. Spectrum of H_{QCD}
3. Answer the question

Problem:

None of the above steps can be done exactly

~~In principle:~~
In practice:

1. ~~Write H_{QCD}~~
   Write H_{model}
2. ~~Spectrum of H_{QCD}~~
   Spectrum of H_{model}
3. Answer the question

Problem:

None of the above steps can be done exactly

~~In principle:~~
In practice:

1. ~~Write H_{QCD}~~
   Write H_{model}
2. ~~Spectrum of H_{QCD}~~
   Spectrum of H_{model}*
3. Answer the question

* Truncated

Problem:

None of the above steps can be done exactly

- Only two quarks and two antiquarks, but...

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- Full set of degrees of freedom of each quark

- Only two quarks and two antiquarks, but...
- Full set of degrees of freedom of each quark
- Only two-body interactions

- Harmonic oscillator a la AdS/QCD

- Harmonic oscillator a la AdS/QCD + complementary longitudinal H.O.

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- Two-meson states!

  Cluster decomposition principle

- Harmonic oscillator a la AdS/QCD + complementary longitudinal H.O.
- Two-meson states!
  Cluster decomposition principle
- One-Gluon-Exchange-like color factors

$$H = P^+(P^-_{\text{free}} + V) - \mathsf{P}^2 \; .$$

Interactions

$$V = \int_{1'2'12} \tilde{\delta}_{1'2'.12} \; U \; \mathrm{BD}_{\mathrm{OGE}}$$

$$H = P^+(P^-_{\text{free}} + V) - \mathrm{P}^2 \ .$$

Interactions

$$V = \int_{1'2'12} \tilde{\delta}_{1'2'.12} \, U \, \mathrm{BD}_{\mathrm{OGE}}$$

$$\tilde{\delta}_{1'2'.12} = 4\pi \, \delta(p^+_{1'} + p^+_{2'} - p^+_1 - p^+_2) \cdot (2\pi)^2 \, \delta^2(\mathrm{p}_{1'} + \mathrm{p}_{2'} - \mathrm{p}_1 - \mathrm{p}_2)$$

$$H = P^+(P^-_{\text{free}} + V) - \mathrm{P}^2 \; .$$

Interactions

$$V = \int_{1'2'12} \tilde{\delta}_{1'2'.12} \, U \, \mathrm{BD}_{\mathrm{OGE}}$$

$$\mathrm{BD}_{\mathrm{OGE}} = \frac{1}{2} \boxed{C_{qq}} \, b^\dagger_{1'} b^\dagger_{2'} b_2 b_1 + \frac{1}{2} \boxed{C_{\bar{q}\bar{q}}} \, d^\dagger_{1'} d^\dagger_{2'} d_2 d_1 + \boxed{C_{q\bar{q}}} \, b^\dagger_{1'} d^\dagger_{2'} d_2 b_1$$

$$\boxed{C_{qq}} = t^a_{1'1} t^a_{2'2} \quad \boxed{C_{\bar{q}\bar{q}}} = t^a_{11'} t^a_{22'} \quad \boxed{C_{q\bar{q}}} = -t^a_{1'1} t^a_{22'}$$

$$H = P^+(P^-_{\text{free}} + V) - \mathsf{P}^2 \ .$$

Interactions

$$V = \int_{1'2'12} \tilde{\delta}_{1'2'.12} \ U \ \mathrm{BD}_{\mathrm{OGE}}$$

$$U = U_{\mathrm{conf},\perp} + U_{\mathrm{conf},z} + U_{\mathrm{OGE}} \ .$$

Transverse confining potential

$$\begin{aligned} U_{\text{conf},\perp} &= \kappa^4\, \delta_{\sigma_{1'},\sigma_1}\delta_{\sigma_{2'},\sigma_2}\, 4\pi\, \delta(x_{12}-x_{1'2'}) \\ &\times\quad (x_{12}x_{21})^2 \left[\frac{\partial^2}{\partial \mathrm{k}_{12}^{\perp 2}}(2\pi)^2\delta^2\left(\mathrm{k}_{12}^{\perp}-\mathrm{k}_{1'2'}^{\perp}\right)\right]\ , \end{aligned}$$

$x_{12} = p_1^+/(p_1^+ + p_2^+)$, $x_{21} = 1 - x_{12}$

$k_{12}^{\perp} = x_{21}p_1^{\perp} - x_{12}p_2^{\perp}$

$$\boxed{C_{q\bar{q}}}\ \kappa^4\, x_{12}x_{21}(\mathrm{r}_1^{\perp}-\mathrm{r}_2^{\perp})^2\psi_M(12) \qquad \mathrm{r}_i^{\perp} = -i\partial/\partial \mathrm{p}_i^{\perp}$$

