Relations between GPDs and TMDs in LFQDM

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Based on: Phys.Rev.D 104 (2021) 7, 076028

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Outline

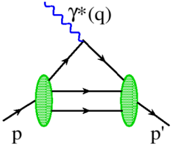
- Introduction: Various aspects of nucleon properties
- LF Quark-Diquark model
- Generalized Parton Distributions (GPDs)
- Tranverse Momentum Dependent Parton Distributions(TMDs)
- Possible relations between GPDs and TMDs
- Orbital Angular Momentum of the quarks
- Conclusion

Introduction: nucleon properties

- Hofstadter and collaborators interpret elastic ep scattering cross section in terms of a form factor: [-R. Hofstadter, Nobel Prize 1961]

$$\sigma(\theta_e) = \sigma_M \left| \int \rho(\vec{r}) e^{i\vec{q}.\vec{r}} d^3\vec{r} \right|^2 = \sigma_M |F(q)|^2$$

- The nucleon EM FFs describe the spatial distributions of electric charge and current inside the nucleon and thus are intimately related to its internal structure.
  - The electromagnetic form factors can be probed through elastic scattering.

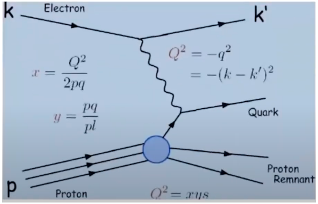


$$\langle N(p')|J^\mu(0)|N(p)\rangle =$$
$$\bar{u}(p')\left[\gamma^\mu \underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu \underbrace{F_2(q^2)}\right] u(p)$$

- The Fourier transformation of these form factors ⇒ spatial distributions.
- Dynamical informations (orbital angular momentum) ⇒ Missing!!

Introduction: nucleon properties

- DIS discovered the existence of quasi-free point-like objects (quarks) inside the nucleon. [Friedman, Kendall, Taylor, Nobel Prize 1990]
- Parton distribution functions (PDFs) are extracted from deep inelastic scattering (DIS) processes.

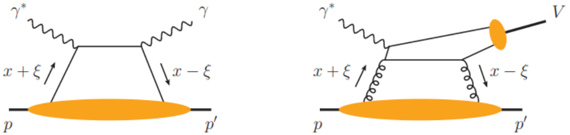


$$\underbrace{q(x)} = \frac{p^+}{4\pi} \int dy^- e^{ixp^+y^-} \times \langle p|\underbrace{\bar{\psi}_q(0)\mathcal{O}\psi_q(y)}|p\rangle\Big|_{y^+=\vec{y}_\perp=0}$$

- PDFs encode the distribution of longitudinal momentum and polarization carried by the constituents
- The PDFs provide no knowledge of spatial locations of parton

Introduction: nucleon properties

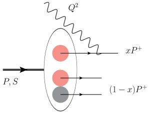
- GPDs appear in the exclusive processes like deeply virtual Compton scattering (DVCS) or vector meson productions.
- In DVCS a highly virtual photon, radiated by incident electron, interacts with a quark and emit a real photon before going back to the nucleon.



- GPDs encode the informations about the three dimensional spatial structure of the nucleon as well as the spin and orbital angular momentum of the constituents.

Light-front quark-diquark Model(LFQDM)

- Proton is considered as a bound state of a quark and a diquark. The diquark can have spin-0 singlet (scalar diquark) or spin-1 triplet (axialvector diquark).



- The proton state is written in the spin-flavor SU(4) structure as [-R. Jakob, P. J. Mulders, j. Rodrigues NPA626(1997)937]

$$|P;\pm\rangle = C_S \left|uS^0\right\rangle^{\pm} + C_V \left|uA^0\right\rangle^{\pm} + C_{VV} \left|dA^1\right\rangle^{\pm} .$$

- Two particle Fock-state expansion for $J_z = \pm\frac{1}{2}$
- The state with spin-0 scalar diquark

$$|uS\rangle^{\pm} = \int \frac{dxd^2\mathrm{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_\lambda \psi_\lambda^{\lambda_N(u)}(x,\mathrm{p}_\perp) \left|\lambda\Lambda_S; xP^+, \mathrm{p}_\perp\right\rangle\Big|_{\Lambda_S=0}$$

