Towards mechanical properties of the proton

What we have learned from data so far

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Gravitational waves observed

- Gravity governs movements of massive structures in the universe.
- Plays a decisive role in neutron stars leading to the most densely packed macroscopic objects in the universe.

The merger of two neutron stars generated gravitational waves that told us much about the equation-of-state of the neutron stars themselves.

Can we use gravitational waves to probe the interior of the proton and learn something about the EoS of the proton and the distribution of the strong force?
Confinement in hadrons becomes manifest in the crossover from the QGP phase to the hadron phase.

The proton emerges as the most fundamental bound-state in nature.

The mechanical properties of the proton are of foremost interest.
Basic questions about the proton

• Proton’s make up nearly 90% of the mass of ordinary matter in the universe. Elementary quarks contribute a fraction to the proton’s mass

  What is the origin of its mass?

• Quarks and gluons formed stable protons as the universe cooled below $10^{13}$ K about few µsec after the Big Bang

  What is the origin of confinement?

• The strong interaction is thought responsible for confinement

  How do distribution of strong forces contribute to the proton’s stability and to confinement?
The structure of strongly interacting particles can be probed by means of the other weaker fundamental forces: \textit{electromagnetic}, \textit{weak}, and \textit{gravity}.

Experimentally we know very little about the mechanical properties. 

\rightarrow \text{Need to probe the proton's energy-momentum tensor.}
Energy Momentum Tensor $T^{\mu\nu}$

The framework for probing the protons EMT was developed in the 1960's.

"... there is very little hope of learning anything about the detailed mechanical structure of a particle, because of the extreme weakness of the gravitational interaction."

(H. Pagels, 1966)

$s_0^Q(r)$: shear stress; $p_0^Q(r)$: normal stress

$j_0, j_2$: 0th, 2nd order spherical Bessel functions

Extract: $s_0^Q(r), p_0^Q(r)$ with Fourier transform of $d_1^Q(t)$ to coordinate space.

- Probing properties of subatomic particles directly with gravity is highly impractical

- Use a substitute that mimics gravity?
GPDs, DVCS, Gravity

D. Müller et al., F. Phys. 42,1994
X. Ji, PRD 55 (1997) 7114
X. Ji, PRL 78 (1997) 610
A. Radyushkin, PRD 56 (1997) 5524

M. Polyakov, PL B555 (2003) 57

4 chiral even GPDs describe soft part. GPD $H$ is important to access gravitational form factor $D(t)$.

DVCS is a suitable probe of mechanical properties of particles

The $2\gamma$ field couples to the EMT as gravity does, with many orders of magnitude greater strength.
Moments of GPD & GFF Relations

Nucleon matrix element of the Energy-Momentum Tensor contains three gravitational form factors (GFF) and can be written as:

\[
\langle p_2 | \hat{T}^{\mu \nu}_q | p_1 \rangle = \bar{U}(p_2) \left[ M_2^q(t) \frac{P_\mu P_\nu}{M} + J^q(t) \frac{(P_\mu \sigma_{\nu \rho} + P_\nu \sigma_{\mu \rho}) \Delta^\rho}{2M} + d_1^q(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu \nu} \Delta^2}{5M} \right] U(p_1)
\]

- \( M_2(t) \): Mass/energy distribution inside the nucleon
- \( J(t) \): Angular momentum distribution
- \( d_1(t) \): Forces and pressure distribution

**GPDs \leftrightarrow GFFs**

In DVCS, GPDs are not directly accessible at all \( x \) and \( \xi \) but constrained to \( x = \pm \xi \)

\[
\int dx \, x \left[ H(x, \xi, t) + E(x, \xi, t) \right] = 2J(t)
\]
\[
\int dx \, x H(x, \xi, t) = M_2(t) + \frac{4}{5} \xi^2 d_1(t)
\]

**From GPD to CFF to GFF**

**Compton Form Factor $\mathcal{H}$ (CFF)**

1) Polarized electron beam – BSA

$$\Delta \sigma_{LU}/d\sigma_U \sim \text{Im}\{F_1 \mathcal{H} + \ldots\} \sin \phi + ..$$