Transverse confining potential

$$
\begin{aligned}
U_{\text{conf},\perp} &= \kappa^4\, \delta_{\sigma_{1'},\sigma_1} \delta_{\sigma_{2'},\sigma_2}\, 4\pi\, \delta(x_{12} - x_{1'2'}) \\
&\times \quad (x_{12} x_{21})^2 \left[\frac{\partial^2}{\partial \mathrm{k}_{12}^{\perp 2}} (2\pi)^2 \delta^2 \left(\mathrm{k}_{12}^{\perp} - \mathrm{k}_{1'2'}^{\perp} \right) \right] ,
\end{aligned}
$$

$$x_{12} = p_1^+ / (p_1^+ + p_2^+),\ x_{21} = 1 - x_{12}$$

$$k_{12}^{\perp} = x_{21} p_1^{\perp} - x_{12} p_2^{\perp}$$

$$\boxed{C_{q\bar{q}}}\ \kappa^4\, x_{12} x_{21} (\mathrm{r}_1^{\perp} - \mathrm{r}_2^{\perp})^2 \psi_M(12) \qquad \mathrm{r}_i^{\perp} = -i\partial / \partial \mathrm{p}_i^{\perp}$$

The kernel of the longitudinal confining potential is,

$$U_{\text{conf},z} = -\kappa^4\, \delta_{\sigma_{1'},\sigma_1}\delta_{\sigma_{2'},\sigma_2}\,(2\pi)^2\delta^2\left(\mathrm{q}^{\perp}_{12} - \mathrm{q}^{\perp}_{1'2'}\right) \times \left[\frac{1}{\sqrt{D_{1'2'}}}\frac{\partial}{\partial x_{1'2'}}\frac{1}{\sqrt{D_{1'2'}}}\frac{1}{\sqrt{D_{12}}}\frac{\partial}{\partial x_{12}}\frac{1}{\sqrt{D_{12}}}\; 4\pi\,\delta(x_{12} - x_{1'2'})\right] ,$$

where

$$\mathrm{q}^{\perp}_{12} = \frac{\mathrm{k}^{\perp}_{12}}{\sqrt{x_{12}x_{21}}} ,$$

$$q^{z}_{12} = m\,\frac{x_{12} - x_{21}}{\sqrt{x_{12}x_{21}}} ,$$

and

$$D_{12} = \frac{dq^{z}_{12}}{dx_{12}} = \frac{m}{2\,[x_{12}(1-x_{12})]^{3/2}} .$$

We get $\longrightarrow -\boxed{C_{q\bar{q}}}\,\kappa^4\,\frac{1}{\sqrt{D_{12}}}\frac{\partial}{\partial x_{12}}\frac{1}{D_{12}}\frac{\partial}{\partial x_{12}}\frac{1}{\sqrt{D_{12}}}\psi_M(12)$

The kernel of OGE term is,

$$U_{\mathrm{OGE}}(1',2';1,2) = -g^2 \frac{\bar{u}_{1'}\gamma_\mu u_1 \bar{u}_{2'}\gamma^\mu u_2}{(x_{12}-x_{1'2'})\mathcal{D}} \sqrt{p_1^+ p_2^+ p_{1'}^+ p_{2'}^+} \,,$$

where $\mathcal{D}$ is the energy denominator,

$$\begin{aligned} \mathcal{D} &= \frac{1}{2}\left[\frac{\mathrm{p}_1^{\perp 2}+m^2}{x_{12}} - \frac{\mathrm{p}_{1'}^{\perp 2}+m^2}{x_{1'2'}} - \frac{\mathrm{p}_2^{\perp 2}+m^2}{x_{21}} + \frac{\mathrm{p}_{2'}^{\perp 2}+m^2}{x_{2'1'}}\right] \\ &- \frac{(\mathrm{p}_1^\perp - \mathrm{p}_{1'}^\perp)^2+\mu^2}{x_{12}-x_{1'2'}} \,, \end{aligned}$$

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Basis light-front quantization approach to positronium

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(Received 1 May 2014; published 12 May 2015)