- The state with spin-1 axialvector diquarks

$$|\nu A\rangle^{\pm} = \int \frac{dxd^2\mathrm{p}_{\perp}}{2(2\pi)^3\sqrt{x(1-x)}} \sum_{\lambda} \sum_{\Lambda_A} \psi^{\lambda N(\nu)}_{\lambda \Lambda_A}(x, \mathrm{p}_{\perp}) \left|\lambda \Lambda_A; xP^+, \mathrm{p}_{\perp}\right\rangle \Big|_{\Lambda_A=1,0,-1}$$

- The light-front wave functions:

$$\psi^{\pm(\nu)}_{\lambda\Lambda}(x, \mathrm{p_T}) = N^{\nu} f(x, \mathrm{p_T}, \lambda, \Lambda)\, \phi^{(\nu)}_i(x, \mathrm{p_T})\,|_{i=1,2}$$

- Modified soft-wall AdS/QCD wave function for two particle bound state: [-Brodsky and Teramond arXiv:1203.4025]

$$\varphi^{(\nu)}_i(x, \mathrm{p}_{\perp}) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\, x^{a^{\nu}_i}(1-x)^{b^{\nu}_i} \exp\left[-\delta^{\nu}\frac{\mathrm{p}^2_{\perp}}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right]$$

with the AdS/QCD scale parameter $\kappa = 0.4 GeV$.

- The parameters $a^{\nu}_i, b^{\nu}_i, \delta^{\nu}$ are fixed by fitting the experimental data of the Dirac $F^{\nu}_1(Q^2)$ and Pauli $F^{\nu}_2(Q^2)$ form factors.

Nucleon Generalized Parton Distributions (GPDs)

- GPDs can be defined through off-forward matrix elements of quarks and gluon operators at a light-cone seprations

$$F^{q[\Gamma]}\left(x,\Delta;\lambda,\lambda'\right)=\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ik\cdot z}\left\langle p';\lambda'\right|\bar{\psi}\left(-\frac{1}{2}z\right)\Gamma\times\mathcal{W}_{GPD}\left(-\frac{1}{2}z;\frac{1}{2}z\right)\psi\left(\frac{1}{2}z\right)|p;\lambda\rangle\Big|_{z^+=0^+,z_T=0_T}$$

- For three different Dirac, $\Gamma=\gamma^+$(vector), $\Gamma=\gamma^+\gamma_5$(axial vector) and $\Gamma=i\sigma^{+j}\gamma_5$(tensor) structures, The GPD correlator

$$F^{q\left[\gamma^+\right]}\left(x,\Delta;\lambda,\lambda'\right)=\frac{1}{2P^+}\bar{u}\left(p',\lambda'\right)\left[H^q\gamma^++E^q\frac{i}{2M}\sigma^{+\alpha}\Delta_\alpha\right]u(p,\lambda)$$

$$F^{q\left[\gamma^+\gamma_5\right]}\left(x,\Delta;\lambda,\lambda'\right)=\frac{1}{2P^+}\bar{u}\left(p',\lambda'\right)\left[\tilde{H}^q\gamma^+\gamma_5+\tilde{E}^q\frac{\gamma_5\Delta^+}{2M}\right]u(p,\lambda)$$

$$F^{q\left[\sigma^{+j}\gamma_5\right]}\left(x,\Delta;\lambda,\lambda'\right)=\frac{1}{2P^+}\bar{u}\left(p',\lambda'\right)\left[H_T^q\sigma^{+j}\gamma_5+\tilde{H}_T^q\frac{\epsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2}+E_T^q\frac{\epsilon^{+j\alpha\beta}\Delta_\alpha\gamma_\beta}{2M}+\tilde{E}_T^q\frac{\epsilon^{+j\alpha\beta}P_\alpha\gamma_\beta}{M}\right]u(p,\lambda)$$

- Project out quark polarizations: Unpolarized:Tr[Fγ^+] $\rightarrow$ (H,E)(x,ζ,t), Longitudinally polarized:Tr[F$\gamma^+\gamma_5$] $\rightarrow$ ($\tilde{H},\tilde{E}$) Transversely polarized[chiral-odd]:Tr[F$\sigma^{\perp+}$] $\rightarrow$ ($H_T,E_T,\tilde{H}_T,\tilde{E_T}$)

