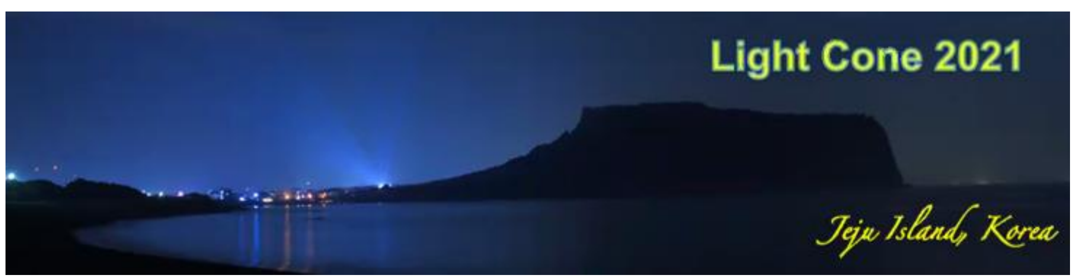
Transverse structure of heavy baryons in momentum space: A light-front Hamiltonian approach

Zhi-Min Zhu[1,2]

zhuzhimin@impcas.ac.cn

[1]Institute of Modern Physics, Chinese Academy of Science

[2]School of Nuclear Physics, University of Chinese Academy of Sciences



Light Cone 2021, December 2, 2021

Outline

- Overview on transverse momentum distributions (TMDs)

- The bound state of heavy baryons in BLFQ framework

- Numerical results

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- **Overview on transverse momentum distributions (TMDs)**

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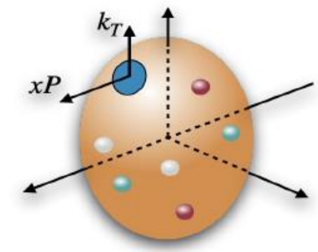
Overview on TMDs (spin 1/2)



Definition:

$$\Phi^{[\Gamma]}\left(P, S, S'; x = \frac{k^+}{P^+}, \boldsymbol{k}_T\right) = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2\boldsymbol{z}^\perp}{2(2\pi)^3} e^{ik\cdot z} \langle P, S'|\bar{\Psi}(0)\Gamma\mathcal{W}(0^\perp, z^\perp)\Psi(z)|P, S\rangle\Big|_{z^+=0}$$

Parameterization:

8 terms leading-twist TMDs

$$\Phi^{[\gamma^+]}(x, \boldsymbol{k}_T; S) = f_1 - \frac{\epsilon_\perp^{ij} k_T^i S_\perp^j}{M} f_{1T}^\perp$$

$$\Phi^{[\gamma^+\gamma^5]}(x, \boldsymbol{k}_T; S) = \Lambda g_{1L} + \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_\perp}{M} g_{1T}$$

$$\Phi^{[i\sigma^{j+}\gamma^5]}(x, \boldsymbol{k}_T; S) = S_\perp^j h_1 + \Lambda \frac{k_T^j}{M} h_{1L}^\perp + S_\perp^i \frac{2k_T^i k_T^j - (\boldsymbol{k}_T)^2 \delta^{ij}}{2M^2} h_{1T}^\perp + \frac{\epsilon_\perp^{ji} k_T^i}{M} h_1^\perp$$

Amsterdam notation:

f → unpolarized quarks
g → longitudinally polarized quarks
h → transversely polarized quarks

1 → the leading twist
L → longitudinally polarized hadron
T → transversely polarized hadron

Meißner, Metz, Schlegel. JHEP08 (2009) 056

[Zhi-Min Zhu, et al. in preparation]

Overview on TMDs (spin 1/2)

