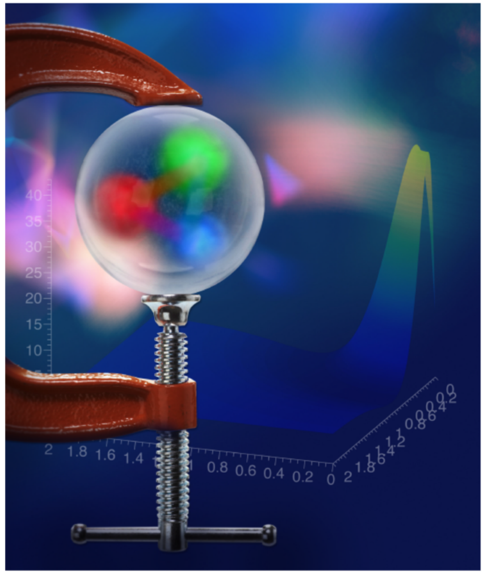
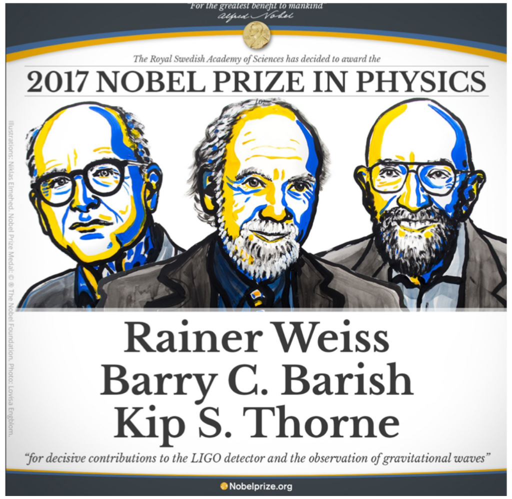
Towards mechanical properties of the proton

What we have learned from data so far



V. Burkert, L. Elouadrhiri, F.X. Girod

Gravitational waves observed

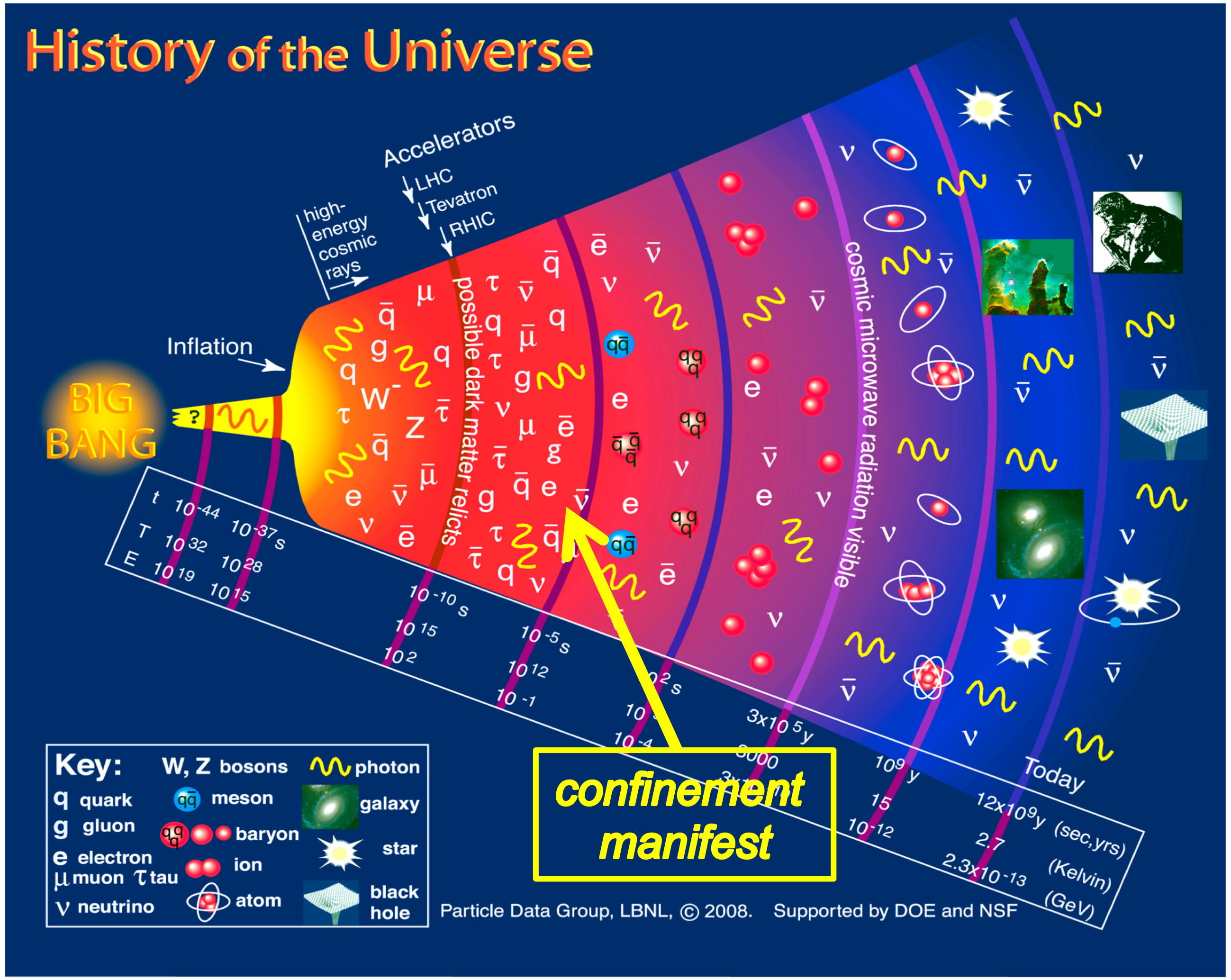


- **Gravity governs movements of massive structures in the universe.**
- **Plays a decisive role in neutron stars leading to the most densely packed macroscopic objects in the universe.**

The merger of two neutron stars generated gravitational waves that told us much about the equation-of-state of the neutron stars themselves.

Can we use gravitational waves to probe the interior of the proton and learn something about the EoS of the proton and the distribution of the strong force?

Focus on the Proton



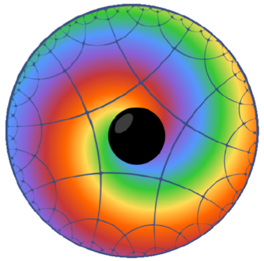
Confinement in hadrons becomes manifest in the crossover from the QGP phase to the hadron phase.

The proton emerges as the most fundamental bound-state in nature.

The mechanical properties of the proton are of foremost interest.

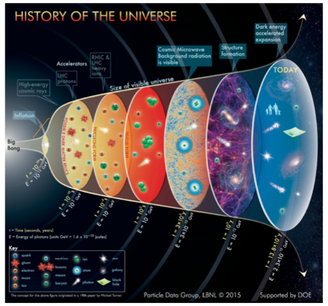
Basic questions about the proton

- Proton's make up nearly 90% of the mass of ordinary matter in the universe. Elementary quarks contribute a fraction to the proton's mass



What is the origin of its mass?

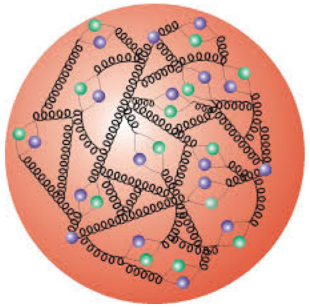
- Quarks and gluons formed stable protons as the universe cooled below 10^{13} K about few μsec after the Big Bang



What is the origin of confinement ?