Three eigenvalue equations

$$H|\psi\rangle \quad = \quad M^2|\psi\rangle$$

$$|\psi\rangle \quad = \quad \int_{12} P_M^+ \tilde{\delta}_{12.P_M} \;\psi_M(12)\; b_1^\dagger d_2^\dagger |0\rangle \longrightarrow \boxed{1}$$

$$|\psi\rangle \quad = \quad \int_{1234} P_T^+ \tilde{\delta}_{1234.P_T} \;\psi_T(1234)\; b_1^\dagger b_2^\dagger d_3^\dagger d_4^\dagger |0\rangle \longrightarrow \boxed{2}$$

$$|\psi\rangle \quad = \quad \int_{13} P_A^+ \tilde{\delta}_{13.P_A}\, \psi_A(13)\, b_1^\dagger d_3^\dagger \int_{24} P_B^+ \tilde{\delta}_{24.P_B}\, \psi_B(24)\, b_2^\dagger d_4^\dagger \;|0\rangle \longrightarrow \boxed{3}$$

1 Single meson

$$\frac{m^2 + \mathrm{k}_{12}^2}{x_1}\,\psi_M(12) + \frac{m^2 + \mathrm{k}_{12}^2}{x_2}\,\psi_M(12) + \sum_{c_{1'},c_{2'}} t^a_{11'}\,t^a_{2'2}\,\kappa^4\,\tilde{U}_{12}\psi_{Mc_{1'}c_{2'}}(12)$$

$$-\int_{1'2'} P_M^+\tilde{\delta}_{1'2'.P_M}\,t^a_{11'}\,t^a_{2'2}\,U_{\mathrm{OGE}}(1,2;1',2')\,\psi_M(1'2') \;=\; M^2\,\psi_M(12)$$

1 Single meson

$$\frac{m^2 + \mathrm{k}_{12}^2}{x(1-x)}\,\psi_M(12) + \vec{T}_1 \cdot \vec{T}_2\,\kappa^4\,\tilde{U}_{12}\psi_M(12) \;=\; M^2\,\psi_M(12)\;,$$

where $x = x_{12}$ and

$$\tilde{U}_{12} \quad = \quad x(1-x)\left(\mathrm{r}_1^\perp - \mathrm{r}_2^\perp\right)^2 - \frac{1}{\sqrt{D_{12}}}\,\partial_x\,\frac{1}{D_{12}}\,\partial_x\,\frac{1}{\sqrt{D_{12}}}$$

$\frac{1}{D_{12}} = \frac{2}{m}[x(1-x)]^{3/2}$ $\qquad -\omega^2\partial_x x(1-x)\partial_x \leftarrow \mathrm{BLFQ}_0$

$$\psi_M(12) \;=\; \frac{\delta_{c_1,c_2}}{\sqrt{N_c}}\,\sqrt{D_{12}}\,\phi(\vec{q}_{12}) \quad\longrightarrow\quad \left(4m^2 + \vec{q}_{12}^{\,2} - C_F\,\kappa^4\,\frac{\partial^2}{\partial\vec{q}_{12}^{\,2}}\right)\phi \;=\; M^2\,\phi$$

2 Tetraquark

$$\sum_{i=1}^{4} \frac{m^2 + \mathrm{k}_i^2}{x_i}\ \psi(1234) + \sum_{i<j} \frac{\vec{T}_i \cdot \vec{T}_j\, \kappa^4}{x_i + x_j}\, \tilde{U}_{ij}\, \psi(1234) \quad = \quad M^2\ \psi(1234)\ ,$$

$$\vec{T}_i \cdot \vec{T}_j \quad = \quad \begin{cases} \sum_{a=1}^{8} T_i^a T_j^a & \text{for } QQ \\ \sum_{a=1}^{8} T_i^a (-T_j^{a*}) & \text{for } Q\bar{Q} \\ \sum_{a=1}^{8} (-T_i^{a*})(-T_j^{a*}) & \text{for } \bar{Q}\bar{Q} \end{cases}$$

$T^a = \lambda^a/2$

3 Two mesons

$$\frac{\mathcal{E}_A \psi_B(24) + \mathrm{k}_{AB}^{\perp 2}}{x_A} + \frac{\mathcal{E}_B \psi_A(13) + \mathrm{k}_{AB}^{\perp 2}}{x_B} \quad = \quad M^2 \psi_A(13)\, \psi_B(24)\ ,$$