GPDs sum rules and relations

- First moment of the GPDs are related to the electromagnetic FF.
- Second moment of GPDs give the gravitational FFs.

$$\int_0^1 dx\, H_v^q(x,t) = F_1^q(t), \quad \int_0^1 dx\, E_v^q(x,t) = F_2^q(t)$$
$$\int_0^1 dx\, xH_v^q(x,t) = A^q(t), \quad \int_0^1 dx\, xE_v^q(x,t) = B^q(t)$$

- At $t = -Q^2 = 0$, Gravitational FFs $\Rightarrow$ Angular momentum.

$$J^q = \frac{1}{2}[A^q(0) + B^q(0)]$$ [X. Ji, PRL78, 610(1997)]

$$J_T^q = \frac{1}{2}[A_T^q(0) + 2\widetilde{A}_T^q(0) + B_T^q(0)]$$ [M. Burkardt, PRD72, 094020(2005)]

- GPDs $\Rightarrow$ not probabilistics!! (in momentum space).
- GPDs in impact parameter space[$\zeta = 0$]:

$$\mathcal{F}(x, \mathrm{b}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\Delta_T \cdot b_T} F(x, 0, \Delta_T)$$

- GPDs in Impact parameter space(IPDs) $\Rightarrow$ Parton distribution in transverse position space.
- Impact parameter space$\rightarrow$ "digonal" matrix elements $\Rightarrow$ Density interpretions!

TMDPDFs

- TMDs give probability of finding a parton with the longitudinal momentum fraction x and the transverse momentum $p_\perp$ inside a nucleon.
- In the light front formalism, the unintegrated quark-quark correlator for SIDIS

$$\Phi^{\nu[\Gamma]}(x, \mathrm{p}_\perp; S) = \frac{1}{2}\int \frac{dz^- d^2z_T}{2(2\pi)^3} e^{ip.z} \left\langle P; S \left| \bar{\psi}^\nu(-\frac{z}{2}) \mathcal{W}_{TMD}(\frac{-z}{2}, \frac{z}{2}) \Gamma \psi^\nu(\frac{z}{2}) \right| P; S \right\rangle \Big|_{z^+=0}$$

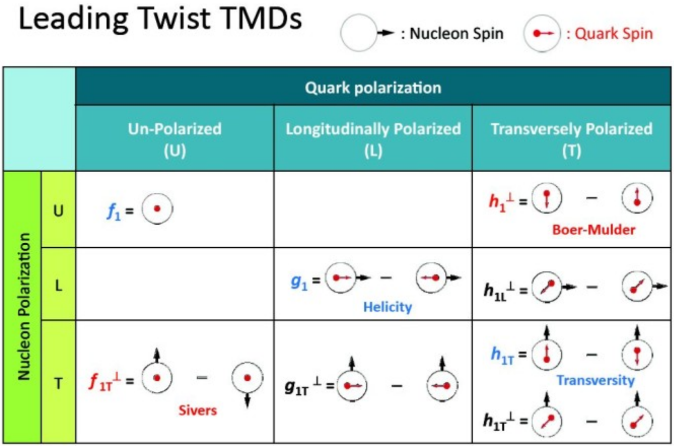
- Leading twist TMDs are projected out using different Dirac structure

$$\Phi^{\nu\left[\gamma^+\right]}(x, \mathrm{p}_\perp; S) = f_1^\nu\left(x, \mathrm{p}_\perp^2\right) - \frac{\epsilon_T^{ij} p_\perp^i S_T^j}{M} f_{1T}^{\perp\nu}\left(x, \mathrm{p}_\perp^2\right)$$

$$\Phi^{\nu\left[\gamma^+\gamma^5\right]}(x, \mathrm{p}_\perp; S) = \lambda_N g_{1L}^\nu\left(x, \mathrm{p}_\perp^2\right) + \frac{\mathrm{p}_\perp \cdot \mathrm{S}_T}{M} g_{1T}^\nu\left(x, \mathrm{p}_\perp^2\right)$$