2) Unpolarized beam:

$$d\sigma_U \sim k\{\mathcal{H}^* + \ldots\} + ..$$

3) Dispersion Relation

$$\text{Re} \mathcal{H}(\xi, t) = \Delta(t) + \frac{1}{\pi} \mathcal{P} \int_0^1 dx \left[ \frac{1}{\xi - x} - \frac{1}{\xi + x} \right] \text{Im} \mathcal{H}(x, t).$$


Sample fits to determine $\mathcal{H}(\xi, t)$

Samples of Beam Spin Asymmetry with fits

$-t = 0.14\text{GeV}^2$

$-t = 0.18\text{GeV}^2$


Samples of differential cross sections with fits

$d^2\sigma/(\text{nb}\cdot\text{GeV}^2)$

Global fit:

$$\text{Im}\mathcal{H}(\xi, t) = \frac{N}{1 + \xi} \left( \frac{2\xi}{1 + \xi} \right)^{-\alpha(t)} \left( \frac{1 - \xi}{1 + \xi} \right)^b \left( 1 - \frac{1 - \xi}{1 + \xi} \frac{t}{M^2} \right)^p$$

$K. \text{ Kumericki et al, EPJ A 52, 159 (2016)}$

Extracting CFF $\text{Im} \mathcal{H}$ & $\text{Re} \mathcal{H}$

Global parameterizations:

BSA to determine $\text{Im} \mathcal{H}$

Differential DVCS cross sections to determine $\text{Re} \mathcal{H}$

$\Delta(0) = 0$

Re$\mathcal{H}$ has strong sensitivity to subtraction term $\Delta$. 

K. Kumericki and D. Müller, 
*EPJ Web Conf. 112* (2016) 01012
Extraction of $\Delta(t)$ and $d_1(t)$ for quark distribution

$\Delta(t)$ from CLAS 6 GeV DVCS data

$\Delta(t) = \Delta(0)(1 - \frac{t}{M^2})^\alpha$

Dimensional scaling: $\alpha = 3$

$\Delta(t) \propto d_1^Q(t)$ in double-distribution parameterization

$\Delta(t) = 2 \int_{-1}^{1} dz \frac{D(z,t)}{1-z}$

$D(z,t) = (1 - z^2) \left[ e_u^2 + e_d^2 \right] \frac{d_1^Q(t)}{2} 3z$

$d_1^Q(t) \approx \frac{9}{10} \Delta(t)$

Estimate of next to leading term in $\chi$QSM: $d_3^Q/d_1^Q \sim 0.3$

$\Rightarrow$ Use as systematic uncertainty in estimating the force/pressure distributions.

$\Delta(0) = -2.27 \pm 0.16 \pm 0.36$

$M^2 = 1.02 \pm 0.13 \pm 0.21$

$\alpha = 2.76 \pm 0.23 \pm 0.48$

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Shear Stress on Quarks in the Proton

Shear stress $r^2 s(r)$

- Peak stress near $r = 0.6$ fm: $4\pi r^2 s(r) = 0.238$ GeV/fm
  \[(38 \pm 20) \times 10^3$ N\]

Uncertainties of fit to world data prior to inclusion of CLAS6.

Uncertainties for analysis with CLAS6 data only.

4 metric tons
**Pressure & Normal Stress on Quarks**

### Pressure \( r^2 p^Q(r) \)

**Normal stress (0.6fm):**

\[
F_N = 4\pi r^2 [\frac{2}{3} s(r) + p(r)]
\approx (20\pm11) \times 10^3 \text{ N}
\]

\( p^Q(0.6\text{fm}) \sim 0 \)

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**Nature 557 (2018) 7705, 396**

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**Uncertainties from fit to world data prior to CLAS6.**

**Uncertainties for analysis with CLAS6 data only.**

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**\( r^2 p_Q(r) \text{ (GeV fm}^{-1}\text{)} \)**

**Data fit**

\( \chi QSM \)

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Normal & Tangential Stress on Quarks

Normal stress: \( F_n = 4\pi r^2 [\frac{2}{3} s(r) + p(r)] \)

Tangential stress: \( F_t = 4\pi r^2 [-\frac{1}{3} s(r) + p(r)] \)

Forces change direction

Tangential stresses change direction near \( r \sim 0.45 \text{ fm} \)
**Proton mechanical Radius**

\( D(t) \) gravitational form factor of the proton.