$x_A = x_1 + x_3$, $x_B = x_2 + x_4$

$$\begin{aligned}
\mathcal{E}_A &= \frac{m^2 + \mathrm{k}_{13}^2}{x_{13} x_{31}}\, \psi_A(13) + \vec{T}_1 \cdot \vec{T}_3\, \kappa^4\, \tilde{U}_{13} \psi_A(13)\ , \\
\mathcal{E}_B &= \frac{m^2 + \mathrm{k}_{24}^2}{x_{24} x_{42}}\, \psi_B(24) + \vec{T}_2 \cdot \vec{T}_4\, \kappa^4\, \tilde{U}_{24} \psi_B(24)\ .
\end{aligned}$$

$$M^2 \quad = \quad (P_{A\mu} + P_{B\mu})(P_A^\mu + P_B^\mu) = \frac{M_A^2 + \mathrm{k}_{AB}^{\perp 2}}{x_A} + \frac{M_B^2 + \mathrm{k}_{AB}^{\perp 2}}{x_B}\ .$$

Problem with negative M^2

- We get negative M^2
- Modifications:

$$
\begin{aligned}
H_{\rm transverse} &= \int_{1'2'12} (p_1^+ + p_2^+)\tilde{\delta}_{1'2'.12}\ U_{{\rm conf},\perp}\ {\rm BD}_{\rm OGE}\ , \\
H_{\rm longitudinal} &= \int_{1'2'12} (p_1^+ + p_2^+)\tilde{\delta}_{1'2'.12}\ U_{{\rm conf},z} \\
&\quad \times \left[a\,{\rm BD}_{\rm OGE} + \frac{4}{3}(a-1)\,{\rm BD}_{\rm CI} \right]\ ,
\end{aligned}
$$

$a = 0.85$

$$
{\rm BD}_{\rm CI} \quad = \quad \delta_{c_{1'},c_1}\delta_{c_{2'},c_2}\left(\frac{1}{2}\, b_{1'}^\dagger b_{2'}^\dagger b_2 b_1 + b_{1'}^\dagger d_{2'}^\dagger d_2 b_1 + \frac{1}{2}\, d_{1'}^\dagger d_{2'}^\dagger d_2 d_1 \right)\ .
$$

CI stands for “color independent.”

- Longitudinal box

$$x^- \in [-L, L]\ , \quad p^+ = \frac{2\pi}{L} k\ , \quad \begin{array}{ll} k = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots & \text{for fermions} \\ k = 0, 1, 2, 3, \ldots & \text{for bosons} \end{array}$$

- Finite resolution $\sum_i k_i = K$, $P^+ = \frac{2\pi K}{L}$

BLFQ

Basis Light Front Quantization

- Transverse basis of harmonic oscillator wave functions

$$\Psi_n^m(\mathrm{q}) = \frac{1}{b}\sqrt{\frac{4\pi n!}{(n+|m|)!}}\, L_n^{|m|}\left(\frac{q^2}{b^2}\right) e^{-\frac{q^2}{2b^2}} \left|\frac{q}{b}\right|^{|m|} e^{im\varphi}$$

$$B_i = \frac{1}{\sqrt{P^+}} \int \frac{d^2q}{(2\pi)^2}\, \Psi_{n_i}^{m_i}(\mathrm{q})^*\ b_i\big|_{\mathrm{p}_i=\sqrt{x_i}\mathrm{q}}$$

$$D_i = \frac{1}{\sqrt{P^+}} \int \frac{d^2q}{(2\pi)^2}\, \Psi_{n_i}^{m_i}(\mathrm{q})^*\ d_i\big|_{\mathrm{p}_i=\sqrt{x_i}\mathrm{q}}$$

$$\langle 0|b\ B^\dagger|0\rangle = \sqrt{P^+}\,\Psi\left(\frac{p^\perp}{\sqrt{x}}\right)$$

HAMILTONIAN LIGHT-FRONT FIELD THEORY IN A . . . PHYSICAL REVIEW C **81**, 035205 (2010)

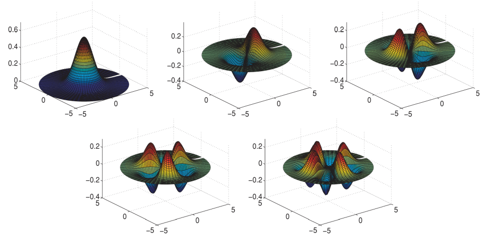

FIG. 2. (Color online) Modes for $n = 0$ of the 2D harmonic oscillator selected for the transverse basis functions. The orbital quantum number m progresses across the rows by integer steps from 0 in the upper left to 4 in the lower right and counts the pairs of angular lobes. Amplitudes as well as x-axis and y-axis coordinates are in dimensionless units.