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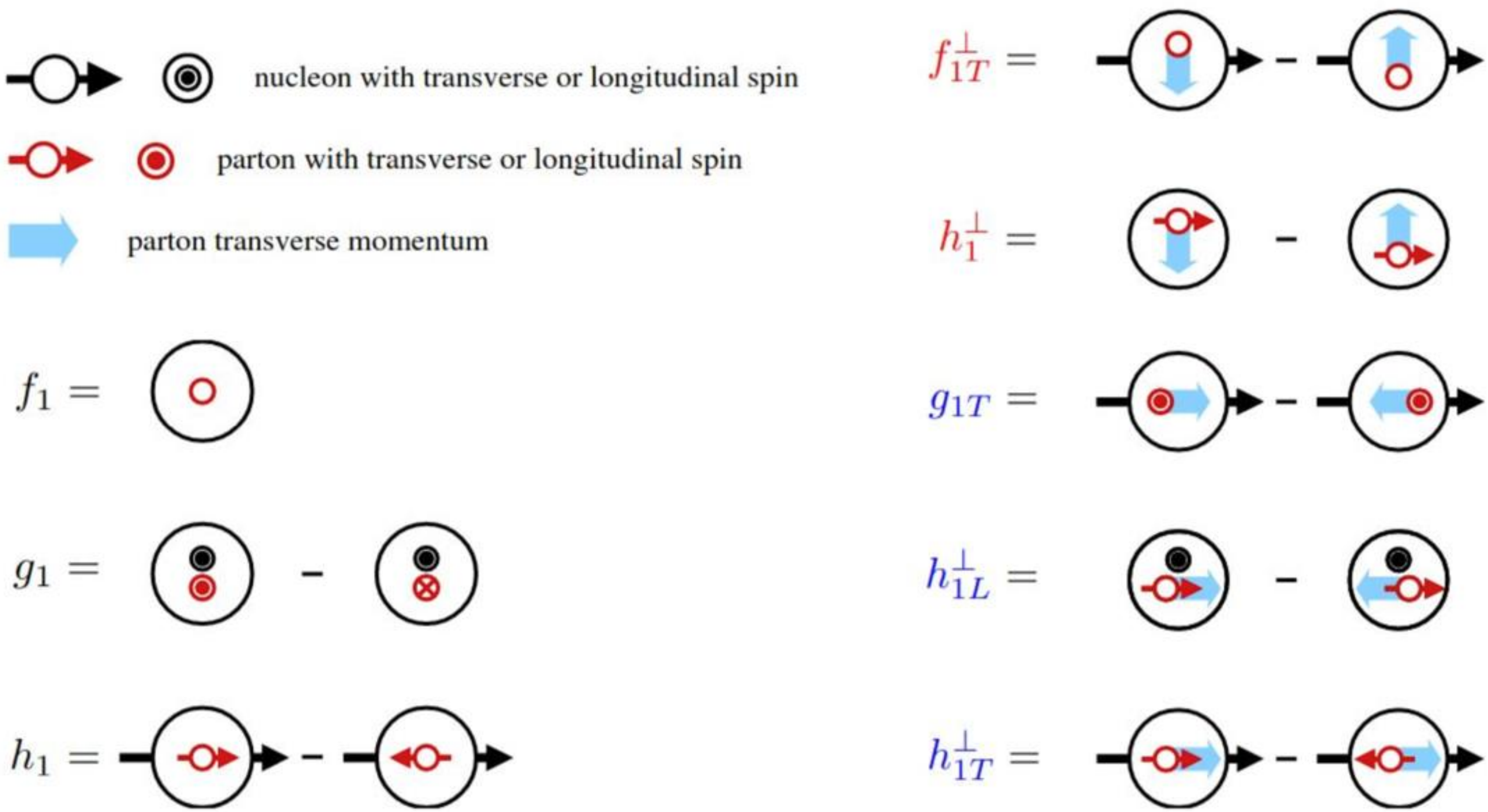

From Bacchetta's "Trento" Lecture notes

[Zhi-Min Zhu, et al. in preparation]

TMDs Evaluation using the overlaps LFWF

TMDs definition:

$\mathcal{W} \to 1$

$$\Phi^{[\Gamma]}\left(P,S,S';x=\frac{k^+}{P^+},\boldsymbol{k}_T\right)=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}^2\boldsymbol{z}^\perp}{2(2\pi)^3}e^{ik\cdot z}\langle P,S'|\bar{\Psi}(0)\Gamma\mathcal{W}(0^\perp,z^\perp)\Psi(z)|P,S\rangle\Big|_{z^+=0}$$

Baryonic bound state:

Λ^0 baryon

$$|\Lambda^0(P,\Lambda)\rangle=\sum_{\lambda_1,\lambda_2,\lambda_3}\int\prod_i^3\frac{[\mathrm{d}x_i\,\mathrm{d}p_i^\perp]}{2(2\pi)^3\sqrt{x_i}}2(2\pi)^3\delta\left(1-\sum_i^3 x_i\right)\delta^2\left(\boldsymbol{P}^\perp-\sum_i^3\boldsymbol{p}_i^\perp\right)\psi_3^\Lambda(\{\lambda_i,p_i\})|u(\lambda_1,p_1)d(\lambda_2,p_2)s(\lambda_3,p_3)\rangle$$

Leading Fock Sector

Overlap form of TMDs:

$$\Phi^{[\Gamma]}_{\Lambda',\Lambda;u}(x,k^\perp)=\sum_{\lambda_1'\lambda_1\lambda_2\lambda_3}\int\mathrm{d}x_2\mathrm{d}^2p_2^\perp\psi^{\Lambda'*}_{3,\lambda_1',\lambda_2,\lambda_3}(k,p_2,P-k-p_2)\psi^{\Lambda}_{3,\lambda_1,\lambda_2,\lambda_3}(k,p_2,P-k-p_2)f^\Gamma_{\lambda_1'\lambda_1}$$

