Higher-Spin Baryon Photoproduction with Twisted Photons

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William & Mary

Light Cone 2021: Physics of Hadrons on the Light Front
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Crim Dell

Talk based on work with Andrei Afanasev plus related work with Asmita Mukherjee, Maria Solyanik-Gorgone, Christian Schmiegelow, Ferdinand Kaler-Schmidt, Jonas Schulz, Hao Wang

Jeongbang Waterfall

Outline

There have been several talks about experiments that will or hope to run in the future.
Here is also a talk about future possibilities, with specialized photon beams—twisted photons—not yet produced at particle/nuclear physics energies, but which are widely produced and used in atomic physics.

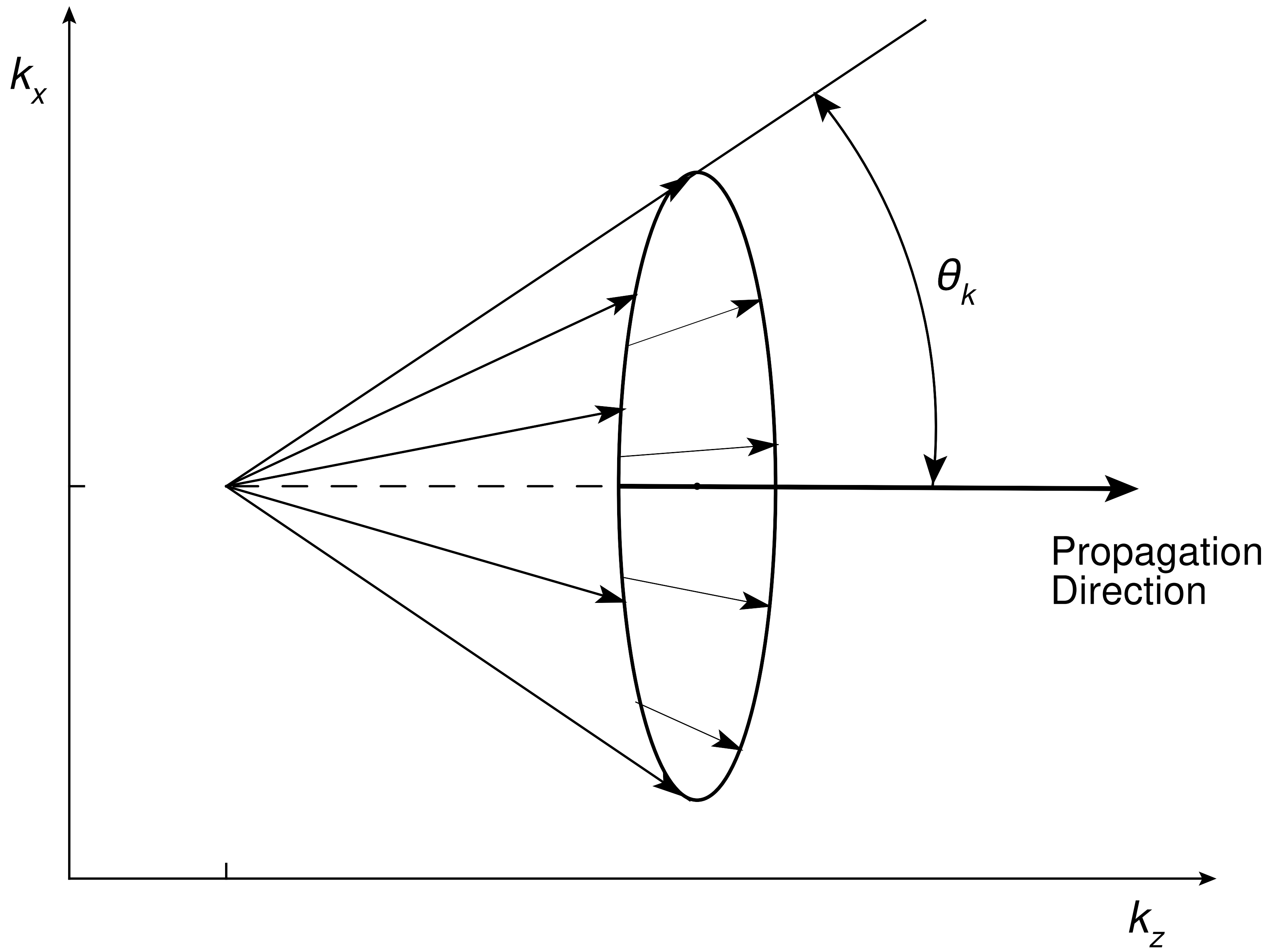
- What are twisted photons?
- Existence in atomic physics.
- Hadronic application photoproduction of higher spins states
  - $\Delta(1232)$
  - Other high spin states

Intro to twisted photons

- Plane wave photons: angular momentum along direction of motion ("helicity") is just spin and just ± 1 (or $\pm \hbar$).
- Not true in general!
- Angular momentum along direction of motion can be any integer (times $\hbar$)
- Not realized until 1992, Allen et al., PRA with 8000 citations (Google Scholar, November 29, 2021)

Intro

- One way to build: in momentum space, make state from plane wave photons with momenta $\vec{k}$ on a cone:



- Algebraically,

$$|\kappa, m_\gamma, k_z, \Lambda\rangle = A_0 \int \frac{d\phi_k}{2\pi}\, i^{-m_\gamma}\, e^{i m_\gamma \phi_k} \qquad \underbrace{|\vec{k}, \Lambda\rangle}_{\textit{plane wave states}}$$

- Phasing crucial!

- In coordinate space,

$$\vec{A}_{\kappa,m.k_z,\Lambda} = A_0\, \hat{\epsilon}\, e^{i(k_z z - \omega t + m_\gamma \phi)} J_{m_\gamma - \Lambda}(\kappa\rho) + \textbf{smaller terms}$$

Intro 3

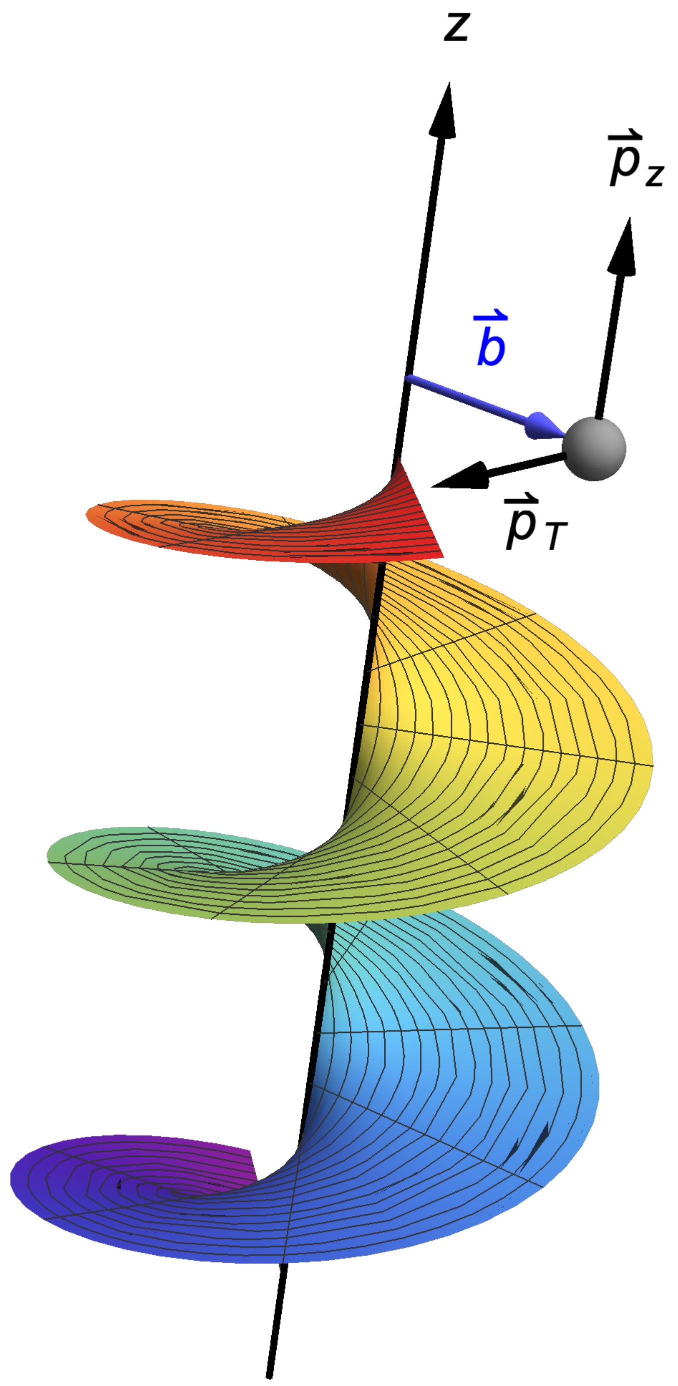
- Can also work purely in coordinate space. Monochromatic (time dependence $e^{-i\omega t}$) and propagating in z-direction (z-dependence $e^{i\beta z}$). Scalar case,

$$0 = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\psi(t, \vec{x}) = \left(-\frac{\omega^2}{c^2} + \beta^2 - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\psi(t, \vec{x})$$

- Solvable: $\psi(t, \vec{x}) = e^{i\beta z - \omega t}\, e^{i\ell\phi}\, J_\ell(\alpha\rho)$ $\left[\alpha^2 = \frac{\omega^2}{c^2} - \beta^2 \,,\;\; \rho^2 = x^2 + y^2\right]$
(Do recall $L_z = -i\partial/\partial\phi$.)