$$\Phi^{\nu\left[i\sigma^{j+}\gamma^5\right]}(x, \mathrm{p}_\perp; S) = S_T^j h_1^\nu\left(x, \mathrm{p}_\perp^2\right) + \lambda_N \frac{p_\perp^j}{M} h_{1L}^{\perp\nu}\left(x, \mathrm{p}_\perp^2\right) + \frac{2p_\perp^j \mathrm{p}_\perp.\mathrm{S}_T - S_T^j \mathrm{p}_\perp^2}{2M^2} h_{1T}^{\perp\nu}\left(x, \mathrm{p}_\perp^2\right) - \frac{\epsilon_T^{ij} p_\perp^i}{M} h_1^{\perp\nu}\left(x, \mathrm{p}_\perp^2\right)$$

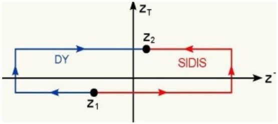
- Depending on the different polarization of nucleon and the interior partons, there are altogether 8 TMDs at the leading twist: 6 T-even and 2 T-odd.
- Project out quark polarizations: Unpolarized: $Tr[\Phi\gamma^{+}] \rightarrow (f_1, f_{1T}^{\perp})(x, p_T^2)$, Longitudinally polarized: $Tr[\Phi\gamma^{+}\gamma_5] \rightarrow (g_{1L}, g_{1T})$, Transversely polarized[chiral-odd]: $Tr[\Phi\sigma^{\perp +}] \rightarrow (h_1, h_{1T}^{\perp}, h_{1L}^{\perp}, h_1^{\perp})$



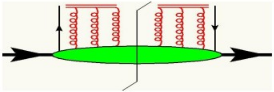
Gauge link for TMDs

- Quark field operators are at two different points $\Rightarrow$ breaks the gauge invarience of the correlator!!
- Gauge invarience restored $\Rightarrow$ Gauge link/ Wilson line $\mathcal{W}[z_1; z_2]$
- Gauge link $\mathcal{W}[z_1; z_2]$ for T-even TMDs $\Rightarrow$ Unity.
- Gauge link for T-odd TMDs $\not\Rightarrow$ Unity[more complicated function].

$$\mathcal{W}[z_1; z_2] = \mathcal{P}e^{-ig\int_{z_1}^{z_2} ds \cdot A(s)}$$



- Describes initial (DY) and Final(SIDIS) state interactions



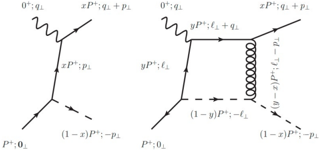
- Time-reversal: switches Wilson-line *ISI* $\rightarrow$ *FSI*

$f_{1T}^{\perp}|_{DIS} = -f_{1T}^{\perp}|_{DY}$, $\quad h_1^{\perp}|_{DIS} = -h_1^{\perp}|_{DY}$ $\Rightarrow$ process dependence!!

Final state interactions for SIDIS and T-odd TMDs

- Gluon exchange between outgoing quark and the target spectator system $\Rightarrow$ FSI

- T-odd TMDs $\Rightarrow$ FSI $\rightarrow$ complex phase!
- Leads SSA at leading twist.



- The modified LFWFs for T-odd TMDs

$$\psi^{\lambda_N}_{\lambda_q} = \tilde{\psi}^{\lambda_N}_{\lambda_{\rm q}} + i\tilde{\phi}^{\lambda_N}_{\lambda_{\rm q}}$$

$$\psi^{\lambda_N}_{\lambda_{\rm q}\lambda_{\rm D}} = \tilde{\psi}^{\lambda_N}_{\lambda_{\rm q}\lambda_{\rm D}} + i\tilde{\phi}^{\lambda_N}_{\lambda_{\rm q},\lambda_{\rm D}}$$

$$g_1 = \int_0^1 d\alpha \frac{-1}{\alpha(1-\alpha)\mathrm{p}_\perp^2 + \alpha m_g^2 + (1-\alpha)B}$$

$$g_2 = \int_0^1 d\alpha \frac{-\alpha}{\alpha(1-\alpha)\mathrm{p}_\perp^2 + \alpha m_{\rm g}^2 + (1-\alpha)B}$$

$$B = x(1-x)\left(-M^2 + \frac{m_q^2}{x} + \frac{m_D^2}{1-x}\right)$$

$$\tilde{\phi}^{\lambda_N}_{\lambda_q \Lambda_D}(x, \mathrm{p_T}) = N^\nu f(x, \mathrm{p_T}, \lambda, \Lambda)\left[\frac{e1e2}{8\pi}\left(p_T^2 + B(x)\right) g_i\right] \phi_i^{(\nu)}(x, \mathrm{p_T})$$