Hadron spin

Light-front wave function (LFWF)

Struck quark spin

For example,

$$f_1^u(x,k^\perp)=\frac{1}{2}\left(\Phi^{[\gamma^+]}_{++;u}+\Phi^{[\gamma^+]}_{--;u}\right)$$

$$f^{[\gamma^+]}_{\lambda',\lambda}=\sigma^0_{\lambda'\lambda}$$

[Zhi-Min Zhu, et al. in preparation]

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A light front Hamiltonion approach

Light front Schrödinger equation:

$$H_{eff}|\Psi\rangle = M^2|\Psi\rangle$$

where

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^{\,2} + m_a^2}{x_a}$$ Kinetic energy

$$+ \frac{1}{2}\sum_{a\neq b}\kappa^4\left[x_a x_b\left(\vec{r}_{\perp a} - \vec{r}_{\perp b}\right)^2 - \frac{\partial_{x_a}\left(x_a x_b \partial_{x_b}\right)}{\left(m_a + m_b\right)^2}\right]$$ Confining potential

$$+ \frac{1}{2}\sum_{a\neq b}\frac{C_F 4\pi\alpha_s}{Q_{ab}^2}\bar{u}_{s_a'}\left(k_a'\right)\gamma^\mu u_{s_a}\left(k_a\right)\bar{u}_{s_b'}\left(k_b'\right)\gamma^\nu u_{s_b}\left(k_b\right)g_{\mu\nu}$$ OGE

Wave function basis expansion:

Xu, Mondal, Lan, et al. Phys. Rev. D 104(9), 094036 (2021)

$$\Psi^\Lambda(\{x_i, \boldsymbol{k}_{\perp i}, \lambda_i\}) = \sum_{\{n_i, m_i\}} \psi^\Lambda(k_i, n_i, m_i, \lambda_i)$$

$$\times \prod_i \frac{1}{b}\sqrt{\frac{4\pi n_i!}{(n_i + |m_i|)!}} e^{-\boldsymbol{k}_{\perp i}^2/(2b^2)} \left(\frac{|\boldsymbol{k}_{\perp i}|}{b}\right)^{|m_i|} L_{n_i}^{|m_i|}\left(\frac{\boldsymbol{k}_{\perp i}^2}{b^2}\right) e^{i m_i \theta_i} \Psi_k(x_i)$$

2D-HO basis in transverse direction

plane-wave basis in longitudinal direction

[Zhi-Min Zhu, et al. in preparation]

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TMDs for the valance quarks of Λ^0

$$|\Lambda^0\rangle = |uds\rangle$$

$$\Phi^{[\gamma^+]}(x, \boldsymbol{k}_T; S) = f_1 - \frac{\epsilon_\perp^{ij} k_T^i S_\perp^j}{M} f_{1T}^\perp$$

$$\Phi^{[\gamma^+\gamma^5]}(x, \boldsymbol{k}_T; S) = \Lambda g_{1L} + \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_\perp}{M} g_{1T}$$