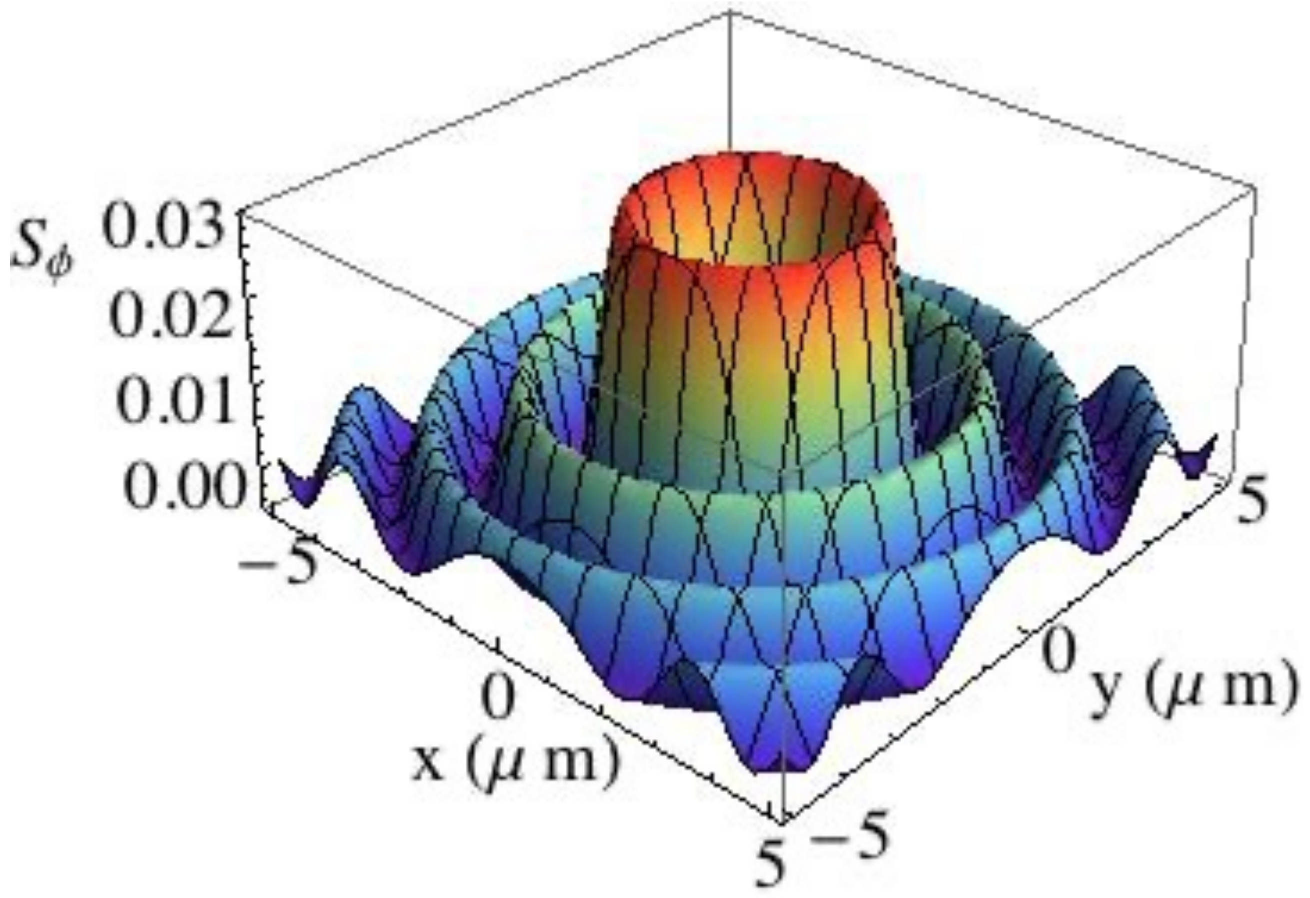
- BTW, Bessel function wavefront does not spread or diffract (interesting, although not material to applications here).

Intro - pictures



$$m_\gamma - \Lambda = -1$$

Surface of constant phase



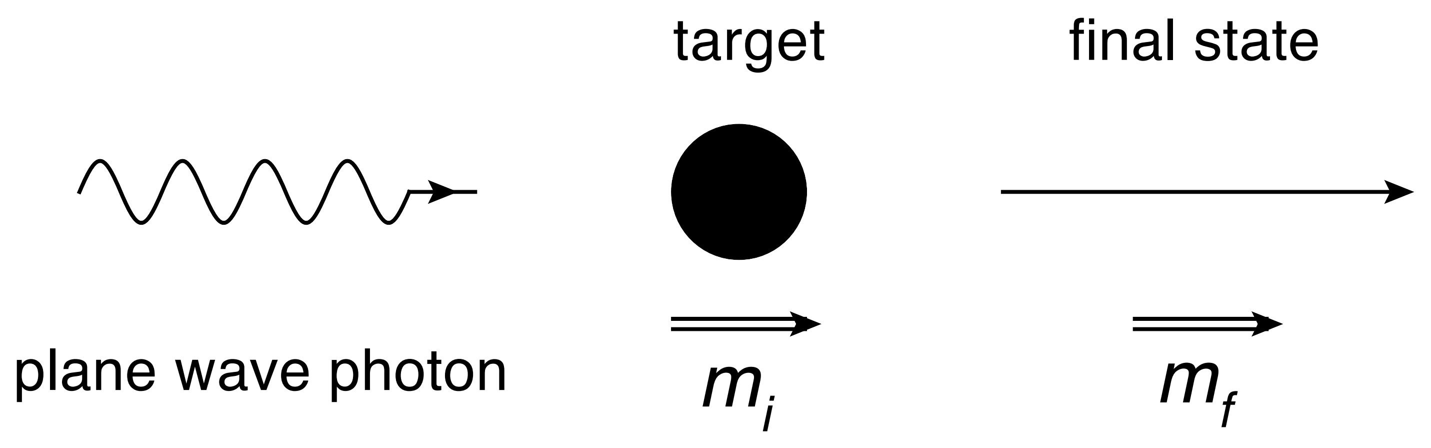
Azimuthal component of
Poynting vector magnitude

Names: Twisted photons, Vortex photons, Structured light

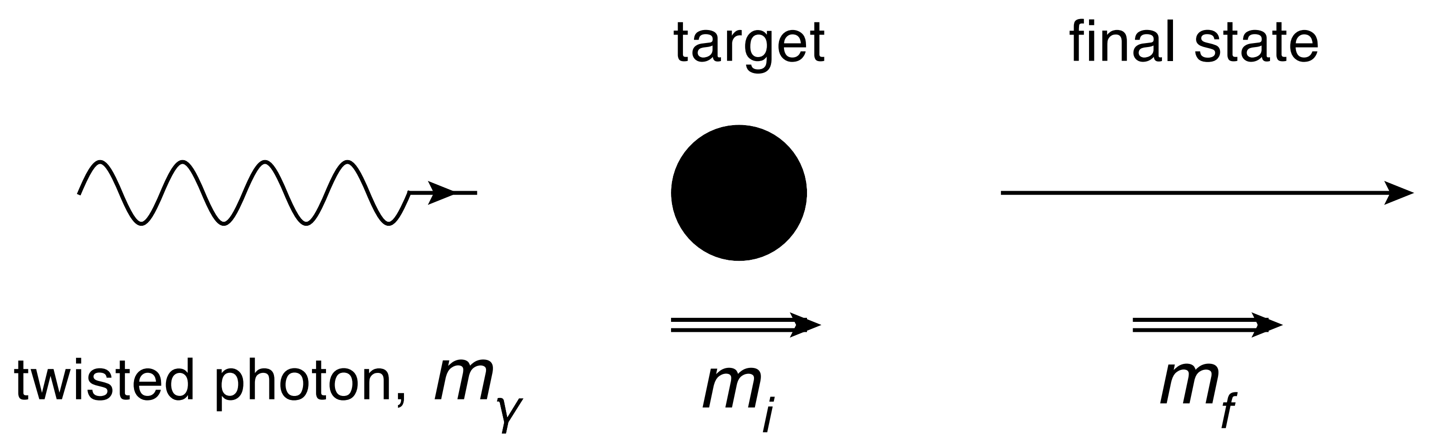
Selection rules

- Special (i.e., non-plane wave) selection rules.