J. P. VARY *et al.* PHYSICAL REVIEW C **81**, 035205 (2010)

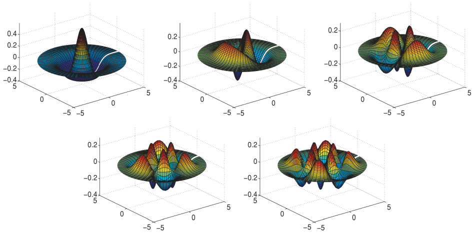

FIG. 3. (Color online) Modes for $n = 1$ of the 2D harmonic oscillator selected for the transverse basis functions. The orbital quantum number m progresses across the rows by integer steps from 0 in the upper left to 4 in the lower right and counts the pairs of angular lobes. Amplitudes as well as x-axis and y-axis coordinates are in dimensionless units.

- Truncation

$$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$$

- M-scheme $M_J = \sum_i (m_i + \sigma_i)$
- Center of mass motion decouples in truncated basis

- $cc\bar{c}\bar{c}$ states:

$$|1234\rangle = B_1^\dagger B_2^\dagger D_3^\dagger D_4^\dagger |0\rangle \ .$$

- Double counting and degenerate states

$$|1234\rangle = -|2134\rangle \ , \quad |1234\rangle = -|2143\rangle \ , \quad |1134\rangle = 0 \ .$$

- Relation of strict order in the set of all possible quantum numbers:

$$1 > 2 \quad \Leftrightarrow \quad \begin{array}{c} k_1 > k_2 \\ \text{or} \\ k_1 = k_2 \text{ and } n_1 > n_2 \\ \text{or} \\ k_1 = k_2 \text{ and } n_1 > n_2 \text{ and } m_1 > m_2 \\ \text{or} \\ k_1 = k_2 \text{ and } n_1 > n_2 \text{ and } m_1 > m_2 \text{ and } \sigma_1 > \sigma_2 \\ \text{or} \\ k_1 = k_2 \text{ and } n_1 > n_2 \text{ and } m_1 > m_2 \text{ and } \sigma_1 > \sigma_2 \text{ and } c_1 > c_2 \end{array}$$

- Keep only states with $1 > 2$ and $3 > 4$

- $C_2|\text{Color singlet}\rangle = 0$
- Casimir operator $C_2 = \sum_{a=1}^{8} \hat{T}^a \hat{T}^a$, where

$$\hat{T}^a \quad = \quad \sum_{12} {}^{knm\sigma}\delta_{1,2} \left(t^a_{c_1 c_2} B^\dagger_1 B_2 - t^a_{c_2 c_1} D^\dagger_1 D_2 \right) \ ,$$

- Case 1: $1 \gg 2$ and $3 \gg 4$

$$\begin{aligned} |1234, S\rangle \quad = \quad & \frac{1}{2\sqrt{6}} \Big(2|rr\bar{r}\bar{r}\rangle + 2|gg\bar{g}\bar{g}\rangle + 2|bb\bar{b}\bar{b}\rangle \\ & + |rg\bar{r}\bar{g}\rangle + |gr\bar{r}\bar{g}\rangle + |gr\bar{g}\bar{r}\rangle + |rg\bar{g}\bar{r}\rangle \\ & + |gb\bar{g}\bar{b}\rangle + |bg\bar{g}\bar{b}\rangle + |bg\bar{b}\bar{g}\rangle + |gb\bar{b}\bar{g}\rangle \\ & + |br\bar{b}\bar{r}\rangle + |rb\bar{b}\bar{r}\rangle + |rb\bar{r}\bar{b}\rangle + |br\bar{r}\bar{b}\rangle \Big) \ , \end{aligned}$$

- Case 1: $1 \gg 2$ and $3 \gg 4$

$$
\begin{aligned}
|1234, A_1\rangle \quad = \quad & \frac{1}{\sqrt{12}}\Big(|rg\bar{r}\bar{g}\rangle - |gr\bar{r}\bar{g}\rangle + |gr\bar{g}\bar{r}\rangle - |rg\bar{g}\bar{r}\rangle \\
& +|gb\bar{g}\bar{b}\rangle - |bg\bar{g}\bar{b}\rangle + |bg\bar{b}\bar{g}\rangle - |gb\bar{b}\bar{g}\rangle \\
& +|br\bar{b}\bar{r}\rangle - |rb\bar{b}\bar{r}\rangle + |rb\bar{r}\bar{b}\rangle - |br\bar{r}\bar{b}\rangle\Big) \ ,
\end{aligned}
$$