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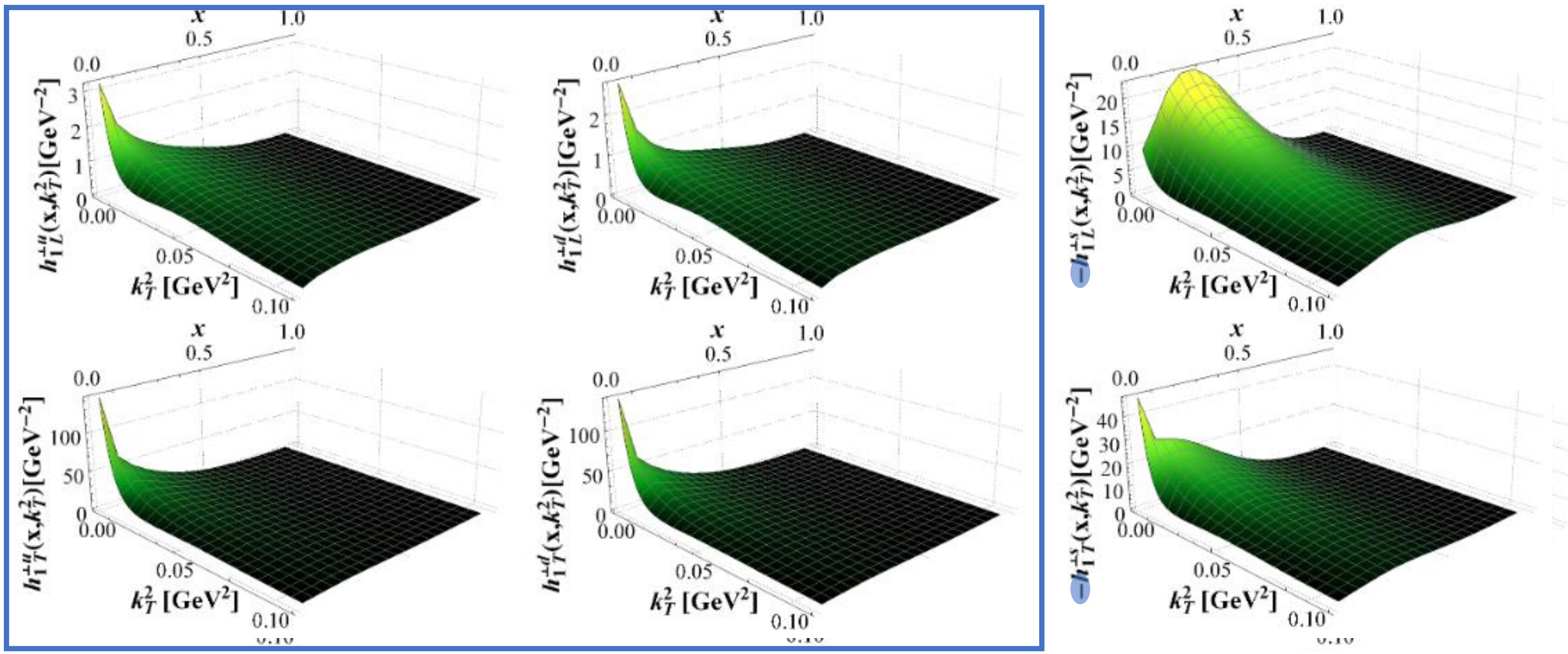


u quark　　d quark　　s quark

TMDs of u and d quark have the same structure.

Heavy quarks concentrate in larger x range.

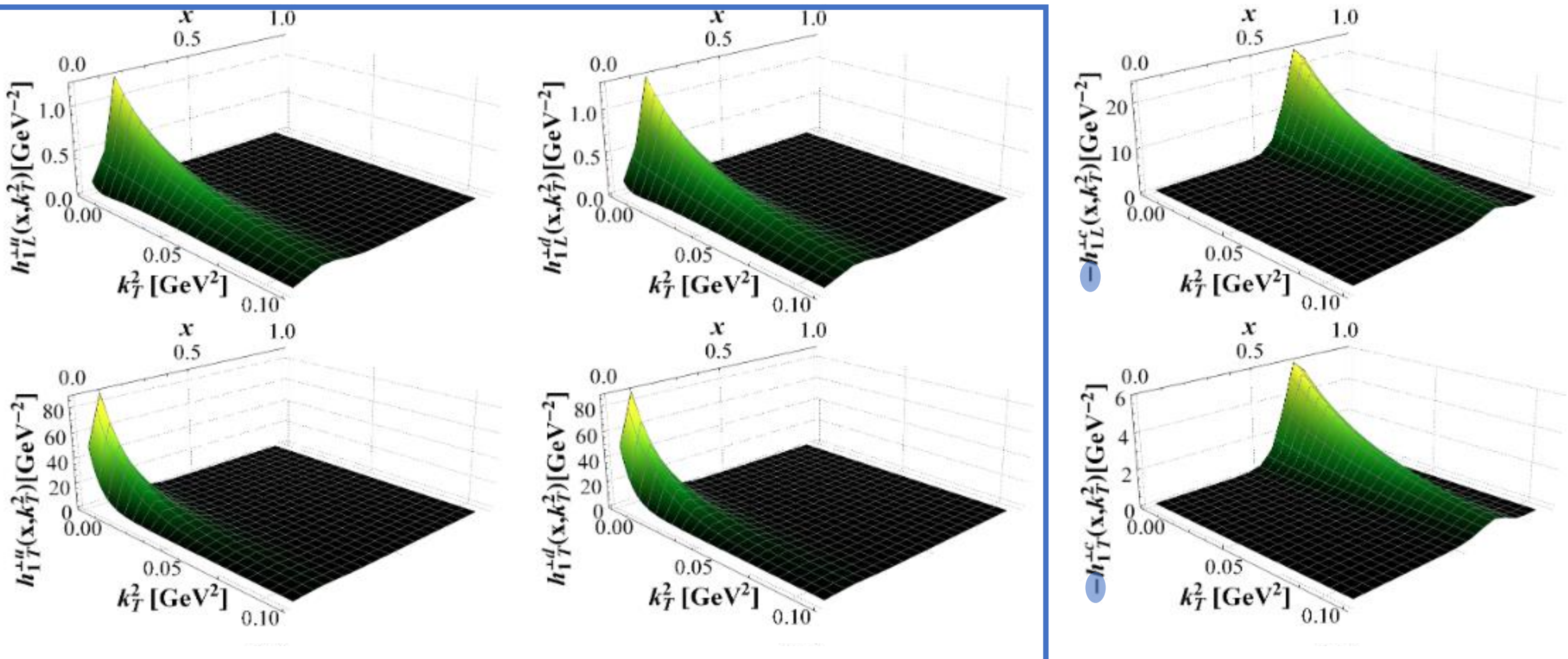
[Zhi-Min Zhu, et al. in preparation]