  E.g., in photoproduction or photo excitation

- 

  $m_f = m_i \pm 1$ only



$m_f = m_i + m_\gamma$ and m_γ can be any integer

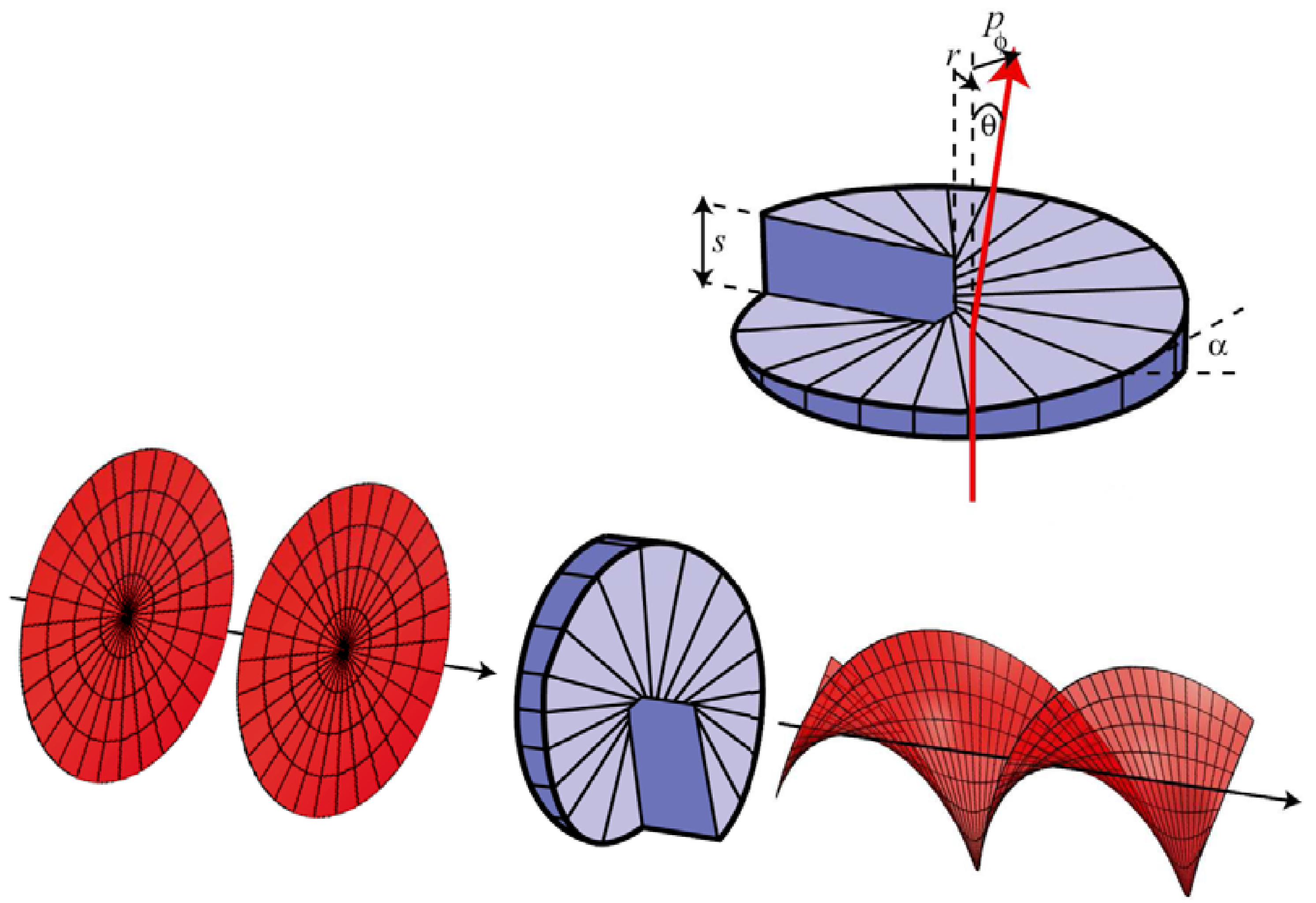
- Target on-axis: all angular momentum must go to internal excitation.
- Off-axis: share angular momentum between internal excitation of target and OAM of CM of final state (relative to vortex line of photon).

Atomic studies

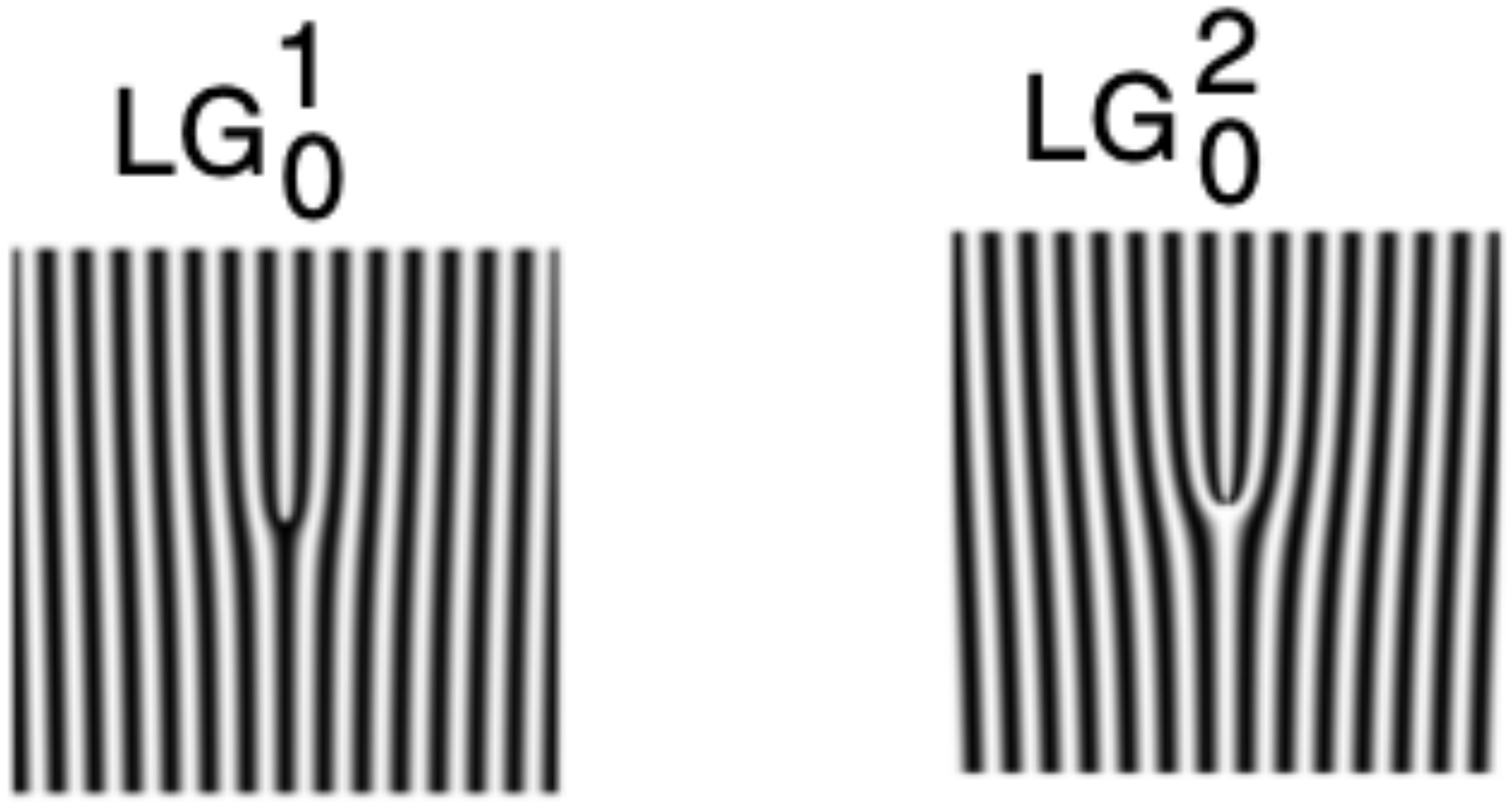
- Can we make twisted photons in the laboratory?
- Can we hold the target acurately enough in place?
- Learn from atomic physics.