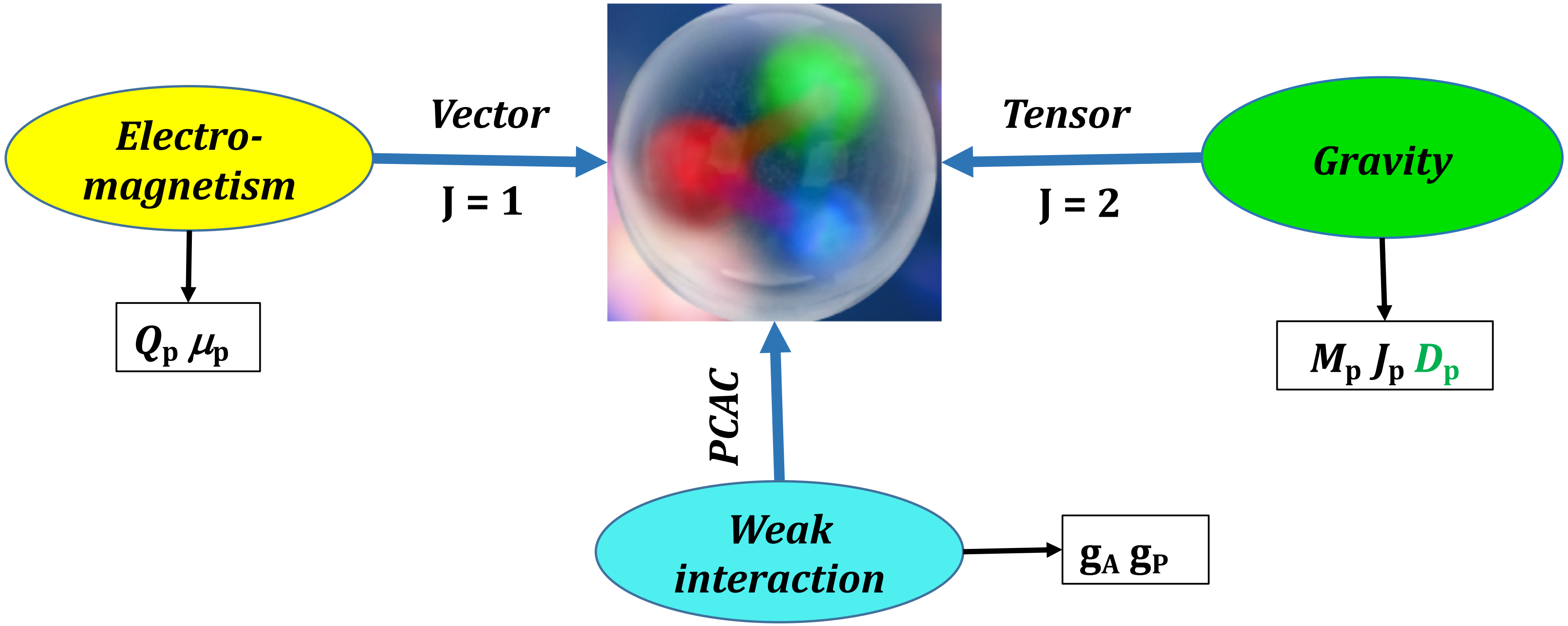
- The strong interaction is thought responsible for confinement

How do distribution of strong forces contribute to the proton's stability and to confinement?



Probing structural properties of the proton

The structure of strongly interacting particles can be probed by means of the other weaker fundamental forces: *electromagnetic, weak,* and *gravity*.

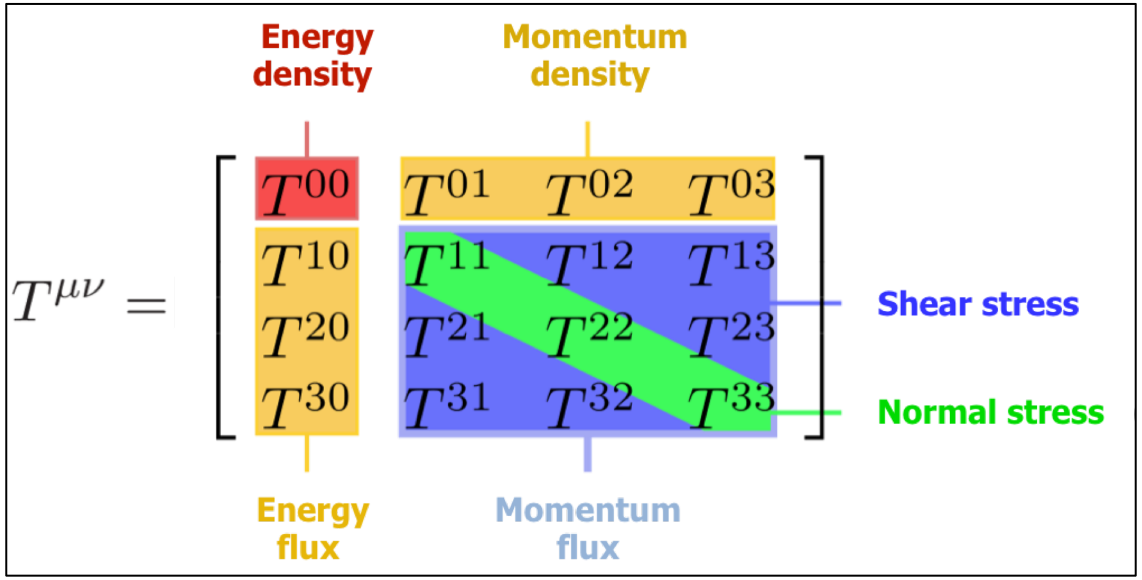


Experimentally we know very little about the mechanical properties.
➔ Need to probe the proton's energy-momentum tensor.

Energy Momentum Tensor $T^{\mu\nu}$

The framework for probing the protons EMT was developed in the 1960's.

Yu. Kobzarev and L.B. Okun, JETP 16, 5 (1963)
H. Pagels, Phys. Rev. 144 (1966) 1250-1260



"… there is very little hope of learning anything about the detailed mechanical structure of a particle, because of the extreme weakness of the gravitational interaction."
(H. Pagels, 1966)

$$T^{ij}(\mathbf{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s^Q(r) + \delta^{ij} p^Q(r)$$

$$d_1^Q(t) = 5M_p \int d^3\mathbf{r} \frac{j_2(r\sqrt{-t})}{t} s^Q(r)$$

$$d_1^Q(t) = 15M_p \int d^3\mathbf{r} \frac{j_0(r\sqrt{-t})}{2t} p^Q(r)$$

$s^Q(r)$: *shear stress;* ***$p^Q(r)$:*** *normal stress*
***j_0, j_2*:** 0^{th}, 2^{nd} order spherical Bessel functions

Extract: ***$s^Q(r)$, $p^Q(r)$*** with Fourier transform of ***$d_1^Q(t)$*** to coordinate space.

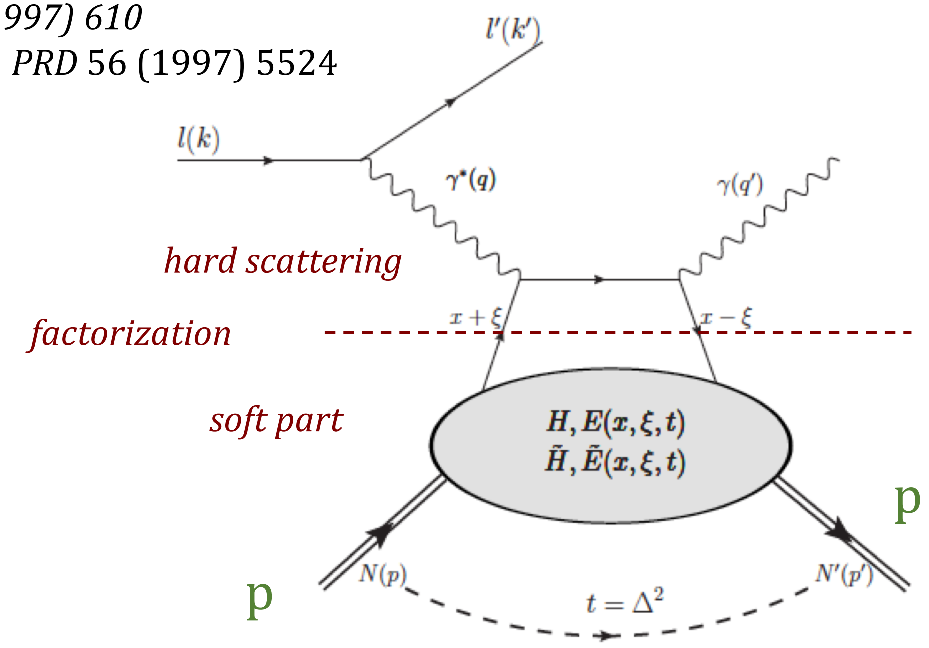
➔ Probing properties of subatomic particles directly with gravity is highly impractical

➔ **Use a substitute that mimics gravity ?**

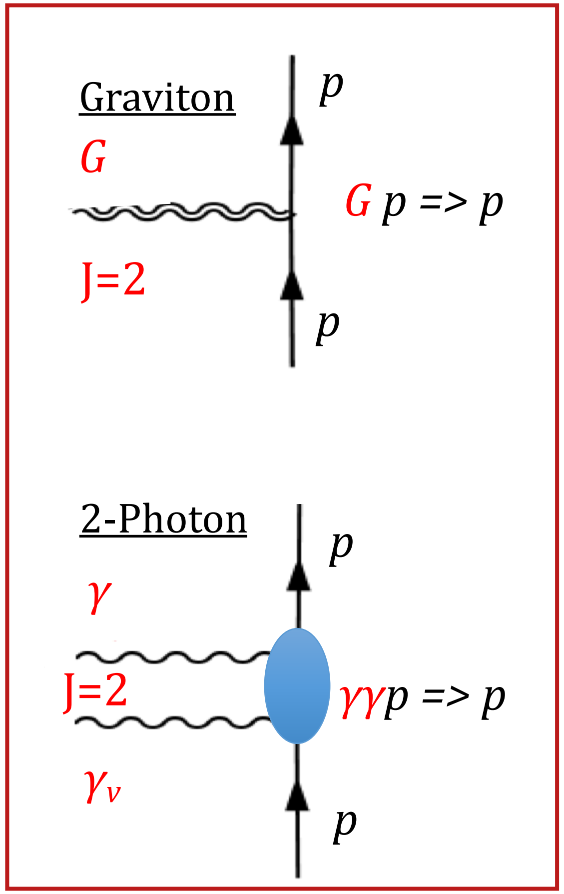
GPDs, DVCS, Gravity

D. Müller et al., F. Phys. 42,1994
X. Ji, PRD 55 (1997) 7114
X. Ji, PRL 78 (1997) 610
A. Radyushkin, PRD 56 (1997) 5524

M. Polyakov, PL B555 (2003) 57



$$\xi = \frac{x_B}{2 - x_B}$$



4 chiral even GPDs describe soft part. GPD ***H*** *is* important to access gravitational form factor ***D(t)***.