- Case 2: $1 \approx 2$ and $3 \gg 4$

$$
|1234, A_2\rangle \quad = \quad \frac{|gr\bar{g}\bar{r}\rangle - |gr\bar{r}\bar{g}\rangle + |br\bar{b}\bar{r}\rangle - |br\bar{r}\bar{b}\rangle + |bg\bar{b}\bar{g}\rangle - |bg\bar{g}\bar{b}\rangle}{\sqrt{6}}
$$

- Case 3: $1 \gg 2$ and $3 \approx 4$

$$
|1234, A_3\rangle \quad = \quad \frac{|gr\bar{g}\bar{r}\rangle - |rg\bar{g}\bar{r}\rangle + |br\bar{b}\bar{r}\rangle - |rb\bar{b}\bar{r}\rangle + |bg\bar{b}\bar{g}\rangle - |gb\bar{b}\bar{g}\rangle}{\sqrt{6}}
$$

- Case 4: $1 \approx 2$ and $3 \approx 4$

$$
|1234, A_4\rangle \quad = \quad \frac{|gr\bar{g}\bar{r}\rangle + |br\bar{b}\bar{r}\rangle + |bg\bar{b}\bar{g}\rangle}{\sqrt{3}} \ .
$$

- Example of a matrix element:

$$\hat{V}_{qq} = \sum_{5',6',5,6} t^a_{5'5} t^a_{6'6} \; V_{5',6';5,6} \; \frac{1}{2} B^\dagger_{5'} B^\dagger_{6'} B_6 B_5$$

$$\langle 1'2'3'4', S | \hat{V}_{qq} | 1234, S \rangle = \frac{1}{2} \left(V_{1',2';1,2} - V_{2',1';1,2} - V_{1',2';2,1} + V_{2',1';2,1} \right) \delta_{e_{3'},e_3} \, \delta_{e_{4'},e_4}$$

Table: Parameters obtained from fit to experimental meson spectrum.

m	κ	α
1.25 GeV	1.21 GeV	0.367

Table: Fitted masses in MeV. $N_{\max} = 6$, $K = 9$.

	$\eta_c(1S)$	J/ψ	χ_{c0}	χ_{c1}	χ_{c2}	h_c	$\eta_c(2S)$	$\psi(2S)$
Fit	3031	3067	3415	3517	3564	3474	3676	3666
Exp.	2984	3097	3415	3511	3556	3525	3637	3686

Numerical artifacts

Numerical artifacts

- Kinetic energy penalty

$$\Delta M^2 = M^{2\,\mathrm{free}}_{cc\bar{c}\bar{c}}(N_{\max}, K) - \min_{N_1, K_1}\left[\frac{M^{2\,\mathrm{free}}_{c\bar{c}}(N_1, K_1)}{K_1/K} + \frac{M^{2\,\mathrm{free}}_{c\bar{c}}(N_2, K_2)}{K_2/K}\right]$$

Numerical artifacts

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All-charm tetraquarks

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(Received 5 November 2003; published 27 July 2004)

Numerical artifacts

- Kinetic energy penalty

$$\Delta M^2 = M^{2\,\mathrm{free}}_{cc\bar{c}\bar{c}}(N_{\mathrm{max}}, K) - \min_{N_1, K_1}\left[\frac{M^{2\,\mathrm{free}}_{c\bar{c}}(N_1, K_1)}{K_1/K} + \frac{M^{2\,\mathrm{free}}_{c\bar{c}}(N_2, K_2)}{K_2/K}\right]$$

$$M^{\mathrm{corrected}}_{cc\bar{c}\bar{c}} = \sqrt{M^{2\,\mathrm{full}}_{cc\bar{c}\bar{c}}(N_{\mathrm{max}}, K) - \Delta M^2(N_{\mathrm{max}}, K)}$$

Numerical artifacts

- Kinetic energy penalty

$$\Delta M^2 = M^{2\,\mathrm{free}}_{cc\bar{c}\bar{c}}(N_{\mathrm{max}}, K) - \min_{N_1, K_1}\left[\frac{M^{2\,\mathrm{free}}_{c\bar{c}}(N_1, K_1)}{K_1/K} + \frac{M^{2\,\mathrm{free}}_{c\bar{c}}(N_2, K_2)}{K_2/K}\right]$$

$$M^{\mathrm{corrected}}_{cc\bar{c}\bar{c}} = \sqrt{M^{2\,\mathrm{full}}_{cc\bar{c}\bar{c}}(N_{\mathrm{max}}, K) - \Delta M^2(N_{\mathrm{max}}, K)}$$