Atomic studies 2

- Acknowledgement: I learned a lot from the QUANTUM group (Ferdinand Schmidt-Kaler and co) at the JGU Mainz
- Yes can make twisted light, using laser light with circular polarization passing through spiral wave plates or tuning fork diffraction grating.



A spiral phase plate can generate a helically phased beam from a Gaussian. In this case $\ell = 0 \to \ell = 2$.



Atomic studies 3

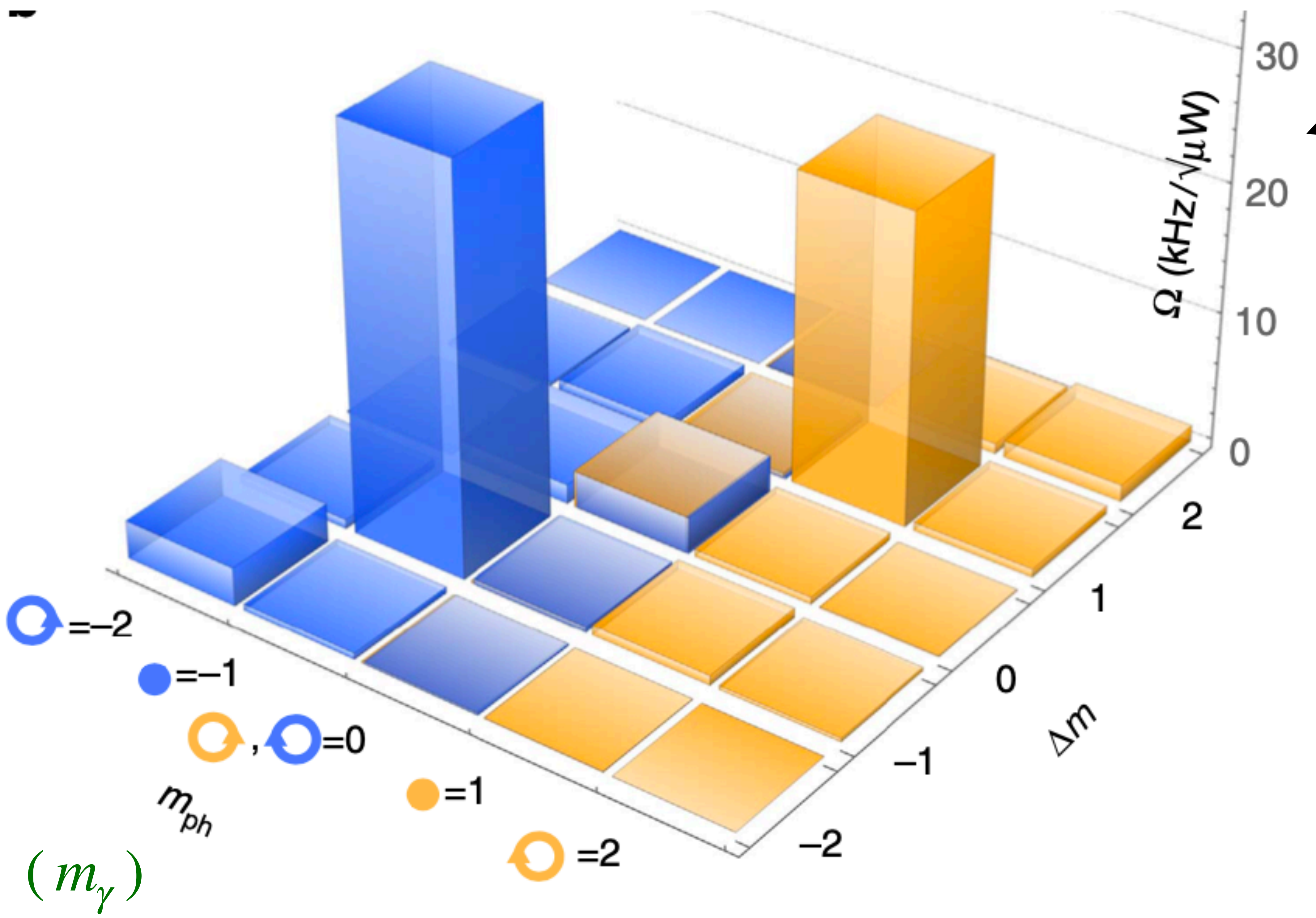
- Yes, can localize atoms in Paul trap to few 10's of nanometers
- "Hole" in middle of vortex photons are few wavelengths

  i.e., few times several hundred nm.
- Hence localization very good.
- Excellent study ion is ^{40}Ca^{+}
  - Ground state has $4s_{1/2}$ valence electron
  - Excite to $3d_{5/2}$ with 729 nm photons
  - $3d_{5/2}$ (of course) has $m_f = -5/2, -3/2, \ldots, 5/2$
- Zeeman split levels by magnetic field, determine m_f from small changes in γ-energy needed to excite.

Atomic studies 4

- Hence $\Delta m = m_f - m_i$ measurable. m_γ known. Plot for on-axis data.

$\Delta m = m_\gamma$ **testable**



(m_γ)

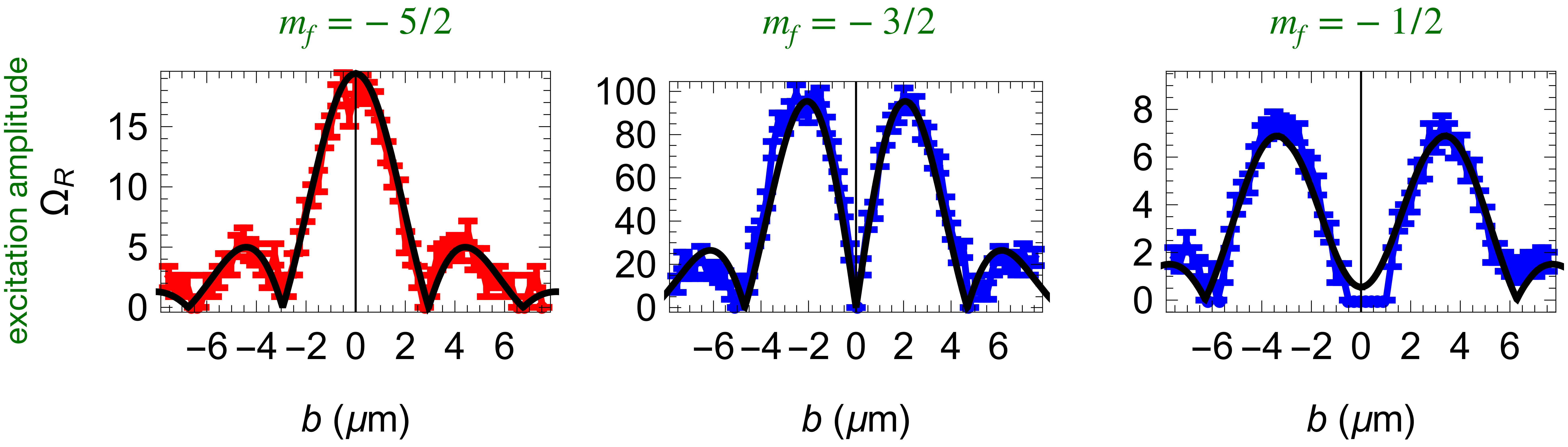
Excitation amplitude

Fig. from Schmiegelow et al., Nat. Comm. (2016)

Atomic studies 5

- In addition there is off-axis data for many m_γ, m_f, m_i

- Experimenters have data for $m_\gamma = \pm 2, \pm 1, 0$ (two versions), $m_f =$ 5 values (one missing), $m_i = \pm 1/2$, and 60 plots total, among them the following plots for $m_\gamma = -2,\ \Lambda = -1,\ m_i = -1/2$



impact parameter

$\theta_k \approx 0.1$

Data from experiment of Schmiegelow et al., Nat. Comm. (2016)
Curves from Afanasev et al., NJP 2018 and Solyanik-Gorgone et al, JOSA-B 2019

Hadronic physics

- Generic: selective excitation of high spin baryon states
- First example: in $\gamma + N \to \Delta$, isolate E2 transitions from dominant M1
- Notes: N is $J^P = (1/2)^+$; Δ is $J^P = (3/2)^+$
- More notes: $N \to \Delta$ photoproduction requires M1 or E2 transitions
  - M1 means angular momentum change $L = 1$, parity $(-1)^{L+1} = +$
  - E2 means angular momentum change $L = 2$, parity $(-1)^{L} = +$

Hadronic physics, esp. $p \to \Delta(1232)$

- M1 transition often very small in atomic physics, not so for hadrons.
- For $N \to \Delta$, both mostly orbital S-states; M1 only needs spin flip and dominates
- The E2 needs two units of orbital angular momentum, involving the (small) D-state of the N or Δ, hence small. But important.