DVCS is a suitable probe of mechanical properties of particles

The 2γ field couples to the EMT as gravity does, with many orders of magnitude greater strength.

Moments of GPD & GFF Relations

Nucleon matrix element of the Energy-Momentum Tensor contains three gravitational form factors (GFF) and can be written as:

$$\langle p_2|\hat{T}^q_{\mu\nu}|p_1\rangle = \bar{U}(p_2)\left[M_2^q(t)\frac{P_\mu P_\nu}{M} + J^q(t)\frac{\imath(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d_1^q(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M}\right]U(p_1)$$

$M_2(t)$: Mass/energy distribution inside the nucleon
$J(t)$: Angular momentum distribution
$d_1(t)$: Forces and pressure distribution

GPDs ⟺ GFFs

$$\int dx\, x\,[H(x,\xi,t)+E(x,\xi,t)] = 2J(t)$$

$$\int dx\, x H(x,\xi,t) = M_2(t) + \frac{4}{5}\xi^2 d_1(t),$$

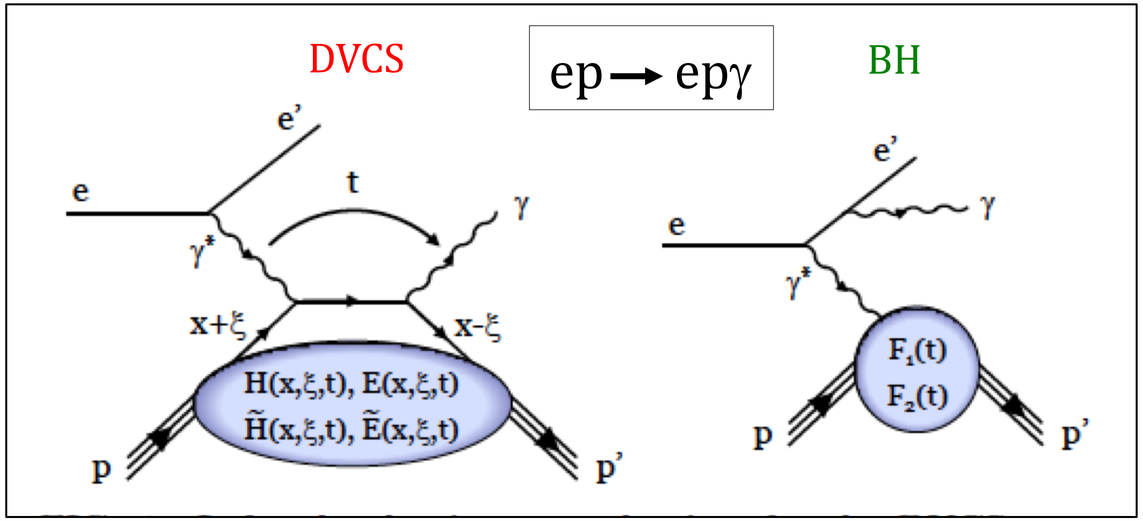
In DVCS, GPDs are not directly accessible at all x and ξ, but constrained to $x = \pm\,\xi$

X. Ji, Phys. Rev. D55, 7114 (1997)

From GPD to CFF to GFF

$$\mathcal{H}(\xi,t) = \int_{-1}^{+1} dx H(x,\xi,t) \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

Compton Form Factor $\mathcal{H}$ (CFF)



1) Polarized electron beam – BSA

$$\Delta\sigma_{LU}/d\sigma_U \sim \mathrm{Im}\{F_1\mathcal{H} + \ldots\}\sin\phi + ..$$

2) Unpolarized beam:

$$d\sigma_U \sim k\{\mathcal{H}\mathcal{H}^* + \ldots\} + ..$$

3) Dispersion Relation

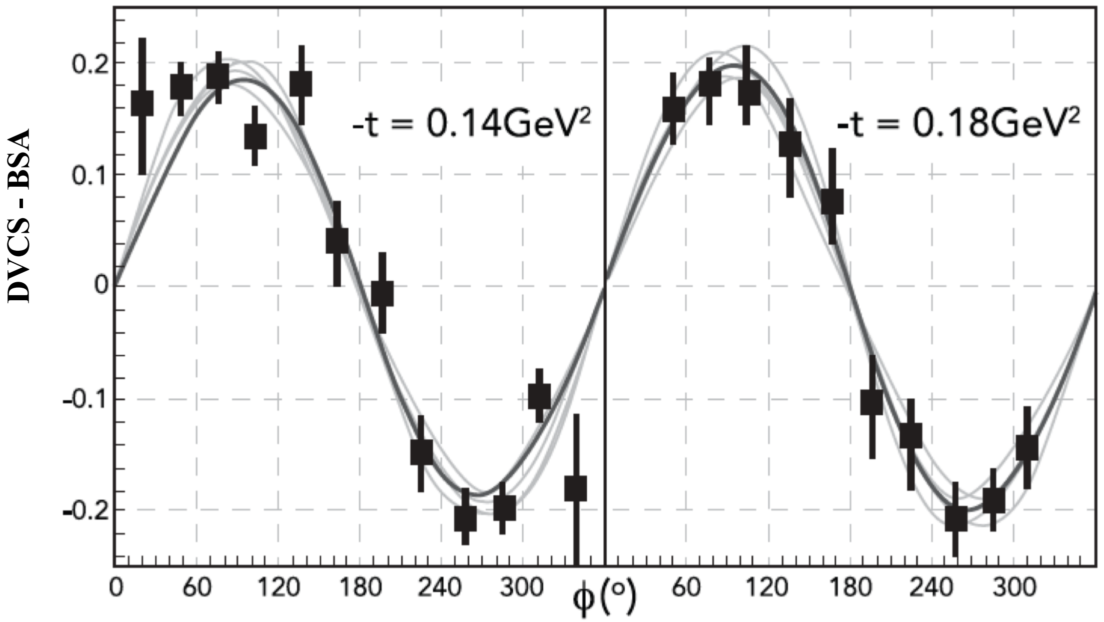
I.V. Anikin and O.V. Teryaev, Phys.Rev.D76, 056007 (2007)
M. Diehl and D.Y. Ivanov, Eur. Phys. J. C52, 919, (2007)

$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta(t) + \frac{1}{\pi}\mathcal{P}\int_0^1 dx \left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right] \mathrm{Im}\mathcal{H}(x,t).$$