TMDs for the valance quarks of Λ_c $|\Lambda_c\rangle = |udc\rangle$

$$\Phi^{[\gamma^+]}(x, \boldsymbol{k}_T; S) = f_1 - \frac{\epsilon_\perp^{ij} k_T^i S_\perp^j}{M} f_{1T}^\perp$$

$$\Phi^{[\gamma^+\gamma^5]}(x, \boldsymbol{k}_T; S) = \Lambda g_{1L} + \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_\perp}{M} g_{1T}$$

$$\Phi^{[i\sigma^{j+}\gamma^5]}(x, \boldsymbol{k}_T; S) = S_\perp^j h_1 + \Lambda \frac{k_T^j}{M} h_{1L}^\perp + S_\perp^i \frac{2k_T^i k_T^j - (\boldsymbol{k}_T)^2 \delta^{ij}}{2M^2} h_{1T}^\perp + \frac{\epsilon_\perp^{ji} k_T^i}{M} h_1^\perp$$



u quark d quark c quark

TMDs of u and d quark have the same structure.

Heavy quarks concentrate in larger x range.

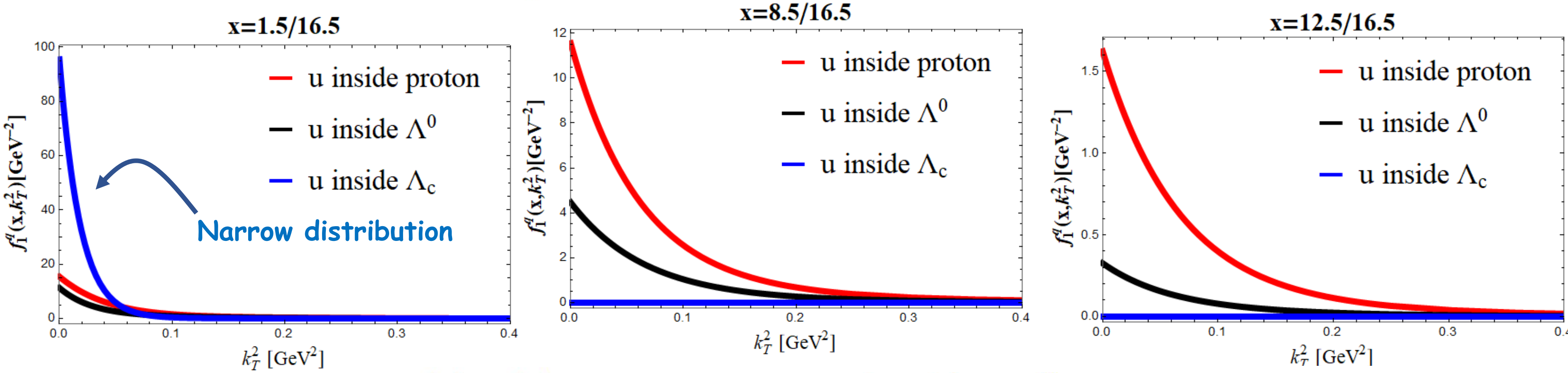
Heavy quarks are dispersed in broader k_T range.