$e_1 e_2 \rightarrow -4\pi C_F \alpha_s$ [-T. Maji, D. Chakrabarti, and A.Mukherjee Phys. Rev. D 97, 014016]

- The non-trivial and non-perturbative correlations between the intrinsic transverse momentum of the partons and the spin of the nucleon $\Rightarrow$ Sivers/Boer Mulders (T-odd) TMDs
- T-odd (Sivers and Boer Mulder) TMDs,

$$f_{1T}^{\perp\nu}\left(x,\mathrm{p}_T^2\right) = \left(C_S^2 N_S^{\nu 2} - C_A^2 \frac{1}{3} N_0^{\nu 2}\right) f^{\nu}\left(x,\mathrm{p}_T^2\right)$$

$$h_1^{\perp\nu}\left(x,\mathrm{p}_T^2\right) = \left(C_S^2 N_S^{\nu 2} + C_A^2 \left(\frac{1}{3} N_0^{\nu 2} + \frac{2}{3} N_1^{\nu 2}\right)\right) f^{\nu}\left(x,\mathrm{p}_T^2\right)$$

$$f^{\nu}\left(x,\mathrm{p}_T^2\right) = \left(-C_F \alpha_s\right) \times \frac{1}{x}\left[\frac{\mathrm{p}_T^2 + B(x)}{\mathrm{p}_T^2}\right] \log\left[\frac{\mathrm{p}_T^2 + B(x)}{B(x)}\right] \times \frac{1}{16\pi^3} A_1^{\nu}(x) A_2^{\nu}(x) \exp\left[-a(x)\mathrm{p}_T^2\right]$$

- Boer Mulders function $\propto$ Sivers function

$$h_1^{\perp\nu}(x,\mathrm{p}_\perp^2) \simeq \lambda^{\nu} f_{1T}^{\perp\nu}(x,\mathrm{p}_\perp^2)$$

λ^{ν}	Our Model	Phenomenological fits
λ^{u}	2.29	2.1 ± 0.1
λ^{d}	-1.08	-1.11 ± 0.02

[-Barone,Melis, Prokudin,PRD81,114026]

For SIDIS process

- The Sivers TMD $\Rightarrow$

$f_{1T}^{\perp,u} < 0$, and $f_{1T}^{\perp,d} > 0$

- The Boer Mulders TMD $\Rightarrow$

$h_1^{\perp,u} < 0$, and $h_1^{\perp,d} < 0$

Confirmed by HERMES, COMPASS data

Non-trivial relations for "T-odd" parton distributions:

- Sivers TMD ⇒ Unpolarized partons inside transversely polarized nucleon. Related to the distortion in impact parameter space through "Lensing function"

$$\left\langle k_T^{q,i}(x)\right\rangle_{UT} = -\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M} f_{1T}^{\perp q}\left(x, \vec{k}_T^2\right)$$
$$\simeq \int d^2\vec{b}_T \mathcal{I}^{q,i}\left(x, \vec{b}_T\right) \frac{\epsilon_T^{jk} b_T^j S_T^k}{M}\left(\mathcal{E}^q\left(x, \vec{b}_T^2\right)\right)'$$

- Boer Mulder TMD ⇒ Transversely polarized partons inside unpolarized nucleon. The spin-orbit correlation of quarks can be understood from $h_1^{\perp}(x, \mathrm{p}_\perp)$

$$\left\langle k_T^{q,i}(x)\right\rangle_{TU} = -\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{j} k_T^j S_T^k}{M} h_1^{\perp q}\left(x, \vec{k}_T^2\right)$$
$$\simeq \int d^2\vec{b}_T \mathcal{I}^{q,i}\left(x, \vec{b}_T\right) \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \times \left(\mathcal{E}_T^q\left(x, \vec{b}_T^2\right) + 2\tilde{\mathcal{H}}_T^q\left(x, \vec{b}_T^2\right)\right)'$$