$\mathcal{H}(\xi,t)$ ⇒ $\Delta(t)$ ⇒ $d_1(t)$

Sample fits to determine $\mathcal{H}(\xi, t)$

Samples of Beam Spin Asymmetry with fits



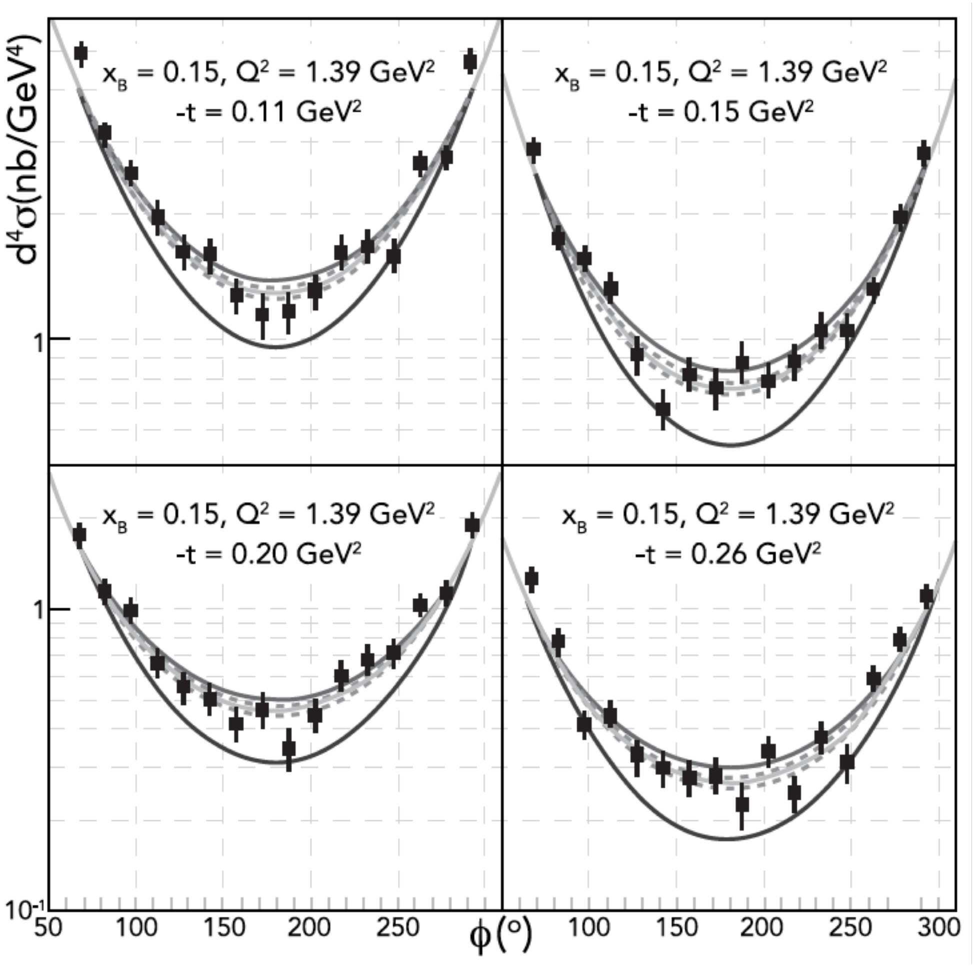
F.X. Girod et al., Phys.Rev.Lett. 100 (2008) 162002

Global fit:

$$\mathrm{Im}\mathcal{H}(\xi, t) = \frac{N}{1+\xi}\left(\frac{2\xi}{1+\xi}\right)^{-\alpha(t)}\left(\frac{1-\xi}{1+\xi}\right)^{b}\frac{1}{\left(1-\frac{1-\xi}{1+\xi}\frac{t}{M^2}\right)^{p}}$$

K. Kumericki et al, EPJ A 52, 159 (2016)

Samples of differential cross sections with fits



H.S. Jo et al., Phys.Rev.Lett. 115 (2015) 212003

Extracting CFF Im$\mathcal{H}$ & Re$\mathcal{H}$

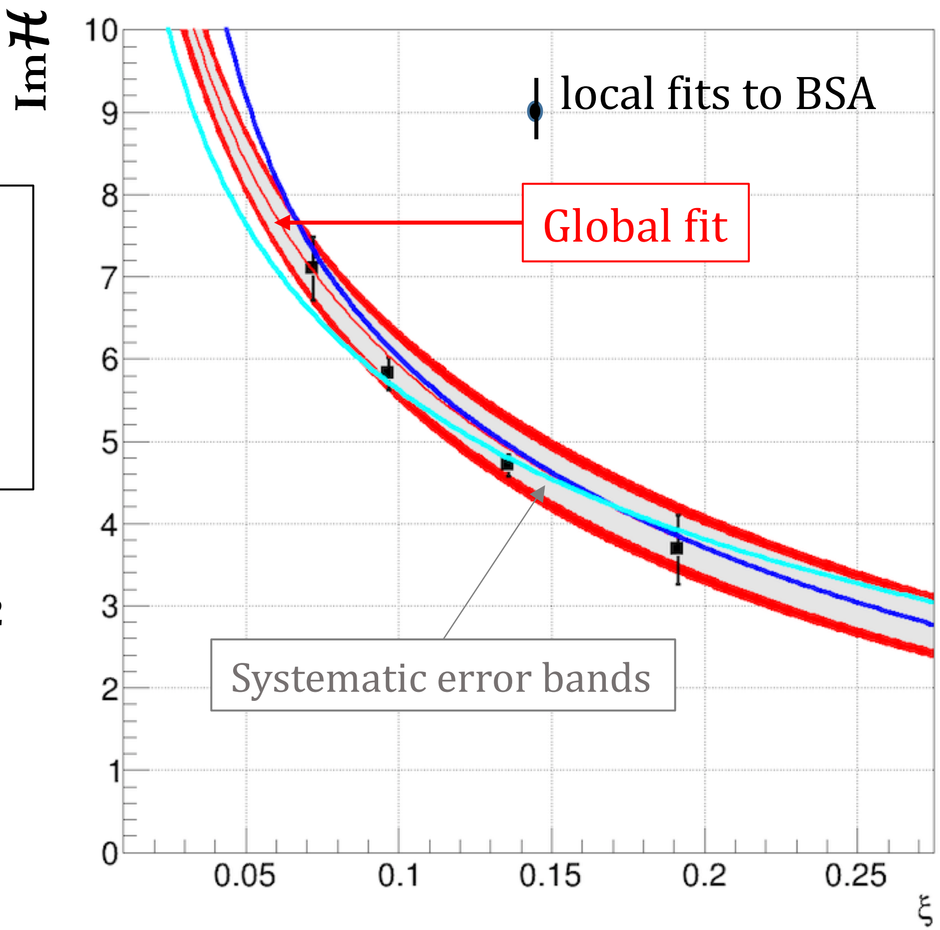
-t= 0.13 – 0.15 GeV²

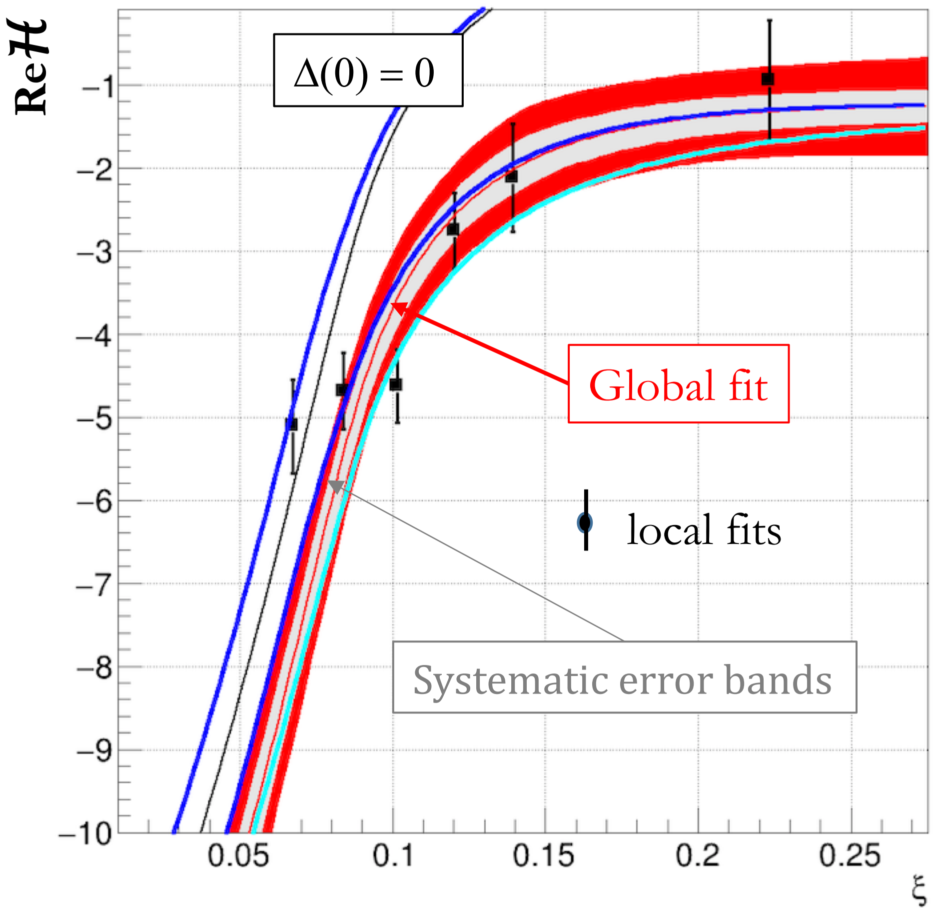
Global parameterizations:

BSA to determine Im$\mathcal{H}$

Differential DVCS cross sections to determine Re$\mathcal{H}$

K. Kumericki and D. Müller, EPJ Web Conf. 112 (2016) 01012

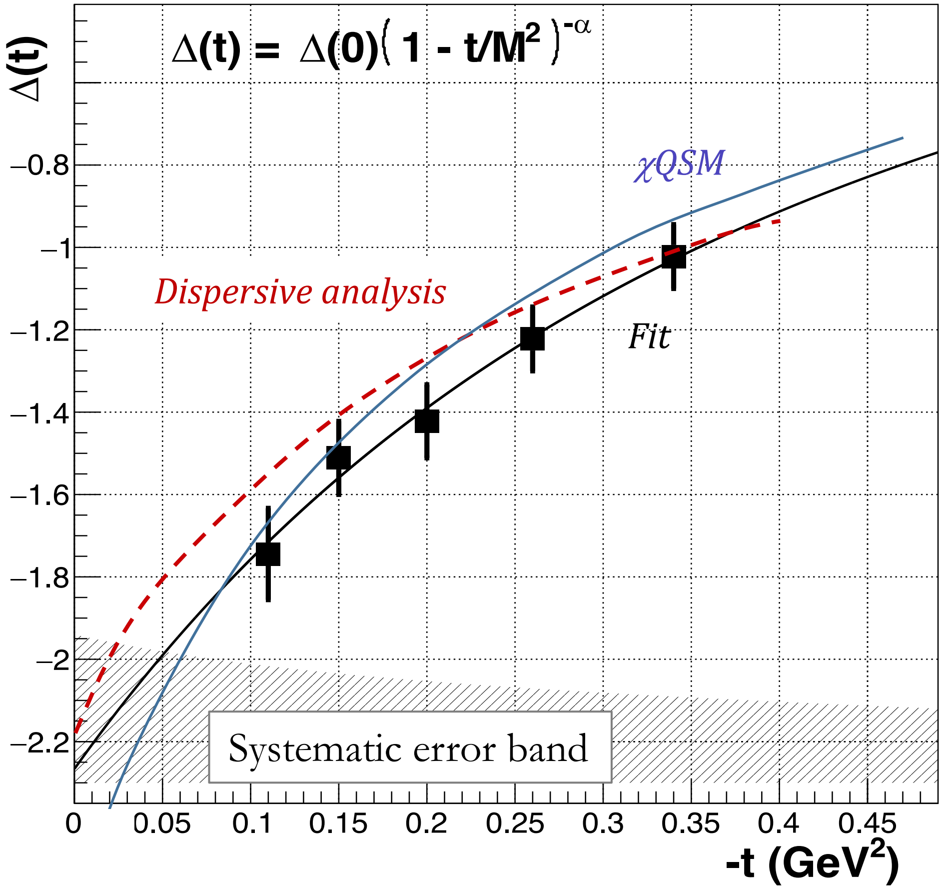




Re$\mathcal{H}$ has strong sensitivity to subtraction term Δ.

Extraction of Δ(t) and $d_1(t)$ for quark distribution

Δ(t) from CLAS 6 GeV DVCS data



V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396-399

$\Delta(0)$	=	**-2.27**	± 0.16	± 0.36
M^2	=	**1.02**	± 0.13	± 0.21
α	=	**2.76**	± 0.23	± 0.48

Dimensional scaling: **α = 3**

$\Delta(t) \propto d_1^Q(t)$ in double-distribution parameterization

$$\Delta(t) = 2\int_{-1}^{1} dz \frac{D(z,t)}{1-z}$$

$-1 < z = x/\xi < 1$

$$D(z,t) = (1-z^2)\left[e_u^2 + e_d^2\right]\frac{d_1^Q(t)}{2}3z,$$

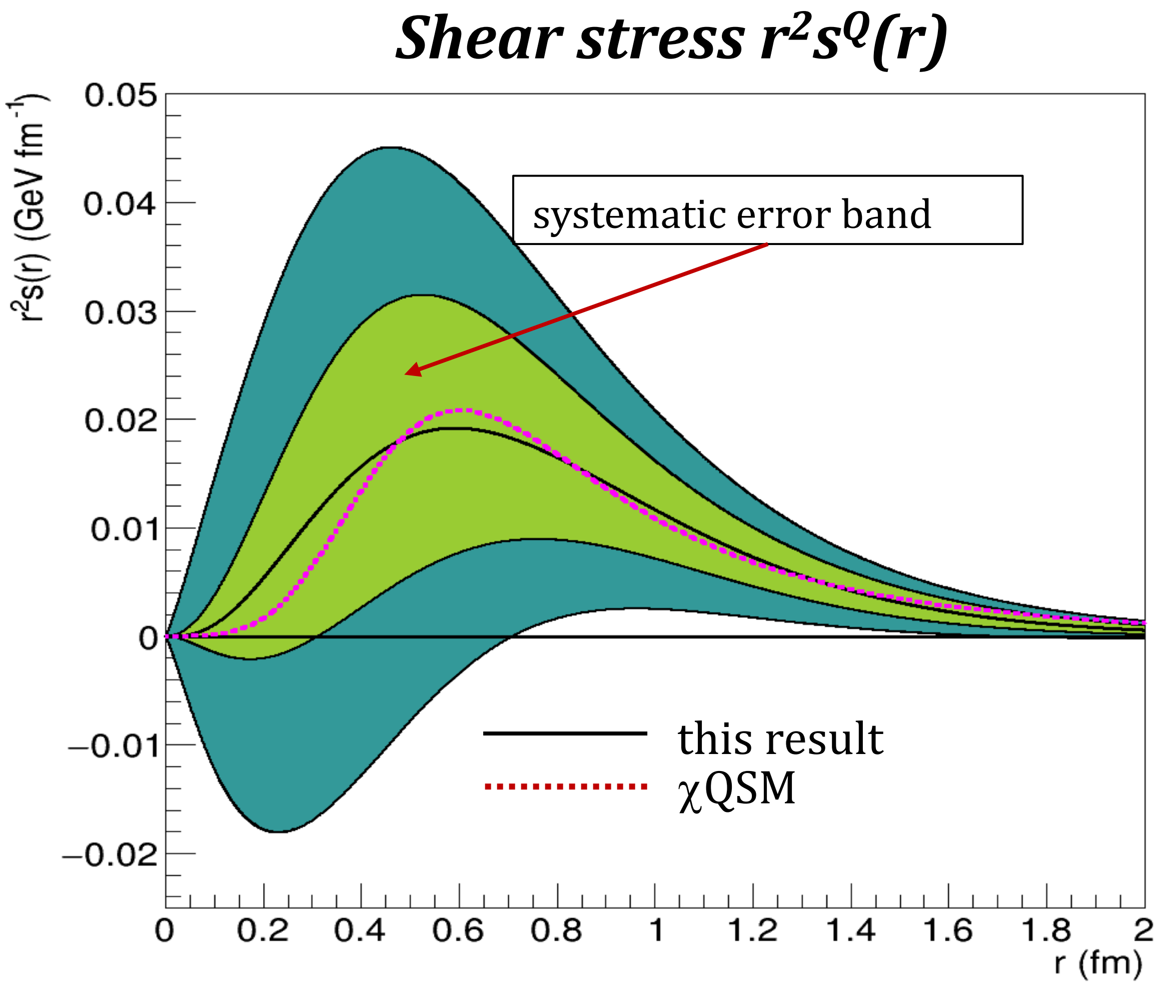
$$d_1^Q(t) \approx \frac{9}{10}\Delta(t)$$

$d_1^Q(t)$ - first coefficient in Gegenbauer polynomial expansion.

Estimate of next to leading term in χQSM: $d_3^Q/d_1^Q \sim 0.3$

➔ **Use as systematic uncertainty in estimating the force/pressure distributions.**

Shear Stress on Quarks in the Proton



Uncertainties of fit to world data prior to inclusion of CLAS6.

Uncertainties for analysis with CLAS6 data only.