[Zhi-Min Zhu, et al. in preparation]

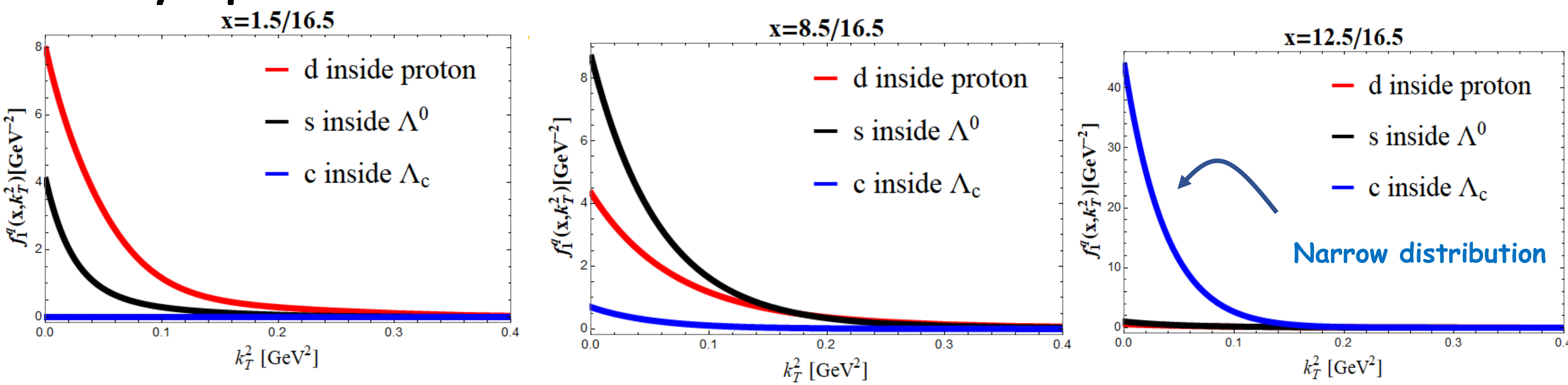
TMDs for proton, Λ^0 and Λ_c

Proton	938MeV
Λ^0	1115MeV
Λ_c	2286MeV

light quark — u inside proton — u inside Λ^0 — u inside Λ_c



"heavy" quark — d inside proton — s inside Λ^0 — c inside Λ_c



[Zhi-Min Zhu, et al. in preparation]

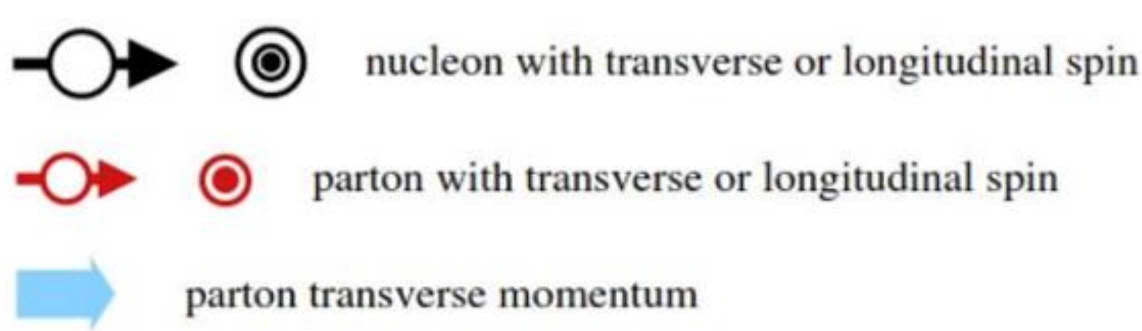
The spin-density

Definition:

Pasquini B, Lorcé C. arXiv:1203.5006, 2012.

$$\rho\left(x,\boldsymbol{k}_T,(\lambda,\boldsymbol{s}_\perp),(\Lambda,\boldsymbol{S}_\perp)\right)=\frac{1}{2}\Big[f_1+S_\perp^i\epsilon^{ij}k_T^j\frac{1}{M}f_{1T}^\perp+\lambda\Lambda g_{1L}+\lambda S_\perp^i k_T^i\frac{1}{M}g_{1T}$$
$$+s_\perp^i\epsilon^{ij}k_T^j\frac{1}{M}h_1^\perp+\Lambda s_\perp^i k_T^i\frac{1}{M}h_{1L}^\perp$$
$$+s_\perp^i S_\perp^i h_1+s_\perp^i\left(2k_T^ik_T^j-\vec{k}_\perp^2\delta^{ij}\right)S_\perp^j\frac{1}{2M^2}h_{1T}^\perp\Big]$$