[-M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]]

Lensing function for LFQDM

- The Lensing function contains final state interaction information.
- The lensing function SDM/QTM model: $\mathcal{I}^{q,i}_{SDM}(x, b_T) = \frac{e_q e_s}{4\pi} \frac{(1-x) b^i_T}{b^2_T}$

[-S. Meissner, A. Metz, K. Goeke PhysRevD.76.034002, 2007]

(1) Axial vector diquark model

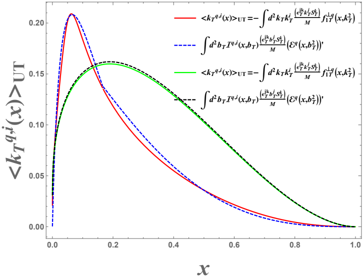
$$\mathcal{I}^{q,i} = \frac{5 F_F \alpha_s}{2\pi} x^{3/2} (1-x)^{-1/5} \log^5(1/x) \frac{b^i_T}{b^2_T}, \; 0.2 > x > 0$$

$$= \frac{2 C_F \alpha_s}{\pi} \sqrt{(1-x)} \log(1/x) \frac{b^i_T}{b^2_T}, \qquad 1 > x > 0.2$$

(2) scalar diquark model

$$\mathcal{I}^{q,i}_{SDM}(x, b_T) = \frac{5 C_F \alpha_s}{\pi} \frac{(1-x) b^i_T}{b^2_T}$$

- The lensing function for our model:



[B.Gurjar, D.Chakrabarti, P.Choudhary, A.Mukherjee and P.Talukdar,PhysRevD.104.076028]

Moment relations between GPDs and TMDs

- Relations between arbitrary moments:

$$\bullet GPD : X^{(n)}(x) = \frac{1}{2M^2}\int d^2\vec{\Delta}_T \left(\frac{\vec{\Delta}_T^2}{2M^2}\right)^{n-1} X\left(x, 0, -\frac{\vec{\Delta}_T^2}{(1-x)^2}\right)$$

$$\bullet TMD : Y^{(n)}(x) = \int d^2\mathrm{k}_T \left(\frac{\mathrm{k}_T^2}{2M^2}\right)^{n} Y\left(x, \mathrm{k}_T^2\right)$$

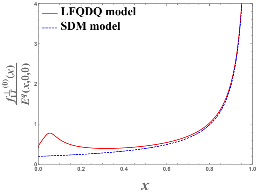
- Moment relations between $f_{1T}^{\perp q}$ and E^q

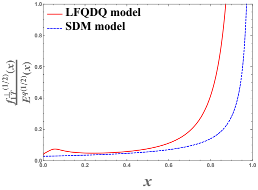
$$f_{1T}^{\perp q(0)}(x) = \frac{\pi e_q e_s}{48(1-x)} E^q(x,0,0)$$

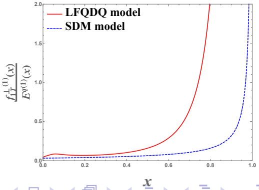
$$f_{1T}^{\perp q(1/2)}(x) = \frac{2 e_q e_s \ln(2)}{(2\pi)^3(1-x)} E^{q(1/2)}(x)$$

$$f_{1T}^{\perp q(1)}(x) = \frac{e_q e_s}{4(2\pi)^2(1-x)} E^{q(1)}(x)$$

[-S. Meissner, A. Metz, K. Goeke PhysRevD.76.034002, 2007]







- Relation between pretzelosity TMD $h_{1T}^{\perp,(n)}$ and GPD $\tilde{H}_T^{(n)}$

