Conformal Field Theory

Jean-François Fortin

Département de Physique, de Génie Physique et d'Optique
Université Laval, Québec, QC G1V 0A6, Canada

July 16, 2020
IPP Town-hall Meeting

Fundamental physics

Two pillars of fundamental physics ($\sim$ 1920s)

- General relativity (GR)
  - Long distance scales
  - Classical
- Quantum field theory (QFT)
  - Short distance scales
  - Quantum

Holy grail of fundamental physics

- Quantum gravity
  - Unification of GR and QFT (string theory?)

Conformal field theory (CFT) $\subset$ QFT

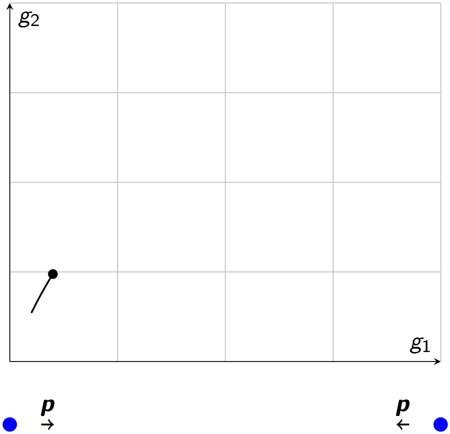
- Why study QFT?
  - Standard model of particle physics $\Rightarrow$ QFT of $SU(3) \times SU(2) \times U(1)$ + matter ($\sim$ 1970s)
  - $3d$ QFT in Euclidean time $\Rightarrow$ Condensed matter systems
- Many questions still left unanswered
  - Non-perturbative control $\Rightarrow$ QFT at strong coupling

Observables $\Leftarrow$ Correlation functions

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{f(m|x-y|,\cdots)}{|x-y|^{2\Delta}}$$

- Poincaré invariance
  - Spacetime translations and rotations $\Rightarrow$ Factor $|x-y|^{-2\Delta}$
  - Dependence on mass scales $\{m,\cdots\}$ $\Rightarrow$ Arbitrary function $f(m|x-y|,\cdots)$
- Computation of $f(m|x-y|,\cdots)$
  - Perturbative control $\Rightarrow$ QFT at weak coupling (*e.g.* QED)

Space of QFTs



QFT $\Rightarrow$ Coupling constants are NOT constant

- Renormalization group (RG) ($\sim$ 1970s)
  - Coupling constants $\Rightarrow$ Functions of energy scale μ
  - Flow in the space of QFTs
  - QFT at weak coupling at energy scale μ_H $\Rightarrow$ QFT at strong coupling at energy scale μ_L
- Computation of $f(m|x-y|,\cdots)$
  - Loss of perturbative control $\Rightarrow$ QFT at strong coupling (*e.g.* QCD)

Conformal field theory

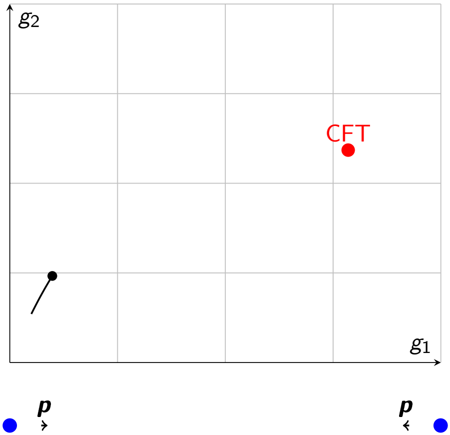
Why study CFT?

- RG flow limiting behaviors
  - Limit cycle $\Rightarrow$ Scale invariance
  - Fixed point $\Rightarrow$ Conformal invariance

$\Rightarrow$ QFTs = Relevant deformations of CFTs!

- Classification of CFTs $\Rightarrow$ Classification of QFTs (QFT phases)
- $3d$ CFT in Euclidean time $\Rightarrow$ Second-order phase transitions
- CFT tractable at strong coupling!

Space of QFTs



Observables $\Leftarrow$ Correlation functions

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{c}{|x-y|^{2\Delta}}$$

- Poincaré invariance
  - Spacetime translations and rotations $\Rightarrow$ Factor $|x-y|^{-2\Delta}$
- Conformal invariance
  - $c =$ constant at arbitrary coupling!

N-point correlation functions

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\rangle$$

- Two- and three-point correlation functions
  - completely fixed by conformal invariance (up to overall constants)

- Four-point correlation functions
  - NOT completely fixed by conformal invariance
  - $\Delta = \sum_{1 \le i \le 4} \Delta_i$
  - u and v conformal cross-ratios

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_4(x_4) \rangle = F(u, v) \prod_{1 \le i < j \le 4} |x_i - x_j|^{-(\Delta_i + \Delta_j) + \Delta/3}$$

$$u = \frac{|x_1 - x_2|^2 |x_3 - x_4|^2}{|x_1 - x_3|^2 |x_2 - x_4|^2} \qquad v = \frac{|x_1 - x_4|^2 |x_2 - x_3|^2}{|x_1 - x_3|^2 |x_2 - x_4|^2}$$

Operator product expansion

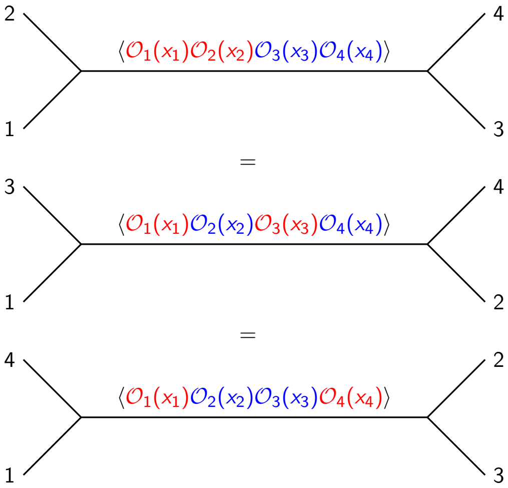
$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) = \sum_k \sum_{a=1}^{N_{12k}} {}_a c_{12}{}^k \, {}_a\mathcal{D}_{12}{}^k(x_1, x_2)\mathcal{O}_k(x_2)$$

- Operator product expansion (OPE)
  - ${}_a c_{ij}{}^k$ OPE coefficients (different for each CFT)
  - ${}_a\mathcal{D}_{ij}{}^k(x_1, x_2)$ conformal differential operators completely fixed by conformal invariance

$\Rightarrow$ Unknown function $F(u, v)$ expressible in terms of conformal blocks $G_k(u, v)$

$$F(u, v) = \sum_k \sum_{a=1}^{N_{12k}} \sum_{b=1}^{N_{34k}} {}_a c_{12}{}^k \, {}_b c_{34}{}^k \, G_k(u, v)$$

Four-point correlation functions constrained by crossing symmetry (conformal bootstrap)



Conformal bootstrap

- Consistency under associativity
  - Same $F(u, v)$ expressed in terms of different conformal blocks and OPE coefficients

$\Rightarrow$ Constraints on OPE coefficients

  - Valid at arbitrary coupling!
  - No experimental observations needed, computation from first principle?!?

Conclusion

Why study CFT?

- Implications
  - Building blocks for QFT
  - Second-order phase transition in condensed matter systems
  - Quantum gravity through string theory and AdS/CFT
- Prospects
  - Non-perturbative control
  - Classifiable *à la* Lie algebra?!?

$\Rightarrow$ Active area of research in Canada and abroad