Mechanical radius:

\[
\langle r^2 \rangle_{\text{mech}} = \frac{\int d^3r \ r^2 \left[ \frac{2}{3} s(r) + p(r) \right]}{\int d^3r \left[ \frac{2}{3} s(r) + p(r) \right]} = \frac{6D}{\int_{-\infty}^{0} dt \ D(t)}
\]

\[
D(t) = D \left[ 1 + \frac{-t}{M^2} \right]^{-\alpha}
\]

For the multipolar form in the fit

\[
\langle r^2 \rangle_{\text{mech}} = 6(\alpha - 1)/M^2
\]

\[
r^2_{\text{mech}} = 0.40 \pm 0.08 \pm 0.16 \text{ fm}^2
\]

\[
r_{\text{mech}} = 0.63 \pm 0.06 \pm 0.13 \text{ fm}
\]

Mechanical radius versus charge radius

<table>
<thead>
<tr>
<th>Source</th>
<th>Mechanical (fm)</th>
<th>Charge (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>$0.62 \pm 0.06 \pm 0.13$</td>
<td>$0.8408 \pm 0.0004$</td>
</tr>
<tr>
<td>Neutron</td>
<td>$r_n \cong r_p$ (isospin)</td>
<td>$r^2 = -0.1161 \pm 0.0022$ (fm$^2$)</td>
</tr>
<tr>
<td>LC SR</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>QCD SR</td>
<td>0.72 - 0.74</td>
<td></td>
</tr>
</tbody>
</table>

• The charge radius is measured in elastic scattering at small $Q^2$, and is influenced by the proton’s peripheral pion cloud.

• The mechanical radius is measured at large $Q^2$, and the probe couples to mass and pressure (D-term) that are concentrated closer to the proton’s center.
JLab 12 GeV upgrade

CLAS12 1\textsuperscript{st} TCS results

\[ \gamma p \to e^+e^- \]

Contiguous coverage in wide $Q^2$, $x_B$, $t$ range enables 3D imaging and study of mechanical structure with controlled systematics.

Hall A

High impact for nucleon 3D imaging program
Precision scaling tests of DVCS cross section
Explore high $x_B$ region for the first time

\[ A_{FB} \]

D-term accounts for large part of the FB-asymmetry, consistent with DVCS.

\[ P. \text{Chatagnon (CLAS), PRL accepted} \]

CLAS12

\[ \text{DVCS-BH BSA on Proton} \]

\[ \Rightarrow \text{Also DVCS BSA results on neutrons} \]
Precision studies of QCD@EIC - example

CFF $\mathcal{H}(x,t)$ extraction at EIC kinematics after $L = 200 \text{ fb}^{-1}$ w/ polarized electrons and protons.

DVCS Kinematics @ $30 \sqrt{s} = 30 \text{ GeV}$

Source: 2021 Workshops PSQ@EIC and IR2@EIC

Courtesy F.X. Girod
First data-based estimate of the stress on quarks in the proton
First determination of the mechanical radius of the proton
First results on TCS in 12 GeV era confirm effect of D-term from DVCS at lower energies
New 12 GeV data extend the kinematic reach and precision with high luminosity experiments
DVCS experiments with positron beam are planned with different sensitivity to gravitational form factors.
There are plans to employ double DVCS to overcome constraint that limit GPD extraction
Current results strongly support this program at the Electron Ion Collider with
  a large increase in kinematic reach
  to study gluon contributions in J/ψ production.