N_{max}	6	8	10	12
ΔM^2 [GeV^2]	1.213	1.013	0.659	0.584

- Two-body threshold estimate

$$T_1' \quad = \quad \sqrt{\min_{N_1,K_1}\left[\frac{M_{c\bar{c}}^{2\,\mathrm{full}}(N_1,K_1)}{K_1/K}+\frac{M_{c\bar{c}}^{2\,\mathrm{full}}(N_2,K_2)}{K_2/K}\right]}$$

- Two-body threshold estimate

$$T_1' = \sqrt{\min_{N_1,K_1}\left[\frac{M_{c\bar{c}}^{2\,\mathrm{full}}(N_1,K_1)}{K_1/K} + \frac{M_{c\bar{c}}^{2\,\mathrm{full}}(N_2,K_2)}{K_2/K}\right]}$$

$$T_1(N_{\mathrm{max}},K) = 2\sqrt{M_{c\bar{c}}^{2\,\mathrm{full}}\left(\frac{N_{\mathrm{max}}}{2},\frac{K}{2}\right)}$$

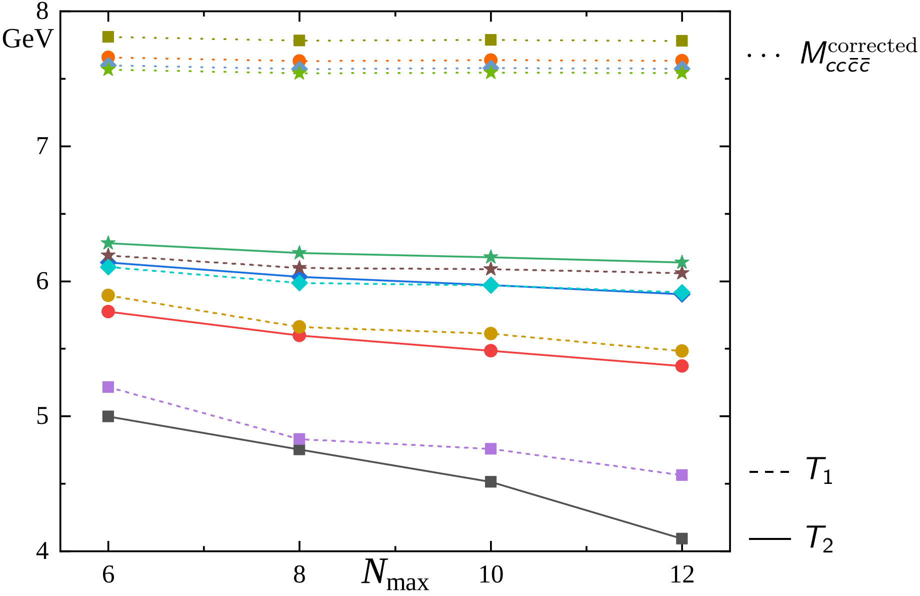
- Two-body threshold estimate

$$T_1' = \sqrt{\min_{N_1,K_1}\left[\frac{M^{2\,\mathrm{full}}_{c\bar{c}}(N_1,K_1)}{K_1/K} + \frac{M^{2\,\mathrm{full}}_{c\bar{c}}(N_2,K_2)}{K_2/K}\right]}$$

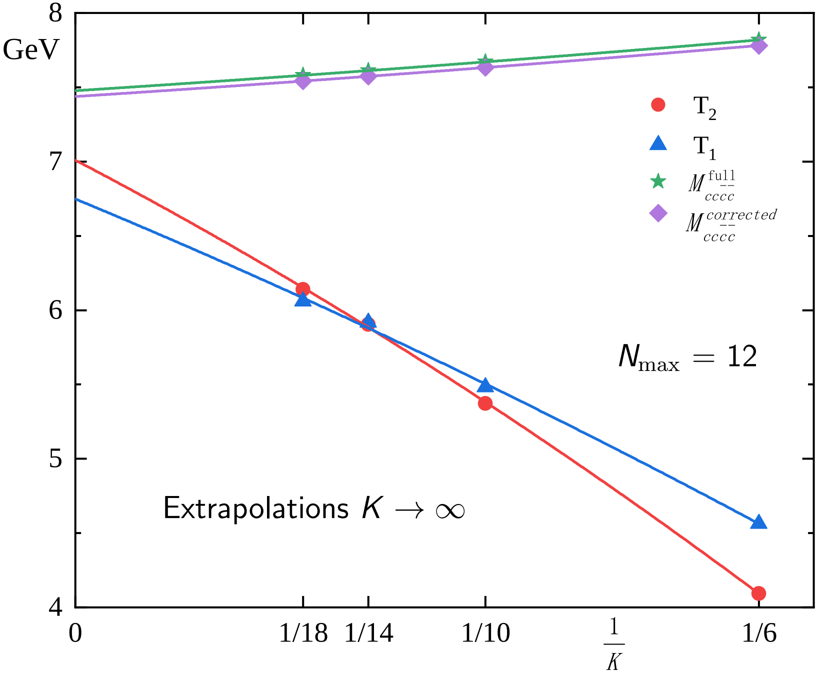
$$T_1(N_{\max},K) = 2\sqrt{M^{2\,\mathrm{full}}_{c\bar{c}}\left(\frac{N_{\max}}{2},\frac{K}{2}\right)}$$