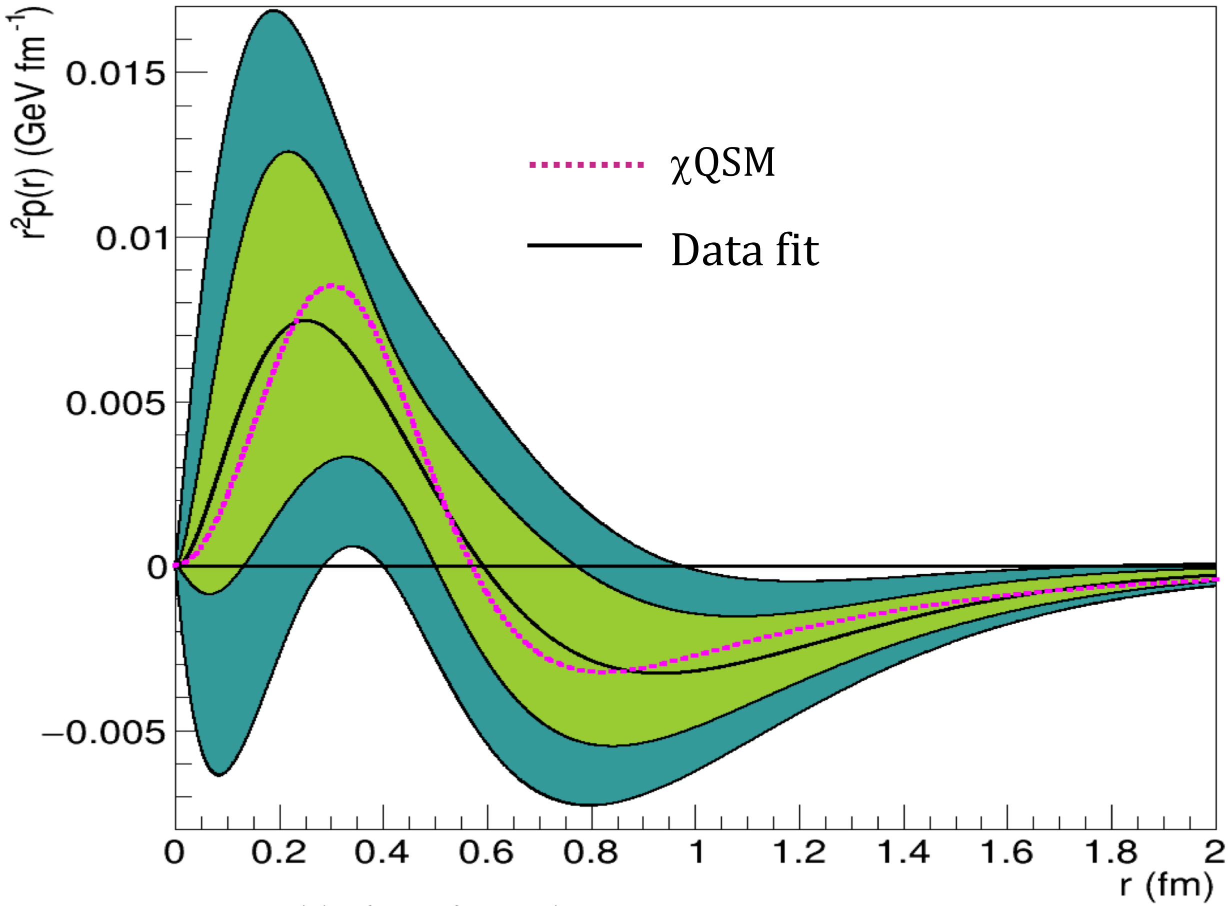
Peak stress near r = 0.6 fm:

$4\pi r^2 s(r) = 0.238$ GeV/fm

$(38 \pm 20) \times 10^3$ N

Pressure & Normal Stress on Quarks

Pressure $r^2 p^Q(r)$



Nature 557 (2018) 7705, 396

Uncertainties from fit to world data prior to CLAS6.

Uncertainties for analysis with CLAS6 data only.

Normal stress (0.6fm):

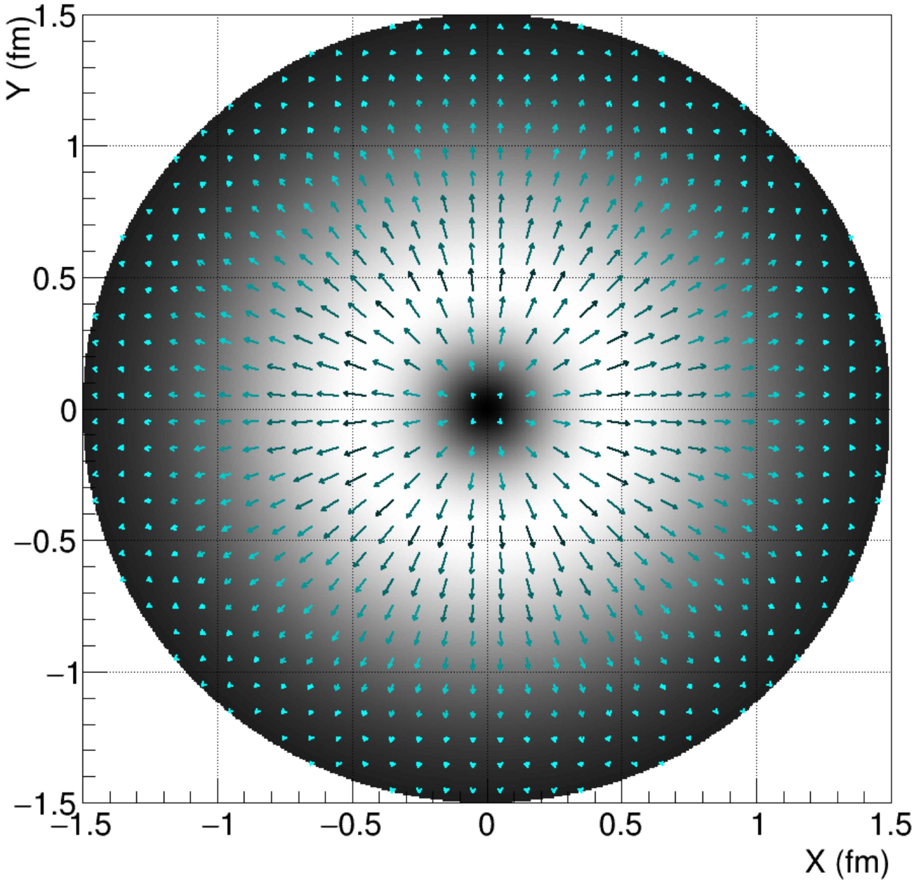
$F_N = 4\pi r^2[2/3\ s(r) + p(r)]$

$\approx (20 \pm 11) \times 10^3$ N

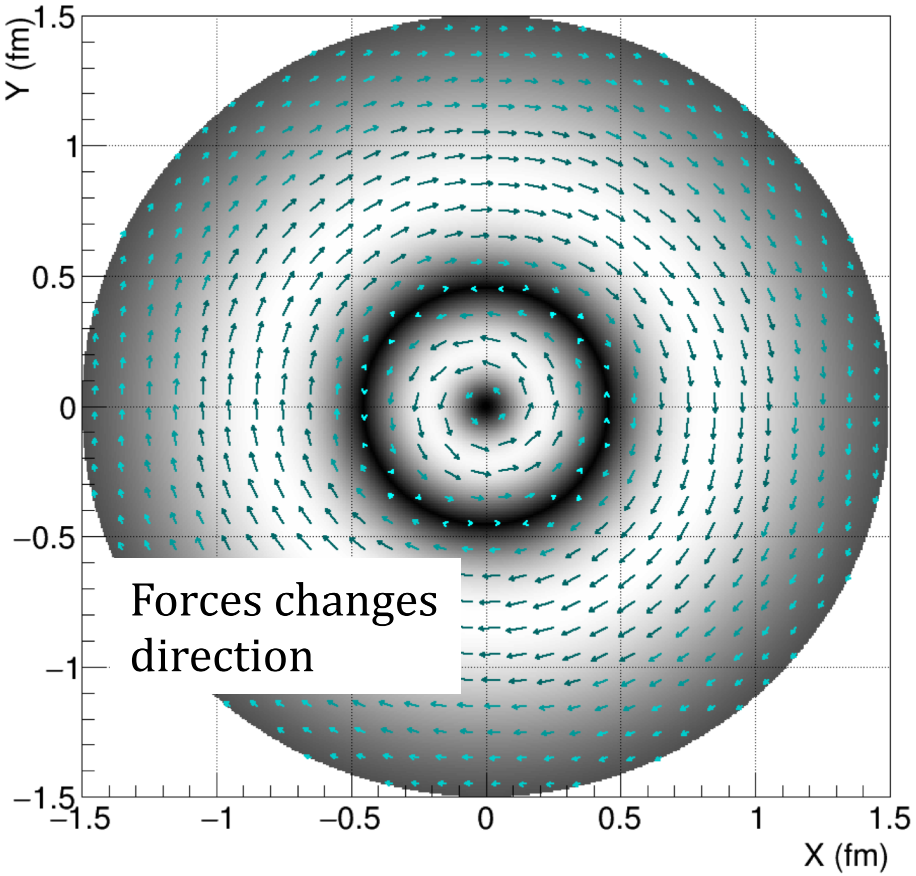
p^Q(0.6fm) ~0

Normal & Tangential Stress on Quarks

Normal stress: $F_n = 4\pi r^2[2/3\ s(r) + p(r)]$



Tangential stress: $F_t = 4\pi r^2[-1/3\ s(r) + p(r)]$



Tangential stresses change direction near r ~ 0.45 fm

Proton mechanical Radius

***D*(*t*)** gravitational form factor of the proton.

M. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A33 (2018) 26, 1830025

Mechanical radius:

$$\langle r^2\rangle_{\rm mech} = \frac{\int d^3r\ r^2\ \left[\frac{2}{3}s(r)+p(r)\right]}{\int d^3r\ \left[\frac{2}{3}s(r)+p(r)\right]} = \frac{6D}{\int_{-\infty}^{0} dt\ D(t)}$$

$$D(t) \ = \ D\left[1+\frac{-t}{M^2}\right]^{-\alpha}$$

For the multipolar form in the fit

$$\langle r^2\rangle_{\rm mech} = 6(\alpha-1)/M^2$$

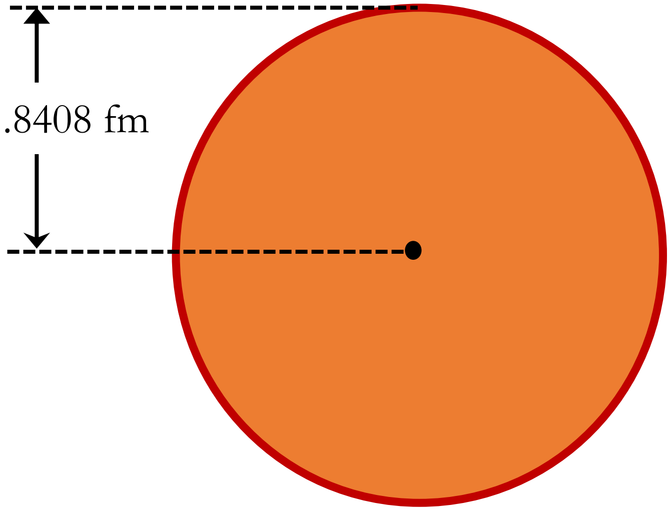
$r^2_{\rm mech} = 0.40 \pm 0.08 \pm 0.16\ {\rm fm}^2$

$\boldsymbol{r_{\rm mech} = 0.63 \pm 0.06 \pm 0.13\ {\rm fm}}$

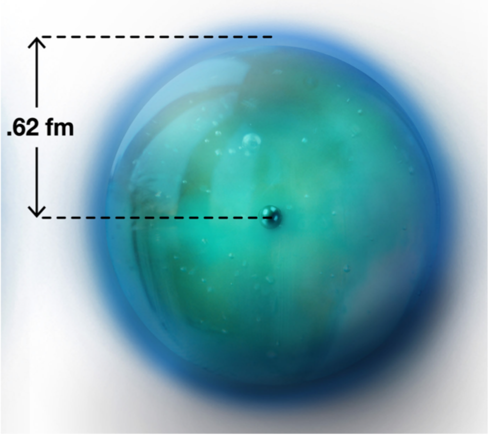
Mechanical radius versus charge radius

Source	Mechanical (fm)	Charge (fm)
Proton	$0.62 \pm 0.06 \pm 0.13$	0.8408 ± 0.0004
Neutron	$r_n \cong r_p$ (isospin)	$r^2 = -0.1161 \pm 0.0022$ (fm^2)
LC SR	0.73	
QCD SR	0.72 - 0.74	

Charge radius



Mechanical radius

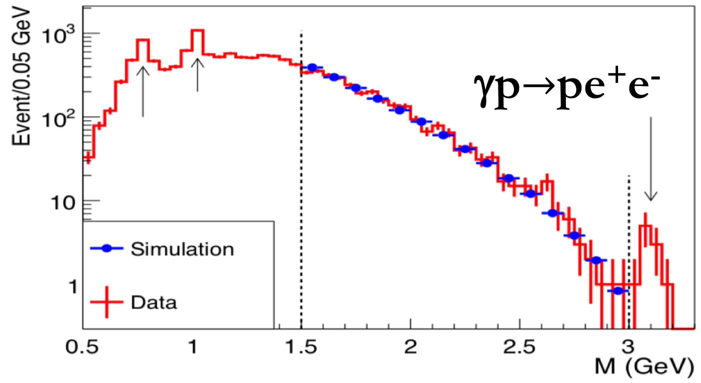


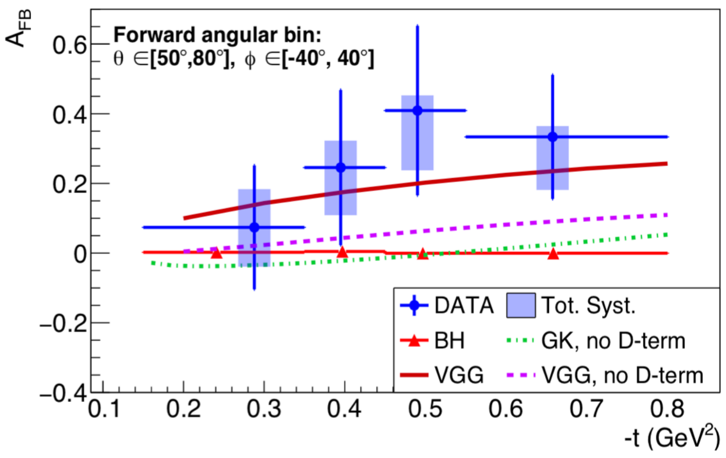
- The charge radius is measured in elastic scattering at **small Q^2**, and is influenced by the proton's peripheral **pion cloud** .

- The mechanical radius is measured at **large Q^2**, and the probe couples to **mass and pressure** (D-term) that are concentrated closer to the proton's center**.**

JLab 12 GeV upgrade

CLAS12 1st TCS results



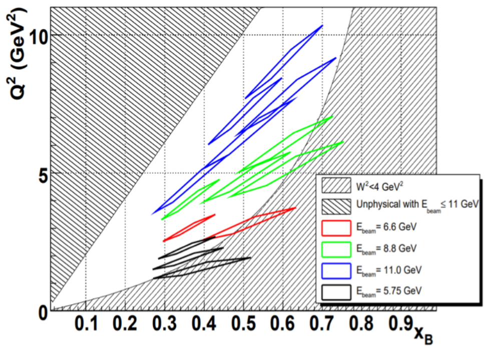


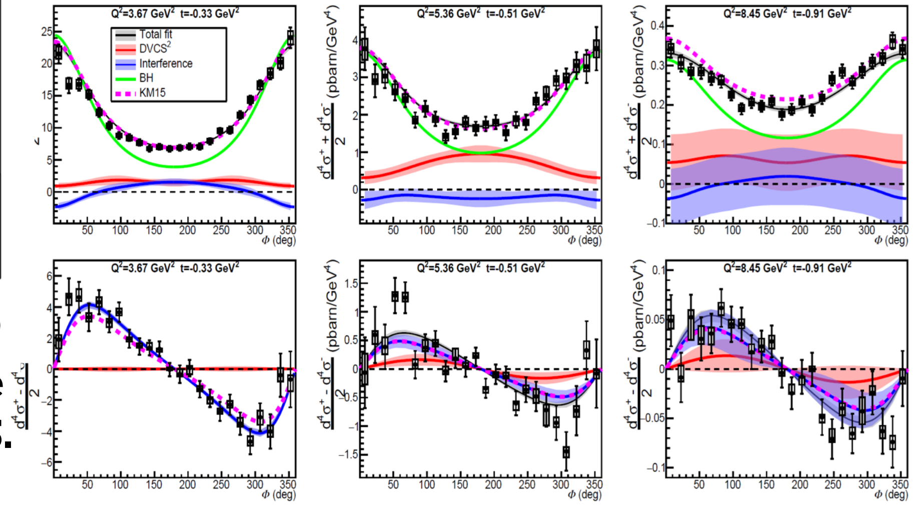
D-term accounts for large part of the FB-asymmetry, consistent with DVCS.