LFQDM

$$h_{1T}^{\perp q(0)}(x) = \frac{2}{(1-x)^2} \tilde{H}_T^q(x,0,0)$$

$$h_{1T}^{\perp q(1/2)}(x) = \frac{1}{2\pi(1-x)^2} \tilde{H}_T^{q(1/2)}(x)$$

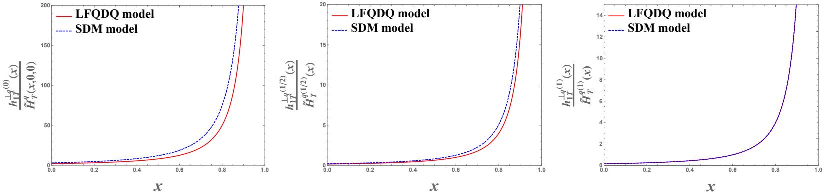
$$h_{1T}^{\perp q(1)}(x) = \frac{1}{2\pi(1-x)^2} \tilde{H}_T^{q(1)}(x)$$

SDM/QTM

$$h_{1T}^{\perp q(0)}(x) = \frac{3}{(1-x)^2} \tilde{H}_T^q(x,0,0)$$

$$h_{1T}^{\perp q(1/2)}(x) = \frac{8}{(2\pi)^2(1-x)^2} \tilde{H}_T^{q(1/2)}(x)$$

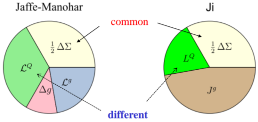
$$h_{1T}^{\perp q(1)}(x) = \frac{1}{2\pi(1-x)^2} \tilde{H}_T^{q(1)}(x)$$



[B.Gurjar, D.Chakrabarti et al.,PhysRevD.104.076028, 2021] & [-S. Meissner, A. Metz, K. Goeke PhysRevD.76.034002, 2007]

Quark OAM

- For many years, the spin of the proton S=1/2 ⇒ spin of the three valence quarks!
- In 1980, the EMC experiment at CERN ⇒ only 13 ± 16 % comes from the spin of the quarks, 'proton spin crisis' or 'proton spin puzzle'.
- Lattice QCD calculations indicates ⇒ 50% proton spin come from quark OAM.
- The orbital angular momentum (OAM) of a quark inside the nucleon plays an important role in the spin sum rule of the nucleon



- Jaffe and Manohar showed the decomposition of the nucleon spin (not gauge invarient!)

$$S_{proton} = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \langle L_q \rangle + \langle L_g \rangle$$

[−R.I.Jaffe, A.Manohar, Nucl.Phys.B337, 509(1990)]

- Ji proposed a gauge invariant decomposition of the nucleon spin as $S^q + L^q + J^g = \frac{1}{2}$

-X.D. Ji, Phys. Rev. Lett. 78, 610 (1997)

Quark OAM in LFQDM

- For the diquark model, the above sum rule can be written as

$$S^q + \ell^q + S^D + \ell^D = \frac{1}{2}$$

- The kinetic orbital angular momentum of the quark can extract in terms of GPDs as

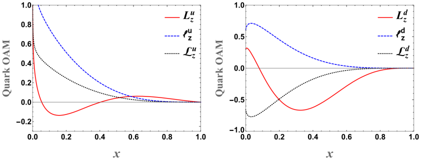
$$L_z^q = \frac{1}{2}\int dx \left[x\left(H^q(x,0,0) + E^q(x,0,0)\right) - \tilde{H}^q(x,0,0)\right]$$

- The quark OAM also can be calculated by using the pretzelosity TMD as

$$\mathcal{L}_z^q = -\int dx d^2p_\perp \frac{\vec{p}_\perp^2}{2M^2} h_{1T}^{\perp q}(x, \vec{p}_T^2)$$

- The Canonical quark OAM can be given through the GTMDs as

$$\ell_z^\nu = -\int dx d^2\mathrm{p}_\perp \frac{\mathrm{p}_\perp^2}{M^2} F_{1,4}^\nu\left(x, 0, \mathrm{p}_\perp^2, 0, 0\right)$$



[B.Gurjar, D.Chakrabarti et al.,PhysRevD.104.076028,

Conclusion

- Using LFQDM for the proton we have presented FFs, both the chiral even and odd GPDs and the T-even/T-odd TMDs at leading twist.
- We investigated moment relation between GPDs and TMDs in momentum space using LFQDM.
- Non-trivial relations between GPDs and (T-odd) TMDs were suggested on the basis of a separation between distortion of parton distribution in the transverse plane + Final state interactions.
- We calculate the quark orbital angular momentum and the results are compared with the results in other similar models.

Thank you!!