Can't calculate red terms in our initial work



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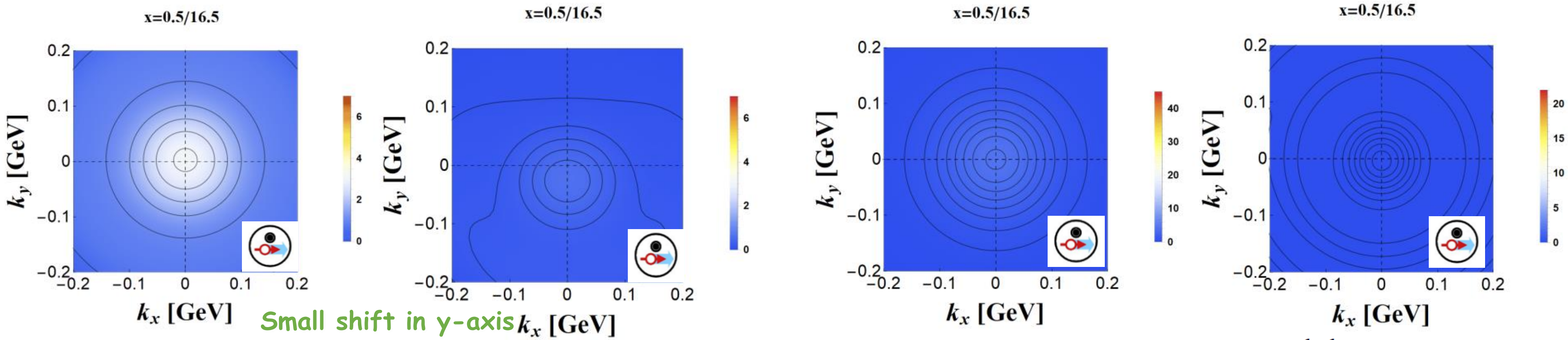
The spin-density for Λ^0 and Λ_c

$|\Lambda^0\rangle = |uds\rangle$ $\qquad$ $|\Lambda_c\rangle = |udc\rangle$

u/d quark | s quark | u/d quark | c quark

T(quark) L(baryon) spin density

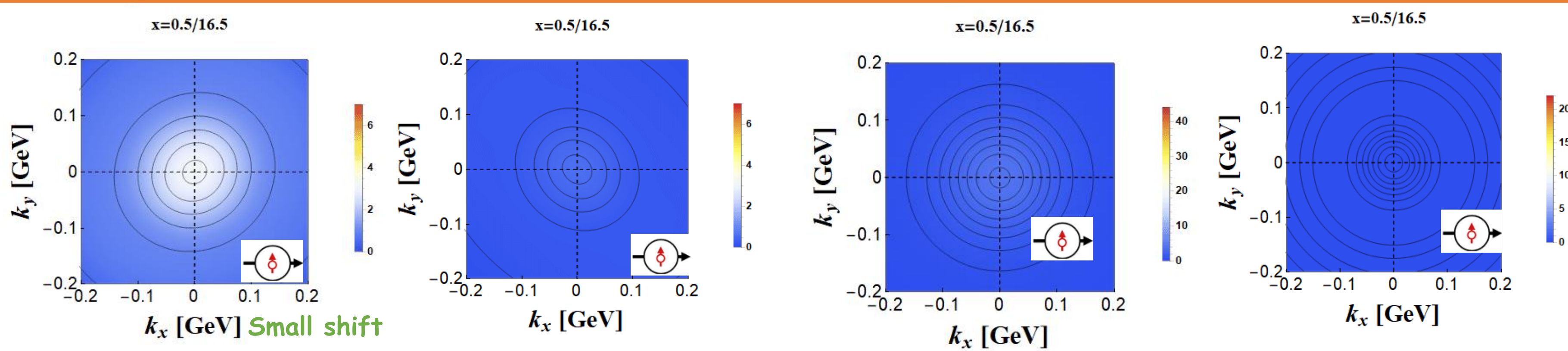
$(f_1^s + \frac{k_y}{M_{\Lambda^0}} h_{1L}^{\perp s})/2[\mathrm{GeV}^{-2}]$ $\qquad$ $(f_1^u + \frac{k_y}{M_{\Lambda_c}} h_{1L}^{\perp u})/2[\mathrm{GeV}^{-2}]$ $\qquad$ $(f_1^c + \frac{k_y}{M_{\Lambda_c}} h_{1L}^{\perp c})/2[\mathrm{GeV}^{-2}]$



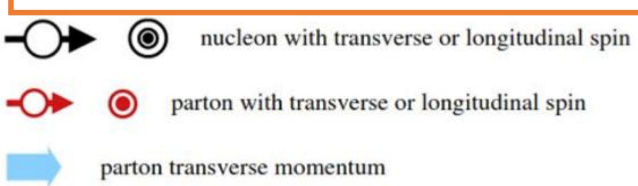
Small shift in y-axis

T-y(quark) T-x(baryon) spin density

$(f_1^s + \frac{k_x k_y}{M_{\Lambda^0}^2} h_{1T}^{\perp s})/2[\mathrm{GeV}^{-2}]$ $\qquad$ $(f_1^u + \frac{k_x k_y}{M_{\Lambda_c}^2} h_{1T}^{\perp u})/2[\mathrm{GeV}^{-2}]$ $\qquad$ $(f_1^c + \frac{k_x k_y}{M_{\Lambda_c}^2} h_{1T}^{\perp c})/2[\mathrm{GeV}^{-2}]$