P. Chatagnon (CLAS), PRL accepted

Hall A

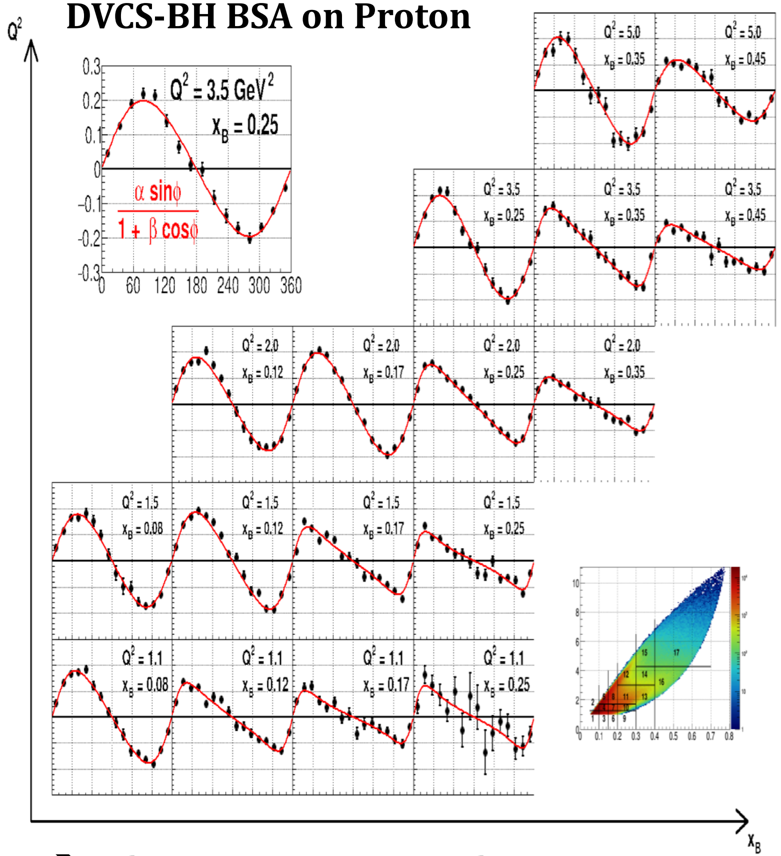
High impact for nucleon 3D imaging program
Precision scaling tests of DVCS cross section
Explore high x_B region for the first time





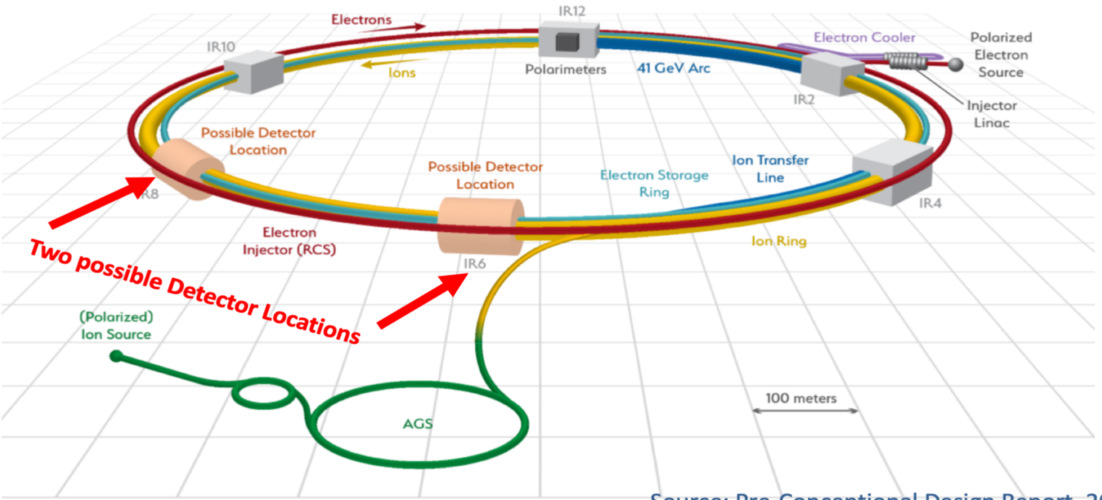
CLAS12

Contiguous coverage in wide Q^2, x_B, t range enables 3D imaging and study of mechanical structure with controlled systematics.

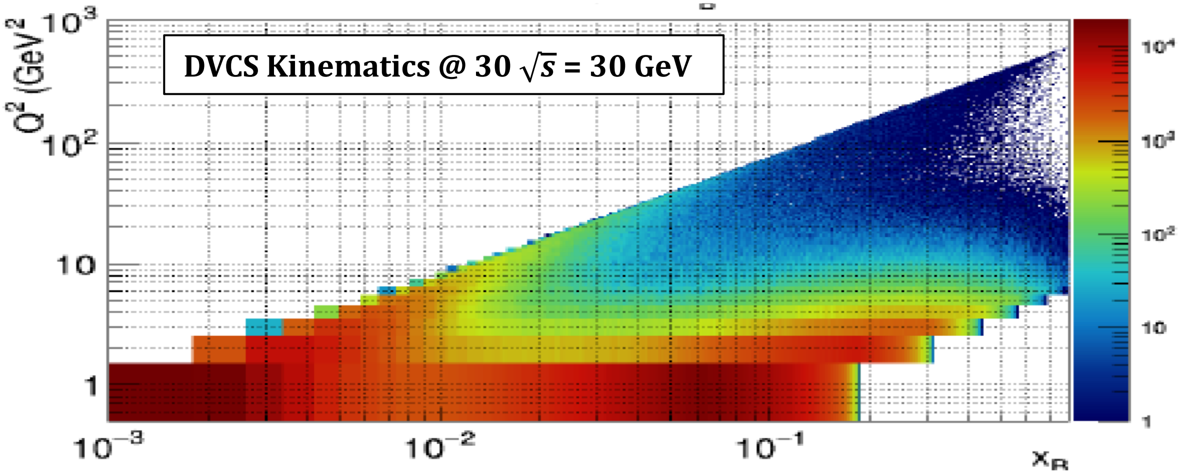


➔ Also DVCS BSA results on neutrons

Precision studies of QCD@EIC - example

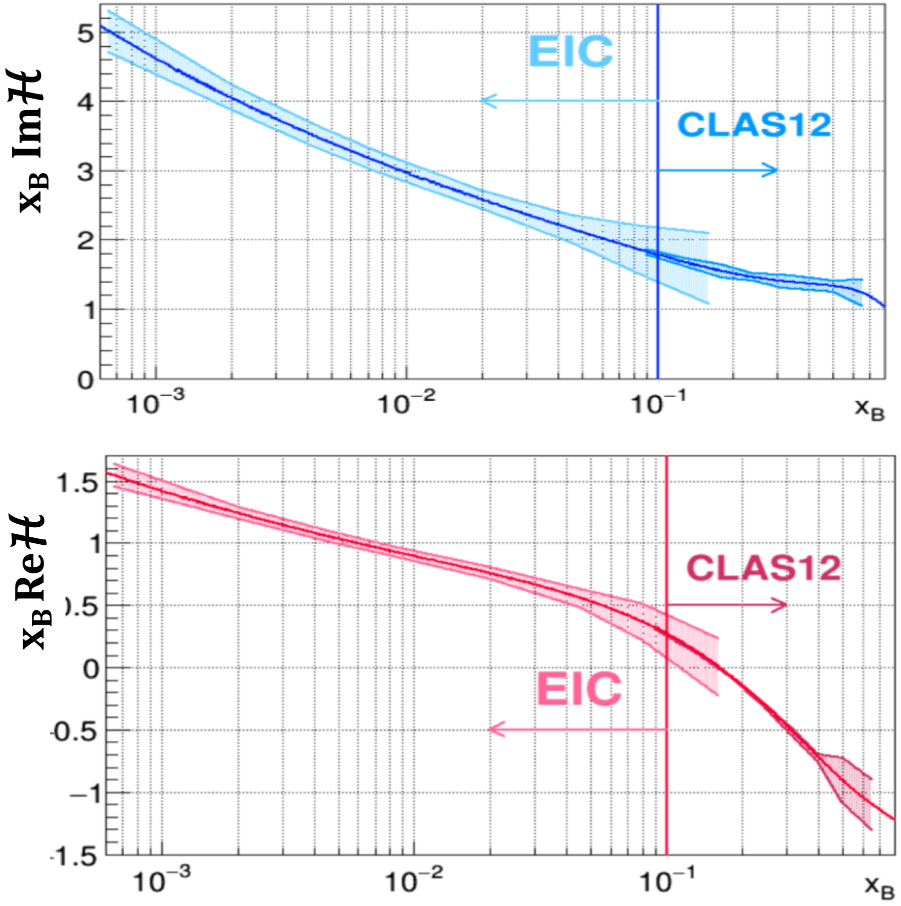


Source: Pre-Conceptional Design Report, 2019



Source: 2021 Workshops PSQ@EIC and IR2@EIC

CFF $\mathcal{H}$(x,t) extraction at EIC kinematics after L = 200 fb^{-1} w/ polarized electrons and protons.



Courtesy F.X. Girod

Summary and Outlook

- First data-based estimate of the stress on quarks in the proton
- First determination of the mechanical radius of the proton
- First results on TCS in 12 GeV era confirm effect of D-term from DVCS at lower energies
- New 12 GeV data extend the kinematic reach and precision with high luminosity experiments
- DVCS experiments with positron beam are planned with different sensitivity to gravitational form factors.
- There are plans to employ double DVCS to overcome constraint that limit GPD extraction
- Current results strongly support this program at the Electron Ion Collider with
  - a large increase in kinematic reach
  - to study gluon contributions in J/ψ production.