Hadronic physics

- How actually to calculate?
- Will give an atomic-like treatment (ignoring recoil), and later correct for recoil (in archived paper if not in talk).
- Method: relate twisted photon amplitudes to plane wave helicity amplitudes, which in turn are related to M1 and E2 amplitudes

Hadronic physics

- Put target at origin with twisted photon offset. Its vortex line will pass through $\vec{b} = b_\perp$ in the x-y plane,

$$|\gamma(\kappa m_\gamma k_z \Lambda \vec{b})\rangle = A_0 \int \frac{d\phi_k}{2\pi} (-i)^{m_\gamma} e^{i m_\gamma \phi_k - i \vec{k} \cdot \vec{b}} |\gamma(\vec{k}, \Lambda)\rangle$$

- Want amplitude

$$\mathcal{M} = \langle \Delta(m_f) \,|\, \mathcal{H}(0) \,|\, N(m_i);\ \gamma(\kappa m_\gamma k_z \Lambda \vec{b})\rangle$$

- $\mathcal{H}$ is interaction Hamiltonian; nucleon is at rest; m_i & m_f are spin projections in z-direction.

Hadronic physics

- Plane wave states obtained by rotating states w/momentum in z-direction,

$$|\gamma(\overrightarrow{k},\Lambda)\rangle = R(\phi_k,\theta_k,0)\,|\gamma(k\hat{z},\Lambda)\rangle = R_z(\phi_k)\,R_y(\theta_k)\,|\gamma(k\hat{z},\Lambda)\rangle$$

  using Wick (1962) phase convention.

- Rewrite the previous amplitude, since $\mathscr{H}$ is rotation invariant. Rotate all plane wave constituent photons to z-direction. Then have to rotate N and Δ states at the same time. Easy.

- Since the nucleon is at rest, rotations are given in terms of Wigner functions

$$R^\dagger(\phi_k,\theta_k,0)\,|N(m_i)\rangle = e^{im_i\phi_k}\sum_{m_i'} d^{1/2}_{m_i,m_i'}(\theta_k)\,|N(m_i')\rangle$$

  (all spins projected along z-axis).

Hadronic physics

- Can do the same for Δ, if we neglect recoil

$$\langle \Delta(m_f) \,|\, R(\phi_k, \theta_k, 0) = \sum_{m_f'} \langle \Delta(m_f') \,|\, e^{-im_f \phi_k} d^{3/2}_{m_f, m_f'}(\theta_k)$$

- Put together. Do ϕ_k integral to obtain Bessel function. Result is

$$\mathscr{M} = A_0 (-i)^{m_f - m_i} e^{i(m_\gamma + m_i - m_f)\phi_b} J_{m_f - m_i - m_\gamma}(\kappa b) \sum_{m_i'} d^{3/2}_{m_f, m_i' + \Lambda}(\theta_k)\, d^{1/2}_{m_i, m_i'}(\theta_k)\, \mathscr{M}^{(\mathrm{pw})}_{m_i', \Lambda}$$

- The plane wave amplitude is

$$\langle \Delta(m_f') \,|\, \mathscr{H}(0) \,|\, N(m_i');\, \gamma(k\hat{z}, \Lambda) \rangle = \mathscr{M}^{(\mathrm{pw})}_{m_i', \Lambda}\, \delta_{m_f', m_i' + \Lambda}$$

$p \to \Delta(1232)$

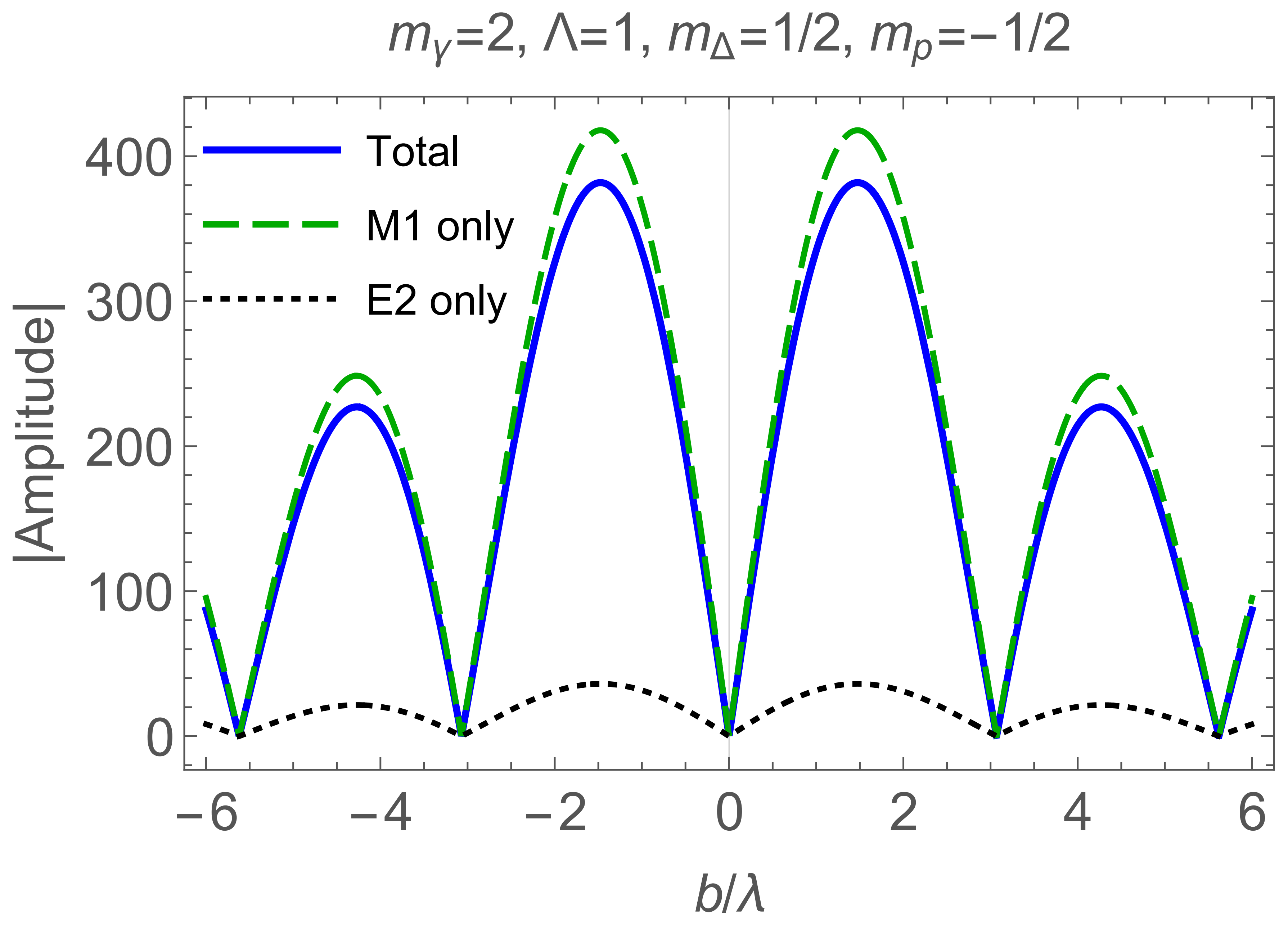
- There are two independent plane wave amplitudes (and others connected by parity):