- Four-body threshold estimate

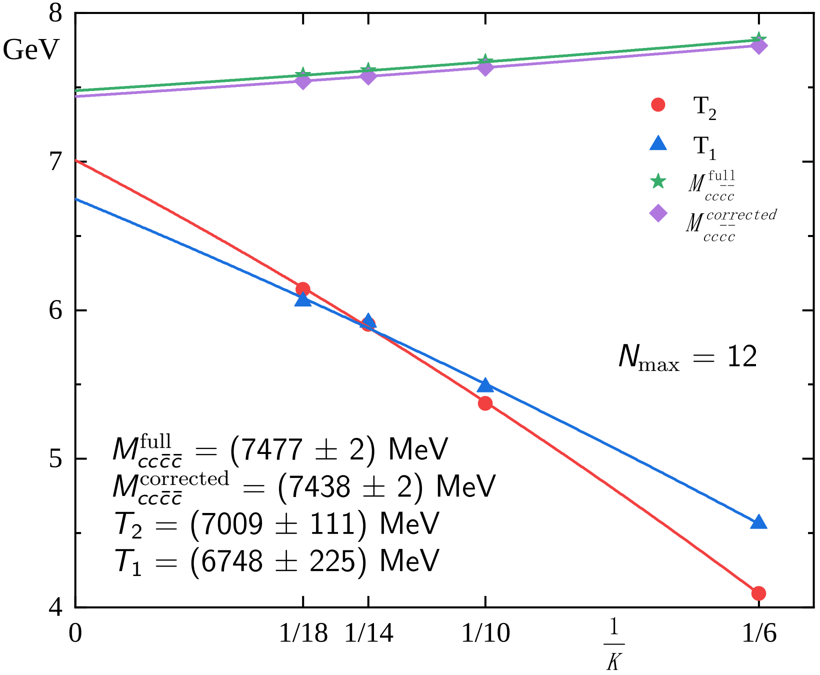
$$T_2 = \sqrt{M^2_{\text{two-meson}}(N_{\max},K) - \Delta M^2(N_{\max},K)}$$



$K = 6$ ■, 10 ●, 14 ♦, 18 ★



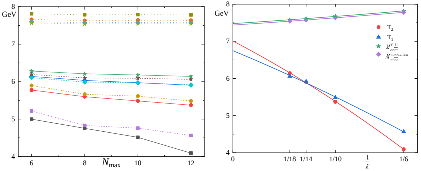
GeV
8
7
6
5
4
0
1/18
1/14
1/10
1/6
$\frac{1}{K}$
T_2
T_1
$M^{\text{full}}_{cc\bar{c}\bar{c}}$
$M^{corrected}_{cc\bar{c}\bar{c}}$
$N_{\text{max}} = 12$
Extrapolations $K \to \infty$



8
GeV
7
6
5
4
0
1/18
1/14
1/10
1/6
$\frac{1}{K}$
T_2
T_1
$M^{\mathrm{full}}_{cc\bar{c}\bar{c}}$
$M^{corrected}_{cc\bar{c}\bar{c}}$
$N_{\max} = 12$
$M^{\mathrm{full}}_{cc\bar{c}\bar{c}} = (7477 \pm 2)$ MeV
$M^{\mathrm{corrected}}_{cc\bar{c}\bar{c}} = (7438 \pm 2)$ MeV
$T_2 = (7009 \pm 111)$ MeV
$T_1 = (6748 \pm 225)$ MeV

Summary

- Cluster decomposition principle
- Identical particles
- Color singlets
- We computed tetraquark mass and thresholds numerically
- Ground-state tetraquark seems unbound



Outlook

- Solve negative M^2 problem and improve the computation
- Apply the lessons to other systems
- $J/\psi\, J/\psi$ scattering?