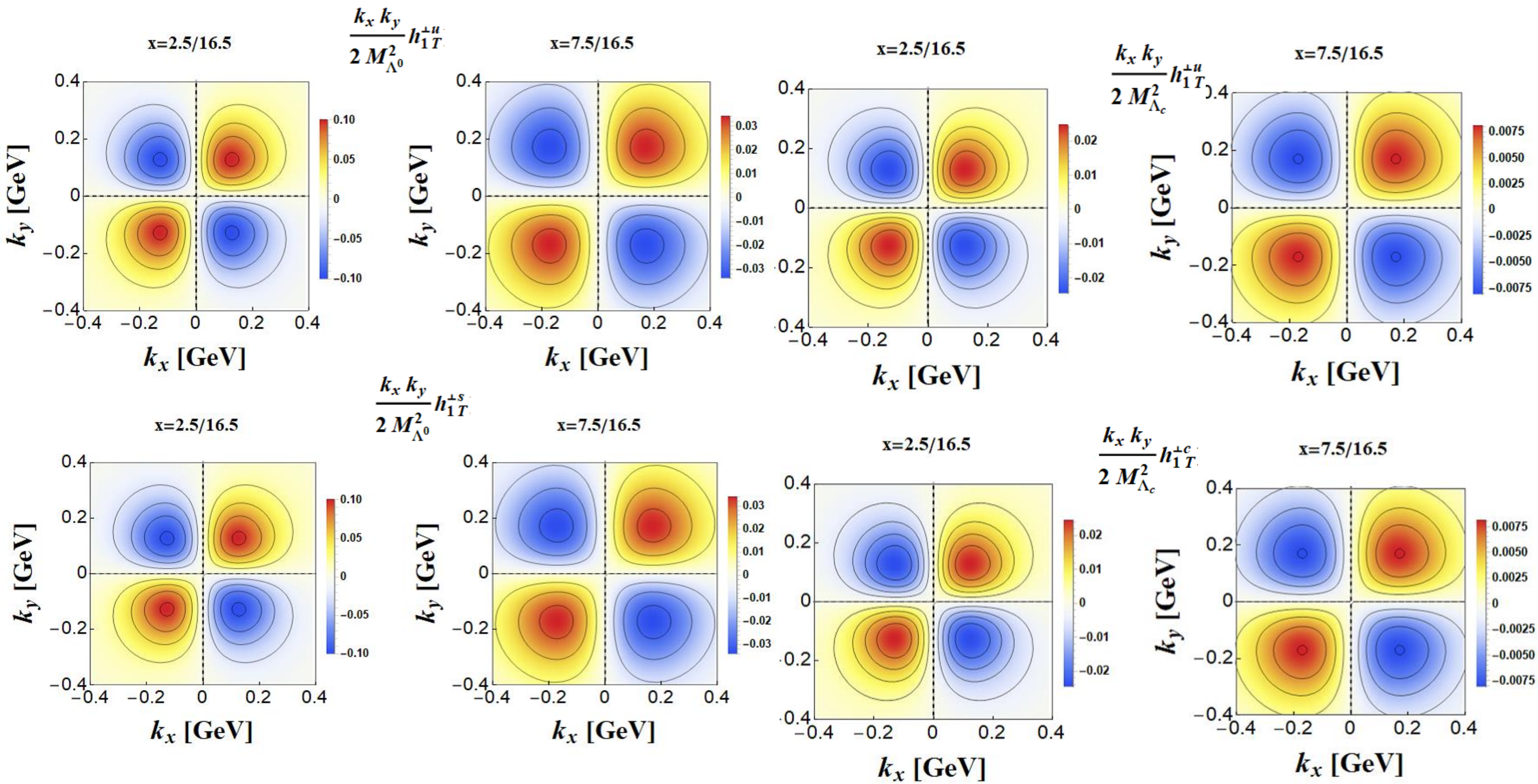
Small shift



Not circular but elliptical

[Zhi-Min Zhu, et al. in preparation]

The spin-density for Λ^0 and Λ_c



The distortion term is small.

[Zhi-Min Zhu, et al. in preparation]

Conclusion

- Heavy quarks concentrate in larger x range and carry more longitudinal momentum than light quark;

- Heavy quarks are dispersed in broader k_T range compared with light quark;

- There is a small shift in the spin-densities of quarks inside Λ^0 and Λ_c with different polarization direction.

Thank You !

[Zhi-Min Zhu, et al. in preparation]