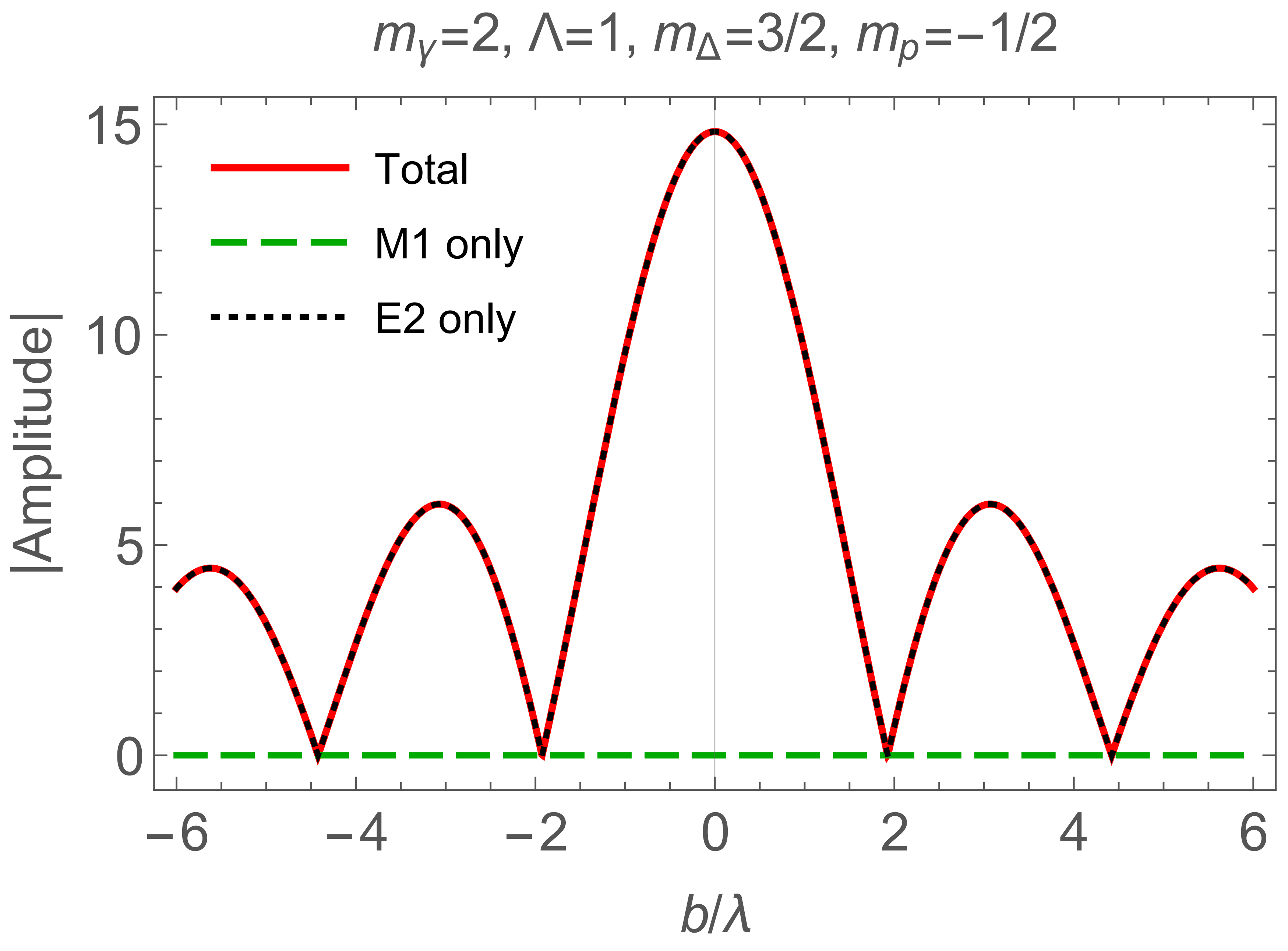
$$\mathcal{M}^{(\mathrm{pw})}_{1/2,1} \propto G^*_M + G^*_E$$
$$\mathcal{M}^{(\mathrm{pw})}_{-1/2,1} \propto \frac{1}{\sqrt{3}}\left(G^*_M - 3G^*_E\right)$$

- The M1 and E2 amplitudes are represented by Jones-Scadron form factors G^*_M and G^*_E , resp.

$p \to \Delta(1232)$

- Can calculate. With $m_\gamma = \pm 2, \pm 1, 0$ (two versions), $m_f =$ 4 values, and $m_i = \pm 1/2$, could make 48 plots. Show 2. Done for $G_E^*/G_M^* = 3\,\%$.

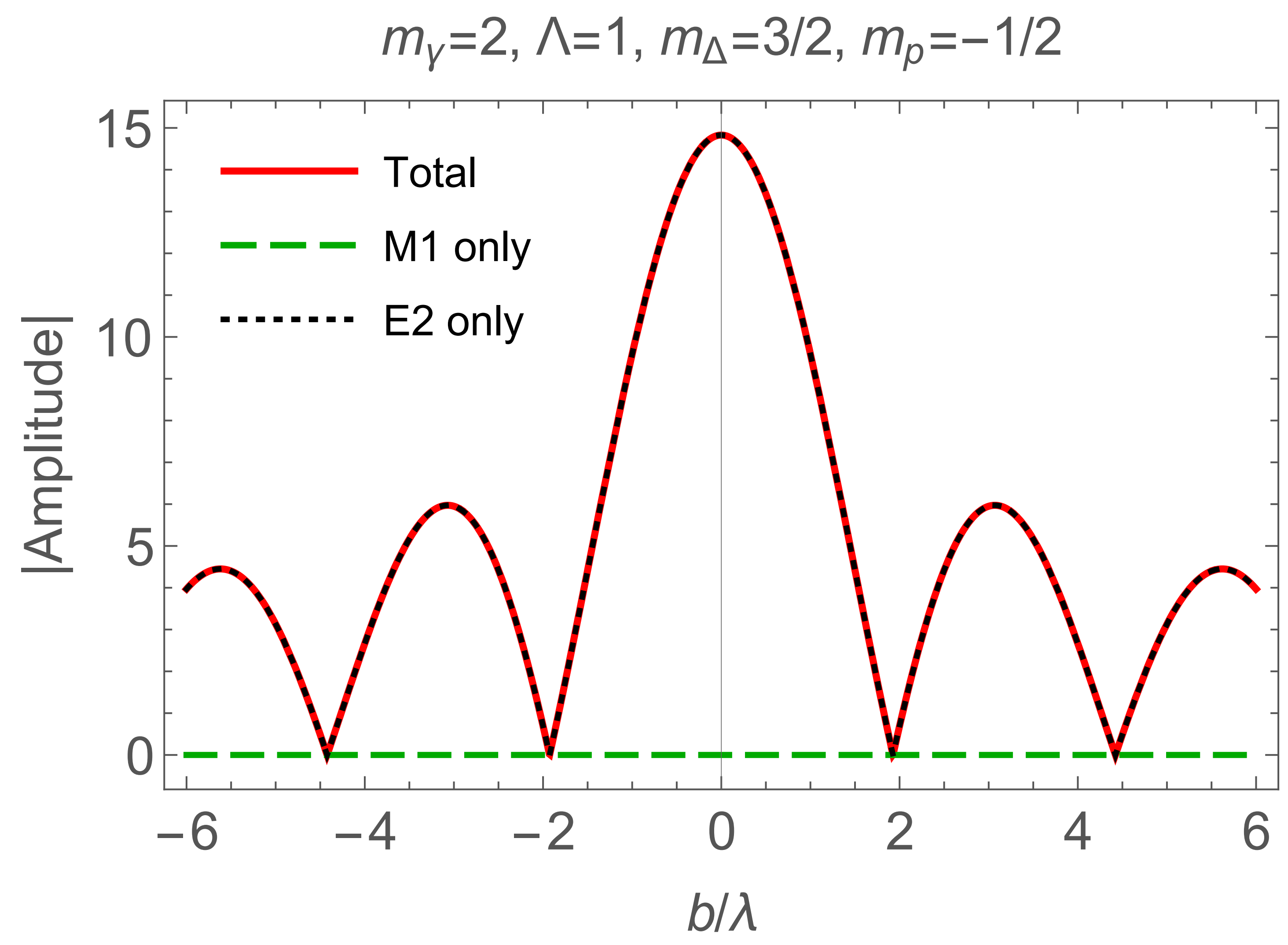




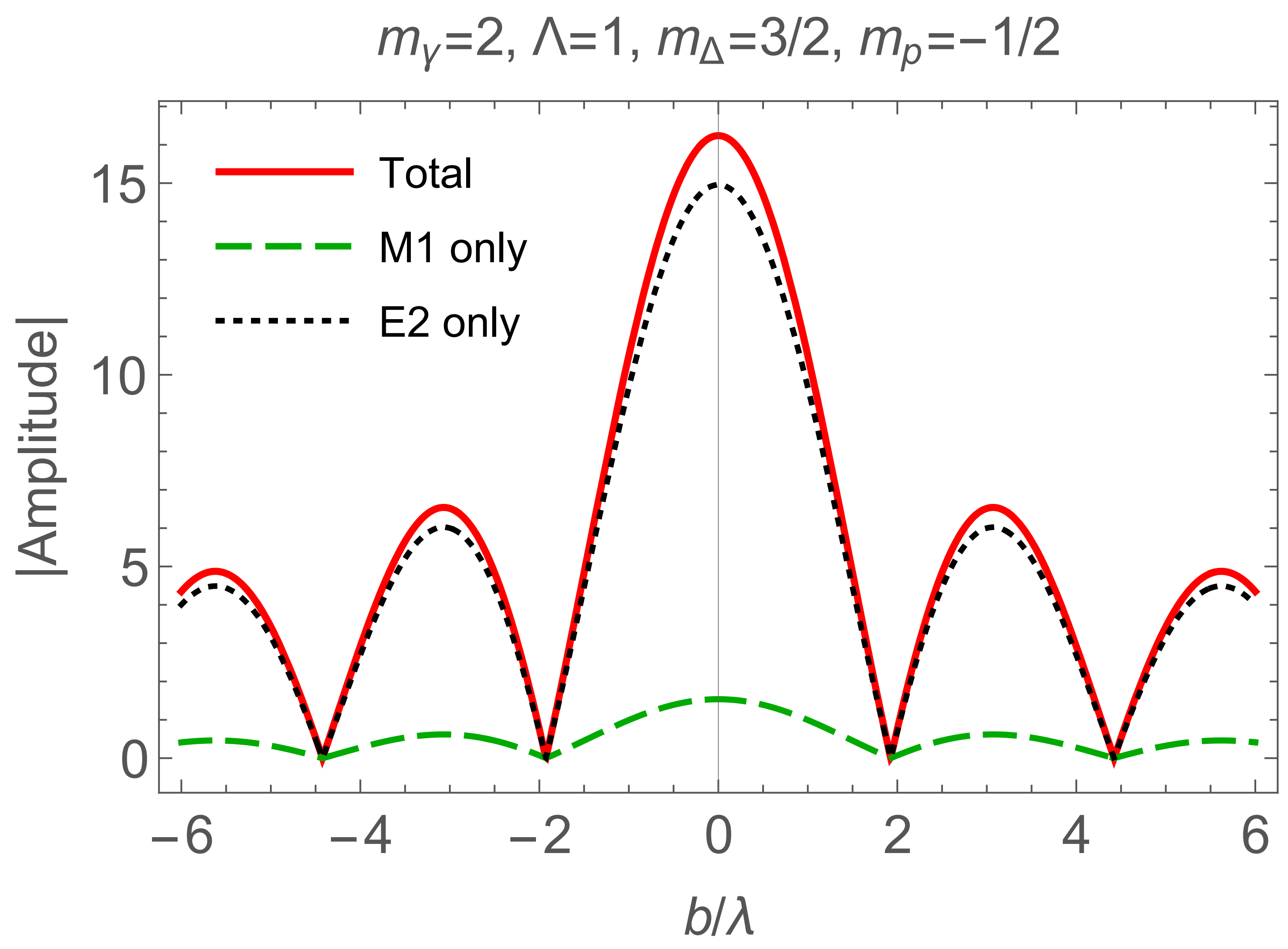
- No M1 contribution for $\Delta m = 2$ case (right hand plot)

Hadronic physics

- Comments (only) about recoil. Corrections worked out in published work. [A. Afanasev and CEC, Ann. Phys. (Berlin) 2021, 2100228; ArXiv:2105.07271] Size depends on small components of Δ wave function squared, nominally $(E_\gamma/2M_\Delta)^2 \approx 1.9\,\%$. Not so serious.



becomes



Higher spin baryons generally

- Generally two independent helicity amplitudes.
- Often called $A_{1/2}$ or $A_{3/2}$, equivalently $M^{(\mathrm{pw})}_{m_i,\Lambda} = M^{(\mathrm{pw})}_{1/2,1}$ or $M^{(\mathrm{pw})}_{-1/2,1}$.
- Or, electric and magnetic multipole amplitudes, EJ and MJ (E2 and M1 for the Δ)
- Again, generally two for each nucleon to resonance transition.

Example of $p \to F_{15}(1680)$

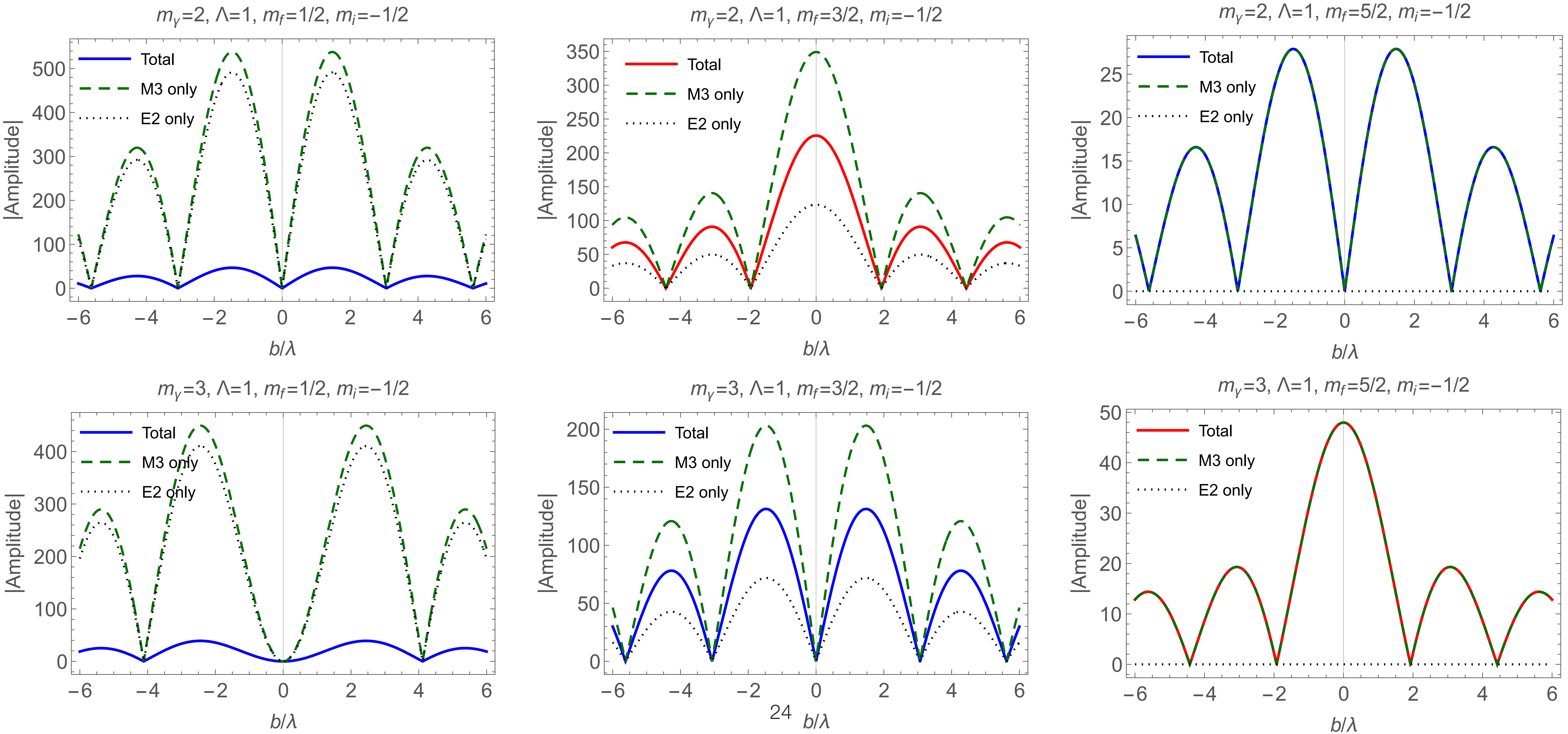
- a.k.a., $N(1680)$ with $I = 1/2$ and $J^P = (5/2)^+$.

- The nonzero multipoles are $E2$ and $M3$. Can work out
$$A_{1/2} \propto E2 + \sqrt{2}\, M3 \,,$$
and
$$A_{3/2} \propto \sqrt{2}\, E2 - M3 \,.$$

- From known data [PDG], $A_{1/2}$ small and $E2 \approx -\sqrt{2}\, M3$.

- Consider especially high $\Delta m = m_f - m_i = m_{N(1680)} - m_p$ transitions, e.g., $m_f = 5/2$ with $m_i = -1/2$.

Plots for $p \to F_{15}(1680)$ (other QN as labeled)



Summary

- Used twisted photons: sticking to plane wave photons has angular momentum along direction of motion $m_\gamma = \pm 1$ only. Very standard.
- Explore what can be done with the extra degree of freedom.
  - Communications.
  - Atomic physics. BTW, can reverse and use final QN and off-axis behavior as diagnostic of structured photon state.
  - Hadronic physics
    - Higher spin states, can use to isolate the highest allowed multipole amplitude.
    - For example, picking out the small E2 in Δ photoproduction.
- Beam requirements not trivial and not currently possible. But there is a future.

Extra

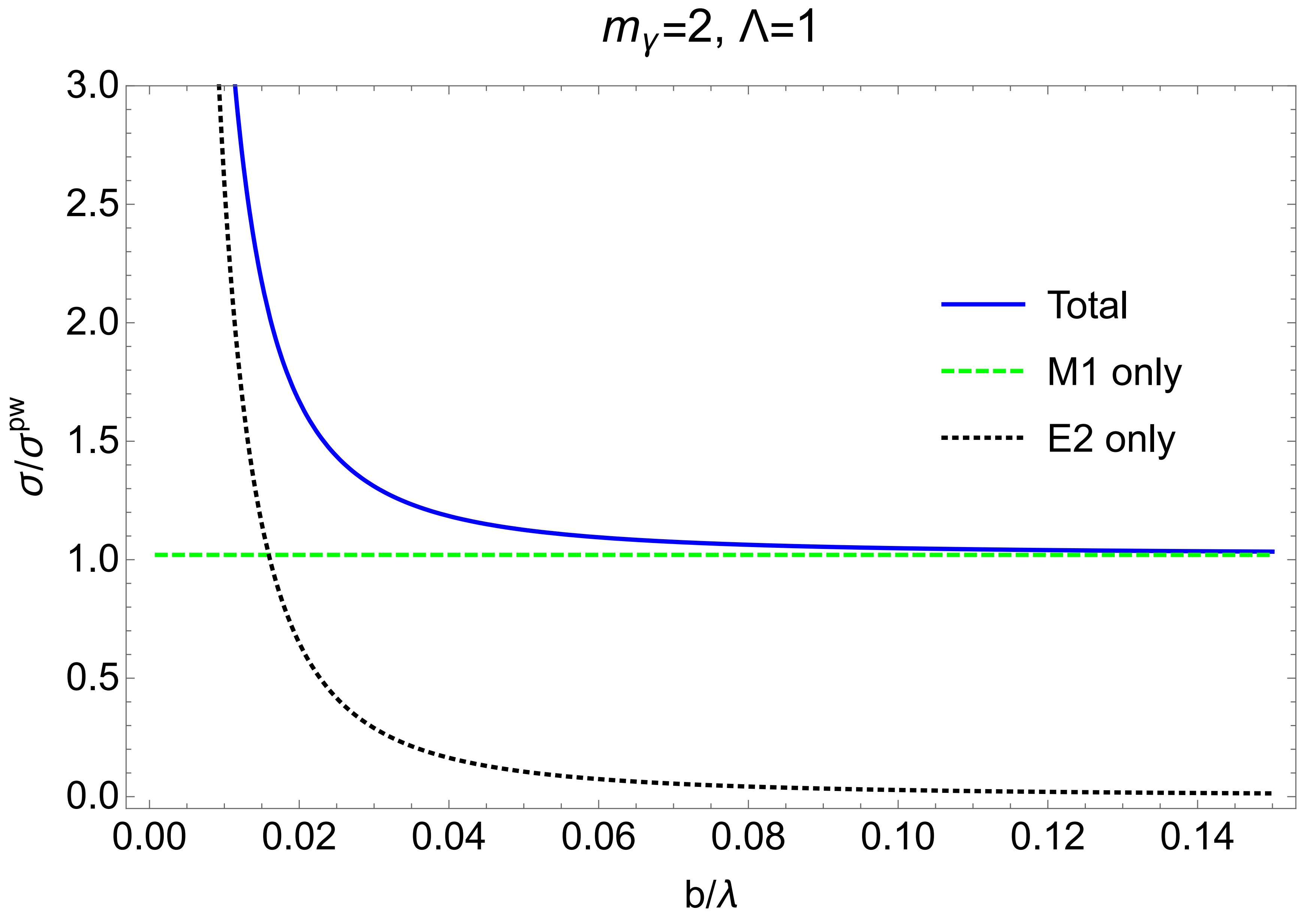
Target localization

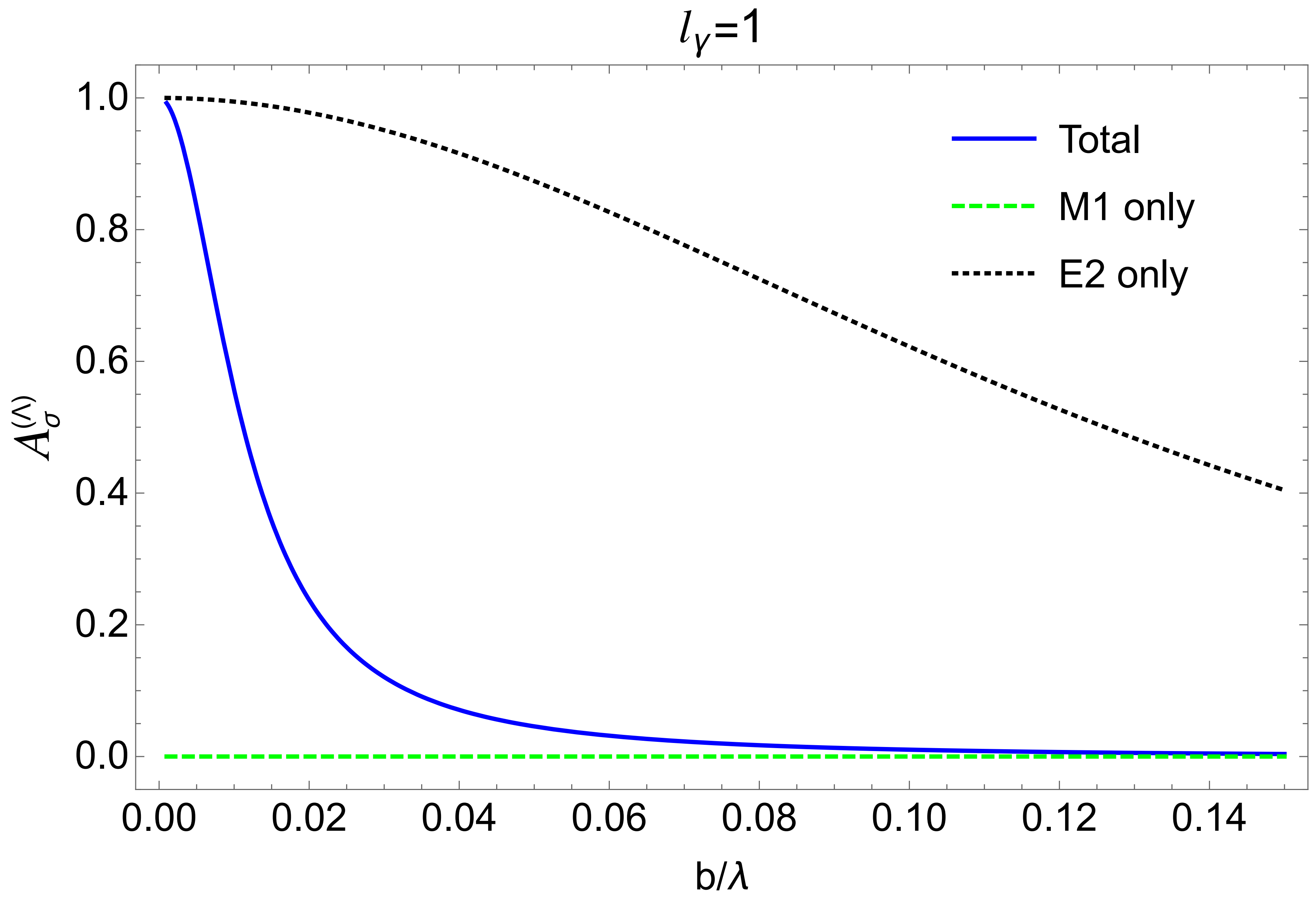
- Want target at rest and stationary. But we know quantum mechanics. Best we can do is

$$\Delta x\,\Delta p \geq \frac{1}{2}\hbar \approx \frac{1}{2}200 \text{ MeV fm}$$

- A minimal possibility: $\Delta x \approx 3$ fm and $\Delta p \approx 30$ MeV. Could be o.k.
- For Δ kinetic energy, $\Delta E_\Delta \approx 2E_\gamma \Delta p/(2M_\Delta) \approx 7$ MeV , small compared to Δ width.
- Amplitude plots have minima a few or several λ, and $\lambda = 3.65$ fm. So 3 fm for Δx acceptable, and can adjust. See also Zheludev et al on super-resolution ideas

Unpolarized results





$$A_\sigma^{(\Lambda)} = \frac{\sigma_{\Lambda=1} - \sigma_{\Lambda=-1}}{\sigma_{\Lambda=1} + \sigma_{\Lambda=-1}}$$

Twisted vector potential

- Components for $(\kappa, m_\gamma, k_z, \Lambda)$ and $b = 0$, Coulomb gauge.
- $A_\rho = i \frac{A_0}{\sqrt{2}} e^{i(k_z z - \omega t + m_\gamma \phi)} \left[\cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa\rho) + \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa\rho)\right]$
- $A_\phi = -\Lambda \frac{A_0}{\sqrt{2}} e^{i(k_z z - \omega t + m_\gamma \phi)} \left[\cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa\rho) - \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa\rho)\right]$
- $A_z = \Lambda \frac{A_0}{\sqrt{2}} e^{i(k_z z - \omega t + m_\gamma \phi)} \sin\theta_k J_{m_\gamma}(\kappa\rho)$